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Center-of-mass corrections and fermions consisting of confined quarks

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This is a continuation of our paper [Phys. Rev. D **29**, 981 (1984)] on magnetic moments. The center-of-mass corrections for the static-model values for the mass and for the charge radius depend strongly on the radius of confinement R_0. If $MR_0 \ll 1$, then $M = m_e$ (electron mass) can be obtained with model parameters only about 30% smaller than those needed to fit $M = M_p$ (proton mass) for $MR_0 > 1$. A composite fermion with $M = m_e$ ("mock electron") can have $\langle r^2 \rangle \sim R_0{}^2$ only if its magnetic moment is 1 Bohr magneton. The center-of-mass corrections for M_p are in qualitative agreement with usual estimates. For the proton charge radius they are small.

I. INTRODUCTION

In a previous paper,[1] we have shown that the center-of-mass corrections (CMC) can significantly improve the results obtained by using static models[2–7] for the confinement of quarks. We have also assumed that the confined fermions can be viewed as constituents of a "mock electron" as conjectured by Halprin and Kerman.[8] Everything discussed in Ref. 1 and in the following text holds qualitatively also for a model in which constituents are a fermion and a boson.[9,10]

The model in which the recoil corrections (RC) and CMC factorize was based on an approximation[11–14] which considers a moving bag as a wave packet with a nonzero momentum p, i.e.,

$$|p\rangle_B = \int d^3k\, W^{-1/2}(k) W^{-1/2}(k-p)\phi(k-p)\,|k\rangle \,. \quad (1.1)$$

Here ϕ is the wave packet and W is the normalization factor. Our notation and formalism follow those of Ref. 1. In order to make this text to some extent self-contained, some details are repeated in the Appendix.

We do not intend to discuss here a justification[1,11–15] of the ansatz (1.1). We intend to take it as a definition of a model and then to see whether such a model can be made self-consistent and how it agrees with experiment. Depending on the confinement radius R_0 and the mass of the composite object (i.e., proton, mock electron) M, there are two regions of interest.

(i) The region $MR_0 > 1$ corresponds to a proton (or an octet baryon). It has been shown[1] that in that region CMC improve the theoretical values for the magnetic moment μ and the axial-vector coupling constant g_A. In this paper we will discuss corrections to the baryon mass and to the charge radius.

(ii) The region $MR_0 \ll 1$ might serve as a model for a mock electron. We have shown[1] that the model leads to a value for the magnetic moment of about 1 Bohr magneton. In Ref. 1 we have supposed[8] that the mass of a mock electron can be obtained somehow by adjusting the model parameters. Here we will demonstrate that due to CMC the mass of a composite object is decreasing with R_0. Finally, we will show that the charge radius stays roughly proportional to R_0 for $MR_0 \ll 1$ if there is no anomalous magnetic moment. (This means $\mu = e/2M$, i.e., 1 Bohr magneton when $M = m_e$.) As this is the result found in Ref. 1, the model seems to show an overall consistency. However, one must be aware that when $MR_0 \ll 1$ the model is pushed to its extreme relativistic limit for which the ansatz (1.1) might be too naive. Nevertheless, we feel that satisfactory results obtained in this simple model are encouraging.

II. THEORETICAL MASS

In order to calculate the mass one starts with the energy-momentum tensor T^{00} (Refs. 3 and 4). The diagonal matrix element of the momentum

$$P^0 = \int d^3x\, T^{00} \quad (2.1)$$

between bag-model states

$$_B\langle 0|P^0|0\rangle_B = E(\text{bag}) = \kappa R_0{}^{-1} \quad (2.2)$$

is the bag-model energy E(bag). It is well known that E(bag) behaves as $R_0{}^{-1}$ (Refs. 2–4) even when all possible corrections are included. Thus, the simple parametrization indicated in (2.2) is sufficient for the present purposes. By using formula (1.1), one finds

$$\begin{aligned} _B\langle 0|P^0|0\rangle_B &= \int d^3k\, d^3k'\, W^{-1}(k) W^{-1}(k')\phi(k)\phi(k') \times \langle k'|P^0|k\rangle \\ &= \int d^3k\, I(k) E(k) \,. \end{aligned} \quad (2.3)$$

The function $I(k)$ is defined in the Appendix. The last result follows from

$$P^0|k\rangle = E(k)|k\rangle \,.$$

Since $E^2(k)=k^2+M^2$, one can write

$$M/E(\text{bag})=Z^{-1}(R_0,M)\ ,$$
$$Z(R_0,M)=\int d^3k(1+k^2/M^2)^{1/2}I(k)\ . \tag{2.4}$$

