

Breaking of isospin symmetry in theories with a dynamical Higgs mechanism

Thomas Appelquist, Mark J. Bowick, Eugene Cohler, and Avi I. Hauser

Department of Physics, Yale University, New Haven Connecticut 06511

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The sensitivity of physical amplitudes to the observed isospin-symmetry breaking in the fermion mass matrix is analyzed in theories where the electroweak symmetry is dynamically broken. As a first step toward discussing dynamical theories in a model-independent way, we consider a strongly interacting Higgs theory. The nonlinear σ model coupled to an $SU(2) \times U(1)$ Yang-Mills theory and to fermions is used to generate the low-dimensional operators induced by the quantum theory. The strengths of these operators, in particular those that break isospin symmetry, are estimated using the Higgs-boson mass of the linear σ model as a regulator. Technicolor models are next considered as a specific example of the strong-interaction physics which leads to electroweak symmetry breaking. Emphasis is placed on the effective four-fermion interactions that are natural partners of those which give mass to the ordinary fermions. New sources of isospin-symmetry breaking are found; in particular there is one which contributes directly to the ρ parameter with strength that is linear in the effective Yukawa coupling of the heaviest ordinary fermion. This effect is qualitatively different from the quadratic dependence found in Higgs theories.

I. INTRODUCTION

The experimental success^{1,2} of the $SU(2)_L \times U(1)$ electroweak theory is both reassuring and challenging. It emphasizes the importance of understanding the mechanism of electroweak symmetry breaking and the attendant mystery of quark and lepton mass generation. The standard device for incorporating these essential features into the theory is the inclusion of a set of scalar Higgs fields with a potential arranged to produce spontaneous symmetry breakdown. Although renormalizable, the resultant theory is widely regarded as being an effective theory valid only at energies below a few TeV. To take it more seriously, with the usual light, weakly coupled Higgs boson, leads to a naturalness problem: the necessity of fine tuning the Higgs-boson mass to remain light in the presence of renormalization corrections which depend quadratically on the cutoff. On a more pragmatic level, the elementary-scalar device offers very little hope for really explaining anything. All the fermion masses, for example, are simply put in by hand in the form of Yukawa coupling constants.

An attractive alternative is that new matter, with strong interactions at energies of a few TeV, dynamically breaks the electroweak symmetry. This possibility has been studied extensively, both by inventing specific models of dynamical symmetry breaking³ and by investigating the low-energy consequences of a wide class of such theories.⁴⁻⁶ While there is no compelling model, and no unambiguous low-energy experimental signatures have been suggested, the idea remains alive and provides the focus of much theoretical work and experimental planning.

One of the most troubling features that must be built into the electroweak theory is the large amount of weak isospin breaking exhibited by the quark and lepton masses. Apart from explaining the large mass differences within the doublets, there is a potentially serious problem of internal consistency. The experimental fact that the ρ

parameter ($\rho \equiv M_W^2/M_Z^2 \cos^2 \theta_W$) is unity to within a few percent⁷ can be explained if the Higgs sector of the theory has an $SU(2)_L \times SU(2)_R$ global symmetry. The spontaneous breakdown of this symmetry to $SU(2)_{L+R}$ then gives $\rho=1$ to lowest order in the gauge coupling. Small deviations from $\rho=1$ will arise from higher-order gauge interactions which respect only the (local) $SU(2)_L \times U(1)$ subgroup of $SU(2)_L \times SU(2)_R$. Additional breaking of $SU(2)_L \times SU(2)_R$ must be introduced in some way to give the correct fermion masses. This might be expected to produce large, and possibly unacceptable, deviations from $\rho=1$ once the mass splitting within a fermion doublet becomes as large as, say, the current lower bound on the t -quark mass.

This issue has been looked at extensively in the conventional model with elementary scalars. The result of several analyses^{8,9} is that, at one loop, $\rho-1$ goes roughly as $G_F/8\sqrt{2}\pi^2$ (up to a color factor) times the square of the intradoublet mass difference and therefore splittings up to several hundred GeV can be tolerated. As long as the t quark is not too heavy, there is no obvious problem with internal consistency.

When the Higgs sector is dynamical, with strong interactions at a mass scale of a few TeV, it is not so clear how the large isospin breaking in the fermion masses will feed back into the ρ parameter or other measurable quantities. The loop expansion is of limited use in analyzing this question. One must also appeal to naturalness arguments and order-of-magnitude estimates. It is the purpose of this paper to investigate this problem by using general effective-Lagrangian techniques and by examining technicolor models.

In Sec. II we review the status of this problem in the standard model with a light weakly coupled Higgs boson. A general analysis of the isospin-breaking problem of a strongly interacting Higgs sector is presented in Sec. III. Using the nonlinear σ model as an effective low-energy description of the full electroweak theory, estimates are made of the expected isospin breaking in various ampli-

tudes of interest. A list of low-dimension operators consistent with $SU(2)_L \times U(1)$ symmetry and incorporating the observed $SU(2)_{L+R}$ splitting in the fermion masses is generated. The natural size of each operator is then estimated and its physical consequences are described. The differences between the dynamical case and the elementary-scalar case are emphasized. Special attention is paid to the ρ parameter.

