Intermediate mass scales in the new SO(10) grand unification in the one-loop approximation

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Chang, Mohapatra, and Parida have recently developed a new approach to SO(10) grand unification where D-parity breaking (at scale M_P) and $SU(2)_R$ breaking are decoupled. We have extended their one-loop analysis of the intermediate mass scales in the new SO(10) grand unification. We derive the general formulas for $\sin^2\theta_W(M_W)$ and $\alpha(M_W)/\alpha_s(M_W)$ for all breakings of SO(10) and examine the constraints these formulas place on the mass scales of the theory for the different chains. We identify only two (marginal) chains that lead to detectable values of the intermediate mass scales when $M_P=M_X$, the grand unification scale.

I. INTRODUCTION

There has recently been renewed interest in the grand unified theories (GUT's) based on SO(10) (Ref. 1). Part of this is due to the lack of experimental confirmation of minimal SU(5) GUT (Ref. 2). However, SO(10) has several features which make it independently interesting. SU(5) and SO(10) are the only GUT's which contain $SU(2)_L$ and $U(1)_Y$ as local symmetries and which do not have exotic or mirror fermions.[3] In SU(5), each generation of fermions is assigned to two representations, 5* and 10. The anomalies vanish due to an "accidental" cancellation between these two representations. In SO(10), each generation is assigned to a single 16-dimensional spinor representation which contains the ordinary quarks and leptons and an additional neutrino. SO(10) is naturally anomaly-free since all representations are (pseudo)real. In SU(5) and the standard electroweak model, $B-L$ is an accidental global symmetry which guarantees the masslessness of the neutrino. SO(10), on the other hand, contains $B-L$ as a local generator.[3] If one takes the view that all global symmetries should be made local, one is naturally led to SO(10) (Ref. 4). The attractive properties of SO(10) have been reviewed in Ref. 5.

There are many ways SO(10) can break to the standard model. In one alternative, SO(10) initially breaks to $SU(5)\times U(1)$. After the breaking of the U(1), this invariant becomes normal SU(5), and hence is ruled out by experiment. We consider the remaining alternatives, which are displayed in Fig. 1. We have introduced the notation

$$G_{224}=SU(2)_L\times SU(2)_R\times SU(4)_C\ ,$$

$$G_{214}=SU(2)_L\times U(1)_Y\times SU(4)_C\ ,$$

$$G_{213}=SU(2)_L\times U(1)_Y\times SU(3)_C\ ,$$

$$G_{2213}=SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C\ ,$$

$$G_{2113}=SU(2)_L\times U(1)_R\times U(1)_{B-L}\times SU(3)_C\ .$$

M_X is the unification scale; M_C is associated with the breaking of $SU(4)_C$ to $U(1)_{B-L}\times SU(3)_C$; M_{R^+} with the breaking of $SU(2)_R$ to $U(1)_R$; and M_{R^0} with the breaking of $U(1)_R\times U(1)_{B-L}$ to $U(1)_Y$. At M_X, there exists a discrete left-right symmetry called D parity.[6] At this scale, all couplings in the theory, and the particle spectrum, are left-right symmetric. Customarily, D parity has been broken at the same time as $SU(2)_R$. However, this is not necessary, as was noted in an important paper by Chang, Mohapatra, and Parida.[7] The only constraints on the scale associated with the breaking of this discrete symmetry are $M_X\geq M_P\geq M_{R^+}$ (M_P is the scale at which D parity is broken). Once D parity is broken, the particle spectrum of the theory need not be left-right symmetric. Now, the evolution of the coupling constants is governed by the renormalization-group equations, which in turn, depend on the particle spectrum of the theory. Hence, one may start with $\alpha_L(M_P)=\alpha_R(M_P)$ but end up with $\alpha_L(M_{R^+})$ significantly larger than $\alpha_R(M_{R^+})$ (Ref. 8).

In contrast to SU(5), SO(10) is compatible with the observed lower limit on the proton decay lifetime.[9] However, it will not merit serious consideration until at least one experiment identifiable with one of its intermediate mass scales is observed. Higgs-boson-mediated neutron oscillation[10] has been emphasized as an experimental test of SO(10) grand unification because, in principle, it probes the highest of the three intermediate mass scales M_C, M_{R^+}, and M_{R^0} in the descent from SO(10) to the standard model. However, any experiment that will shed light on M_C, e.g., the detection of the gauge-driven process $K_L\rightarrow\mu\bar{e}$, is of equal interest. Further, the charged right-handed weak boson R^+ plays a role in a variety of physical phenomena, such as neutrinoless double-β decay, $\mu\rightarrow e\gamma$, muon conversion into electrons, etc.,[11] and R^0 determines the masses of the Majorana neutrinos, and its coupling to right-handed neutral weak currents could be observed in suitable experiments. Indeed, any experiment that can be related to one of the three intermediate mass scales M_C, M_{R^+}, or M_{R^0} will provide a test of SO(10) grand unification.

Chang, Mohapatra, and Parida[7] have examined five different breakings of SO(10) down to the standard model in which D parity is decoupled from $SU(2)_R$. In view of the importance of finding symmetry-breaking chains in SO(10) that give rise to detectable C, R^+, or R^0 phenomena, we have extended their analysis to all chains in which

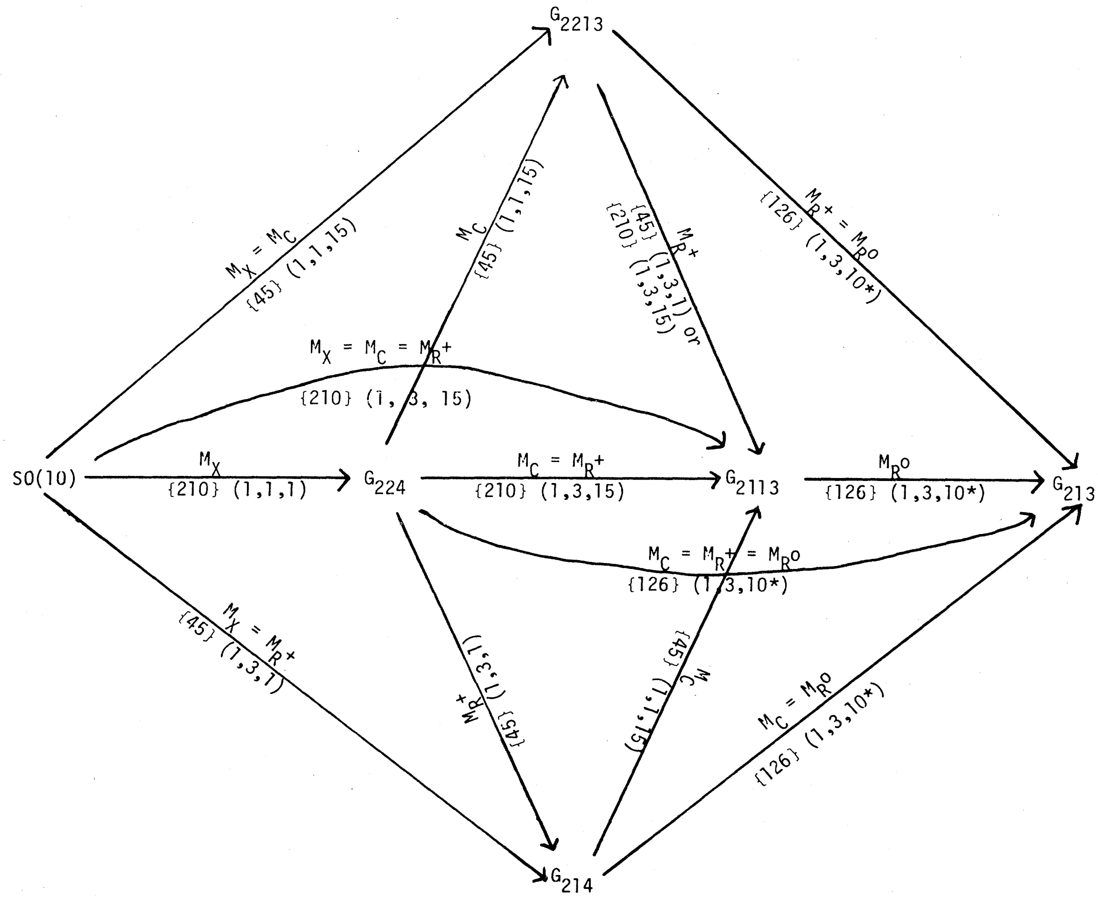

FIG. 1. Symmetry-breaking chains in SO(10) with $M_P = M_X$. M_C is the mass scale of $SU_C(4)$ unification, M_{R^+} the mass scale of $SU_R(2)$ unification, and M_{R^0} the scale at which $U_R(1)\times U_{B-L}(1)$ breaks.

the scalars can be formed out of fermion bilinears. One consequence of this assumption is that D parity is broken at M_X. This is the opposite extreme from the conventional scenario. In Sec. II, we review the renormalization-group equations and derive the general formulas for $\sin^2\theta(M_W)$ and $\alpha(M_W)/\alpha_s(M_W)$. There exist a dozen different SO(10) chains having between one and three intermediate mass scales. The results of our analysis for these chains are presented graphically in Figs. 2–13. We summarize our results and make some concluding remarks in Sec. III.

