Composite invisible axion

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(Received 27 December 1984; revised manuscript received 21 February 1985)

A new non-Abelian gauge group which becomes strong at 10^8-10^{12} GeV is suggested for a confining force of a composite invisible axion. This scheme naturally explains why the invisible-axion order parameter is between the grand unification scale and the electroweak scale.

One unsatisfactory feature of the invisible axion[1] is why its order parameter $v_{\rm PQ}$ is much smaller than the grand unification (GU) scale of $10^{15}-10^{17}$ GeV (Refs. 2 and 3),

$$10^8 < v_{\rm PQ} < 10^{12}\ \text{GeV}\ . \tag{1}$$

Even though one can raise $v_{\rm PQ}$ to the GU scale by coupling the invisible axion in many steps to the anomalous quarks,[4] it is challenging to find a simpler reason to have the range (1).

If the axion is fundamental, there are two reasonable scales for the order parameter, 250 GeV (Ref. 5) and 10^{15} GeV. To have the allowed region Eq. (1) without a severe fine tuning, it is logical to suppose that the invisible axion is composite.

A good theory to have composite bosons is a confining gauge theory with broken chiral symmetries, such as quantum chromodynamics and the hypercolor model.[6] The hypercolor idea works beautifully in the effective Higgs sector, but inclusion of quarks and leptons does not lead to satisfactory models. However, a confining gauge theory can be a successful one for the invisible axion.

Therefore, let us introduce a new confining non-Abelian gauge group, for example SU(N), which we will call *axicolor.* The axicolor scale must be different from the hypercolor scale in view of Eq. (1). Probably the group SU(N) is larger than $SU(3)_c$ so that it becomes strong much above the QCD scale. Let us first present the idea of making the invisible axion complex in the simplest form, and then discuss other cosmological problems. We introduce axiquarks $Q^{A\alpha}$ and q^A, where $\alpha=1,2,3$ is the color index and $A=1,2,\ldots,N$ is the axicolor index. Both axiquarks do not have any mass term so that the kinetic term has $U(1)^4$ global symmetry. Two of these correspond to the Q-number conservation and the q-number conservation. The remaining two symmetries are axial $\widetilde{U}(1)_A$ and $U(1)_A$, and the corresponding currents are

$$\widetilde{J}^5_\mu=\tfrac{1}{2}\overline{Q}\gamma_\mu\gamma_5 Q+\tfrac{1}{2}\overline{q}\gamma_\mu\gamma_5 q\ , \tag{2}$$

$$J^5_\mu=\frac{1}{\sqrt{12}}\overline{Q}\gamma_\mu\gamma_5 Q-\frac{3}{\sqrt{12}}\overline{q}\gamma_\mu\gamma_5 q\ . \tag{3}$$

The divergences of these currents are zero except for the axicolor and color anomalies,

$$\partial^\mu \widetilde{J}^5_\mu=\frac{1}{32\pi^2}(Ng^2F^\alpha_{\mu\nu}\widetilde{F}^{\alpha\mu\nu}+4g'^2F'^A_{\mu\nu}\widetilde{F}'^{A\mu\nu})\ , \tag{4}$$

$$\partial^\mu J^5_\mu=\frac{1}{32\pi^2}\frac{1}{\sqrt{3}}Ng^2F^\alpha_{\mu\nu}\widetilde{F}^{\alpha\mu\nu}\ , \tag{5}$$

where $F^\alpha_{\mu\nu}$ ($\widetilde{F}^A_{\mu\nu}$) and $F'^A_{\mu\nu}$ ($\widetilde{F}'^A_{\mu\nu}$) are the field strengths (their dual) of color and axicolor. The two Goldstone bosons resulting from the chiral-symmetry breaking $\langle \overline{Q}_L Q_R\rangle=\langle \overline{q}_L q_R\rangle\neq 0$ acquire masses through the above anomalies. The $\widetilde{U}(1)$ Goldstone boson is superheavy, presumbly around 10^{11} GeV, as the QCD U(1) problem is solved by the anomaly. This dynamical degree settles the axicolor angle θ_a to $2m\pi$ ($m=0,\pm1,\ldots$). The other Goldstone boson corresponding to the U(1) symmetry [Eq. (3)] is the invisible axion. This is the *composite invisible axion* whose axiquark content is

$$(\overline{Q}\gamma_5 Q-3\overline{q}\gamma_5 q)/\sqrt{12}\ .$$

This composite invisible axion fulfills the required dynamical degree to settle down an arbitrary QCD angle θ_c to $2n\pi$ ($n=0,\pm1,\ldots$). If the axicolor becomes strong at Λ_a, the composite axion-gluon-gluon coupling is given by[7]

$$\frac{1}{32\pi^2}\frac{Ng^2}{\sqrt{3}}\frac{a}{\Lambda_a}F^\alpha_{\mu\nu}\widetilde{F}^{\alpha\mu\nu}\ . \tag{6}$$