One model for dynamical symmetry breaking is that of technicolor. Although there is no realistic technicolor model and existing models generally predict either excessive flavor-changing neutral currents or excessively light pseudo-Goldstone bosons, the technicolor mechanism serves as an explicit example which we explore in Sec. IV. After a brief introduction, fermion mass generation in these models will be described with special emphasis on isospin violation. It will be shown how to obtain the low-energy operators and physical effects discussed in Sec. III by integrating out the technifermions. The major result is that technicolor theories naturally contain more sources of isospin-symmetry breaking than required for fermion mass generation and that these produce qualitatively new effects in the low-energy theory.

II. THE STANDARD MODEL

In this section we review how strong isospin violation in the fermion mass matrix is compatible with the weak isospin relation⁷

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 \pm 0.03. \quad (1)$$

Relation (1) follows from charge conservation and the existence of an $SU(2)$ symmetry of both the Lagrangian and the vacuum under which the generators of $SU(2)_L$ transform as a triplet.¹⁰ These two conditions force the gauge-boson mass matrix, in the $SU(2)_L \times U(1)_Y$ basis, to have the structure

$$\begin{pmatrix} g_2^2 & & & \\ & g_2^2 & & \\ & & g_2^2 & -g_1 g_2 \\ & & -g_1 g_2 & g_1^2 \end{pmatrix} \langle \Phi \rangle^2, \quad (2)$$

where g_2 [g_1] is the $SU(2)_L$ [$U(1)$] gauge coupling constant and $\langle \Phi \rangle \simeq 250$ GeV is the Higgs-field vacuum expectation value. This gives a mixing angle $\tan \theta_W = g_1/g_2$ and $\rho=1$ at the tree level. The $SU(2)$ symmetry is evident in the diagonal g_2^2 structure of the W sector of the matrix. It can be traced to a symmetry of the scalar potential.

Writing the complex doublet Φ as

$$\Phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_0 + i\pi_3 \\ -\pi_1 + i\pi_2 \end{pmatrix}, \quad (3)$$

and noting that $V(\Phi)$ is a function only of $\Phi^\dagger \Phi$ we see that $V(\Phi)$ has a global $O(4) \simeq SU(2)_L \times SU(2)_R$ symmetry corresponding to the preservation of the length of the four-vector $(\sigma_0, \boldsymbol{\pi})$. When $\langle \phi^0 \rangle \equiv \langle \sigma_0 \rangle / \sqrt{2} \neq 0$, this $O(4)$

is spontaneously broken to $O(3) \simeq SU(2)$. To consistently couple the Higgs sector to the gauged weak-interaction sector its global symmetry need only be $SU(2)_L \times U(1)$. The extra symmetry, which is hereafter identified with isospin, inescapably leads to the mass relation $\rho=1$. It is consequence of using a single Higgs doublet and is sometimes called, for this reason, the Higgs $\Delta I_w = \frac{1}{2}$ rule. It is worth noting here that the $O(4)$ symmetry of the Higgs potential arises simply because, for a single doublet, the only possible potential terms of any dimensionality that are $SU(2)_L \times U(1)$ symmetric are also $SU(2)_L \times SU(2)_R$ symmetric.⁶ This does not necessarily hold for more than one Higgs doublet.

We now discuss the higher-order corrections to $\rho=1$. First there are $O(\alpha)$ corrections since the gauge symmetry is only $SU(2)_L \times U(1)$. These corrections have been studied extensively.^{8,11} They include a term that grows like $\ln(M_H)$ as $M_H \rightarrow \infty$,^{6,8} where M_H is the mass of the Higgs boson. With $M_H \leq 1$ TeV, however, this term is not especially important experimentally.

The other source of isospin breaking is the fermion mass matrix, which is parametrized by the Yukawa couplings. At one loop, $\Delta\rho$ ($\equiv \rho-1$) receives contributions from fermion loops inserted in the gauge-boson propagator (Fig. 1). The result is given by,^{8,9}

$$\Delta\rho = \xi \frac{1}{16\pi} \frac{\alpha_w}{M_W^2} \left[\frac{2m_u^2 m_d^2}{m_u^2 - m_d^2} \ln \frac{m_d^2}{m_u^2} + m_u^2 + m_d^2 \right], \quad (4)$$

where m_u and m_d are the masses of the individual fermions within a doublet and ξ is a color factor (which is 1 for leptons and 3 for quarks). Expanding in the mass splitting, this becomes

$$\Delta\rho = \xi \frac{1}{8\pi} \frac{\alpha_w}{M_W^2} (m_u^2 + m_d^2) \left[\frac{1}{3} \left[\frac{m_u^2 - m_d^2}{m_u^2 + m_d^2} \right]^2 + \dots \right], \quad (5a)$$

while for $m_u \gg m_d$, it reduces to

$$\Delta\rho = \xi \frac{\alpha_w}{16\pi} \frac{m_u^2}{M_W^2}. \quad (5b)$$

The quadratic dependence on the fermion mass splitting is easily understood. The Yukawa interactions in the standard model may be written in the form

$$\mathcal{L}_Y = -\frac{1}{2} \bar{f}_L M [(y_u + y_d) + (y_u - y_d)\tau_3] f_R + \text{H.c.}, \quad (6)$$

where $f = (u, d)$. The matrix M is defined by

$$M = \sqrt{2} \begin{pmatrix} \phi^0 & -\phi^+ \\ \phi^- & \phi^{0*} \end{pmatrix} \quad (7)$$