II. GENERAL RENORMALIZATION-GROUP ANALYSIS OF NEW SO(10) GRAND UNIFICATION

The renormalization-group equation relates the effective coupling constants of a theory at two different energies. This equation evaluated at one loop is[12]

$$\alpha^{-1}(\mu)=\alpha^{-1}(M)+b\ln(M/\mu),\quad \mu \ll M\ , \tag{1}$$

where $\alpha=g^2/4\pi$ and b depends on the particle content of the theory. In an $SU(N)$ gauge theory with chiral fermions and complex scalars

$$b=-\frac{1}{2\pi}[11N/3-2C(F)/3-C(T)/3]\ , \tag{2}$$

where $C(F)$ and $C(T)$ are the sums of the indices of fermion and scalar multiplets with masses less than μ. The index of the fundamental representation is normalized to $\frac{1}{2}$. The first term on the left-hand side of (2) is missing for a U(1) gauge group.

For comparison with the SO(10) case, we recapitulate the renormalization-group analysis of minimal SU(5). In this model there are only two mass scales: the electroweak scale M_W and the unification scale M_5. All particles are assumed to be light, $M \le M_W$, or superheavy, $M \simeq M_5$. The three couplings are unified at M_5:

$$\alpha_L^{-1}(M_5)=\alpha_s^{-1}(M_5)=\tfrac{3}{5}\alpha_Y^{-1}(M_5)\ . \tag{3}$$

Using

$$\alpha^{-1}(M_W)=\alpha_L^{-1}(M_W)+\alpha_Y^{-1}(M_W)\ , \tag{4}$$

the definition

$$\sin^2\theta(M_W)\equiv\alpha(M_W)/\alpha_L(M_W) \tag{5}$$

and the renormalization-group equations, one finds

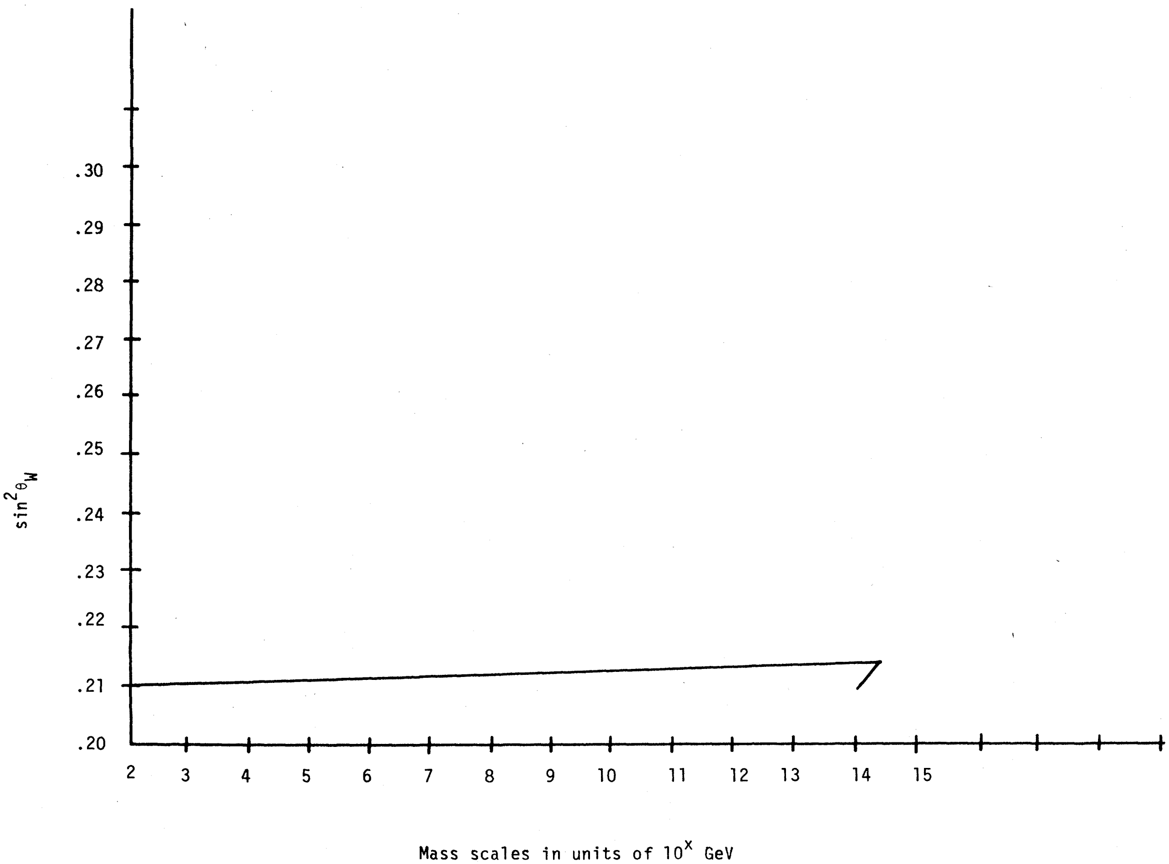

FIG. 2. Intermediate mass scale M_{R^0} and unification scale M_X for the symmetry-breaking chain $SO(10) \underset{M_X}{\rightarrow} G_{2113} \underset{M_{R^0}}{\rightarrow} G_{213}$.

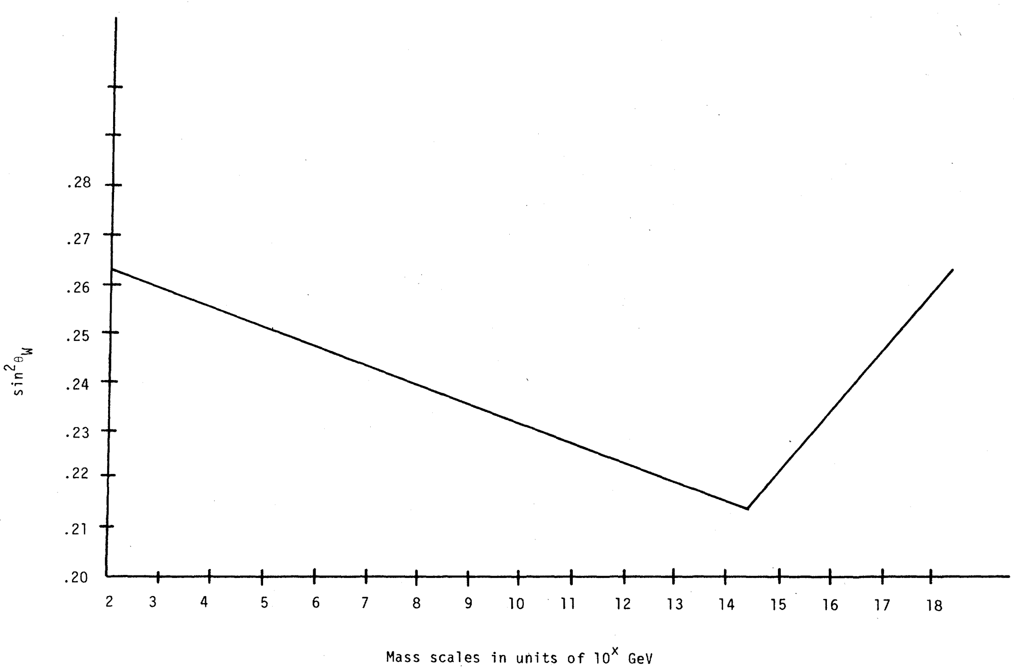

FIG. 3. Intermediate mass scale M_{R^+} and unification scale M_X for the symmetry-breaking chain $SO(10) \underset{M_X}{\rightarrow} G_{2213} \underset{M_{R^+}}{\rightarrow} G_{213}$.

