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Experimental tests of new SO(10) grand unification

D. Chang and R. N. Mohapatra
Department of Physics, University of Maryland, College Park, Maryland 20742

J. M. Gipson, R. E. Marshak, and M. K. Parida
Physics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
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A new approach to SO(10) grand unification was recently proposed by three of the authors (D.C., R.N.M., and M.K.P.), where the mass scale M_P at which the D-parity symmetry present in the SO(10) group breaks was decoupled from the scale M_{W_R} of the right-handed currents. In contrast with the conventional treatment of the SO(10) model, $SU(2)_L \times SU(2)_R \times G$ [G is $SU(4)_C$ or $SU(3)_C \times U(1)_{B-L}$] can appear as an intermediate symmetry with $g_L \neq g_R$. Calculations in the one-loop approximation lead to a substantially different picture of intermediate mass scales for SO(10)-symmetry breaking than before. In this paper, this analysis is extended to include two-loop contributions which are significant for several symmetry-breaking chains. All possible chains descending to the standard group $SU(2)_L \times U(1)_Y \times SU(3)_C$ are examined. A unique chain emerges if one imposes a minimality condition (using the lowest-dimensional Higgs multiplet at each symmetry-breaking stage) and the phenomenological requirements of $\tau_{p \to e^+ \pi^0} \geq 2 \times 10^{32}$ yr, $\sin^2\theta_W(M_W) = 0.22 \pm 0.02$, and $\alpha_s(M_W) = 0.10-0.12$. This chain is $SO(10) \to SU(2)_L \times SU(2)_R \times SU(4)_C \times P \to SU(2)_L \times SU(2)_R \times SU(4)_C \to SU(2)_L \times U(1)_R \times U(1)_{B-1} \times SU(3)_C \to SU(2)_L \times U(1)_Y \times SU(3)_C$ and allows for the following detectable consequences: (a) neutron oscillations with $\tau_{n\bar{n}} \sim 10^8-10^9$ sec; (b) a branching ratio for $K_L \to \mu\bar{e}$ of $7 \times (10^{-8}-10^{-12})$; (c) a second neutral Z_R boson in the $\frac{1}{2}$-to-10-TeV range; (d) a proton lifetime $\tau_p = 6.5 \times 10^{35.0 \pm 0.9} (\Lambda_{\overline{\rm MS}}/160 \text{ MeV})^4$ yr ($\overline{\rm MS}$ denotes the modified minimal subtraction scheme), which, given the theoretical uncertainties, may barely be within experimental reach; (e) a Majorana mass for the electron neutrino in the range of electron volts. This experimentally interesting chain also predicts $M_P \simeq 10^{14.3 \pm 1.0}$ GeV, which satisfies all cosmological constraints. All other symmetry-breaking chains that satisfy the phenomenological requirements do not have experimentally testable consequences at low energies.

I. INTRODUCTION

The concept of grand unification[1,2] has had a profound impact on particle theory in recent years. However, the simplest grand unified model, based on the SU(5) group of Georgi and Glashow,[2] appears to be in conflict with recent experiments on proton decay. This negative result also rules out the "desert"—that there is no new physics between 100 and 10^{14} GeV—a dreadful prospect for experimental physicists. The disagreement with the SU(5) model certainly does not disprove the grand unification hypothesis. In fact, since SU(5) and SO(10) (Ref. 3) are the only two simple groups that can embed the $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard group of quarks and leptons, if we forbid mirror fermions (fermions with $V+A$ interactions), it is important to look for possible experimental tests of SO(10) models that are consistent with known results on proton decay.

From the aesthetic point of view, SO(10) (Refs. 4 and 5) is a much more desirable grand unification group in the sense that one spinor representation contains all the known fermions (plus the right-handed neutrino ν_R), in contrast with two representations for SU(5) (without ν_R). Moreover, the SO(10) model leads to parity conservation at the grand-unified-theory (GUT) scale if the theory conserves CP, thereby providing an explanation for the origin of parity violation and a connection of CP with P violation. It has also been shown[6] that a longer τ_p [than that predicted by the SU(5) model] requires that SO(10) pass through an intermediate stage with $SU(2)_R$ as a good symmetry rather than through the stage $SO(10) \to SU(5) \times U(1)$. This raises the hope that the right-handed scale M_{W_R} may be low enough within the SO(10) model to be visible in the next generation of accelerators, say ≤ 10 TeV. In the conventional approaches[4-7] to SO(10), this would require, however, a larger $\sin^2\theta_W$ ($\simeq 0.275$) than is consistent with present data. Furthermore, there are several cosmological difficulties if SO(10) is allowed to pass through the left-right-symmetry group $SU(2)_L \times SU(2)_R \times G$ [hereafter $G \equiv SU(4)_C$ or $U(1)_{B-L} \times SU(3)_C$] with $g_L = g_R$, as was done before D-parity breaking was considered (see below). It was pointed out by Kuzmin and Shaposhnikov[8] that unbroken D parity, a discrete symmetry, that guarantees $g_L = g_R$, also requires the Universe to have zero baryon number in the symmetric phase. Consequently, in order to understand the baryon asymmetry of the Universe, one must have the breaking scale of the D parity (called M_P) close to the scale of grand unification, M_U. Since, in conventional approaches, $M_{W_R} = M_P$, this implies that while there may be an intermediate $SU(2)_L \times SU(2)_R \times G$ symmetry, its breaking scale cannot be much below M_U.

Another cosmological difficulty associated with the existence of an intermediate left-right symmetry was noted by Kibble, Lazarides, and Shafi.[9] They point out that the breaking of D parity at an intermediate stage creates massive domain walls bounded by strings which do not disappear in the course of evolution of the Universe. This will lead to a huge mass density in the present Universe, in conflict with observation. Two possible ways to avoid this cosmological problem are (i) breaking left-right symmetry at M_U and/or (ii) accepting the inflationary-universe scenario with the desirable result that any walls formed above the reheating temperature of 10^{12} GeV or so will have no effect on the mass density in the present Universe. This would imply $M_P > 10^{12}$ and thus an extremely high scale for right-handed W_R bosons in the conventional treatment of SO(10).

Finally, some years ago, it was suggested[10] that CP violation in $K \rightarrow 2\pi$ decays would owe its origin to the existence of right-handed currents. An interesting outcome of this hypothesis is that the CP-violating parameter ϵ is proportional to the ratio $(M_{W_L}/M_{W_R})^2$, which connects the breakdown of CP symmetry to the breakdown of parity symmetry. It is then clear that this picture of CP violation can arise out of a grand unification scenario only if the latter can accommodate an $M_{W_R} \lesssim 100$ TeV (Ref. 11). Clearly, conventional treatments of SO(10) grand unification exclude this approach to CP violation.

In a recent series of papers, three of the authors[12] (D.C., R.N.M., and M.K.P.) have suggested a new approach to left-right-symmetric unified theories, in which the breaking scale M_P of the D parity and that of the local $SU(2)_R$ symmetry (M_{W_R}) are decoupled from one another. Applying this idea to SO(10) models, it was pointed out that certain Higgs multiplets such as {210} and {45} contain singlets under the $SU(2)_L \times SU(2)_R \times G$ group, which are odd under D parity [i.e., (1,1,1) of {210} and (1,1,15) of {45}]. An immediate consequence is that the mass of W_R can be much lower than M_P, which is constrained by cosmology to be above 10^{12} GeV. The only phenomenological constraints on M_{W_R} are then those required by the values of $\sin^2\theta_W$ and α_s at low energies. These constraints were analyzed in the one-loop approximation, using the method of Georgi, Quinn, and Weinberg,[13] and a number of symmetry-breaking patterns were found where the W_R bosons and/or the M_C bosons [associated with the breaking of $SU(4)_C$] could be visible at low energies, without making unnatural adjustment of parameters. This new picture of intermediate scales in SO(10) was a direct consequence of the broken D parity which implied $g_L \neq g_R$ at scales where $SU(2)_L \times SU(2)_R \times G$ symmetry was valid.[14]

To emphasize the difference between the intermediate-mass-scale picture in conventional SO(10) and in the new approach, we note that previously $U(1)_{I_{3R}}$ [denoted by $U(1)_R$] was the only symmetry that could be present at intermediate energies consistent with low-energy physics.[6,14–16] On the other hand, in our approach, in addition to the second neutral Z_R boson (also denoted by R^0) in the TeV range, it appeared possible to have a right-handed charged W_R boson (also denoted by R^+) in the 10-TeV range as well as $\Delta B = 2$ phenomena,[17,18] such as n-$\bar{n}$ oscillations, as a consequence of a sufficiently low M_C. This new approach to SO(10) makes it not only of great cosmological appeal but also opens up the possibility of experimental tests of SO(10) in the near future.

In this paper, we extend the analysis of Ref. 12 to include the effects of two-loop contributions to the intermediate mass scenarios considered in those papers. We find that, whenever $SU(4)_C$ survives as an intermediate symmetry to lower energies, the two-loop effects are significant due to high-dimensional Higgs-multiplet contributions, which we include in the renormalization-group equations in accordance with the minimal-fine-tuning and extended survival hypotheses.[19] Since the dimensionality of the Higgs multiplet is important, we first use the lowest-dimensional Higgs multiplet that suffices to break the symmetry at each stage and call this the Higgs-minimality condition. As a result, we find, that *all* symmetry-breaking chains with detectable[20] $M_{W_{R^+}}$ are ruled out by our two-loop analysis while only the following chain[21] with detectable M_C is consistent with acceptable values of $\sin^2\theta_W(M_W)$ and $\alpha_s(M_W)$:

$$\mathrm{SO}(10) \underset{\{54\}}{\overset{M_U}{\longrightarrow}} G_{224P} \underset{\{210\}}{\overset{M_P}{\longrightarrow}} G_{224} \underset{\{201\}}{\overset{M_C=M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213} \,.$$

As in conventional SO(10), detectable values of M_{R^0} are possible for several descents, including the one above that gives rise to detectable n-$\bar{n}$ oscillation and $K_L \rightarrow \mu\bar{e}$ decay. The two-loop corrections are not so significant for the chains which do not contain $SU(4)_C$ as an intermediate symmetry; in these cases, low mass W_{R^+} is ruled out because of the restrictions on admissible values of $\sin^2\theta_W$ and α_s, except for one marginal chain:

$$\mathrm{SO}(10) \underset{\{210\}}{\overset{M_U=M_P}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{45\}}{\overset{M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}$$

(see Table II). If we lift the "minimality" requirement and permit the {210} Higgs representation in place of the {45} [this corresponds to using the (1,3,15) decomposate of {210}, instead of the (1,3,1) decomposate of {45}, for the symmetry-breaking stage $G_{2213} \rightarrow G_{2113}$], two other acceptable chains are identified, namely,

$$\mathrm{SO}(10) \underset{\{210\}}{\overset{M_U=M_P}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{210\}}{\overset{M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}$$

and another with the additional intermediate stage [between SO(10) and G_{224}] of G_{224P}. The first chain allows detectable R^+ phenomena and the second both C *and* R^+ phenomena (see below).