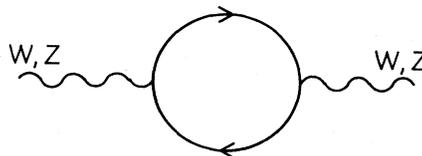


FIG. 1. Fermion-loop correction to the gauge-boson self-energy diagram.

and transforms according to $SU(2)_L \times SU(2)_R$ as

$$M \rightarrow U_L M U_R^\dagger, \quad (8)$$

where the U matrices are $SU(2)$ transformations. For $y_u = y_d = y$ this is $SU(2)_L \times SU(2)_R$ symmetric. When $y_u \neq y_d$ this symmetry is explicitly broken and the strength of the breaking goes smoothly to zero as y_u approaches y_d . We thus expect the correction to ρ to be proportional to $y_u - y_d \propto m_u - m_d (= \Delta m)$ to some power. The quadratic dependence follows simply because the gauge couplings in Fig. 1 conserve helicity and therefore an even number of mass insertions is required.

The fermion loop contribution to $\Delta\rho$ [Eq. (4)] can be combined with Eq. (1) to constrain the fermion mass splittings. It is known that the $O(\alpha)$ radiative corrections reduce ρ by approximately 0.02 (Ref. 11) and therefore the expression in Eq. (4) can be no larger than about 0.05. For the case of a heavy lepton (L) with a massless neutrino partner, this implies $m_L \leq 700$ GeV. For a heavy quark ($m_u \gg m_d$), it implies $m_u \leq 400$ GeV.

One may also arrive at Eq. (4) by considering solely Goldstone-boson dynamics.⁹ By current conservation the gauge-boson two-point function has the transverse form $(g_{\mu\nu} - q_\mu q_\nu / q^2) \Pi(q^2)$. The gauge-boson diagrams already considered (Fig. 1) give the $g_{\mu\nu}$ part. The diagram shown in Fig. 2, the two-point function for the Goldstone bosons, gives the $q_\mu q_\nu / q^2$ part.

The fermion-mass-generation mechanism in the standard model thus allows for rather strong isospin violation in the fermion mass matrix without upsetting the necessary relation $\rho \simeq 1$. It is only necessary that isospin partners have mass splittings less than a few hundred GeV.

We conclude this section with some important remarks about the higher-order corrections to $\Delta\rho$. Here we consider those contributions to $\Delta\rho$ that are nonvanishing only if there is isospin violation in the Yukawa couplings. Equation (4) is then the one-loop contribution to $\Delta\rho$ and is quadratic in the Yukawa couplings. Two-loop corrections could, *a priori*, be (1) quartic in the Yukawa couplings, (2) quadratic in the Yukawa couplings $\times O(g^2/4\pi^2)$, where g is the $SU(2)_L$ or $U(1)$ coupling constant, or (3) quadratic in the Yukawa couplings $\times O(\lambda/4\pi^2)$, where λ is the Higgs-boson self-coupling. We will argue that the last of these corrections is *not* present. This is important because, were this correction present, the one-loop result [Eq. (4)] would be completely unreliable in the limit when the Higgs sector becomes heavy and strongly interacting ($M_H \rightarrow 2$ TeV, $\lambda/4\pi^2 \equiv M_H^2/8\pi^2 \langle \phi \rangle^2 \rightarrow 1$).

Each of the above corrections must be ultraviolet finite since there are no dimension four or lower operators that

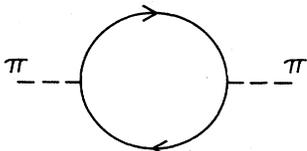


FIG. 2. Fermion-loop contribution to the Goldstone-boson propagator.

contribute to $\Delta\rho$. This was of course true of the one-loop computation. There, since mass insertions are necessary to produce isospin-symmetry breaking, the result could have been at most logarithmically divergent. Explicit calculation showed that $\Delta\rho$ was in fact finite [Eq. (4)] in agreement with the above general argument. At two loops there are many diagrams that, without any mass insertions, contain Yukawa couplings that break the isospin symmetry (see, for example, Fig. 3). Thus it would seem that mass insertions are not required to produce a contribution to $\Delta\rho$. It therefore looks as though there could be quadratically divergent contributions to $\Delta\rho$ at two loops. These divergences must of course cancel once all the graphs are added since there are no renormalizable counterterms to accommodate them.

What can the residual logarithmic divergences and finite pieces then look like? Any logarithmic divergence must be absorbed into renormalizations of the various masses or the gauge or Yukawa couplings. The question then is whether, among the finite pieces, there can be a contribution of type 3 above. Since there are no explicit Higgs-boson self-couplings among the two-loop isospin-violating graphs, it could only arise from a finite contribution proportional to $M_H^2 = 2\lambda \langle \phi \rangle^2$ from the graphs with internal Higgs-boson propagators. Since the divergent contributions of these graphs to $\Delta\rho$ must cancel, however, so too must the finite piece proportional to M_H^2 and quadratic in the Yukawa couplings cancel. This is because the isospin breaking is in the coupling constants; the isospin structure of the divergent pieces and the finite pieces is precisely the same. Only if fermion mass insertions are included can the finite pieces of these graphs have a different isospin structure. But mass insertions here clearly lead to terms quartic in the Yukawa couplings. (Since the self-energy diagrams considered always have an ultraviolet quadratic divergence two mass insertions will lead at most to a logarithmic infrared divergence at zero external momentum. Infrared mass terms thus cannot appear in the denominator to cancel explicit mass insertions in the numerator.) All possible contributions of type 3 above, therefore, must cancel among the diagrams.