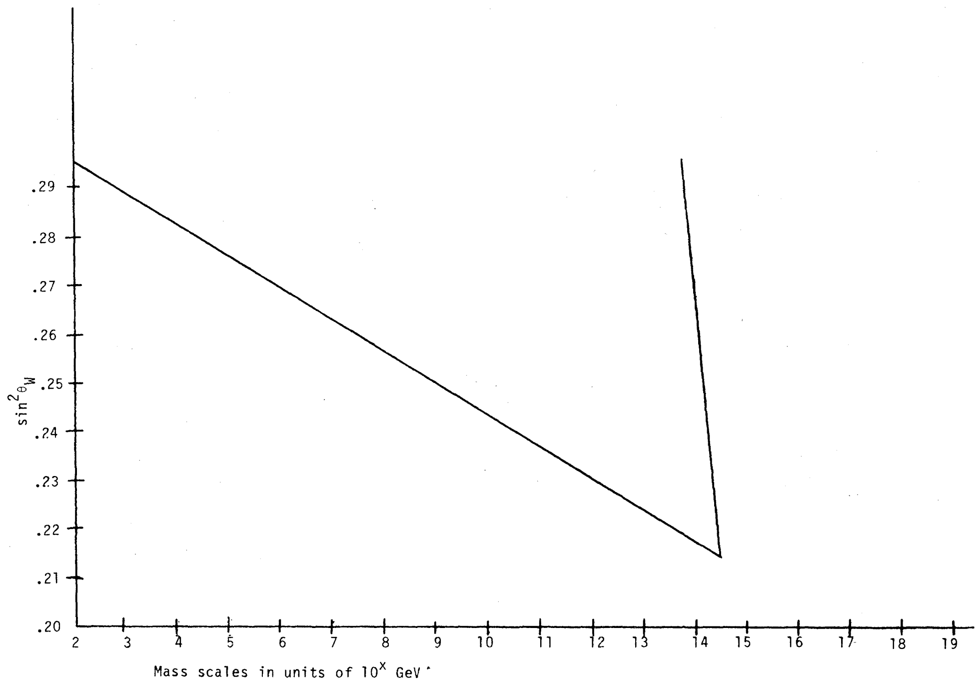

FIG. 4. Intermediate mass scale M_C and unification scale M_X for the symmetry-breaking chain $SO(10) \underset{M_X}{\rightarrow} G_{214} \underset{M_C}{\rightarrow} G_{213}$.

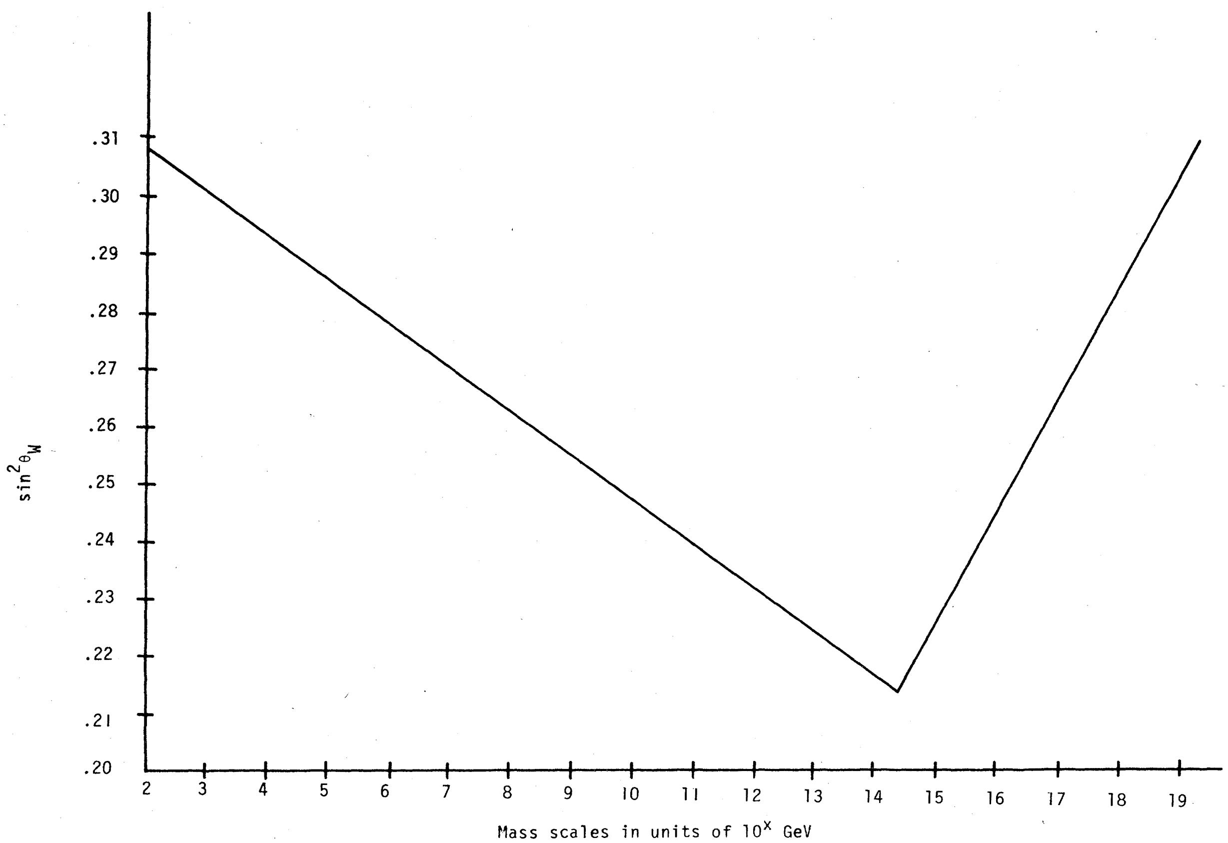

FIG. 5. Intermediate mass scale M_C and unification scale M_X for the symmetry-breaking chain $SO(10) \underset{M_X}{\rightarrow} G_{224} \underset{M_C}{\rightarrow} G_{213}$.

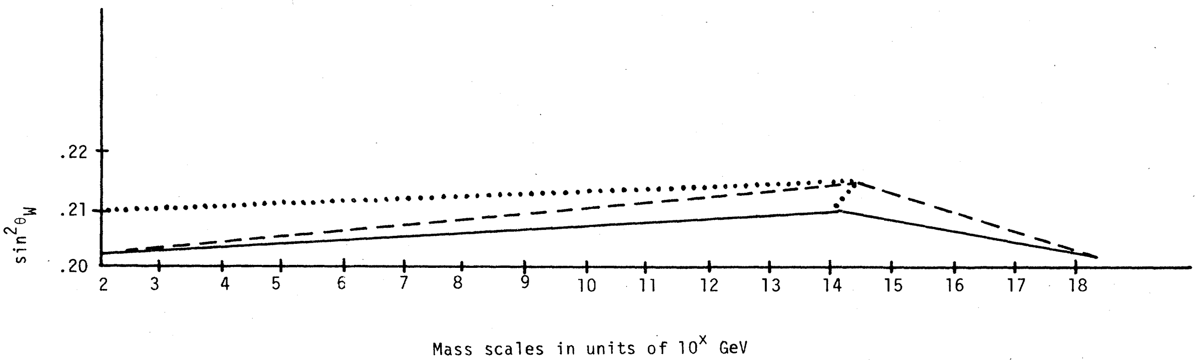

FIG. 6. Intermediate mass scales M_C, M_{R^0} and unification scale M_X for the symmetry-breaking chain $SO(10)\underset{M_X}{\rightarrow} G_{224}\underset{M_C}{\rightarrow} G_{2113}\underset{M_{R^0}}{\rightarrow} G_{213}$. The solid line is $M_{R^0}=M_W$; the dashed line is $M_{R^0}=M_C$; the dotted line is $M_C=M_X$.

$$\sin^2\theta_5(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{48\pi}(110+3T_Y-5T_L)\times\ln(M_5/M_W)\,,$$
$$\alpha(M_W)/\alpha_s(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}(66+T_L+T_Y)\times\ln(M_5/M_W)\,. \quad (6)$$

Here T_L and T_Y denote the contributions of the light scalars to the β function. In the minimal model with one light doublet, $T_L=T_Y=\frac{1}{2}$. Hence one finds

$$\sin^2\theta_5(M_W)=\tfrac{23}{134}+\tfrac{109}{201}\alpha(M_W)/\alpha_s(M_W)\simeq 0.215\,, \quad (7)$$

where we have used $\alpha(M_W)=\frac{1}{128}$ and $\alpha_s(M_W)\simeq 0.11$.