This paper is organized as follows: in Sec. II, we briefly review the concept of D parity; in Sec. III, we summarize the situation with respect to intermediate mass scales in the conventional discussions of the SO(10) model prior to the work of Ref. 12; in Sec. IV, we give the two-loop coefficients for various chains and discuss the allowed mass scales for the different chains. We conclude with a discussion of our results in Sec. V.

II. *D* PARITY

D parity has been discussed in Ref. 12. Here we review the basic ideas for completeness. As noted before, the SO(10) grand unification group is the only rank-5 group that contains the standard G_{213} group at low energies without mirror fermions. It contains as a maximal subgroup G_{224}. There exists an element of the SO(10) group, call it $D=\Sigma_{23}\Sigma_{67}$ [where $\Sigma_{\mu\nu}(\mu,\nu,\ldots,10)$ are the 45 totally antisymmetry generators of SO(10)] which plays a role very similar to charge conjugation on the fermions, i.e.,

$$f_L \underset{D}{\rightarrow} f_L^C=(C\bar{f}^{\,T})_L \;. \tag{2.1}$$

It may be identified with charge conjugation[22] if we demand further that under charge conjugation the $\Delta_L\equiv(3,1,10)$ submultiplet of the {126}-dimensional representation changes to $\Delta_R^*\equiv(3,1,\overline{10})$. (This deviates from the usual notion of charge conjugation being a particle-antiparticle symmetry precisely because one is dealing here with a left-right symmetric model; the term *D* parity has been invented to emphasize this special situation.) *D* parity, however, differs from the operator *CP*. To see this, it suffices to point out that, if there are complex Yukawa couplings, the SO(10) model can lead to *CP* violation whereas SO(10) invariance implies that *CP*-violating Yukawa interactions respect *D* parity. Of course, for real Yukawa couplings, *D* parity is equivalent to *CP*.[12]

It was also noted in Ref. 12 that there are certain Higgs multiplets in SO(10), which contain submultiplets which are singlets under $SU(2)_L\times SU(2)_R\times G$ but are odd under *D* parity. Such representations can lead to intermediate $SU(2)_L\times SU(2)_R$ local symmetry with $g_L\neq g_R$. As shown in Ref. 12, this difference comes about due to asymmetric Higgs boson contributions to g_L and g_R in the renormalization-group equations. Examples of such representations are {210}, which contains a *D*-odd G_{224} singlet (1,1,1) and {45}, which contains a *D*-odd G_{2213} singlet (1,1,15). On the other hand, the {54}-dimensional representation of SO(10) contains a *D*-even G_{224} singlet (1,1,1) and {210} a *D*-even G_{2213} singlet (1,1,15). It is therefore clear that depending on which submultiplets we use to break the symmetry, we will have *D* parity broken or not at a given stage of symmetry breaking.

III. INTERMEDIATE MASS SCALES IN CONVENTIONAL SO(10) MODEL

In this section, we remind the reader about the intermediate mass scales in the conventional SO(10) models, where *D* parity and $SU(2)_R$ are broken together. We will compare our results with the situation in SU(5). Denoting all minimal-SU(5) predictions by a subscript 5, we can write

$$\sin^2\theta_5(M_W)=\frac{3}{8}-109\frac{\alpha(M_W)}{48\pi}\ln\left[\frac{M_5}{M_W}\right], \tag{3.1}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)}=\frac{3}{8}-\frac{67\alpha(M_W)}{16\pi}\ln\left[\frac{M_5}{M_W}\right]. \tag{3.2}$$

In the case of SO(10) unification, we can distinguish two different classes of symmetry-breaking chains:

$$\text{(A) SO(10)}\underset{M_U}{\overset{\{54\}}{\longrightarrow}}G_{224P}\underset{M_C}{\overset{\{210\}}{\longrightarrow}}G_{2213P}\underset{M_{R^+}}{\overset{\{45\}}{\longrightarrow}}G_{2113}\underset{M_{R^0}}{\overset{\{126\}}{\longrightarrow}}G_{213}\;,$$

$$\text{(B) SO(10)}\underset{M_U}{\overset{\{54\}}{\longrightarrow}}G_{224P}\underset{M_{R^+}}{\overset{\{45\}}{\longrightarrow}}G_{214}\underset{M_C}{\overset{\{45\}}{\longrightarrow}}G_{2113}\underset{M_{R^0}}{\overset{\{126\}}{\longrightarrow}}G_{213}\;.$$

The corresponding equations for $\sin^2\theta_W$ and α_s can be written down for the following two chains:

$$\sin^2\theta_W(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{48\pi}\left[(110+3T_Y-5T_L)\ln\left[\frac{M_{R^0}}{M_W}\right]+(110+3T_{R^0}+2T_{BL}-5T_L)\ln\left[\frac{M_{R^+}}{M_{R^0}}\right]\right.$$
$$\left.+(44+3T_R+2T_{BL}-5T_L)\ln\left[\frac{M_C}{M_{R^+}}\right]+(-44+3T_R+2T_4-5T_L)\ln\left[\frac{M_U}{M_C}\right]\right], \tag{3.3}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)}=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}\left[(66+T_L+T_Y-\tfrac{8}{3}T_s)\ln\left[\frac{M_{R^0}}{M_W}\right]+(66+T_L+T_{R^0}+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln\left[\frac{M_{R^+}}{M_{R^0}}\right]\right.$$
$$\left.+(44+T_L+T_R+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln\left[\frac{M_C}{M_{R^+}}\right]+(44+T_L+T_R-2T_4)\ln\left[\frac{M_U}{M_C}\right]\right]. \tag{3.4}$$

The T_i's in Eqs. (3.3) and (3.4) denote the contributions of the Higgs-boson multiplets to the β functions for the ith symmetry group. Their contributions have to be included in accordance with the minimal-fine-tuning and extended survival hypothesis.[19] In Table I, we give the formulas for these coefficients. Already a very important conclusion can be drawn looking at Eqs. (3.3) and (3.4). Note that if we ignore the Higgs-boson contributions to these equations in the range $M_{R^0}<\mu<M_{R^+}$, then M_{R^0} drops out of both equations. (In fact, all the T_i's are small, of order unity, in this mass range.) Thus, an important feature of SO(10) grand unified models is that M_{R^0} can be as light as possible without affecting the

TABLE I. Higgs-boson coefficients in β functions. In this table, $T(s_1)$ for a given representation is defined as follows: $\mathrm{Tr}(\theta^a\theta^b)=T(s_1)\delta^{ab}$ where θ^a are generators of the group on the representation space. $d(s_2)$ is the extra dimensionality of the representation s_2.

$T_Y=\left[\frac{Y}{2}\right]^2 d(s_2)$
$T_L=T(s_1)d(s_2)$
$T_{R^0}=I_{3R}{}^2 d(s_2)$
$T_{BL}=\left[\frac{3}{2}\right]\left[\frac{B-L}{2}\right]^2 d(s_2)$
$T_4=T(s_1)d(s_2)$

values of $\sin^2\theta_W(M_W)$ and $\alpha_s(M_W)$. This point has been emphasized before in Refs. 6 and 14. In fact, as was noted in Ref. 6, if all Higgs-boson contributions are ignored, then Eqs. (3.2) and (3.4) imply that

$$M_U=M_5\left[\frac{M_5}{M_{R^+}}\right]^{1/2} \tag{3.5}$$

which is even independent of the scale M_C. From this one can conclude that, if M_{R^+} is lower than the SU(5) scale, M_U for SO(10) becomes higher, making the proton lifetime longer, i.e.,

$$\tau_{10}=\tau_5\left[\frac{M_5}{M_{R^+}}\right]^2 . \tag{3.6}$$

Thus, a longer proton lifetime in the context of SO(10) grand unification implies that there must exist an intermediate left-right-symmetric scale. Tosa, Branco and Marshak[6] have also shown that SU(5)$\times$U(1) breaking chains are ruled out by the proton lifetime result.

We can derive a further equation involving $\sin^2\theta_{10}$ and $\sin^2\theta_5$ using Eqs. (3.1) and (3.3). Again, ignoring the Higgs-boson contributions, we find

$$\Delta(\sin^2\theta_W)=\sin^2\theta_{10}-\sin^2\theta_5 =\frac{11\alpha(M_W)}{24\pi}\left[\ln\left[\frac{M_U{}^2M_5{}^5}{M_C{}^4M_{R^+}{}^3}\right]\right] . \tag{3.7}$$

We easily conclude from Eq. (3.7) that if $M_C=M_U$ and $M_{R^+}\simeq 1$ TeV, then $\Delta(\sin^2\theta_W)\simeq 0.06$, which, on using the Marciano-Sirlin[23] SU(5) result, implies $\sin^2\theta_{10}\simeq 0.275$, in conflict with experimental data.

Let us now look at the symmetry-breaking pattern (B) where $M_{R^+}>M_C$. The equations for $\sin^2\theta_W$ and α_s can be written as

$$\sin^2\theta_W(M_W)=\frac{3}{8}-\alpha\frac{(M_W)}{48\pi}\Bigg[(110+3T_Y-5T_L)\ln\left[\frac{M_{R^0}}{M_W}\right]+(110+3T_{R^0}+2T_{BL}-5T_L)\ln\left[\frac{M_C}{M_{R^0}}\right] + (22+3T_{R^0}+2T-5T_L)\ln\left[\frac{M_{R^+}}{M_C}\right]+(-44+3T_R+2T_4-5T_L)\ln\left[\frac{M_U}{M_{R^+}}\right]\Bigg] , \tag{3.8}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)}=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}\Bigg[(66+T_L+T_Y-\tfrac{8}{3}T_s)\ln\left[\frac{M_{R^0}}{M_W}\right]+(66+T_L+T_{R^0}+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln\left[\frac{M_C}{M_{R^0}}\right] + (66+T_L+T_{R^0}-2T_4)\ln\left[\frac{M_{R^+}}{M_C}\right]+(44+T_L+T_R-2T_4)\ln\left[\frac{M_U}{M_{R^+}}\right]\Bigg] . \tag{3.9}$$

Again, as in chain (A), we note that, ignoring the Higgs-boson contributions, the equations become independent of M_{R^0} and therefore, a low M_{R^0} is allowed by the second symmetry-breaking chain without conflicting with low-energy data. However, as far as the proton lifetime is concerned, the result is sensitive to the Higgs-boson contributions and no unambiguous prediction can be made; nevertheless, no low values of M_C or M_{R^+} can be obtained.