The remaining contributions to $\Delta\rho$ are of types 1 and 2. These will presumably include terms which grow like $\ln(M_H)$ as M_H is increased, indicating a sensitivity to a strongly coupled Higgs sector at this level. This mixing of strong Higgs-boson and heavy-fermion mass effects is, however, screened from the leading effect [Eq. (4)] by extra powers of the gauge or Yukawa couplings. To study in more detail the interplay of isospin-symmetry breaking associated with fermion masses and strong Higgs-boson effects, we next make use of the nonlinear σ model.

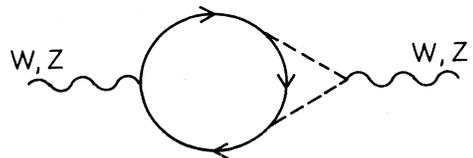


FIG. 3. Two-loop contribution to the gauge-boson self-energy. Dashed lines represent the Higgs fields.

III. ISOSPIN-SYMMETRY BREAKING WITH A DYNAMICAL HIGGS SECTOR

If electroweak-symmetry breaking arises from some strong dynamics at a mass scale of a few TeV, the theory at $E < 1$ TeV is conveniently described by the gauged nonlinear σ model.^{5,6} Corrections to the leading low-energy theory can then be summarized in the form of higher-dimension operators that exhibit the $SU(2)_L \times SU(2)_R$ symmetry [reduced to $SU(2)_L \times U(1)$ by gauge and Yukawa couplings]. These operators will equivalently be generated as counterterms in the loop expansion of the nonrenormalizable theory. This technique—listing and then estimating the size of higher-dimension operators—has been used, neglecting fermion masses, to describe the sensitivity of the current generation of experiments to a TeV Higgs-boson sector.⁶

It is the purpose of this section to extend this sort of analysis to the full theory, including the Yukawa couplings needed to describe the fermion masses. The nonlinear theory, which emerges as the Higgs-boson mass of the linear theory is pushed up, can be described by the M matrix [Eq. (7)] subject to the nonlinear constraint

$$MM^\dagger = M^\dagger M = f^2, \quad (9)$$

where $f/\sqrt{2}$ can be identified with $\langle \Phi \rangle$ introduced in Eq. (3). It will be convenient to express the theory in terms of the unitary matrix field

$$U = M/f. \quad (10)$$

The interaction of the Goldstone fields with the gauge fields is described by the operator

$$\mathcal{L}_{\text{NL}} = \frac{f^2}{4} \text{Tr}(D_\mu U)^\dagger (D^\mu U), \quad (11)$$

where

$$D_\mu U = \partial_\mu U + ig_2 \tau \cdot \mathbf{W}_\mu U / 2 - ig_1 B_\mu U \tau_3 / 2. \quad (12)$$

If U is replaced by its vacuum value $\mathbb{1}$, corresponding to the spontaneous breakdown of $SU(2)_L \times SU(2)_R$ to $SU(2)_{L+R}$, the gauge-boson mass matrix [Eq. (3)] is obtained. For further details of the gauge field dynamics the interested reader can consult Ref. 5.

In the gauged nonlinear σ model, fermion masses must be introduced by the same Yukawa couplings used in the linear (light, elementary-scalar) theory. For a single generation of quarks (u, d), the appropriate operator is

$$\mathcal{L}_Y = -f \bar{f}_L [U(y + y_3 \tau_3)] f_R + \text{H.c.}, \quad (13)$$

where $f_{L,R} = (u, d)_{L,R}$ and y and y_3 are the Yukawa coupling constants, related to those of the previous section by $y = \frac{1}{2}(y_u + y_d)$ and $y_3 = \frac{1}{2}(y_u - y_d)$. The relative strength of y and y_3 determines the strength of explicit isospin-symmetry breaking. The mass of each member of the doublet is given by

$$m_u = f(y + y_3) = f y_u, \quad (14)$$

$$m_d = f(y - y_3) = f y_d.$$

The above notation is easily generalized to include mul-

iple generations and leptons. The parameters y and y_3 become matrices which can be diagonalized. For the case of three generations, this leads to a flavor-diagonal quark mass matrix together with the Kobayashi-Maskawa angles in the charge-changing interactions of the quarks.

The operators \mathcal{L}_{NL} and \mathcal{L}_Y , together with the standard interactions between the gauge bosons and the fermions and among the gauge bosons themselves, are those that emerge from taking the infinite-Higgs-boson-mass limit of the usual renormalizable theory. They do not constitute, however, a complete list of low-dimension operators consistent with the $SU(2)_L \times U(1)$ symmetry of the theory. By extending the list, we shall be able to enumerate the low-energy ($E < 1$ TeV) amplitudes allowed by the theory and estimate their likely strength. This will be done with special attention paid to those amplitudes that respect $SU(2)_L \times U(1)$, but not $SU(2)_L \times SU(2)_R$, symmetry.

In the absence of fermions this task has been taken far enough for our purposes by Longhitano.⁶ To summarize his results it is convenient to classify operators by dimension, using the dimensionless U matrix. Because of the nonlinear constraint [Eq. (9)] it is best not to attach dimensionality to the Goldstone-boson matrix field. The operator \mathcal{L}_{NL} then has dimension two. It turns out that there is one additional, independent, dimension-two operator and then a rather large number of dimension-four operators. If the nonlinear theory is analyzed in the loop expansion, the dimension-two and dimension-four operators are the counterterms necessary to absorb all the infinities through one loop. Two loops then require dimension-six operators and so on.