We now turn to the SO(10) analysis. We find attractive the possibility that scalars arise as composites of two fundamental fermions.[13] Such bilinears are

$$16\times 16=10+120+126\,,$$
$$16\times 16^*=1+45+210\,. \quad (8)$$

The $SU(2)_L\times SU(2)_R\times SU(4)_C$ decomposition of these scalars is

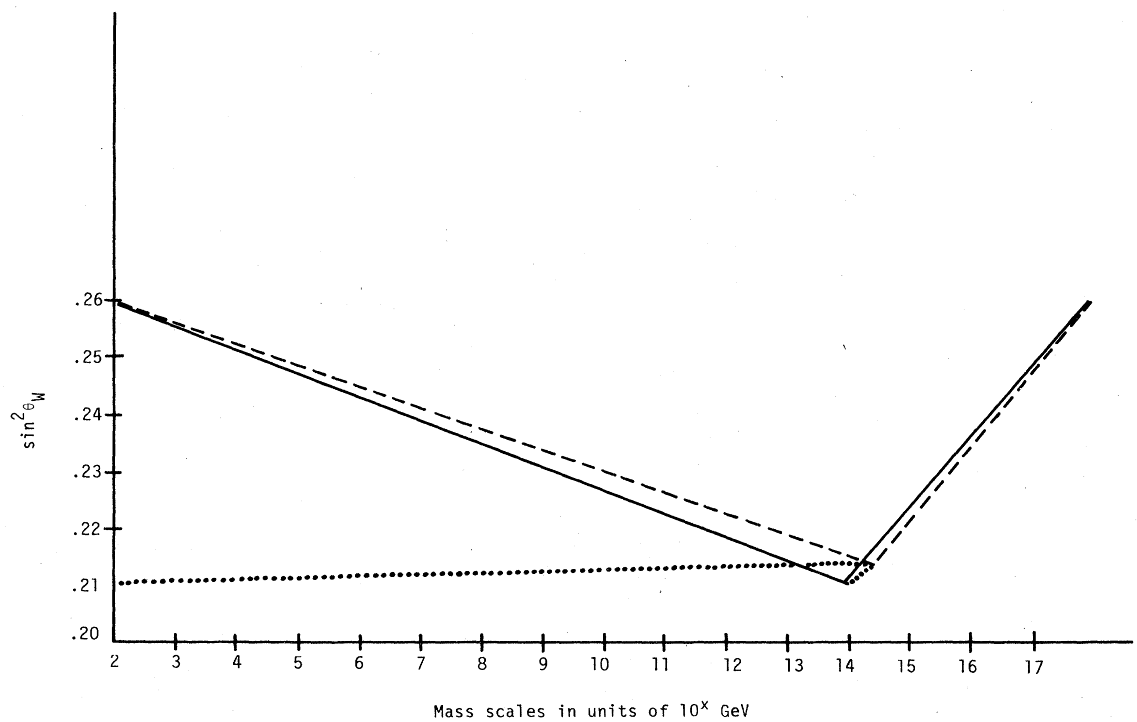

FIG. 7. Intermediate mass scales M_{R^+}, M_{R^0} and unification scale M_X for the symmetry-breaking chain $SO(10)\underset{M_X}{\rightarrow} G_{2213}\underset{M_{R^+}}{\rightarrow} G_{2113}\underset{R^0}{\rightarrow} G_{213}$. The solid line is $M_{R^0}=M_W$; the dashed line is $M_{R^0}=M_{R^+}$; the dotted line is $M_{R^+}=M_X$.

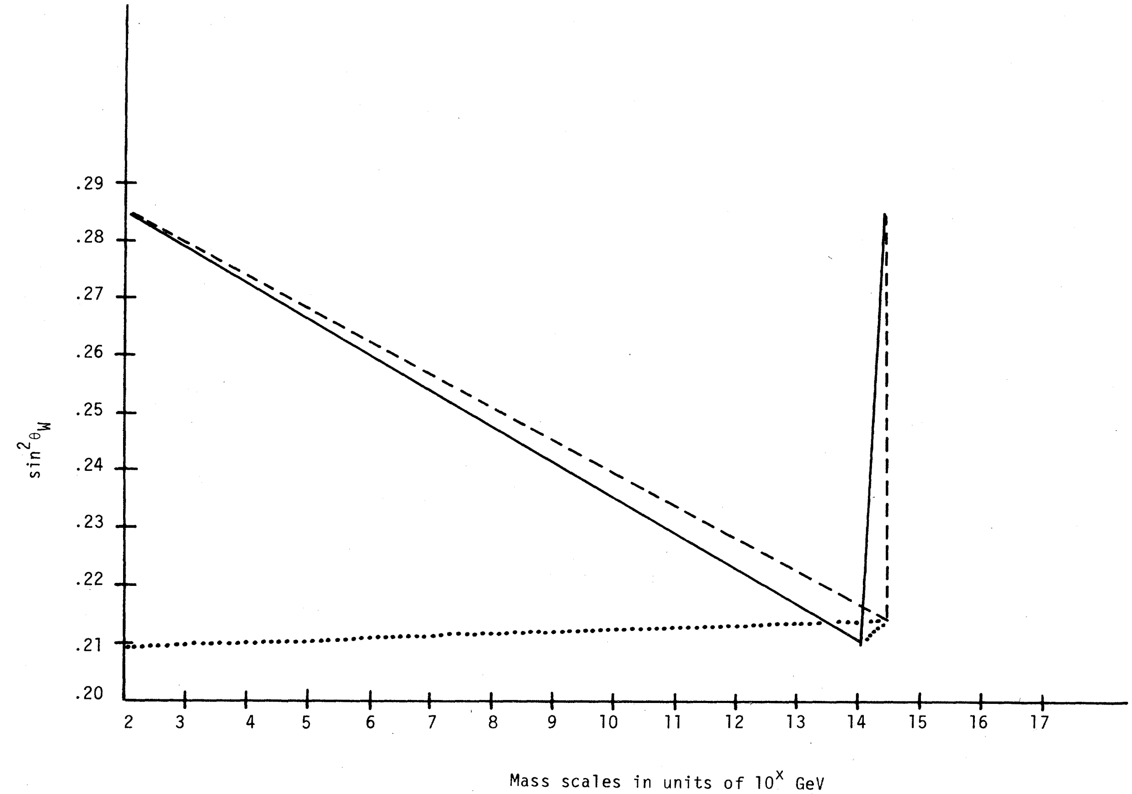

FIG. 8. Intermediate mass scales M_C, M_{R^0} and unification scale M_X for the symmetry-breaking chain $SO(10) \underset{M_X}{\rightarrow} G_{214} \underset{M_C}{\rightarrow} G_{2113} \underset{M_{R^0}}{\rightarrow} G_{213}$. The solid line is $M_{R^0}=M_W$; the dashed line is $M_{R^0}=M_C$; the dotted line is $M_C=M_X$.