We conclude that, within the framework of conventional SO(10) grand unification, the only interesting physics, beyond the predictions of the standard subgroup G_{213}, is a low-mass ($\approx$300 GeV to 1 TeV) right-handed R^0 (neutral Z_R boson).

IV. EFFECTS OF *D*-PARITY BREAKING ON INTERMEDIATE MASS SCALES IN THE ONE-LOOP APPROXIMATION

In this section, we study the impact of *D*-parity breaking on the intermediate mass scales in SO(10) models. An extensive study of this was done in Ref. 12 and, subsequently, additional calculations were reported in Ref. 24. We will summarize the general situation by extending the discussion of Sec. III. We first note that *D*-parity breaking can occur at any stage above or starting from where $SU(2)_L\times SU(2)_R\times G$ appears as the intermediate symmetry. If we denote this scale by M_P (to conform with the notation of Ref. 12), then the equations for $\sin^2\theta_W$ and α_s are modified depending on the stage at which M_P appears. The general rule for writing down the equations is that, for $M_{R^+} < \mu < M_P$, the Higgs-multiplets contribute to the β function in a left-right-asymmetric manner; right-handed multiplets contribute to the β function and left-handed multiplets, which acquire the mass at the scale M_P, do not contribute.[25] We illustrate this point by choosing case (A) and keeping $M_C < M_P < M_U$. We therefore get

$$\sin^2\theta_W(M_W) = \frac{3}{8} - \frac{\alpha(M_W)}{48\pi}\left[(110+3T_Y-5T_L)\ln\left[\frac{M_{R^0}}{M_W}\right] + (110+3T_{R^0}+2T_{BL}-5T_L)\ln\left[\frac{M_{R^+}}{M_{R^0}}\right]\right.$$
$$+(44+3T'_R+2T'_{BL}-5T'_L)\ln\left[\frac{M_C}{M_{R^+}}\right]+(-44+3T'_R+2T'_4-5T'_L)\ln\left[\frac{M_P}{M_C}\right]$$
$$\left.+(-44+3T_R+2T_4-5T_L)\ln\left[\frac{M_U}{M_P}\right]\right], \tag{4.1}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)} = \frac{3}{8} - \frac{\alpha(M_W)}{16\pi}\left[(66+T_L+T_Y-\tfrac{8}{3}T_s)\ln\left[\frac{M_{R^0}}{M_W}\right] + (66+T_L+T_{R^0}+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln\left[\frac{M_{R^+}}{M_{R^0}}\right]\right.$$
$$+(44+T'_L+T'_R+\tfrac{2}{3}T'_{BL}-\tfrac{8}{3}T'_s)\ln\left[\frac{M_C}{M_{R^+}}\right]+(44+T'_L+T'_R-2T'_4)\ln\left[\frac{M_P}{M_C}\right]$$
$$\left.+(44+T_L+T_R-2T_4)\ln\left[\frac{M_U}{M_P}\right]\right]. \tag{4.2}$$

The primed T_i's stand for the fact that in the indicated mass range, left and right Higgs multiplets contribute in an asymmetric manner.

The first conclusion we draw from Eqs. (4.1) and (4.2) is that both $\sin^2\theta_W$ as well as α_s are independent of the value of the M_{R^0} scale (where $m_W < M_{R^0} < M_{R^+}$), once we ignore the Higgs-boson contributions. We have checked that $3T_Y-5T_L$, $3T_{R^0}+2T_{BL}-5T_L$, $T_L+T_Y-\frac{8}{3}T_s$, and $T_L+T_{R^0}+\frac{2}{3}T_{BL}-\frac{8}{3}T_s$ are of order 1, so that including Higgs bosons does not alter this conclusion. This result is the same as for the conventional SO(10) model. However, as noted in Ref. 12, due to the asymmetric Higgs-boson contributions, there exist symmetry-breaking chains where M_C or M_{R^+} can also remain in the experimentally testable range.

An analysis of all possible descents—allowing for one to four intermediate mass scales (as shown in Fig. 1)—has been made on the basis of the one-loop approximation in Refs. 12 and 24, under the following assumptions: $M_U \gtrsim 10^{14.5}$ GeV (to explain the absence of proton decay), $M_P \gtrsim 10^{12}$ GeV (to satisfy the cosmological constraints), $\sin^2\theta_W(M_W) = 0.22 \pm 0.02$ (Ref. 23), and $\alpha_s(M_W) = 0.10$–0.12. We consider the chains to be of experimental interest if the predicted values of the C, R^+, and R^0 intermediate mass scales are in the following ranges:[20] $10^5 \lesssim M_C \lesssim 10^7$ GeV *or* $10^3 \lesssim M_{R^+} \lesssim 10^5$ GeV *or* $10^{2.5} \lesssim M_{R^0} \lesssim 10^4$ GeV, in order to ensure the possibility of *detectable processes in the near future* and observable ν_e mass.[26]

With the assumption of minimal Higgs multiplets (see above), it turns out that only two chains are acceptable in that they allow detectable M_C (and M_{R^0}) phenomena but that no chains exist with detectable M_{R^+} phenomena. The two testable chains listed in Table II with the corresponding ranges of $\sin^2\theta(M_W)$ and M_U are chains III and IV:

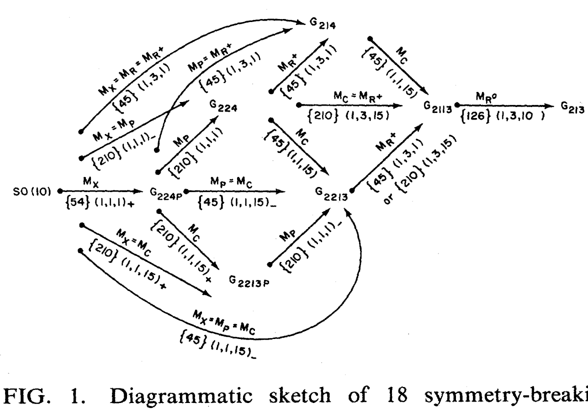

FIG. 1. Diagrammatic sketch of 18 symmetry-breaking chains in SO(10). The SO(10) representations as well as the G_{224} submultiplets responsible for the particular symmetry breaking are shown. Any intermediate group will descend to G_{213} via the {126} representation, adding six more chains. The decomposate (1,1,1) of {54} and (1,1,15) of {210} do not break D parity (hence the subscript P) whereas (1,1,1) of {210} and (1,1,15) of {45} do.

TABLE II. Acceptable and marginal chains in the new SO(10) grand unification in the one- and two-loop approximations with $\alpha_s(M_W)=0.10$.

	Chain	Range of intermediate mass scale	Predicted ranges (a) denotes the one-loop numbers, (b) the two-loop numbers $\sin^2\theta_W(M_W)$	$\log_{10}[M_U\ (\text{GeV})]$
I	$SO(10) \xrightarrow[\{210\}]{M_U=M_P} G_{224} \xrightarrow[\{126\}]{M_C=M_R} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.286–0.270	18.0–17.3
			(b) 0.272–0.260	17.6–16.9
II	$SO(10) \xrightarrow[\{45\}]{M_U=M_P=M_C} G_{2213} \xrightarrow[\{126\}]{M_R} G_{213}$	$3\le\log_{10}[M_R\ (\text{GeV})]\le 5$	(a) 0.263–0.255	18.1–17.6
			(b) 0.262–0.254	17.5–17.3
III	$SO(10) \xrightarrow[\{210\}]{M_U=M_P} G_{224} \xrightarrow[\{210\}]{M_C=M_{R^+}} G_{2113} \xrightarrow[\{126\}]{M_{R^0}} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.203–0.206	17.4–16.7
			(b) 0.177–0.188	16.8–16.4
IV	$SO(10) \xrightarrow[\{54\}]{M_U} G_{224P} \xrightarrow[\{210\}]{M_P} G_{224} \xrightarrow[\{210\}]{M_C=M_{R^+}} G_{2113} \xrightarrow[\{126\}]{M_{R^0}} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.232$\{\log_{10}[M_P\ (\text{GeV})]=15.5\}$	17.0$\{\log_{10}[M_P\ (\text{GeV})]=15\}$
		$(M_{R^0}\simeq M_W)$	(b) 0.227–0.225	16.6–16.1
		$\log_{10}[M_P\ (\text{GeV})]=14.0$ (see Table III)		
V	$SO(10) \xrightarrow[\{45\}]{M_U=M_P=M_C} G_{2213} \xrightarrow[\{45\}]{M_{R^+}} G_{2113} \xrightarrow[\{126\}]{M_{R^0}} G_{213}$	$3\le\log_{10}[M_{R^+}\ (\text{GeV})]\le 5$	(a) 0.257–0.250	17.6–17.2
			(b) 0.258–0.250	17.2–16.6
VI	$SO(10) \xrightarrow[\{210\}]{M_U=M_P} G_{224} \xrightarrow[\{45\}]{M_C} G_{2213} \xrightarrow[\{126\}]{M_R} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.275–0.262	19.5–18.4
			(b) 0.260–0.251	19.2–18.1
VII	$SO(10) \xrightarrow[\{210\}]{M_U=M_P} G_{224} \xrightarrow[\{45\}]{M_R} G_{214} \xrightarrow[\{126\}]{M_C} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.278–0.263	17.8–16.8
			(b) 0.267–0.257	17.4–16.7
VIII	$SO(10) \xrightarrow[\{210\}]{M_U=M_P} G_{224} \xrightarrow[\{45\}]{M_C} G_{2213} \xrightarrow[\{45\}]{M_{R^+}} G_{2113} \xrightarrow[\{126\}]{M_{R^0}} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.275–0.255	19.3–17.6
			(b) 0.251–0.243	18.9–17.7
IX	$SO(10) \xrightarrow[\{210\}]{M_U=M_P} G_{224} \xrightarrow[\{45\}]{M_{R^+}} G_{214} \xrightarrow[\{45\}]{M_C} G_{2113} \xrightarrow[\{126\}]{M_{R^0}} G_{213}$	$5\le\log_{10}[M_C\ (\text{GeV})]\le 7$	(a) 0.270–0.257	18.8–17.8
			(b) 0.263–0.252	20.0–18.5

TABLE II. (*Continued*).