The expression (2.4) is the required nonlinear equation for the mass M, as both E(bag) and $I(k)$ are entirely determined by the model parameters. Without CMC

$$I(k)=\delta^3(\mathbf{k}),\quad Z=1\ ,$$

while for any other model wave-function normalization of $I(k)$ guarantees that $Z>1$. The model does not contain any intrinsic mass scales. The only quantity with the dimension of energy is the inverse bag radius $R_0{}^{-1}$. One can parametrize mass M by the dimensionless constant η, which is always smaller than the constant κ which symbolizes the bag-model parameters. One finds

$$\eta Z=\kappa,\quad Z\geq 1, M=\eta/R_0\ . \tag{2.5}$$

For example, when $M=M_p$ one finds without CMC

$$M_p=E(\text{bag})=\frac{\kappa}{R_0},\quad R_0=0.65\text{ fm},\quad \kappa=3.088\ . \tag{2.6}$$

Using the full formula (2.4) and Table I, one obtains

$$M_p=E(\text{bag})Z^{-1}(R_0,M_p),\quad R_0=0.65\text{ fm}\ ,$$
$$\kappa=3.778,\quad \eta=3.088\ . \tag{2.7}$$

This increase in the value of κ is in concert with usual expectations.[4,13]

In the general case one can solve (2.4) numerically in order to adjust model parameters in such a way as to obtain the desired M. Two limits are of the particular interest. The sector $MR_0>1$ corresponds to the nonrelativistic limit in which

$$\langle k^2\rangle \ll M^2\ ,$$
$$\langle k^2\rangle=\int d^3k\, I(k)k^2\ . \tag{2.8}$$

By expanding the square root in (2.4), one finds the well-known[4,13] approximate formula [by replacing $M\to E$(bag) in the denominator]

$$M\cong E(\text{bag})-\frac{\langle k^2\rangle}{2E(\text{bag})}\ . \tag{2.9}$$

In the mock-electron case $MR_0\ll 1$ and one has to use the full formula (2.4), or try an expression for the case when $\langle k^2\rangle \gg M^2$. One finds

$$M^2\cong 2[E(\text{bag})-W_1]W_2{}^{-1}\ ,$$
$$W_1=\int d^3k\, I(k)k\ , \tag{2.10}$$
$$W_2=\int d^3k\, I(k)k^{-1}\ .$$

In order for the model to be convincing a very small (mock-) electron mass has to be reproduced with κ_e, which is of the same order of magnitude as κ found in (2.7). Indeed, by using Table I, one finds

$$m_e{}^2=2\left[\frac{\kappa_e}{0.618R_0{}^2}-\frac{3.331}{R_0{}^2}\right]\ ,$$
$$\kappa_e=2.0586\ . \tag{2.11}$$

This is also consistent with the complex formula (2.4), which gives

$$m_e=\frac{\kappa_e m_e R_0}{2.0586R_0}\ . \tag{2.12}$$

If one feels optimistic, one can even speculate that the value κ_e [Eq. (2.11)], by being of the same order of magnitude as (2.7), supports the conjecture[8] that the composite electron model can be built from the constituents bonded by some QCD-type interactions.

III. CHARGE RADIUS

In order for the model to be acceptable, the CMC has to lead to the charge radius which is proportional to R_0 for any MR_0. This is essential if the mock electron is to be comparable with the real-world electron. Interestingly enough, it turns out that when $MR_0\ll 1$ one has

$$\langle r^2\rangle \sim R_0{}^2 \tag{3.1}$$

only if there is no anomalous magnetic moment.

The approach is completely analogous to the one employed by Ref. 1. One introduces the time component J_0 of the current in the equation

TABLE I. Center-of-mass correction and energy.

R_0 (fm)	$MR_0Z(R_0,M)$ $M=M_p$	$M=m_e$	R_0W_1	$R_0{}^{-1}W_2$
0.001	2.0586	2.0586	2.0586	0.617 99
0.1	2.1246	2.0586	2.0586	0.617 99
0.2	2.3005	2.0586	2.0587	0.617 98
0.4	2.8627	2.0586	2.0587	0.617 96
0.65	3.7780	2.0585	2.0510	0.618 66
1.0	5.2557	2.0586	2.0576	0.618 00

$$\int d^3x\, e^{i\mathbf{q}\cdot\mathbf{x}}{}_B\left\langle \frac{\mathbf{q}}{2} \left| J_0(x) \right| -\frac{\mathbf{q}}{2} \right\rangle_B = (2\pi)^3 \int d^3k \left[W(k)W(k+q)W^2\left[k+\frac{q}{2}\right]\right]^{-1/2} \phi^2(k+q)\langle k+q \mid J_0(0) \mid k \rangle \cdot \quad (3.2)$$

This corresponds to formula (2.5) in Ref. 1. The right-hand-side (RHS) of (3.2) is of the form

$$\mathrm{RHS} = \int d^3k\, I(k+q)\bar{u}(k+q)\left[\gamma_0 F_1(q^2) + i\frac{\sigma_{0k}q_k}{2M}F_2(q^2)\right]u(k)\,. \quad (3.3)$$