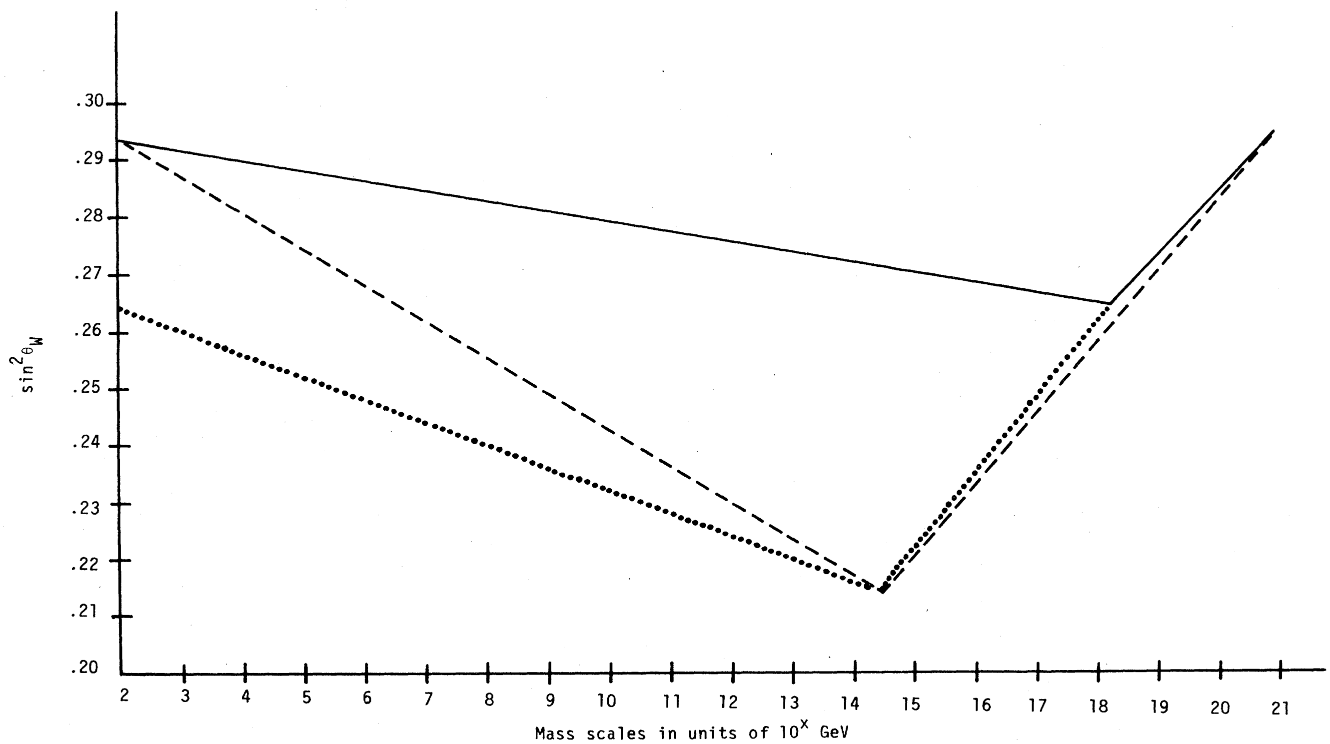

FIG. 9. Intermediate mass scales M_C, M_{R^+} and unification scale M_X for the symmetry-breaking chain $SO(10) \underset{M_X}{\rightarrow} G_{224} \underset{M_C}{\rightarrow} G_{2213} \underset{M_{R^+}}{\rightarrow} G_{213}$. The solid line is $M_{R^+}=M_W$; the dashed line is $M_{R^+}=M_C$; the dotted line is $M_C=M_X$.

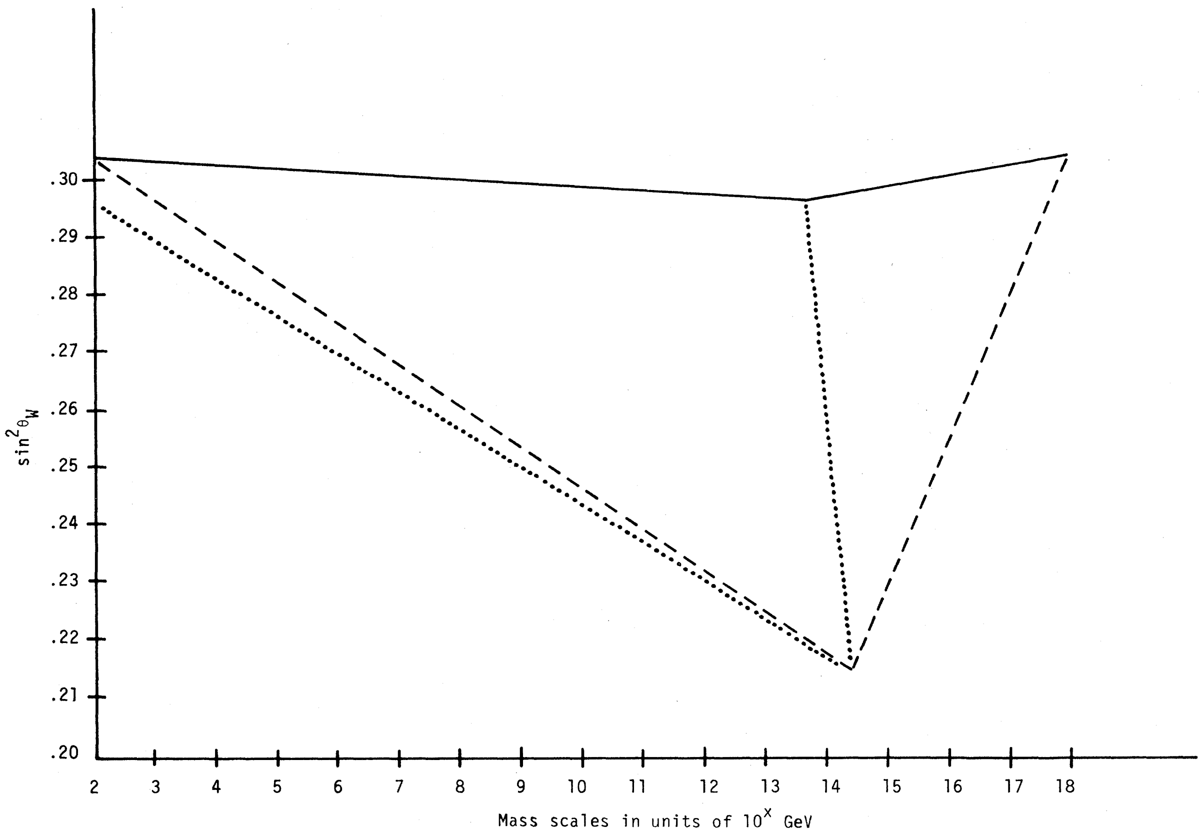

FIG. 10. Intermediate mass scales, M_{R^+}, M_C and unification scale M_X for the symmetry-breaking chain $SO(10)\underset{M_X}{\rightarrow}G_{224}\underset{M_{R^+}}{\rightarrow}G_{214}\underset{M_C}{\rightarrow}G_{213}$. The solid line is $M_C=M_W$; the dashed line is $M_C=M_{R^+}$; the dotted line is $M_{R^+}=M_X$.

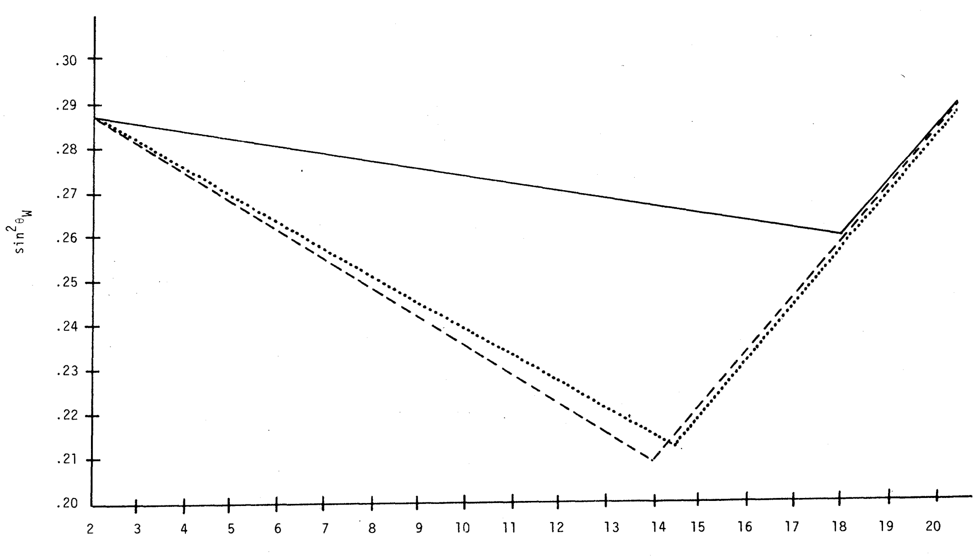

FIG. 11. Intermediate mass scales M_C, M_{R^+}, and M_{R^0}, and unification scale M_X for the symmetry-breaking chain $SO(10)\underset{M_X}{\rightarrow}G_{224}\underset{M_C}{\rightarrow}G_{2213}\underset{M_{R^+}}{\rightarrow}G_{2113}\underset{M_{R^0}}{\rightarrow}G_{213}$, where the breaking of $SU(2)_R$ is done by a 45. The solid line is $M_{R^0}=M_{R^+}=M_W$; the dashed line is $M_{R^0}=M_W$, $M_{R^+}=M_C$; the dotted line is $M_{R^0}=M_{R^+}=M_C$.