The additional dimension-two operator is

$$\mathcal{L}_2 = \frac{af^2}{4} (\text{Tr} \tau_3 U^\dagger D_\mu U)^2, \quad (15)$$

where a is a dimensionless parameter. The dimension-four operators not involving fermion fields, to which we shall return, are listed in Ref. 6. The operator \mathcal{L}_2 has two important features: (1) It is $SU(2)_L \times U(1)$, but not $SU(2)_L \times SU(2)_R$, symmetric. (2) It is the only operator, in addition to \mathcal{L}_{NL} , that contributes to the gauge-boson two-point function at zero momentum. The operators \mathcal{L}_{NL} and \mathcal{L}_2 therefore determine, up to finite radiative corrections, the W and Z “masses,” and the corresponding ρ parameter, as measured in low-momentum-transfer ($q \ll M_W, M_Z$) neutrino scattering experiments. A simple computation reveals that

$$\rho = 1 - 2a + \text{corrections finite in the nonlinear limit } (M_H \rightarrow \infty). \quad (16)$$

The operator \mathcal{L}_2 is induced at the one-loop level by radiative corrections involving the $U(1)$ gauge field. The logarithmically divergent one-loop computation can be sensibly cut off at a few TeV and the order of magnitude is $(g_1^2/4\pi^2) \ln(M_H)$, where g_1 is the $U(1)$ gauge coupling constant and M_H serves as the cutoff. A further analysis of higher-order effects shows that the coefficient of $g_1^2/4\pi^2$ cannot be reliably computed with a heavy, strongly interacting Higgs sector but that it is naturally of

order unity.¹² One concludes therefore that

$$\begin{aligned} a_{\text{gauge-induced}} &= \frac{g_1^2}{4\pi^2} \times O(1) \\ &= \frac{g_2^2}{4\pi^2} \tan^2 \theta_W \times O(1), \end{aligned} \quad (17)$$

a value comfortably within the experimental bound.

We now bring the fermions back into the discussion. The operator of lowest dimension involving the fermion fields is the Yukawa interaction \mathcal{L}_Y , which has dimension three. The isospin breaking in this interaction will be transmitted to the ρ parameter most directly by the one-loop process described in the previous section. The fermion masses arising from \mathcal{L}_Y enter the fermion-loop correction to $\pi_{\mu\nu}$ and give the result Eq. (4). This contribution to $\Delta\rho$ is finite even in the nonlinear limit; it is independent of whether or not the Higgs sector is strongly interacting. Since it is insensitive to TeV physics, unless the fermion mass splitting becomes of this order, we do not regard it as generating the new local operator \mathcal{L}_2 . As noted in the previous section, the splitting is constrained by experiment to be no larger than a few hundred GeV.

The next operators involving the fermion fields are of dimension four. We will list these and then discuss their order of magnitude and the way in which they feed into \mathcal{L}_2 and the ρ parameter. We will then conclude this section with a brief discussion of the remaining dimension-four operators (those not involving the fermion fields) listed in Ref. 6. The first fermion operators of dimension four are the standard gauge-fermion interactions,

$$\begin{aligned} \mathcal{L}_L &= i\bar{f}_L \not{D} f_L \\ &= i\bar{f}_L (\partial_\mu + ig_2 \tau \cdot \mathbf{W}_\mu / 2 + ig_1 B_\mu Y / 2) \gamma^\mu f_L, \end{aligned} \quad (18)$$

and

$$\mathcal{L}_R = \bar{f}_R \not{D} f_R = \bar{f}_R (\partial_\mu + ig_1 B_\mu Y / 2) \gamma^\mu f_R, \quad (19)$$

where Y is the weak-hypercharge operator with eigenvalues $y_{u_L} = y_{d_L} = \frac{1}{3}$, $y_{u_R} = \frac{2}{3}$, and $y_{d_R} = -\frac{2}{3}$.

At the dimension-four level additional nonlinear operators can be constructed, essentially by inserting the dimensionless U and U^\dagger matrix into the expressions \mathcal{L}_L and \mathcal{L}_R . Because of the constraint $UU^\dagger = 1$, however, there is a quickly saturated limit to this procedure. The possible operators are

$$\mathcal{L}_4^1 = i\delta_1 \bar{f}_L [U (\not{D} U)^\dagger] f_L, \quad (20a)$$

$$\mathcal{L}_4^2 = i\delta_2 \bar{f}_R [U^\dagger (\not{D} U)] f_R, \quad (20b)$$

$$\mathcal{L}_4^3 = i\delta_3 \bar{f}_L [(\not{D} U) \tau_3 U^\dagger] f_L + \text{H.c.}, \quad (20c)$$

$$\mathcal{L}_4^4 = i\delta_4 \bar{f}_L [U \tau_3 U^\dagger (\not{D} U) \tau_3 U^\dagger] f_L, \quad (20d)$$

$$\mathcal{L}_4^5 = i\delta_5 \bar{f}_R [\tau_3 U^\dagger (\not{D} U)] f_R + \text{H.c.}, \quad (20e)$$

$$\mathcal{L}_4^6 = i\delta_6 \bar{f}_R [\tau_3 U^\dagger (\not{D} U) \tau_3] f_R, \quad (20f)$$

$$\mathcal{L}_4^7 = i\delta_7 \bar{f}_L [U \tau_3 U^\dagger] \not{D} f_L + \text{H.c.} \quad (20g)$$

All the δ_i coefficients are real. The reality of δ_1 , δ_2 , δ_4 , and δ_6 is ensured by hermiticity. The coefficients δ_3 and δ_5 can only be nonreal if CP violation is incorporated.