Chain	Range of intermediate mass scale	Predicted ranges (a) denotes the one-loop numbers, (b) the two-loop numbers	
X $SO(10) \underset{\{210\}}{\overset{M_U=M_P}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{210\}}{\overset{M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}$	$3 \le \log_{10}[M_{R^+}\,(\text{GeV})] \le 5$ $[\log_{10} M_C(\text{GeV})=13.0]$ (see Table IV)	(b) 0.227–0.223	17.7–17.0
XI $SO(10) \underset{\{54\}}{\overset{M_U}{\longrightarrow}} G_{224P} \underset{\{210\}}{\overset{M_P}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{210\}}{\overset{M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}$	(b) corresponds to $M_{R^0}=M_{R^+}=M_C$ $=10^5$ GeV for $\log_{10}[M_P\,(\text{GeV})]=14.0$	(b) 0.231	17.5
	(b′) corresponds to $M_{R^0}=M_W$; $M_{R^+}=M_C=10^5$ GeV for $\log_{10} M_P(\text{GeV})=14.0$ (see Table V)	(b′) 0.228	17.0

Chain III:
$$SO(10) \underset{\{210\}}{\overset{M_U=M_P}{\longrightarrow}} G_{224} \underset{\{210\}}{\overset{M_C=M_{R^+}}{\longrightarrow}} G_{213} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}\ , \tag{4.3}$$

Chain IV:
$$SO(10) \underset{\{54\}}{\overset{M_U}{\longrightarrow}} G_{224P} \underset{\{210\}}{\overset{M_P}{\longrightarrow}} G_{224} \underset{\{210\}}{\overset{M_C=M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}\,. \tag{4.4}$$

If we relax the minimality condition—by using the component (1,3,15) Higgs decomposate of {210} at the M_{R^+} scale—*and* allow M_C to reach high values, one chain (chain X in Table II) emerges as an acceptable chain for detectable M_{R^+} phenomena:

Chain X:
$$SO(10) \underset{\{210\}}{\overset{M_U=M_P}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{210\}}{\overset{M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213}\ . \tag{4.5}$$

With the lifting of the minimality condition and the insertion of G_{224P} [between SO(10) and G_{224}] in chain X, we can expect—on the basis of the one-loop approximation—that a chain (listed as chain XI in Table II) will emerge that allows simultaneously detectable M_C, M_{R^+}, and M_{R^0} phenomena. It is important to compute the two-loop contributions to chains III, IV, X, and XI to determine whether their acceptability is maintained. It will turn out that only chain IV survives as the really interesting chain.

Several other chains—identified as marginal by the one-loop analysis in Refs. 12 and 24—are also listed in Table II with their corresponding ranges of parameters. These marginal chains have been subjected to the two-loop analysis to check whether any of these chains becomes acceptable as a consequence. The result is essentially negative with only one chain, chain VIII, almost gaining acceptability. The other permissible chains not listed in Table II have been rejected on the basis of the one-loop approximation (see Refs. 12 and 24) because it is unlikely that the two-loop contributions can convert them into acceptable chains. We proceed to the two-loop analysis.

V. TWO-LOOP ANALYSIS OF MASS HIERARCHIES

In this section, we include the effects of two-loop contributions to $\sin^2\theta_W(M_W)$ and $\alpha(M_W)$. We will use the formulas for the evolution of coupling constants given by Jones:[27]

$$\mu\frac{\partial \alpha_i}{\partial \mu} = \frac{a_i \alpha_i^2}{2\pi} + \frac{1}{8\pi^2}\sum_j b_{ij}\alpha_i^2\alpha_j\ . \tag{5.1}$$

$\alpha_i(\mu)$ is the coupling constant $(g_i^2/4\pi)$ for the group G_i at the mass scale μ, a_i (b_{ij}) are related to the one- (two-) loop coefficient of the β function (the formula for b_{ij} is given in the Appendix), and j runs over all symmetry groups G_j (including G_i) present at the given mass range between μ and M. The coefficients a_i and b_{ij} are given in the Appendix where we also comment on the incorporation of threshold effects. For all chains, we have integrated equation (5.1) numerically. We comment briefly on the algorithm employed in the Appendix.

In general, the differential equation (5.1) has no solution in closed term. An exception occurs when b_{ij} is diagonal, in which case

$$\frac{1}{\alpha_i(\mu)}-\frac{1}{4\pi}\frac{b_{ii}}{a_i}\ln\left[\frac{1}{\alpha_i(\mu)}+\frac{1}{4\pi}\frac{b_{ii}}{a_i}\right] =\frac{1}{\alpha_i(M)}-\frac{1}{4\pi}\frac{b_{ii}}{a_i}\ln\left[\frac{1}{\alpha_i(M)}+\frac{1}{4\pi}\frac{b_{ii}}{a_i}\right] +\frac{a_i}{2\pi}\ln\frac{M}{\mu}\ . \tag{5.2}$$

In this case it is clear that two-loop effects are significant only when $b_{ii}/4\pi a_i$ is significantly different from 0 and $1/\alpha_i(\mu)$ is much different from $1/\alpha_i(M)$. We expect these qualitative factors to hold true in the case when b_{ij} is not diagonal. In particular, a factor which is likely to enhance the two-loop contributions would be the presence of Higgs multiplets with large dimensions. For instance, if $SU(4)_C$ is an intermediate symmetry, the presence of multiplets such as $(1,3,\overline{10})$ and (1,3,15) generates large contributions to b_{ij}. By the same token, if the intermediate symmetry group is G_{2213} at $\mu=M_R$, by the survival hypothesis, the Higgs multiplets that contribute above M_R have smaller dimension and therefore, the two-loop effects are smaller.

These general observations seem to be consistent with our two-loop calculations carried out for eleven symmetry-breaking chains listed in Table II. We summarize our results.

Chain I. This chain is listed in Table II despite its bare marginality because its minimal character [only one intermediate symmetry between SO(10) and G_{213}] makes it useful to illustrate the method. Using Eqs. (A6)—(A8) of the Appendix for this chain, we integrate the β function numerically. For chain I, the mass scales M_C and M_U are shown in Fig. 2 as functions of $\sin^2\theta_W(M_W)$. The two-loop corrections are appreciable but not sufficient to make this chain acceptable. The lowest allowed value of M_C is $\sim 3\times 10^{10}$ GeV corresponding to $\sin^2\theta\simeq 0.24$, which rules out the possibility of observable $\Delta B=2$ processes.

Chain II. Examination of Table II reveals that this case is very marginal but we have calculated the two-loop effects because this chain has been studied in connection with the problem of the baryon asymmetry of the universe.[12] Using Eqs. (A7) and (A9), we obtain two sets of equations in the two mass ranges M_W-M_R and M_R-M_U for the coupling constants $\alpha_Y(M_W)$, $\alpha_{2L}(M_W)$, $\alpha_{3C}(M_W)$, $\alpha_{BL}(M_R)$, $\alpha_{2L}(M_R)$, $\alpha_{2R}(M_R)$, and $\alpha_3(M_R)$ and solve them as before. The solutions are displayed in Fig. 3. As expected, the two-loop corrections are small. For $\sin^2\theta_W=0.24$, the lowest allowed value of M_R is 10^6 GeV. Thus, the two-loop effects for this chain have not improved the situation to give observability to the effects of right-handed currents or neutrino masses in the near future. However, its attractiveness as a model for CP violation[10] and for understanding the baryon asymmetry of the Universe remains.

Chain III. This is one of the two acceptable, and therefore potentially very interesting, chains identified within the framework of the one-loop approximation (see Table III). With G_{224} as the first intermediate stage and G_{2113} as the only other intermediate stage before G_{213}, this is the minimal model that could allow neutron oscillations and detectable R^0 processes. Actually, this was the model to which the two-loop approximation was first applied in order to check whether the interesting one-loop predictions would be sustained.

Following the procedure that we have already outlined, three sets of equations for the coupling constants in the mass ranges $M_W-M_{R^0}$, $M_{R^0}-M_C$, and M_C-M_U are obtained, using (A7), (A10), and (A11). The solutions are displayed in Fig. 4. The two-loop corrections, unfortunately, are large and reduce further the one-loop values

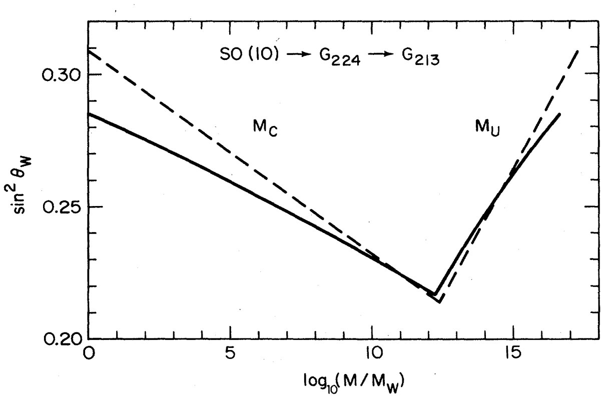

FIG. 2. Intermediate mass scale M_C and unification scale M_U for the symmetry-breaking chain

$$SO(10)\underset{M_U}{\rightarrow}G_{224}\underset{M_C}{\rightarrow}G_{213}\ .$$

General notation for Figs. 2—10: Broken lines denote result of one loop. Solid lines denote 2-loop results. Left arm of the graphs denotes the intermediate scale whereas the right arm denotes the unification scale.

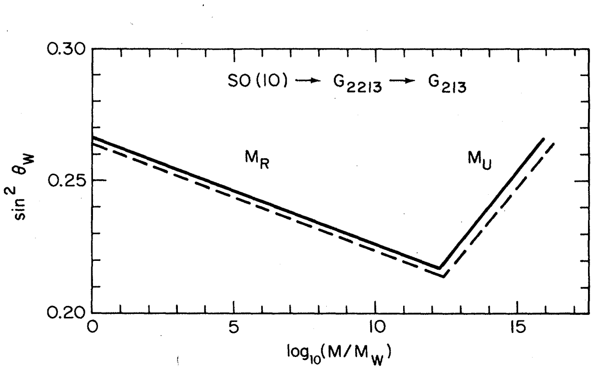

FIG. 3. Intermediate mass scale M_R and the unification scale M_U for the chain

$$SO(10)\underset{M_U}{\rightarrow}G_{2213}\underset{M_R}{\rightarrow}G_{213}\ .$$

See caption to Fig. 2.