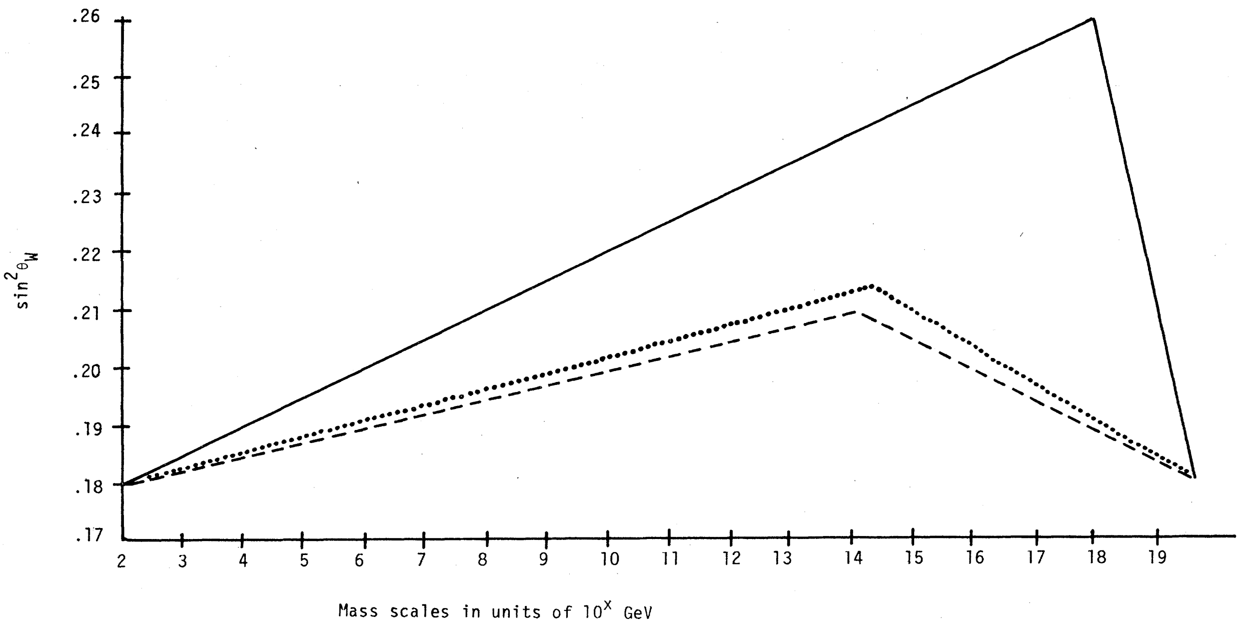

FIG. 12. Intermediate mass scales M_C, M_{R^+}, and M_{R^0} and unification scale M_X for the symmetry-breaking chain $SO(10)\underset{M_X}{\rightarrow}G_{224}\underset{M_C}{\rightarrow}G_{2213}\underset{M_{R^+}}{\rightarrow}G_{2113}\underset{M_{R^0}}{\rightarrow}G_{213}$, where the breaking of $SU(2)_R$ is done by a 210. The solid line is $M_{R^0}=M_{R^+}=M_W$; the dashed line is $M_{R^0}=M_W$, $M_{R^+}=M_C$; the dotted line is $M_{R^0}=M_{R^+}=M_C$.

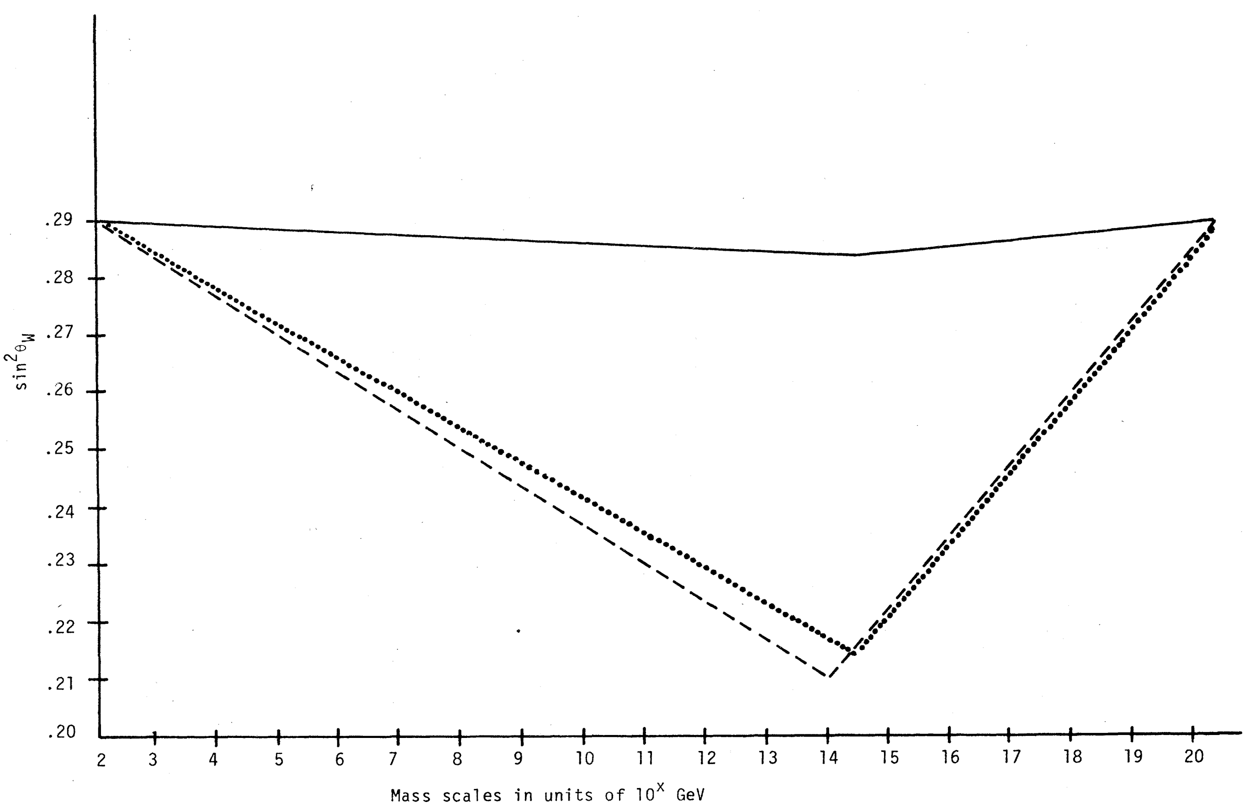

FIG. 13. Intermediate mass scales M_{R^+}, M_C, and M_{R^0}, and unification scale M_X for the symmetry-breaking chain $SO(10)\underset{M_X}{\rightarrow}G_{224}\underset{M_{R^+}}{\rightarrow}G_{214}\underset{M_C}{\rightarrow}G_{2113}\underset{M_{R^0}}{\rightarrow}G_{213}$. The solid line is $M_{R^0}=M_C=M_W$; the dashed line is $M_{R^0}=M_W$, $M_C=M_{R^+}$; the dotted line is $M_{R^0}=M_C=M_{R^+}$.

$$10=(2,2,1)+(1,1,6)\,,$$
$$120=(2,2,1)+(1,1,10)+(1,1,10^*)+(3,1,6)+(1,3,6)+(2,2,15)\,,$$
$$126=(1,1,6)+(3,1,10)+(1,3,10^*)+(2,2,15)\,, \tag{9}$$
$$45=(1,1,15)+(3,1,1)+(1,3,1)+(2,2,6)\,,$$
$$210=(1,1,1)+(1,1,15)+(2,2,6)+(3,1,15)+(1,3,15)+(2,2,10)+(2,2,10^*)\,.$$

If we restrict ourselves to these scalars, there exist 13 different breakings of SO(10) to the standard model which do not pass through SU(5). One chain involves no intermediate step—and is uninteresting—but the other twelve have between one and three intermediate mass scales. These different breakings are summarized in Table I and in Fig. 1. The twelve chains can be divided into two general classes depending on whether $SU(4)_C$ breaks before (class A) or after (class B) $SU(2)_R$.