Since we are neglecting CP violation in this paper, these too must be real.

Two questions must now be addressed:

(1) What is the natural size of each of these operators and is this size compatible with experiment?

(2) How do these operators feed back, through higher-order corrections, to lower-dimensional operators such as \mathcal{L}_2 [Eq. (15)] and to measurable quantities such as $\Delta\rho$?

Each of the operators in Eqs. (20), except \mathcal{L}_4^4 , is generated at the one-loop level with a logarithmically divergent coefficient. These are simplest to compute using the fermion-fermion- π vertex generated by the graphs shown in Fig. 4. Each coefficient δ_i is quadratic in the Yukawa couplings y and y_3 , and with the linear theory used as a cutoff, each is proportional to $\ln(M_H)$. The one-loop value of the cutoff-dependent part of each coefficient is

$$\begin{aligned} \delta_1 &= -(y^2 + y_3^2)C, \\ \delta_2 &= -y^2C, \\ \delta_3 &= yy_3C, \\ \delta_4 &= 0, \\ \delta_5 &= -yy_3C, \\ \delta_6 &= -y_3^2C, \\ \delta_7 &= yy_3C, \end{aligned} \quad (21)$$

where $C = (1/64\pi^2) \ln(M_H/M_W)$. This was obtained using dimensional regularization and identifying the pole $1/\epsilon$ as $\ln(M_H/M_W)$, where M_W is the subtraction point.

If f_L and f_R represent one of the known fermion doublets, then these operators will give M_H -dependent corrections to the measured weak interactions. Since y and y_3 are very small for the known fermions, however, only very tiny deviations from the standard model are predicted. These corrections are just those parts of the standard one-loop radiative corrections that grow (logarithmically) with M_H .

It is worth emphasizing here that in a purely phenomenological application of the nonlinear σ model, we would not be trying to estimate the coefficients in front of the operators \mathcal{L}_2 and \mathcal{L}_4 . They would be free

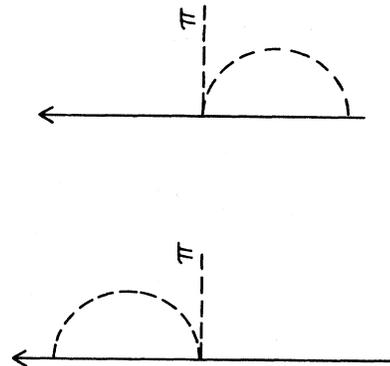


FIG. 4. One-loop divergent diagrams used to evaluate \mathcal{L}_4 . Here the dashed lines are the π fields of the nonlinear σ model.

parameters, necessary to absorb quantum divergences, and finally fit to experiment. The exercise of this section, estimating their natural size by using the linear model to cut the integrals off at $E \approx 1$ TeV, implicitly assumes that this is a reasonable way to describe at low energies whatever new physics lurks around 1 TeV. In using this approach, for example, to estimate the relative strength of isospin-breaking and isospin-conserving operators, one is assuming that there is no new source of isospin breaking at these high energies. In the next section we will in fact argue that technicolor theories do contain new sources of isospin breaking, to which the present analysis is insensitive.

With that proviso we now address the second question above. If f is a doublet containing a fermion as heavy as the top quark, then the most direct effect of these operators is through a heavy-fermion loop (see Fig. 5) formed using one of these operators together with one standard interaction [Eqs. (18) and (19)]. These corrections to the gauge-boson propagator could contribute to \mathcal{L}_2 and therefore to $\Delta\rho$. Since each δ_i is quadratic in the Yukawa couplings, one might worry that there are corrections to \mathcal{L}_2 and $\Delta\rho$ of the same order as that of Eq. (4). It is easy to see, however, that to get nonvanishing contributions to \mathcal{L}_2 from any of these loops, it is necessary to make at least two mass insertions on the fermion propagators. If \mathcal{L}_4^1 , \mathcal{L}_4^2 , \mathcal{L}_4^3 , \mathcal{L}_4^5 , or \mathcal{L}_4^7 is being used, then the insertions are necessary to produce sufficient isospin violation to generate \mathcal{L}_2 . In the case of \mathcal{L}_4^6 , the insertions are necessary to flip helicity to connect to an ordinary left-handed gauge coupling. The one operator that could cause trouble, \mathcal{L}_4^4 , is simply not generated at the one-loop level, either with a logarithmically divergent or finite coefficient [its absence is due to a special feature of the \mathcal{L}_4^4 operator—it has four Goldstone-boson (U) matrices]. Thus these contributions to \mathcal{L}_2 must be at least quartic in the Yukawa couplings.