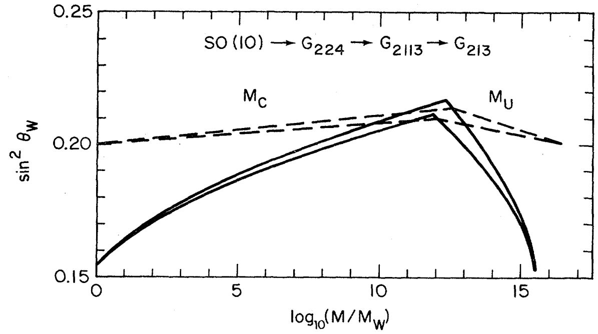

FIG. 4. Intermediate mass scales M_C, M_{R^0} and unification scale M_U for the symmetry-breaking chain

$$\mathrm{SO}(10)\underset{M_U}{\rightarrow}G_{224}\underset{M_C}{\rightarrow}G_{2113}\underset{M_{R^0}}{\rightarrow}G_{213}\ .$$

Top line in each set corresponds to $M_{R^0}=M_C$ and bottom line to $M_{R^0}\simeq M_W$. See caption to Fig. 2.

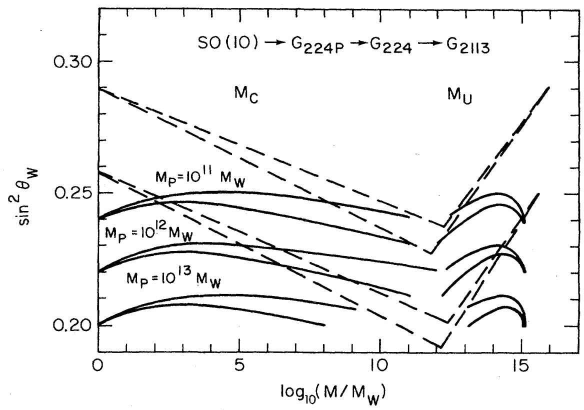

FIG. 5. Mass scales M_C and M_U for various values of M_P for the chain

$$\mathrm{SO}(10)\underset{M_U}{\rightarrow}G_{224P}\underset{M_P}{\rightarrow}G_{224}\underset{M_C}{\rightarrow}G_{2113}\underset{M_{R^0}}{\rightarrow}G_{213}\ .$$

Top line denotes $M_{R^0}=M_C$ and bottom line $M_{R^0}=M_W$. See caption to Fig. 2.

of $\sin^2\theta_W$ which were already marginal ($\simeq 0.205$), by 15–20% in the region $M_C\simeq 10^5$–10^7 GeV; the possibility of observable $\Delta B=2$ transitions is thus ruled out although low values of the R^0 mass are still possible. As noted at the beginning of this section, such significant corrections to $\sin^2\theta_W$ arise because of the large Higgs-multiplet scales which result in $\alpha(\mu)$ being significantly different from $\alpha(\mu)$ at the one-loop level.

Chain IV. Within the framework of the minimal *D*-parity SO(10) model, chain IV is the other acceptable chain in the one-loop approximation listed in Table II. It was noted very early[12] as a promising candidate for observable $\Delta B=2$ processes, $K_L\rightarrow\mu\bar{e}$, and a low mass R^0 boson. This chain differs from chain III only in the presence of the additional intermediate symmetry stage G_{224P} that allows M_P to be smaller than M_U (in chain III, $M_P=M_U$). This feature turns out to be decisive in ensuring that the conclusions drawn from the one-loop calculations are unaltered by the two-loop corrections.

Again, following our procedures, the four sets of equations in the mass ranges M_W–M_{R^0}, M_{R^0}–M_C, M_C–M_P, and M_P–M_U are obtained, using Eqs. (A7) and (A10)–(A12). Their solutions are shown in Fig. 5. At the one-loop level, the value of $\sin^2\theta_W$ is sensitive to the M_P scale.[12] Corrections to $\sin^2\theta_W$ which tend to bring it below the admissible range can be compensated by adjusting M_P to a lower value. Table III shows how M_P alters the values of $\sin^2\theta_W(M_W)$ and M_U for the detectable ranges of M_C and M_{R^0}.

This chain is the only acceptable chain among all those considered using the minimality condition (chains I–IX) and has a very interesting set of experimental consequencies, to wit:

(a) Since $M_C=M_{R^+}\approx 10^5$–10^6 GeV, this would yield (i) a branching ratio for $K_L^0\rightarrow\mu\bar{e}\approx 7\times(10^{-8}$–$10^{-12})$ and (ii) an n-$\bar{n}$ oscillation time, $\tau_{n\bar{n}}\approx 10^8$–$10^9$ sec. As the uncertainty in the determination of $\sin^2\theta_W$ narrows, the prediction for $B(K_L^0\rightarrow\mu\bar{e})$ becomes more precise. Also, as we see from Table III, the prediction cannot be adjusted arbitrarily by varying M_P since $\sin^2\theta_W$ depends sensitively on M_P. This chain is also consistent with a hydrogen-antihydrogen oscillation time of about 10^{12} yr, which implies the exotic double proton decay[28] mode $pp\rightarrow e^+e^+$ with an expected lifetime of the order of 10^{32}–10^{33} yr.

(b) A proton lifetime of $\tau_p\approx 6.5\times 10^{35.0\pm 0.9}\,(\Lambda_{\overline{\mathrm{MS}}}/160\ \mathrm{MeV})^4$ yr ($\overline{\mathrm{MS}}$ denotes minimal-subtraction scheme) is predicted which, if $\Lambda_{\overline{\mathrm{MS}}}$ is 80 MeV, would imply $\tau_p\simeq 4\times 10^{33}$ yr. Given the uncertainties in our approximations, this may barely be within the reach of some ongoing experiments. It is also worth noting that, if τ_p is close to its lower end, then the SO(10) character and especially the intermediate right-handed scale can be tested by looking at the following relation between branching ratios:

$$B(p\rightarrow e^+\pi^0)=B(p\rightarrow\bar{\nu}\pi^+)\ . \tag{5.3}$$

For comparison, the corresponding SU(5) formula has a factor $\frac{2}{5}$ on the left-hand side of the equation. Also, it should be mentioned that, if we wish to make proton decay more visible, the M_C scale becomes higher and n-$\bar{n}$ oscillation as well as $K_L^0\rightarrow\mu\bar{e}$ are suppressed.

(c) The neutral Z_R boson is expected to be in the range of 300 to 1000 GeV and could be detected at the planned superconducting supercollider (SSC).[29] This would also lead to measurable neutrino masses (~ 1 eV).

(d) The parameter M_P is narrowed to the range of $10^{14.3\pm 1.0}$ GeV, thereby satisfying the cosmological con-

TABLE III. Dependence of ranges of $\sin^2\theta_W(M_W)$ and M_U (corresponding to $5\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7$) on M_P for chain IV in two-loop approximation with $M_{R^0}=M_W$ (see Fig. 5).

$\log_{10}[M_P\ (\mathrm{GeV})]$	$\sin^2\theta_W(M_W)$	$\log_{10}[M_U\ (\mathrm{GeV})]$
13.0	0.246–0.244	16.3–15.7
14.0	0.227–0.225	16.2–15.8
15.0	0.208–0.206	16.3–15.9

straints. Table III shows the variation of $\sin^2\theta_W$ with M_P.

It is also interesting that, if we impose the *CP*-violation constraint that $M_{W_R} \leq 10^5$ GeV, then this chain becomes a unique and predictive chain, which can be tested by n-$\bar{n}$-oscillation and $K \to \mu \bar{e}$ experiments. Should the Kobayashi-Maskawa model fail to explain the *CP*-violating parameters ϵ and ϵ', this result will be of great interest.

Chain V. We have succeeded in identifying one chain starting with SO(10) that can give detectable C and R^0 boson processes. As part of our program to suggest experimental tests of the SO(10) model with D parity decoupled from $SU(2)_R$ breaking, we have searched for at least one chain that will give detectable R^+ processes. Examination of Table II indicates that the most promising candidate in this respect is chain V and the question is whether the two-loop corrections will change the status of this chain from "marginal" to "acceptable" for detectable R^+ processes (let us recall our condition for acceptability is $10^3 \leq M_{R^+} \leq 10^5$).

Chain V differs from chain II only in containing one more intermediate symmetry G_{2113}. The equations for the two-loop computations are obtained using (A7), (A10), and (A13). Solutions including one- and two-loop effects are displayed in Fig. 6. The detectability of W_{R^+} processes is still marginal for the near future.

Chains VI—IX listed in Table II will be discussed very briefly because they are highly marginal in the one-loop approximation and the two-loop corrections turn out to be insufficient to alter the status of any one of them in a significant way (except for chain VIII—see below).

Chain VI. Three sets of equations for the coupling constants in the mass range $M_W - M_R$, $M_R - M_C$, and $M_C - M_U$ are obtained, using Eqs. (A7), (A9) and (A14); the solutions are shown in Fig. 7. The lowest allowed value of M_C (or M_R) is $\sim 5\times 10^9$ GeV for $\sin^2\theta_W = 0.24$. Thus, this case is still uninteresting in the two-loop approximation.

Chain VII. For this chain, a right-handed charged gauge boson mass larger than the leptoquark or the R^0-boson mass ($M_{R^+} > M_C = M_{R^0}$) is permitted. Three different sets of equations in the mass ranges $M_W - M_C$, $M_C - M_{R^+}$, and $M_{R^+} - M_U$ are obtained, using (A7), (A15), and (A16); their solutions are displayed in Fig. 8. Interesting solutions relevant for low-energy phenomenology are still absent in this chain.