Let us assume that the following condensates are formed:

$$\langle \overline{Q}_{A1}Q^{A1}\rangle=\langle \overline{Q}_{A2}Q^{A2}\rangle=\langle \overline{Q}_{A3}Q^{A3}\rangle=\langle \overline{q}_A q^A\rangle\cong{\Lambda_a}^3\ , \tag{7}$$

so that QCD is not broken. The order parameter for the axion is Λ_a. Following the standard estimate of the axion mass, we obtain from Eq. (5)

$$m_a\cong\frac{f_\pi m_\pi}{\Lambda_a}\frac{N}{\sqrt{12}}\frac{\sqrt{Z}}{1+Z}\ , \tag{8}$$

where f_π and m_π are the pion decay constant and the pion mass, and

$$Z=m_u\langle \overline{u}u\rangle/m_d\langle \overline{d}d\rangle\ .$$

The axicolor confining scale Λ_a is similar to the Peccei-Quinn symmetry-breaking scale.

Does this composite invisible axion interact with ordinary quarks and leptons? For this discussion, we focus our attention on the model presented in this paper even though other generalizations can be possible. The composite invisible axion does not interact with leptons at the level of our interest. On the other hand, it interacts with light quarks through the color-gluon exchanges as shown in Fig. 1. The blob represents the axion-F-$\widetilde{F}$ coupling, Eq. (6). Because the composite invisible axion arises from the chiral-symmetry breaking, we know that its coupling to a light quark is proportional to the light-quark mass and inversely proportional to the axicolor scale Λ_a,

$$\sim i\frac{m_u}{\Lambda_a} a\bar{u}\gamma_5 u + (d\text{-quark term}) . \tag{9}$$

Therefore, by identifying Λ_a as $v_{\rm PQ}$, we can obtain the lower bound of Eq. (1) from the study of the evolution of red giants. Also, the upper bound of Eq. (1) derives from the axion mass, and Eq. (1) is the condition on Λ_a.

The only implication of the axiquarks Q and q in the low-energy sector is the composite invisible axion. All the other composite particles such as vector aximesons and axibaryons are superheavy (10^{11} GeV) and do not lead to observable effects.

We note that three families and one Higgs doublet of the standard model can be added to the above model without changing the aspect of the strong-CP solution.

Finally, let us discuss briefly the other cosmological implications of the invisible-axion model. So far we have not worried about the number density[8] in axibaryons and Sikivie's well-known domain-wall problem.[9] These problems can be cured elegantly in the inflationary-universe scenario.[10] The low reheating temperature of order 10^{11} GeV for our purpose of the strong-CP solution is acceptable[11] for cosmological baryon-number generation in ordinary grand unification models. However, conflict may arise if we extend to supersymmetric models.

We will also show that the above problems can be solved in the standard big-bang cosmology if we extend the model slightly. For definiteness, let us fix the axicolor group as $SU(3)_a$. We intend to show the existence proof.

To solve the domain-wall problem, it is easier to consider one $U(1)_A$ symmetry. For this purpose, let us introduce some scalar particles (ϕ's) above 10^{12} GeV. The fermions are

$$Q_B^{A\alpha},\ Q_{B\alpha}^A,\ 3Q^{A\alpha},\ 3Q_{A\alpha},\ 3q_B^A,\ \nu , \tag{10}$$

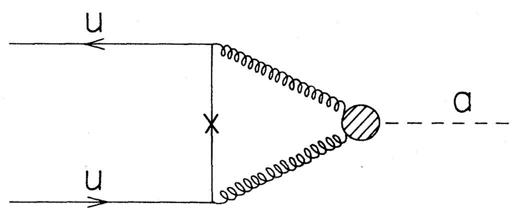

FIG. 1. The Feynman diagram for the effective coupling of a light quark and the composite invisible axion. The blob is Eq. (6).

which are all left-handed, and they transform under $SU(3)_a \times SU(3)_c$ as (8,3) (8,3*), (3,3) (3*,3*), (8,1), and (1,1), respectively. The Yukawa couplings we introduce are

$$\begin{aligned} &Q_B^{A\alpha} Q_{A\alpha}\phi_i^B,\ Q_{A\alpha}^B Q^{A\alpha}\phi_{iB} ,\\ &Q_B^{A\alpha} q_A^B \phi_{j\alpha},\ Q_{B\alpha}^A q_B^A \phi_j^\alpha ,\\ &Q^{A\alpha}\nu\phi_{A\alpha},\ Q_{A\alpha}\nu\phi^{A\alpha} , \end{aligned} \tag{11}$$

where $i=1,2,3$ and $j=1,2,3$ denote family indices for axicolored and colored scalars. Couplings of the Higgs fields are