This is of course the conclusion reached in Sec. II in the context of the standard Glashow-Weinberg-Salam model, i.e., the gauged linear σ model. To summarize, the gauge-induced contribution to $\Delta\rho$ is $(g_2^2/4\pi^2)\tan^2\theta_W$ times a coefficient that we cannot compute when there is a strongly interacting heavy-Higgs-boson sector, but which is expected to be $O(1)$ [Eq. (17)]. The Yukawa-coupling-induced contribution at one loop is given by Eq. (4) and is insensitive to the Higgs sector. The two-loop corrections to this result are either quartic in the Yukawa couplings or quadratic in the Yukawa couplings times $O(g^2/4\pi^2)$. In either case the correction is multiplied by a coefficient expected to be of $O(1)$, with a heavy-Higgs-boson sector, since a logarithmic sensitivity to the Higgs-boson mass appears at this level.

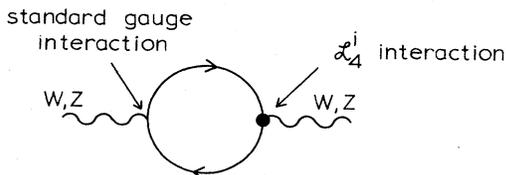


FIG. 5. Fermion-loop correction to the gauge-boson self-energy diagram, utilizing an effective \mathcal{L}_4^i interaction.

The dimension-four operators not involving fermion fields have been discussed in Ref. 6. Many of them are generated at the one-loop level with logarithmically divergent coefficients, and they have direct physical consequences describing the sensitivity of various processes to the heavy-Higgs-boson sector. It would be useful to recompute these operators and effects in the presence of Yukawa couplings, especially large ones associated with new, heavy fermions.¹³ They would describe, for example, effects analogous to Eq. (4) in gauge-boson production amplitudes.

IV. TECHNICOLOR

If one envisages the low-energy theory discussed above as arising from strong dynamics, then one simple possibility is that the Goldstone bosons are in fact bound states of fermions. These fermions, known as technifermions, are assumed to interact via an asymptotically free gauge force, analogous to QCD, and known as technicolor.³ It is expected that at energies above the characteristic scale Λ_{TC} , the appropriate description is one of strongly interacting technifermions, while the low-energy spectrum is that of scalar bound states having the nonlinear interactions described previously.

If the technifermions U and D form a weak doublet and condense at energy Λ_{TC} with $\langle \bar{U}U \rangle = \langle \bar{D}D \rangle \approx \Lambda_{TC}^3$, then the global $SU(2)_L \times SU(2)_R$ symmetry is broken to the diagonal subgroup $SU(2)_{L+R}$ and the local electroweak symmetry $SU(2)_L \times U(1)_Y$ is broken to $U(1)_{EM}$. The technimeson condensates now play the role of the Goldstone bosons in the standard model.

For ordinary fermions to acquire mass in technicolor theories there must be introduced yet another interaction to feed the chiral-symmetry breaking in the technifermion sector to the ordinary fermion sector. This interaction must therefore link ordinary fermions and technifermions and give rise, at energies of order Λ_{TC} , to effective four-fermion interactions between the technifermions and the ordinary quarks and leptons. At higher energies, these nonrenormalizable interactions must of course evolve into something else with an acceptable high-energy behavior. Various ideas have been proposed for this very-high-energy dynamics, from that of extended technicolor³ (ETC), in which there are further gauge interactions between the technifermions and the ordinary fermions, to those of composite models,¹⁴ in which the usual fermions as well as the technifermions are taken to be composite.

The attitude toward fermion mass generation adopted here is that, whatever the underlying dynamics, the nonrenormalizable four-fermion interactions between the technifermions and the ordinary fermions serve as an effective description at energies of order Λ_{TC} . Since the primary concern of this paper is the breaking of isospin symmetry, we will consider only one isodoublet of technifermions $T=(U,D)$. The expressions we write can easily be generalized to include several generations of technifermion doublets and, of course, several generations of ordinary fermions. The simplest mass-generating interaction is then

$$\begin{aligned} \mathcal{L}_m = & \frac{1}{2} G \bar{f}_L [\bar{T}T + i\tau \cdot (\bar{T}i\gamma_5 \tau T)] f_R \\ & + \frac{1}{2} G' \bar{f}_L [\bar{T}T + i\tau \cdot (\bar{T}i\gamma_5 \tau T)] \tau_3 f_R + \text{H.c.} \end{aligned} \quad (22)$$

We naturally assume here that the technicolor interactions themselves respect the isospin symmetry; if that were not the case, there would be strong [$O(\alpha_{TC})$] isospin violation, in contradiction with observation.

A simple, one-loop estimate of the mass-generating graph (Fig. 6) gives

$$\Delta m = m_u - m_d \simeq \frac{G'}{8\pi^2} \Lambda_{TC}^3. \quad (23)$$

It can be helpful to view this mass generation in two steps. First the technicolor dynamics is integrated out to yield the effective Yukawa coupling \mathcal{L}_Y [Eq. (13)] along with the nonlinear Goldstone Lagrangian \mathcal{L}_{NL} of Eq. (11). The vacuum value of $U=1$ then leads to Eq. (23) above, with the identification $y_u - y_d = 2y_3 = G' \Lambda_{TC}^2 / 8\pi^2$ and $f \simeq \Lambda_{TC}$.

In addition to \mathcal{L}_m there may be other four-fermion interactions (fermion is being used generically in this section to denote both ordinary fermions and technifermions) with typical coupling strengths of the same order as G and G' . From these we will identify new sources of isospin-symmetry breaking that contribute to low-energy physics.