In addition to Eq. (4), we have the following relations among the coupling constants:

$$\alpha_Y^{-1}(M_{R^0})=\alpha_{R^0}^{-1}(M_{R^0})+\tfrac{2}{3}\alpha_{BL}^{-1}(M_{R^0})\,,\quad \alpha_{R^0}^{-1}(M_{R^+})=\alpha_{R^+}^{-1}(M_{R^+})\,,$$
$$\alpha_s^{-1}(M_C)=\alpha_{BL}^{-1}(M_C),\quad \alpha_L^{-1}(M_X)=\alpha_R^{-1}(M_X)=\alpha_4^{-1}(M_X)\,. \tag{10}$$

It is straightforward to find the general expressions for $\alpha(M_W)/\alpha_s(M_W)$ and $\sin^2\theta(M_W)$ using Eq. (10) and the renormalization-group equations. For path A, we find

$$\sin^2\theta(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{48\pi}[(110+3T_Y-5T_L)\ln(M_{R^0}/M_W)+(110+3T_{R^0}+2T_{BL}-5T_L)\ln(M_{R^+}/M_{R^0})$$
$$+(44+3T_R+2T_{BL}-5T_L)\ln(M_C/M_{R^+})+(-44+3T_R+2T_4--5T_L)\ln(M_X/M_C)]\,, \tag{11a}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)}=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}[(66+T_L+T_Y-\tfrac{8}{3}T_s)\ln(M_{R^0}/M_W)+(66+T_L+T_{R^0}+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln(M_{R^+}/M_{R^0})$$
$$+(44+T_L+T_R+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln(M_C/M_{R^+})+(44+T_L+T_R-2T_4)\ln(M_X/M_C)]\,. \tag{11b}$$

These equations do not depend on the number of generations of fermions. The T's denote the scalar contributions to the β function.

We emphasize that the only scalars which contribute at each scale are those whose masses are less than that scale. The scalar masses are, *a priori*, unknown. Mohapatra and Senjanović, and others[14] have derived a set of rules based on explicit calculation in several specific models which fix the mass spectrum. These assumptions are (1) minimal fine tuning (do no more fine tuning than is phenomenologically necessary), and (2) extended survival hypothesis (all particles that can become heavy do). With the above hypothesis one can determine the particle spectrum. In paths A, $SU(4)_C$ is broken by a (1,1,15), $SU(2)_R$ by either (1,3,1) or a (1,3,15), $U(1)_{R^0}\times U(1)_{B-L}$ by a (1,3,10*) and $SU(2)_L\times U(1)_Y$ by a (2,2,1). The scalars which contribute to the β functions, and their decomposition with respect to the appropriate gauge groups, are presented below:

$$M_C\le\mu\le M_X:\ \phi(2,2,1),\ \ \Delta(1,3,10^*),\ \ F(1,1,15),\ \ D_1(1,3,1)\ \text{ or }\ D_{15}(1,3,15)\,.$$
$$M_{R^+}\le\mu\le M_C:\ \phi(2,2,0,1),\ \ \Delta(1,3,-2,1),\ \ D(1,3,0,1)\,.$$
$$M_{R^0}\le\mu\le M_{R^+}:\ \phi(2,\tfrac{1}{2},0,1),\ \ \Delta(1,1,-2,1)\,.$$
$$M_W\le\mu\le M_{R^0}:\ \phi(2,\tfrac{1}{2},1)\,.$$

Note that only one of D_1 or D_{15} is present. Equations (11a) and (11b) become

$$\sin^2\theta(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{48\pi}[(110-\phi)\ln(M_{R^0}/M_W)+(110+6\Delta-\phi)\ln(M_{R^+}/M_{R^0})$$
$$+(44+11\Delta+3D-2\phi)\ln(M_C/M_{R^+})$$
$$+(-44+78\Delta+3D_1+57D_{15}+4F-2\phi)\ln(M_X/M_C)]\,, \tag{12a}$$

$$\alpha(M_W)/\alpha_s(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}[(66+\phi)\ln(M_{R^0}/M_W)+(66+2\Delta+\phi)\ln(M_{R^+}/M_{R^0}) +(44+5\Delta+D+2\phi)\ln(M_C/M_{R^+}) +(44+2\Delta+D_1+3D_{15}-4F+2\phi)\ln(M_X/M_C)]\ . \tag{12b}$$

We have written the above equations in such a way that the Higgs-boson contributions can be extracted by setting the corresponding constant equal to 1. For example, the coefficient of Δ is the contribution of $(1,3,10^*)$. One can obtain the equations appropriate to the chains in which $M_C \geq M_{R^+}$ (I a, I b, I d, II a, II b, II d, II e, III a, and III b) by setting the constants Δ, $\phi,\ldots$ equal to one if the particular chain contains this Higgs boson, and zero otherwise. These results are summarized in graphical form in Figs. 2, 3, 5—7, and 9—12.

Comparing Eq. (6) with (12b) allows us to relate the SU(5) unification scale M_5 with the various SO(10) scales. Setting $T_L=T_Y=\phi/2$ in Eq. (6), one finds

$$M_X=M_5(M_5^{\,22+\delta_1+\delta_2+\delta_3}M_{R^0}^{\,-\delta_1}M_{R^+}^{\,-22-\delta_2}M_C^{\,-\delta_3})^{1/(44+\delta_4)}\ . \tag{13}$$

The δ's are due to the Higgs-boson contributions:

$$\delta_1=-2\Delta,\ \ \delta_2=-3\Delta-D-\phi,\ \ \delta_3=3\Delta-2D_{15}+4F,\ \ \delta_4=2\Delta+D_1+3D_{15}-4F+2\phi\ , \quad \delta_1+\delta_2+\delta_3+\delta_4=\phi\ . \tag{14}$$

Ignoring the scalar contributions ($\delta=0$), one recovers the result of Ref. 9:

$$M_X=M_5(M_5/M_{R^+})^{1/2}\ . \tag{15}$$

Repeating the same analysis for a general descent along path B, one finds

$$\sin^2\theta(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{48\pi}[(110+3T_Y-5T_L)\ln(M_{R^0}/M_W)+(110+3T_{R^0}+2T_{BL}-5T_L)\ln(M_C/M_{R^0}) +(22+3T_{R^0}+2T_4-5T_L)\ln(M_{R^+}/M_C)+(-44+3T_R+2T_4-5T_L)\ln(M_X/M_{R^+})]\ , \tag{16a}$$

$$\alpha(M_W)/\alpha_s(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}[(66+T_L+T_Y-\tfrac{8}{3}T_S)\ln(M_{R^0}/M_W)+(66+T_L+T_{R^0}+\tfrac{2}{3}T_{BL}-\tfrac{8}{3}T_s)\ln(M_C/M_{R^0}) +(66+T_L+T_{R^0}-2T_4)\ln(M_{R^+}/M_C)+(44+T_L+T_R-2T_4)\ln(M_X/M_{R^+})]\ . \tag{16b}$$

These equations are independent of the number of generations of fermions. We shall assume that $SU(2)_R$ is broken by a (1,3,1), $SU(4)_C$ by a (1,1,15), and $U(1)_{R^0}\times U(1)_{B-L}$ by a $(1,3,10^*)$. Then the scalars present at the various energy scales, and their decomposition with respect to the appropriate gauge group, are as follows:

$M_{R^+}\leq\mu\leq M_X$: $\phi(2,2,1)$, $\Delta(1,3,10^*)$, $F(1,1,15)$, $D(1,3,1)$.

$M_C\leq\mu\leq M_{R^+}$: $\phi(2,\frac{1}{2},1)$, $\Delta(1,1,10)$, $F(1,1,15)$.

$M_{R^0}\leq\mu\leq M_C$: $\phi(2,\frac{1}{2},0,1)$, $\Delta(1,1,-2,1)$.

$M_W\leq\mu\leq M_{R^0}$: $\phi(2,\frac{1}{2},1)$.