Chain VIII. Compared to chain VI, this chain has one more intermediate symmetry G_{2113} which provides further scope for creating larger left-right asymmetry when D parity is decoupled from $SU(2)_R$ breaking. Four sets of equations for the coupling constants in the mass ranges $M_W - M_{R^0}$, $M_{R^0} - M_{R^+}$, $M_{R^+} - M_C$, and $M_C - M_U$ are obtained, using (A7), (A10), (A13), and (A17); the solutions are shown in Fig. 9. The solutions become marginal with $\sin^2\theta_W(M_W)$ slightly in excess of 0.24 and the unification mass hovering above 10^{18} GeV. If the symmetry-breaking from G_{2213} to G_{2113} is instead implemented by using a 210-dimensional Higgs multiplet, the scale of M_{R^+} becomes of the order of a few TeV and could lead to an experimental signature in the planned Superconducting Super-Collider (this is chain X—see below). We have also checked that for $\sin^2\theta_W \leq 0.24$, for no value of M_C can M_{R^+} be lower than 10^6 GeV. In this case, we further find that if $\alpha_s(M_W) \simeq 0.12$, for $\sin^2\theta_W \simeq 0.238 - 0.24$, M_C is in the range $10^{6.5}$ to 10^7 GeV, which is of interest in connection with n-$\bar{n}$ oscillation.

Chain IX. Similar to chain VII, this chain allows the R^+ mass to be heavier than the leptoquark-gauge-boson mass M_C, which, in turn, is heavier than the R^0 mass. The four sets of equations for the coupling constants in the mass ranges $M_W - M_{R^0}$, $M_{R^0} - M_C$, $M_C - M_{R^+}$, and $M_{R^+} - M_U$ are obtained, using (A7), (A10), (A18), and

TABLE IV. Dependence of ranges of $\sin^2\theta_W(M_W)$ and M_U (corresponding to $3 \leq \log_{10}[M_{R^+}\ (\text{GeV})] \leq 5$) on M_C for chain X in two-loop approximation with $M_{R^0} = M_W$ (see Fig. 11).

$\log_{10}[M_P\ (\text{GeV})]$	$\sin^2\theta_W(M_W)$	$\log_{10}[M_U\ (\text{GeV})]$
11.0	0.211—0.208	17.9—17.3
13.0	0.227—0.223	17.7—17.0
14.0	0.234—0.230	17.6—16.9
15.0	0.241—0.238	17.5—16.8

TABLE V. Dependence of $\sin^2\theta_W(M_W)$ and M_U on M_P for chain XI in two-loop approximation for (i) $M_{R^0} = M_{R^+} = M_C = 10^5$ GeV and (ii) $M_{R^0} = M_W$; $M_{R^+} = M_C = 10^5$ GeV (see Fig. 12).

$\log_{10}[M_P\ (\text{GeV})]$	$\sin^2\theta_W(M_W)$	$\log_{10}[M_U\ (\text{GeV})]$
13.0	(i) 0.252	17.7
	(ii) 0.248	17.8
14.0	(i) 0.231	17.5
	(ii) 0.228	17.3
15.0	(i) 0.211	17.7
	(ii) 0.208	17.5

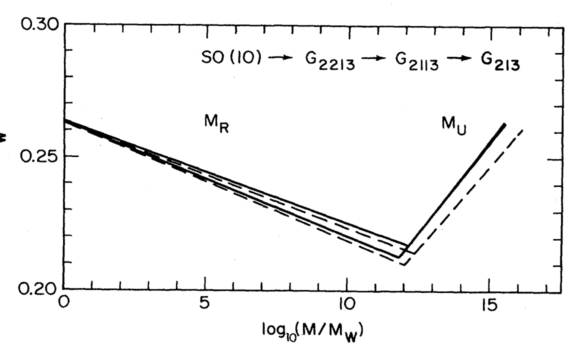

FIG. 6. Mass scales M_R and M_U for the symmetry-breaking chain

$$SO(10) \underset{M_U}{\longrightarrow} G_{2213} \underset{M_{R^+}}{\longrightarrow} G_{2113} \underset{M_{R^0}}{\longrightarrow} G_{213}\ .$$

Top line in figure corresponds to $M_{R^0} \simeq M_{R^+}$ and bottom line $M_{R^0} \simeq M_W$.

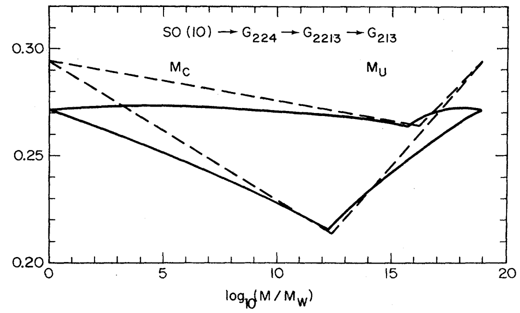

FIG. 7. Mass scales M_C and M_U for the symmetry-breaking chain

$$\mathrm{SO}(10) \underset{M_U}{\rightarrow} G_{224} \underset{M_C}{\rightarrow} G_{2213} \underset{M_R}{\rightarrow} G_{213} .$$

Bottom line corresponds to $M_C \sim M_R$ and top line to $M_R \sim M_W$.

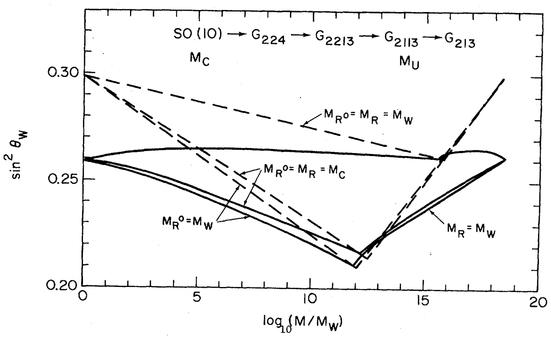

FIG. 9. Mass scales M_C and M_U for the symmetry-breaking chains

$$\mathrm{SO}(10) \underset{M_U}{\rightarrow} G_{224} \underset{M_C}{\rightarrow} G_{2213} \underset{M_{R^+}}{\rightarrow} G_{2113} \underset{M_{R^0}}{\rightarrow} G_{213} .$$

The top line in each graph corresponds to $M_R \simeq M_W$ and bottom line to $M_{R^0} \simeq M_C$.

(A19) and the solutions are shown in Fig. 10. For this chain, the lowest allowed value of M_C is 10^9 GeV corresponding to $\sin^2\theta_W \simeq 0.24$. An interesting aspect of this chain is the connection between the n-$\bar{n}$ oscillation amplitude $\delta m_{n\text{-}\bar{n}}$ and the scale M_C: the dominant graph involves two weak-isospin-zero (i.e., $I_{3R}=0$) components of the diquark fields and yields[17]

$$\delta m_{n\text{-}\bar{n}} \simeq \frac{\lambda h^3 V_{R^0}}{{M_C}^4 {M_{R^+}}^2} \cdots . \tag{5.4}$$

In principle, this could give rise to observable n-$\bar{n}$ oscillations for high M_{R^+}, if M_C is sufficiently low. However, our analysis does not allow M_C to be low enough for this purpose, consistent with $\sin^2\theta_W$. The same remarks apply to chain VII.

Chain X. As remarked before, we have deviated from the minimality principle by choosing a {210}-dimensional Higgs-boson representation to break G_{2213} to G_{2113}. We find that this leads to the interesting result that for $10^{12}\ \mathrm{GeV} \le M_C \le 10^{14}$ GeV and $10^{17}\ \mathrm{GeV} \le M_U \le 10^{18}$ GeV, we find low-mass R^+ and R^0 ($M_{R^+} \simeq 1$ TeV and $M_{R^0} \simeq 1$ TeV) for $\sin^2\theta_W \simeq 0.21-0.23$. In fact, this and the subsequent chain to be discussed are the only two where R^+ physics is in the detectable range. The solutions are shown in Fig. 11 and more results are given in Table IV.

Chain XI. This chain also deviates from the minimality principle in the same way as chain X and, like chain IV, includes the intermediate symmetry G_{224P}. Like chain X, it allows us to obtain simultaneously low mass W_R, Z_R bosons as well as $M_C \simeq 10^4-10^5$ GeV for $\sin^2\theta_W \simeq 0.23$, $M_U \simeq 10^{16}$ GeV, and $M_P \simeq 10^{14}$ GeV. The solutions are shown in Fig. 12 and more detailed results are given in Table V.

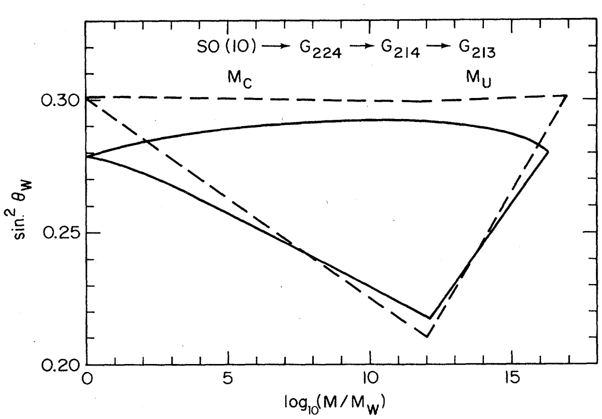

FIG. 8. Mass scale M_R and M_U for the symmetry-breaking chain

$$\mathrm{SO}(10) \underset{M_U}{\rightarrow} G_{224} \underset{M_R}{\rightarrow} G_{214} \underset{M_C}{\rightarrow} G_{213} .$$

Top line corresponds to $M_C \sim M_W$ and bottom line to $M_C \sim M_R$ in each set.

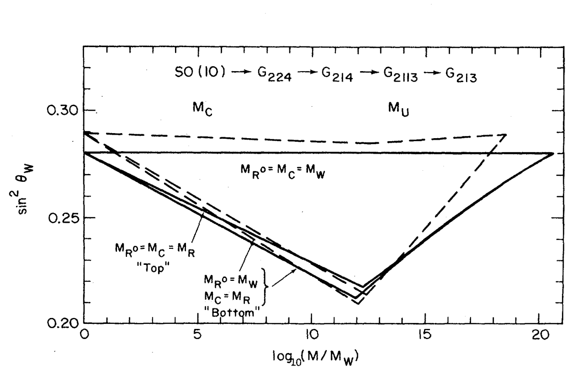

FIG. 10. Mass scales M_C and M_U for the symmetry-breaking chain

$$\mathrm{SO}(10) \underset{M_U}{\rightarrow} G_{224} \underset{M_{R^+}}{\rightarrow} G_{214} \underset{M_C}{\rightarrow} G_{2113} \underset{M_{R^0}}{\rightarrow} G_{213} .$$

Top line corresponds to $M_{R^0} \simeq M_W$ and bottom line to $M_{R^0} \simeq M_C$.