Here we have introduced free-particle spinors u and nucleon form factors $F_i(q^2)$. Expanding to the leading order in q^2 the left-hand side (LHS) of (3.2) is

$$\mathrm{LHS} \cong 1 - \tfrac{1}{6}q^2\langle r^2\rangle_{\mathrm{bag}}\,. \quad (3.4)$$

Here we have replaced $|\mathbf{q}/2\rangle_B$ by $|0\rangle_B$, thus omitting RC. In order to use the definition

$$\langle r^2\rangle = -6\frac{\partial^2}{\partial q^2}G_E(q^2)\,|_{q^2=0}\,, \qquad G_E(q^2) = F_1(q^2) - g^2F_2(q^2)/4M^2\,, \quad (3.5)$$

one has also to expand the RHS of (3.2). The end result is of the form

$$\langle r^2\rangle_{\mathrm{bag}} = \langle r^2\rangle + (-6)\left[F_1(0)(P+Q) + F_2(0)\left[\frac{1}{4M^2} - R\right]\right]. \quad (3.6)$$

The term $F_2(0)(4M^2)^{-1}$ can become unacceptably large for small M. It appeared because the term $q^2F_2/4M^2$ had to be added (and subtracted) in order to use the expression (3.5). Thus, when $MR_0 \ll 1$, the model requires that $F_2(0)\to 0$, as has been already discussed. The function P is defined by

$$P = \int d^3k\, I(k)\left[\frac{k^2}{3E^4}(n_1d_1+n_2+d_2) + \frac{1}{E^2}(n_3+d_3) + \frac{k^2}{6E^3(E+M)}(n_1+d_1)\right]. \quad (3.7a)$$

Here

$$d_1 = -\frac{E}{E+M},\quad n_1 = -\frac{M}{2(E+M)}\,,$$
$$d_2 = \frac{E^2}{(E+M)^2} + \frac{1}{2}\frac{E}{E+M}\,,$$
$$n_2 = \frac{3}{4}\frac{M}{E+M} - \frac{1}{8}\frac{M^2}{(E+M)^2}\,, \quad (3.7b)$$
$$d_3 = -\frac{1}{2}\frac{E}{E+M},\quad n_3 = -\frac{M}{4(E+M)}\,,$$
$$E = E(k) = (k^2+M^2)^{1/2}\,.$$

The function R is

$$R = \int d^3k\, I(k)\left[\frac{1}{6ME(k)} + \frac{1}{12E^2(k)}\right]. \quad (3.8)$$

The integral Q is of a somewhat different type:

$$Q = (-)\int dk\frac{k^3}{2E(k)}I_1(k)\,, \qquad I_1(k) = (-)\frac{1}{3\pi}\int dr\, r^3 j_1(kr)I_3(r)\,. \quad (3.9)$$

Here j_1 is the spherical Bessel function and I_3 is defined in the Appendix.

As can be seen in Table II, the integrals P and Q are both proportional to R_0^2. The integral R which is proportional to R_0M^{-1} is eliminated for $MR_0 \ll 1$ if $F_2(0)\to 0$. Only this condition leads to the satisfactory mock-electron model.

In Ref. 1 we have found $\mu_e = 0.97\mu_B$. This can be easily adjusted to $1\mu_B$ by a very small, and thus quite acceptable, change in the model parameters. For example, by using Duck's[11] quark wave functions, employed in the calculation of $I_c(R_0)$ (Ref. 1), one finds

TABLE II. Center-of-mass correction and charge radius.

R_0 (fm)	$(-)P$ $M=M_p$	$(-)P$ $M=m_e$	R $M=M_p$	R $M=m_e$	Q $M=m_p$	Q $M=m_e$	Q_A
0.001	0.9909×10^{-7}	0.9969×10^{-7}	0.2173×10^{-4}	0.3977×10^{-1}	0.4995×10^{-7}	0.4995×10^{-7}	0.4995×10^{-7}
0.01	0.8111×10^{-5}	0.8442×10^{-5}	0.2064×10^{-3}	0.3717	0.4238×10^{-5}	0.4242×10^{-5}	0.4249×10^{-5}
0.2	0.2440×10^{-2}	0.3832×10^{-2}	0.4298×10^{-2}	7.955	0.1521×10^{-2}	0.1998×10^{-2}	0.1998×10^{-2}
0.4	0.6192×10^{-2}	1.592×10^{-2}	0.6977×10^{-2}	15.92	0.3994×10^{-2}	0.7990×10^{-2}	0.7992×10^{-2}
0.65	0.9734×10^{-2}	4.183×10^{-2}	0.8712×10^{-2}	25.87	0.6374×10^{-2}	2.110×10^{-2}	2.112×10^{-2}
1.0	1.256×10^{-2}	8.961×10^{-2}	0.9803×10^{-2}	39.81	0.8309×10^{-2}	4.990×10^{-2}	4.996×10^{-2}

$$\mu_H^s = \frac{2M\beta}{1+\frac{3}{2}\beta^2} R_0 \frac{e}{2M} . \qquad (3.10)$$