With this input, Eqs. (16a) and (16b), become

$$\sin^2\theta(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{48\pi}[(110-\phi)\ln(M_{R^0}/M_W)+(110+6\Delta-\phi)\ln(M_C/M_{R^0}) +(22+36\Delta+4F-\phi)\ln(M_{R^+}/M_C)+(-44+78\Delta+3D+4F-2\phi)\ln(M_X/M_{R^+})]\ , \tag{17a}$$

$$\alpha(M_W)/\alpha_s(M_W)=\frac{3}{8}-\frac{\alpha(M_W)}{16\pi}[(66+\phi)\ln(M_{R^0}/M_W)+(66+2\Delta+\phi)\ln(M_C/M_{R^0})$$
$$+(66+4\Delta-4F+\phi)\ln(M_{R^+}/M_C)(44+2\Delta+D-4F+2\phi)\ln(M_X/M_{R^+})] \; . \quad \text{(17b)}$$

We have again written these equations in such a way that the Higgs-boson contribution is manifest. The results for chains Ic, IIc, and IIIc, are presented in Figs. 4, 8, and 13. One can again relate M_5 to the various SO(10) mass scales. The result can be cast in the same form as Eq. (13) with different δ's:

$$\delta_1=-2\Delta, \;\; \delta_2=2\Delta-D-\phi \; ,$$
$$\delta_3=4F-2\Delta, \;\; \delta_4=2\Delta+D-4F+2\phi \; , \quad (18)$$
$$\delta_1+\delta_2+\delta_3+\delta_4=\phi \; .$$

VI. SUMMARY AND CONCLUSIONS

We have computed in the one-loop approximation the values of the intermediate mass scales M_C, M_{R^+}, and M_{R^0}, as well as the values of M_X and $\sin^2\theta_W$, for the dozen symmetry-breaking chains in SO(10) for which the D-parity-breaking scale M_P is identical to the GUT scale M_X. The last assumption eliminated M_P as an independent parameter—consistent with the cosmological constraints[15]—and can be lifted if desired. The results are given in the form of curves (Figs. 2—13) without any regard for observational tests and may be useful in this form for model builders. For our purposes, which is to identify symmetry-breaking chains that can give rise to detectable C, R^+, and R^0 physical processes in the near future, it is more convenient to list the ranges of the aforementioned parameters deduced from Figs. 2—13 under a plausible set of constraints on the values of M_C, M_{R^+}; we have not extracted the information concerning acceptable values of R^0 from Figs. 2—13 for inclusion in Table II, because low masses of R^0 are easy to come by even in conventional SO(10).

Examination of Table II reveals that, while decoupling D-parity breaking from $SU(2)_R$ breaking results in lower intermediate mass scales, the number of symmetry-breaking chains in SO(10) that gives rise to detectable C and/or R^+ phenomena is quite small. Indeed, if we take $\sin^2\theta_W=0.22\pm0.02$ (a range that seems to be required by the observed masses of W_L and Z_L bosons[16]), and insist that M_X does not exceed $\frac{1}{10}$ the Planck mass, we find that only two symmetry-breaking chains in SO(10) (chains IIa and IIIa′) survive and they are only marginal (chain IIIa′ only becomes marginal for $M_C\simeq10^{12}$ GeV). It thus appears that SO(10)—even when the breaking of D parity is

TABLE I. Symmetry-breaking chains of SO(10) to G_{213} ($M_P=M_X$) (one to three intermediate mass scales).

	One intermediate mass scale
Ia:	$SO(10)\underset{210}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$
Ib:	$SO(10)\underset{45}{\rightarrow}G_{2213}\underset{126}{\rightarrow}G_{213}$
Ic:	$SO(10)\underset{45}{\rightarrow}G_{214}\underset{126}{\rightarrow}G_{213}$
Id:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{126}{\rightarrow}G_{213}$
	Two intermediate mass scales
IIa:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{210}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$
IIb:	$SO(10)\underset{45}{\rightarrow}G_{2213}\underset{210}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$
IIc:	$SO(10)\underset{45}{\rightarrow}G_{214}\underset{45}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$
IId:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{45}{\rightarrow}G_{2213}\underset{126}{\rightarrow}G_{213}$
IIe:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{45}{\rightarrow}G_{214}\underset{126}{\rightarrow}G_{213}$
	Three intermediate mass scales
IIIa:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{45}{\rightarrow}G_{2213}\underset{45}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$
IIIb:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{45}{\rightarrow}G_{2213}\underset{210}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$
IIIc:	$SO(10)\underset{210}{\rightarrow}G_{224}\underset{45}{\rightarrow}G_{214}\underset{45}{\rightarrow}G_{2113}\underset{126}{\rightarrow}G_{213}$

TABLE II. Possible chains in SO(10) with D-parity breaking decoupled from $\mathrm{SU}(2)_R$ breaking in one-loop approximation $[\alpha_S(M_W)=0.10]$. (M_R stands for $M_{R^+}=M_{R^0}$.)

	Chain	$\log_{10}[M_X\ (\mathrm{GeV})]$	$\sin^2\theta_W(M_W)$
I a	$\mathrm{SO}(10) \xrightarrow[M_X=M_P=M_C=M_{R^+}]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$	13.5—13.6 14.1—14.2	0.210—0.211 0.211—0.212
I b	$\mathrm{SO}(10) \xrightarrow[M_X=M_P=M_C]{} G_{2213} \xrightarrow[M_R]{} G_{213}$ $(2.5\le\log_{10}[M_R\ (\mathrm{GeV})]\le 4)$	18.1—17.6	0.262—0.256
I c	$\mathrm{SO}(10) \xrightarrow[M_X=M_P=M_{R^+}]{} G_{214} \xrightarrow[M_C=M_{R^0}]{} G_{213}$ $(2.5\le\log_{10}[M_{R^0}\ (\mathrm{GeV})]\le 4)$	13.7—13.8	0.293—0.283
I d	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_C=M_R]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7)$	18.5—17.3	0.294—0.271
II a	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_C=M_{R^+}]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7,\ M_{R^0}=M_W)$	17.6—16.5	0.204—0.206
II b	$\mathrm{SO}(10) \xrightarrow[M_X=M_P=M_C]{} G_{2213} \xrightarrow[M_{R^+}]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$ $(2.5\le\log_{10}[M_{R^+}\ (\mathrm{GeV})]\le 4,\ M_{R^0}=M_W)$	17.8—17.3	0.259—0.252
II c	$\mathrm{SO}(10) \xrightarrow[M_X=M_P=M_{R^+}]{} G_{214} \xrightarrow[M_C]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7;\ M_{R^0}=M_W)$	14.3—14.2	0.273—0.254
II d	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_C]{} G_{2213} \xrightarrow[M_R]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7,\ M_R=M_C)$	19.8—18.2	0.281—0.262
II e	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_R]{} G_{214} \xrightarrow[M_C]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7,\ M_R=M_C)$	18.2—17.1	0.289—0.268
III a	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_C]{} G_{2213} \xrightarrow[M_{R^+}]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7,\ M_{R^+}=M_C;\ M_{R^0}=M_W)$	19.4—17.8	0.276—0.256
III b	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_C]{} G_{2213} \xrightarrow[M_{R^+}]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7,\ M_{R^+}=M_{R^0}=M_W)$ $(7\le\log_{10}[M_C\ (\mathrm{GeV})]\le 12,\ M_{R^+}=M_{R^0}=M_W)$	 19.4—19.1 19.1—18.6	 0.189—0.205 0.205—0.230
III c	$\mathrm{SO}(10) \xrightarrow[M_X=M_P]{} G_{224} \xrightarrow[M_{R^+}]{} G_{214} \xrightarrow[M_C]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213}$ $(4\le\log_{10}[M_C\ (\mathrm{GeV})]\le 7,\ M_{R^+}=M_C,\ M_{R^0}=M_W)$	19.4—15.8	0.276—0.26

decoupled from the breaking of $\mathrm{SU}(2)_R$—is remarkably resistant to providing detectable intermediate mass scales M_C and/or M_{R^+} (we reiterate that this is not the case for M_{R^0}) for laboratory experiments in the near future. It appears possible, from the work of Chang, Mohapatra, and Parida,[7] to improve the situation considerably for both M_C and M_{R^+} by allowing M_P to be smaller by a factor of 10^2—10^3 than M_X. Further work, including the computation of the two-loop corrections, is called for if we are to provide the experimentalists with incisive tests of SO(10) grand unification.

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