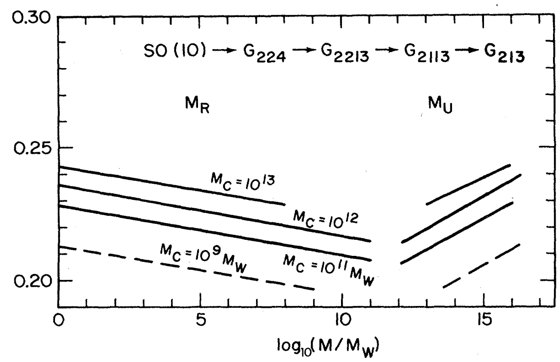

FIG. 11. Mass scales for the symmetry-breaking chain:

$$\mathrm{SO}(10) \underset{M_U}{\longrightarrow} G_{224} \underset{M_C}{\longrightarrow} G_{2213} \underset{M_{R^+}}{\longrightarrow} G_{2113} \underset{M_{R^0}}{\longrightarrow} G_{213} ,$$

where the 210-dimensional Higgs representation has been used between G_{2213} to G_{2113} instead of the 45-dimensional Higgs representation. (See Fig. 9.)

VI. CONCLUSIONS

The concept of D-parity breaking seems to be crucial for opening up interesting experimental tests for SO(10) grand unification. In this paper, we have carried out an exhaustive two-loop analysis of various SO(10) breaking chains that involve G_{224} and D parity as intermediate symmetries in order to see if any of them yield detectable M_C, M_{R^+}, and/or M_{R^0} scales. Although, in the figures, we have presented solutions only for $\alpha_s(M_W)\simeq 0.1$, we have also computed them for $\alpha_s(M_W)\simeq 0.12$ and we find that in all but chain VIII, there is little change in the results. Of all possible chains examined, where minimal Higgs multiplets are used, we have identified a unique symmetry-breaking chain: $\mathrm{SO}(10)\rightarrow G_{224P}\rightarrow G_{224}\rightarrow G_{213}$, which leads to $M_C\simeq M_{R^+}\simeq 10^5$ GeV and $M_{R^0}\simeq 1$ TeV for acceptable values of $\sin^2\theta_W(M_W)$ and $\alpha_s(M_W)$. The key experiments that will test this unique SO(10) model are (i) neutron-antineutron oscillation with a transition time of order 10^8-10^9 sec, which will test the Higgs sector of the model; (ii) $K_L\rightarrow\mu\bar{e}$ decay with a branching ratio $\simeq 7\times(10^{-8}-10^{-12})$, which will test the $\mathrm{SU}(4)_C$ gauge sector of the theory; and (iii) detection of a "live" Z_R boson with a mass in the range 300 GeV to 1 TeV.

This unique chain has several other attractive features that should be noted. The grand unification mass falls in the narrow range $10^{16.0\pm0.2}$ GeV so that the predicted value of $\tau_p\simeq 6.5\times 10^{35.0\pm0.9}$ $(\Lambda_{\overline{\mathrm{MS}}}/160\ \mathrm{MeV})^4$ yr may barely be within reach of experiment. The value of M_P falls in the range $10^{14.3\pm1.0}$ GeV, completely consistent with cosmological constraints.[8,9] Moreover, this chain requires $M_{R^+}\simeq M_C$ so that if n-$\bar{n}$ oscillations and/or $K_\mu\rightarrow\mu\bar{e}$ are seen, it is supportive of the idea that the observed CP-violating effects in $K_L\rightarrow 2\pi$ decay are due to right-handed currents in a left-right symmetric model.[10]

Thus, for the first time, it has been possible to suggest experimental tests of SO(10) grand unification which can be carried out in the not-too-distant future. These tests are associated with a relatively low $\mathrm{SU}(4)_C$ scale and if this is borne out, it would be a significant step in our understanding of the basic quark-lepton symmetry that is implied by the "replication" of the three generations of quarks and leptons. Furthermore, in view of the fact that SO(10) is the only (simple) grand unification group other than SU(5) that does not require mirror and exotic fermions and contains the standard group $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{SU}(3)_C$ as a subgroup, our results ought to provide new impetus to experimental searches for n-$\bar{n}$ oscillation and $K_L^0\rightarrow\mu\bar{e}$ decay. The low mass scale for $\mathrm{SU}(4)_C$ also raises the tantalizing possibility that there may exist leptoquark Higgs bosons[30] in the low-mass range (possibly as low as 100 GeV) which could be looked for in existing and planned colliders. Finally, the importance of measuring $\sin^2\theta_W$ to an accuracy of 0.005 is emphasized by our work.

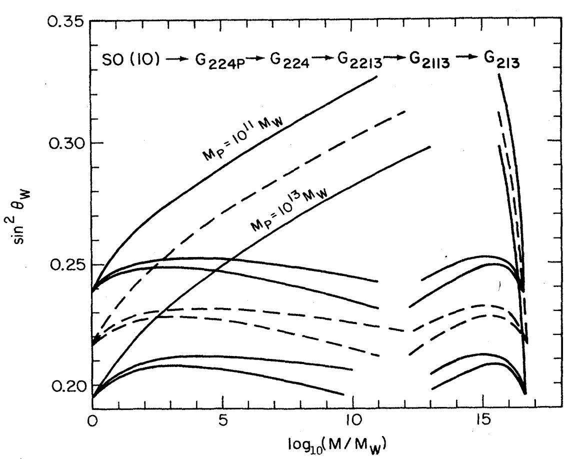

FIG. 12. Same as Fig. 11 except that an intermediate G_{224P} symmetry is introduced at the first stage of breaking.

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APPENDIX

In this appendix, we obtain two-loop equations for nine different cases of SO(10)-symmetry-breaking patterns reported in Secs. III and IV. Including one- and two-loop effects in the β_i function for a given symmetry group G_i occurring in association with others in the intermediate symmetry $G=G_1\times G_2\times G_3\times\cdots\times G_n$, the renormalization-group equations can be written as[27]

$$\mu\frac{\partial\alpha_i}{\partial\mu}=\frac{a_i\alpha_i^2}{2\pi}+\frac{1}{8\pi^2}\sum_j b_{ij}\alpha_i^2\alpha_j , \tag{A1}$$

where g_i is the coupling constant of the group G_i. The one-loop coefficient a_i for the $G_i\equiv\mathrm{SU}(N)$ group with $N>2$ is defined as

$$a_i = \tfrac{4}{3} n_g + \tfrac{1}{3} T(S_i) - \frac{11N}{3} \ . \tag{A2}$$

[In (A2), $N=0$ for any U(1) group.] The coefficients b_{ij} have been defined in Ref. 27. Here, the first, second, and third terms represent the fermion, complex Higgs-boson, and gauge-boson contributions, respectively. The formula for b_{ij} reads as $(i \neq j)$

$$b_{ii} = [\tfrac{10}{3} C_2(G_i) + 2C_2(R_i)] T(R_i) d(R_j) + [\tfrac{2}{3} C_2(G_i) + 4C_2(s_i)] T(s_i) d(s_j) - \tfrac{34}{3} [C_2(G_i)]^2 \ , \tag{A3a}$$

$$b_{ij} = [2C_2(R_j) T(R_i) + 4C_2(s_j) d(s_j) T(s_i)] \ , \tag{A3b}$$

where $\sum_a \theta_R^a \theta_R^a = C_2(R) I$, with θ_R^a the generators of the group in a particular representation R. $\mathrm{Tr}(\theta_R^a \theta_R^b) = T(R) \delta_{ab}$ and $d(R)$ is the dimension of R. Also, we note the relation $C_2(R) d(R) = T(R) n$, where n is the number of group generators. Given the coefficients a_j and b_{ij}, and the coupling constants $\alpha_j(M)$, one can integrate Eq. (A1) numerically to obtain the coupling constants $\alpha_j(\mu)$, where $\mu \leq M$. We use fifth-order Runge-Kutta techniques to perform the numerical integration.

In the one-loop approximation, one equates the coupling constants across regions where different symmetry groups are good. In the two-loop approximation, one must modify the matching condition.[31] In particular, if G_j breaks to G_i, where G_j is simple,

$$\frac{1}{\alpha_j(M)} - \frac{C_j}{12\pi} = \frac{1}{\alpha_i(M)} - \frac{C_i}{12\pi} \ . \tag{A4}$$

C_j and C_i are the values of the quadratic Casimir operator evaluated in the adjoint representation. For SU(N), it is N.

For example, the U(1), SU(2), SU(3), and SU(5) couplings are related across the SU(5) threshold M_X by

$$\frac{1}{\alpha_1(M_X)} = \frac{1}{\alpha_2(M_X)} - \frac{1}{6\pi} = \frac{1}{\alpha_3(M_X)} - \frac{1}{4\pi} = \frac{1}{\alpha_5(M_X)} - \frac{5}{12\pi} \ . \tag{A5}$$

Our strategy for evaluating the two renormalization-group equations for SO(10) follows. First we fix all the mass scales but M_U. Numerically integrating the renormalization-group equations in the various energy regions, and correctly matching coupling constants across energy thresholds, we attempt to find values of M_U and the unification coupling which yield $\alpha_e = \frac{1}{128}$ and $\alpha_s = 0.10$ or 0.12. Any such solution then predicts $\sin^2\theta_W$ and M_U.