With $\beta=0.36$, $R_0=10^{-3}$ fm, and $M=m_e$ we had $\mu_H^s(\text{corr})=\mu_H^s I_c^{-1}=0.97\mu_B$. By changing to $\beta=0.374$ we obtained $\mu_H^s(\text{corr})=1.004\mu_B$. With $\beta=0.374$ and $M=M_p$ we find for $R_0=10^{-3}$ fm, $R_0 I_c^{-1}=0.3418$ and for $R_0=0.65$ fm, $R_0 I_c^{-1}=0.7839$. For $R_0=10^{-3}$ fm and $M=m_e$, $R_0 I_c^{-1}=6.27\times 10^2$. (Compare with Table I of Ref. 1.)

In the case of the proton ($R_0=0.65$, $M=M_p$), one obtains a very small positive correction

$$\langle r^2\rangle_p = \langle r^2\rangle_{\text{bag}} + 0.53\times 10^{-2} .$$

There were numerous cancellations among terms in (3.6). The major contribution came from the term proportional to $F_2(0)$. Without it the correction would be negative ($\sim 2.02\times 10^{-2}$). This has to be kept in mind if any comparison with Refs. 12 and 15, which lack the $F_2(0)$ term, is attempted. It seems that Ref. 12 included RC also.

IV. CONCLUSION

The model[1,11–15] allows relatively simple estimates of the CMC's connected with mass and charge radius of the proton. The function Z appearing in (2.4) can be easily calculated for a given set of model parameters (β, R_0) and for the proton mass ($M=M_p$). Other model parameters (symbolized by κ) can then be adjusted so as to satisfy (2.4). In that way one can determine sets of the model parameters which lead to the correct proton mass. The same holds for any octet or decouplet baryon. The correction for $\langle r^2\rangle$ is given by (3.6) in a closed form. It differs from other estimates[12,15] which started from the same physical assumptions, but did not use algorithm (3.2).

All corrections have also been evaluated for systems with small mass and confinement radius ($MR_0 \ll 1$). The model seems to lead to acceptable descriptions of the mass and charge radius of the mock electron. In an analogous way it did justify[1] the conjecture about the magnetic moment.[8] We hope that such an outcome might stimulate further investigations based on more exact methods.

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APPENDIX

The function $\phi(k)$ is spherically symmetric. It is given by[11,13]

$$\phi^2(k)=(2\pi)^3 W(k) I(k) , \qquad W(k)=E(k)/M , \qquad (A1)$$

$$I(k)=(2\pi)^{-3}\int d^3r\, e^{i\mathbf{k}\cdot\mathbf{r}} I_3(r) ,$$

$$I_3(r)=\left[e^{-(r^2/4R_0^2)} \left[1 - C\frac{r^2}{R_0^2} \right] \right]^3 , \qquad (A2)$$

$$C=\frac{\beta^2}{4+6\beta^2} .$$

The value of the parameter $\beta=0.36$ is such that the harmonic-oscillator wave functions approximate a $1s$ massless quark wave function in a bag of radius $R=1.5R_0$ ($R_0=0.65$ fm means that $R=4.97$ GeV^{-1}). The identity

$$1=\int d^3k\, I(k) , \qquad (A3)$$

which corresponds to the formula (2.8) from Ref. 1 can be used as a useful check for numerical accuracy.

The CMC for the magnetic moment is determined by

$$I_C(R_0)=\int d^3k \frac{M}{E(k)} I(k) . \qquad (A4)$$

The functions $Z(R_0,M)$, W_1, W_2, P, and R, which are defined in the main text are closely related. In those integrals the factor ME^{-1} from (A4) is replaced by factors of similar type. However, care has to be exercised in numerical integration, especially for the integral P, where there are many terms with alternating signs. There is also a check for the integral Q which operates in the region $MR_0 \ll 1$. In that region $E(k)\sim k$. By using the relation for the spherical Bessel functions

$$rj_1(kr)=\frac{\partial}{\partial k} j_0(kr) ,$$

and by partial integration, one finds

$$Q \underset{MR_0\ll 1}{\longrightarrow} Q_A = \tfrac{1}{12}\int d^3k\, I(k) k^{-2} . \qquad (A5)$$

The numerical values of Q_A are given in Table II. In the same region (i.e., $MR_0 \ll 1$) one can find

$$I_C \to MW_2, \quad Z \to M^{-1}W_1 , \qquad P \to (-2)Q_A, \quad R \to (6M)^{-1}W_2 . \qquad (A6)$$

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