$$\begin{aligned} &M^2{}_{ij}\phi_{iA}\phi_j^A,\ M_{ijk}\phi_i^A\phi_j^B\phi_k^C\epsilon_{ABC} ,\\ &M^2{}_{ij}\phi_{i\alpha}\phi_j^\alpha,\ M_{ijk}\phi_i^\alpha\phi_j^\beta\phi_k^\gamma\epsilon_{\alpha\beta\gamma} ,\\ &M^2\phi_{A\alpha}\phi^{A\alpha},\ M\phi_{A\alpha}\phi_{B\beta}\phi_{C\gamma}\epsilon^{ABC}\epsilon^{\alpha\beta\gamma} , \end{aligned} \tag{12}$$

where M's denote merely the mass parameters and their magnitudes can be different. Because of Eq. (12), ϕ's do not carry global quantum numbers. Because of Eq. (11), six phases of Eq. (10) are related, leading to only one global symmetry. The charges of the fermions under this symmetry are

$$\begin{aligned} &Q_B^{A\alpha}:\ 1,\ \ Q_{B\alpha}^A:\ 1,\ \ Q^{A\alpha}:\ -1 ,\\ &Q_{A\alpha}:\ -1,\ \ q_B^A:\ -1,\ \ \nu:\ 1 , \end{aligned} \tag{13}$$

which shows the axial U(1) symmetry. The QCD domain-wall number[9] is

$$N_{\rm DW}^c = |\,{\rm Tr} Q_c N_c l_c\,| = 1 , \tag{14}$$

while the axicolor domain-wall number is

$$N_{\rm DW}^a = |\,{\rm Tr} Q_a N_a l_a\,| = 0 . \tag{15}$$

Therefore, this example dynamically settles θ_c to zero, but cannot settle θ_a to zero. This is not problematic because we do not know the experimental bound on θ_a. From Eq. (14), we know that there is no domain-wall problem.

Let us further introduce an additional Higgs doublet H' which is different from the other Higgs doublet H giving masses to e, d, and u. The vacuum expectation value of H' doublet is of order of the electroweak scale. Its $U(1)_A$ quantum number is

$$H':\ -1 , \tag{16}$$

and its hypercharge is $\frac{1}{2}$. Therefore, we have the Yukawa coupling

$$l\nu H' , \tag{17}$$

where l is the ordinary lepton doublet. When H' acquires a vacuum expectation value, and Dirac mass term of order GeV between ν and the ordinary neutrino is generated. When $U(1)_A$ is broken by condensates, ν aquires a mass of order $f^2\Lambda_a \approx 10^9$ GeV. The seesaw mechanism takes place, and the light-neutrino mass is of order eV. The heavy ν decays to $\bar{l}+\bar{H}'$ by Eq. (17), and we would not have a cosmological heavy-ν problem.

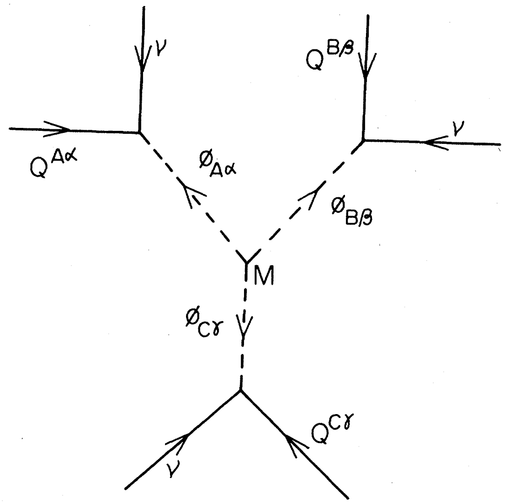

FIG. 2. The axibaryon decay diagram.

Let us turn to the axibaryon problem. One axicolor and color singlet

$$Q^{A\alpha}Q^{B\beta}Q^{C\gamma}\epsilon_{ABC}\epsilon_{\alpha\beta\gamma}$$

is assumed to be the lightest axibaryon. Note that this is possible because we introduced three $Q^{A\alpha}$ in Eq. (10). The spin of this axibaryon can be $\frac{3}{2}$ or $\frac{1}{2}$. This axibaryon will decay to three ν's [$SU(2)\times U(1)$ singlet] by the diagram shown in Fig. 2. Taking $M \simeq M_\phi = 10^{12}$ GeV and $\Lambda_a = 10^{11}$ GeV, the lifetime of the lightest axibaryon is of order 10^{-23} sec, which will not cause cosmological embarrassments such as too much dilution of baryon number already generated. This is because the axibaryons decay before they dominate the mass density of the Universe at the cosmic time 5×10^{-19} sec, i.e., at $T \approx 10^6$ GeV.

In conclusion, a new non-Abelian gauge group which becomes strong at 10^8–10^{12} GeV can be responsible for making the invisible axion composite. For this purpose, we needed at least two massless axiquarks so that both θ_c and θ_a are transformed by the two axions, one of which is invisible and has a mass $\sim \Lambda_c{}^2/\Lambda_a$. But a solution to the domain-wall problem in the standard cosmology may require a baroque structure.

This research has been supported in part by the Korean Ministry of Education through the Research Institute of Basis Sciences, Seoul National University, and also by the Korean Science and Engineering Foundation.

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