The a_i and the b_{ij}'s for the various chains follow. For convenience, we give B_{ij},

$$B_{ij} = b_{ij} / a_j \ . \tag{A6}$$

Chain I. For the range M_W—M_C,

$$a_{1Y} = \tfrac{41}{10} \ , \quad a_{2L} = -\tfrac{19}{6} \ , \quad a_{3C} = -7 \ ,$$

$$B_{ij} = \begin{array}{c} \\ 1Y \\ 2L \\ 3C \end{array} \begin{array}{c} \begin{array}{ccc} 1Y & 2L & 3C \end{array} \\ \begin{bmatrix} \frac{104}{205} & -\frac{81}{95} & -\frac{44}{35} \\ \frac{9}{41} & -\frac{35}{19} & -\frac{12}{7} \\ \frac{11}{41} & -\frac{27}{19} & \frac{26}{7} \end{bmatrix} \end{array} . \tag{A7}$$

For the range M_C—M_U,

$$a_{2L} = -3 \ , \quad a_{2R} = \tfrac{11}{3} \ , \quad a_{4C} = -\tfrac{23}{3} \ ,$$

$$B_{ij} = \begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array} \begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \begin{bmatrix} -\frac{8}{3} & \frac{9}{26} & -\frac{135}{34} \\ -1 & \frac{584}{11} & -\frac{2295}{46} \\ -\frac{3}{2} & \frac{459}{22} & -\frac{643}{46} \end{bmatrix} \end{array} . \tag{A8}$$

Chain II. For the range M_W—M_R, same as in Eqs. (A7). For the range M_R—M_U,

$$a_{BL} = \tfrac{11}{2} \ , \quad a_{2L} = -3 \ , \quad a_{2R} = -\tfrac{7}{3} \ , \quad a_{3C} = -7 \ ,$$

$$B_{ij} = \begin{array}{c} \\ 1BL \\ 2L \\ 2R \\ 3C \end{array} \begin{array}{c} \begin{array}{cccc} 1BL & 2L & 2R & 3C \end{array} \\ \begin{bmatrix} \frac{61}{11} & -\frac{3}{2} & -\frac{243}{14} & -\frac{4}{7} \\ \frac{3}{11} & -\frac{8}{3} & -\frac{9}{7} & -\frac{12}{7} \\ \frac{27}{11} & -1 & -\frac{80}{7} & -\frac{12}{7} \\ \frac{1}{11} & -\frac{3}{2} & -\frac{27}{14} & \frac{26}{7} \end{bmatrix} \end{array} . \tag{A9}$$

Chain III. For the range M_W—M_{R^0}, same as in (A7). For the range M_{R^0}—M_C,

$$a_{BL} = \tfrac{9}{2} \ , \quad a_{1R} = \tfrac{9}{2} \ , \quad a_{2L} = -\tfrac{19}{6} \ , \quad a_{3C} = -7 \ ,$$

$$B_{ij} = \begin{array}{c} \\ 1BL \\ 1R \\ 2L \\ 3C \end{array} \begin{array}{c} \begin{array}{cccc} 1BL & 1R & 2L & 3C \end{array} \\ \begin{bmatrix} \frac{25}{9} & \frac{5}{3} & -\frac{27}{38} & -\frac{4}{7} \\ \frac{5}{3} & \frac{5}{3} & -\frac{9}{19} & -\frac{12}{7} \\ \frac{1}{3} & \frac{1}{9} & -\frac{35}{19} & -\frac{12}{7} \\ \frac{1}{9} & \frac{1}{3} & -\frac{27}{19} & \frac{27}{7} \end{bmatrix} \end{array} . \tag{A10}$$

For the range M_C—M_U,

$$a_{2L} = -3 \ , \quad a_{2R} = \tfrac{26}{3} \ , \quad a_{4C} = -\tfrac{17}{3} \ ,$$

$$B_{ij} = \begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array} \begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \begin{bmatrix} -\frac{8}{3} & \frac{9}{26} & -\frac{135}{34} \\ -1 & \frac{502}{13} & -\frac{3735}{34} \\ -\frac{3}{2} & \frac{747}{52} & -\frac{1315}{34} \end{bmatrix} \end{array} . \tag{A11}$$

Chain IV. In the mass ranges M_W—M_{R^0}, M_{R^0}—M_C, and M_C—M_P, the coefficients are the same as in (A7), (A10), and (A11), respectively. For the range M_P—M_U,

$$a_{2L}=\tfrac{26}{3}\ ,\quad a_{2R}=\tfrac{26}{3}\ ,\quad a_{4C}=-\tfrac{2}{3}\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \left[\begin{array}{ccc} \frac{502}{13} & \frac{9}{26} & -\frac{3735}{4} \\ \frac{9}{26} & \frac{502}{13} & -\frac{3735}{4} \\ \frac{747}{52} & \frac{747}{52} & -\frac{3103}{4} \end{array}\right] \end{array} . \tag{A12}$$

Chain V. For the range $M_W-M_{R^0}$, same as in (A7). For the range $M_{R^0}-M_{R^+}$, same as in (A10). For the range $M_{R^+}-M_U$,

$$a_{BL}=\tfrac{11}{2}\ ,\quad a_{2L}=-3\ ,\quad a_{2R}=-2\ ,\quad a_{3C}=-7\ ,$$

$$B_{ij}=\begin{array}{c} \\ 1BL \\ 2L \\ 2R \\ 3C \end{array}\begin{array}{c} \begin{array}{cccc} 1BL & 2L & 2R & 3C \end{array} \\ \left[\begin{array}{cccc} \frac{61}{11} & -\frac{3}{2} & -\frac{81}{4} & -\frac{4}{7} \\ \frac{3}{11} & -\frac{8}{3} & -\frac{3}{2} & -\frac{12}{7} \\ \frac{27}{11} & -1 & -20 & -\frac{12}{7} \\ \frac{1}{11} & -\frac{3}{2} & \frac{9}{4} & \frac{26}{7} \end{array}\right] \end{array} . \tag{A13}$$

Chain VI. For the range M_W-M_R, same as in (A7). For the range M_R-M_C, same as in (A9). For the range M_C-M_U,

$$a_{2L}=-3\ ,\quad a_{2R}=\tfrac{11}{3}\ ,\quad a_{4C}=-7\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{8}{3} & \frac{9}{11} & -\frac{45}{14} \\ -1 & \frac{584}{11} & -\frac{465}{14} \\ -\frac{3}{2} & \frac{459}{22} & -\frac{867}{42} \end{array}\right] \end{array} . \tag{A14}$$

Chain VII. For the range M_W-M_C, same as in (A7). For the range $M_C-M_{R^+}$,

$$a_{2L}=-\tfrac{19}{6}\ ,\quad a_{1R}=\tfrac{15}{2}\ ,\quad a_{4C}=-\tfrac{29}{3}\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 1R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 1R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{35}{19} & \frac{1}{15} & -\frac{135}{58} \\ -\frac{9}{19} & \frac{28}{5} & -\frac{229}{116} \\ -\frac{27}{19} & \frac{13}{10} & \frac{1019}{232} \end{array}\right] \end{array} . \tag{A15}$$

For the range $M_{R^+}-M_U$,

$$a_{2L}=-3\ ,\quad a_{2R}=4\ ,\quad a_{4C}=-\tfrac{23}{3}\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{8}{3} & \frac{3}{4} & -\frac{135}{46} \\ -1 & 51 & -\frac{2295}{46} \\ -\frac{3}{2} & \frac{153}{8} & -\frac{643}{46} \end{array}\right] \end{array} . \tag{A16}$$

Chain VIII. In the mass ranges $M_W-M_{R^0}$, $M_{R^0}-M_{R^+}$, and $M_{R^+}-M_C$, the a and B coefficients are the same as in (A7), (A10), and (A13), respectively. For the range M_C-M_U,

$$a_{2L}=-3\ ,\quad a_{2R}=4\ ,\quad a_{4C}=-7\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{8}{3} & \frac{3}{4} & \frac{15}{14} \\ -1 & 51 & -\frac{465}{14} \\ -\frac{3}{2} & \frac{153}{8} & -\frac{867}{43} \end{array}\right] \end{array} . \tag{A17}$$

Chain IX. For the range $M_W-M_{R^0}$, same as in (A7). For the range $M_{R^0}-M_C$, same as in (A10). For the range $M_C-M_{R^+}$,

$$a_{2L}=-\tfrac{19}{6}\ ,\quad a_{1R}=\tfrac{15}{2}\ ,\quad a_4=-9\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 1R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 1R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{35}{19} & \frac{1}{15} & -\frac{5}{2} \\ -\frac{9}{19} & \frac{28}{5} & \frac{85}{4} \\ -\frac{27}{19} & \frac{13}{10} & \frac{123}{216} \end{array}\right] \end{array} . \tag{A18}$$

For the range, $M_{R^+}-M_U$,

$$a_{2L}=-3\ ,\quad a_{2R}=4\ ,\quad a_{4C}=-7\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 2R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 2R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{8}{3} & \frac{3}{4} & \frac{45}{14} \\ -1 & 51 & -\frac{465}{14} \\ -\frac{3}{2} & \frac{153}{8} & -\frac{867}{42} \end{array}\right] \end{array} . \tag{A19}$$

Chain X. In the mass ranges $M_W-M_{R^0}$, $M_{R^0}-M_{R^+}$, and $M_{R^+}-M_C$, the a and B coefficients are the same as in (A7), (A10), and (A13), respectively. For the range M_C-M_U,

$$a_{2L}=-3\ ,\quad a_{2R}=\tfrac{26}{3}\ ,\quad a_4=-5\ ,$$

$$B_{ij}=\begin{array}{c} \\ 2L \\ 1R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 1R & 4C \end{array} \\ \left[\begin{array}{ccc} -\frac{8}{3} & \frac{9}{26} & -\frac{9}{2} \\ -1 & \frac{502}{13} & -\frac{249}{2} \\ -\frac{3}{2} & \frac{747}{52} & -\frac{513}{10} \end{array}\right] \end{array} . \tag{A20}$$

Chain XI. In the mass ranges $M_W-M_{R^0}$, $M_{R^0}-M_{R^+}$, $M_{R^+}-M_C$, and M_C-M_P, the a and B coefficients are the same as in (A7), (A10), (A13), and (A20). For the range M_P-M_U,

$$a_{2L}=\tfrac{26}{3}\ ,\quad a_{2R}=\tfrac{26}{3}\ ,\quad a_{4C}=0$$

(since $a_{4C}=0$, we give b_{ij} rather than B_{ij} for this case),

$$b_{ij}=\begin{array}{c} \\ 2L \\ 1R \\ 4C \end{array}\begin{array}{c} \begin{array}{ccc} 2L & 1R & 4C \end{array} \\ \left[\begin{array}{ccc} \frac{1004}{3} & 3 & \frac{1245}{2} \\ 3 & \frac{1004}{3} & \frac{1245}{2} \\ \frac{249}{2} & \frac{249}{2} & \frac{3508}{6} \end{array}\right] \end{array} .$$

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$G_{224P} = SU(2)_L \times SU(2)_R \times SU(4)_C \times P$;
$G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_C$;
$G_{2213} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$;
$G_{214} = SU(2)_L \times U(1)_R \times SU(4)_C$;
$G_{2113} - SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$;
$G_{213} = SU(2)_L \times U(1)_Y \times SU(3)_C$.

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