To begin, notice that by iterating the interaction in \mathcal{L}_m , as shown in Fig. 7, new four-fermion interactions are induced. If the loop integrations are not cut off at energies below the unitarity bound, $\approx G^{-1}, G'^{-1}$, then these new interactions will naturally be of the same strength as \mathcal{L}_m . It might be, of course, that the integrations are damped below the unitarity bound. In ETC models, for example, \mathcal{L}_m could be the result of an ETC-boson exchange with a small dimensionless coupling constant. The higher-order interactions will then be small. It is natural in such models, however, that the tree-level exchange of an ETC boson contributes to these additional four-fermion interactions. Unless there is some suppression, this contribution will be of comparable strength to \mathcal{L}_m .

The following four-fermion interactions may be generated:

$$\mathcal{L}_{ff}^1 = G_1 (\bar{f}_L \gamma^\mu f_L) (\bar{f}_L \gamma_\mu f_L), \quad (24a)$$

$$\mathcal{L}_{ff}^2 = (\bar{f}_L \gamma^\mu f_L) [\bar{f}_R \gamma_\mu (G_2 + G'_2 \tau_3) f_R], \quad (24b)$$

$$\mathcal{L}_{ff}^3 = (\bar{f}_R \gamma^\mu f_R) [\bar{f}_R \gamma_\mu (G_3 + G'_3 \tau_3) f_R], \quad (24c)$$

$$\mathcal{L}_{ff}^4 = G_4 (\bar{f}_R \gamma^\mu \tau_3 f_R) (\bar{f}_R \gamma_\mu \tau_3 f_R), \quad (24d)$$

$$\mathcal{L}_{fT}^1 = G_5 (\bar{f}_L \gamma^\mu T_L) (\bar{T}_L \gamma_\mu f_L), \quad (24e)$$

$$\mathcal{L}_{fT}^2 = (\bar{f}_L \gamma^\mu T_L) [\bar{T}_R \gamma_\mu (G_6 + G'_6 \tau_3) f_R] + \text{H.c.}, \quad (24f)$$

$$\mathcal{L}_{fT}^3 = (\bar{f}_R \gamma^\mu T_R) [\bar{T}_R \gamma_\mu (G_7 + G'_7 \tau_3) f_R] + \text{H.c.}, \quad (24g)$$

$$\mathcal{L}_{fT}^4 = G_8 (\bar{f}_R \gamma^\mu \tau_3 T_R) (\bar{T}_R \gamma_\mu \tau_3 f_R), \quad (24h)$$

$$\mathcal{L}_{fT}^5 = (\bar{f}_L \gamma^\mu f_L) [\bar{T}_R \gamma_\mu (G_9 + G'_9 \tau_3) T_R], \quad (24i)$$

$$\mathcal{L}_{fT}^6 = [\bar{f}_R \gamma_\mu (G_{10} + G'_{10} \tau_3) f_R] (\bar{T}_L \gamma^\mu T_L), \quad (24j)$$

$$\mathcal{L}_{TT}^1 = G_{11} (\bar{T}_L \gamma^\mu T_L) (\bar{T}_L \gamma_\mu T_L), \quad (24k)$$

$$\mathcal{L}_{TT}^2 = (\bar{T}_L \gamma^\mu T_L) [\bar{T}_R \gamma_\mu (G_{12} + G'_{12} \tau_3) T_R], \quad (24l)$$

$$\mathcal{L}_{TT}^3 = (\bar{T}_R \gamma^\mu T_R) [\bar{T}_R \gamma_\mu (G_{13} + G'_{13} \tau_3) T_R], \quad (24m)$$

$$\mathcal{L}_{TT}^4 = G_{14} (\bar{T}_R \gamma^\mu \tau_3 T_R) (\bar{T}_R \gamma_\mu \tau_3 T_R). \quad (24n)$$

Additional interactions having a Lorentz-tensor structure are typically not generated in technicolor models and for simplicity have not been included. Furthermore, for each of the above isoscalar current interactions involving purely left-handed or purely right-handed fields, there could be a corresponding isovector current term. In the following, such interactions lead to results similar to those of the isoscalar case and are not explicitly examined.

The interactions \mathcal{L}_{ff}^i describe direct couplings among the ordinary fermions. They are unusual interactions, including terms with both right-handed fermions and isospin-symmetry violation, but if the coupling constants G_1, G_2, G_3, G_4 are on the order of or smaller than G and G' there will probably be no conflict with experiment except in the neutral sector where, once multiple families are included, there can be flavor-changing neutral currents.⁵ In order for technicolor theories to be viable, a Glashow-Iliopoulos-Maiani (GIM) mechanism¹⁶ must be operative in these interactions. Whether the fundamental origin of these interactions is ETC exchange, composite structure, or something else, it is not yet clear if a GIM mechanism can be incorporated. For the purposes of this paper we will assume that this problem can be solved and disregard it, in effect by disregarding family labels.

At the one-loop level it is possible to use an interaction in \mathcal{L}_{ff}^i together with a standard gauge interaction and integrate over the ordinary fermion loop. This will, first of all, renormalize the basic gauge interactions [Eqs. (18) and (19)]. After spontaneous symmetry breaking the operators in \mathcal{L}_4^i will also be generated. In order to obtain a contribution to \mathcal{L}_4^i from \mathcal{L}_{ff}^i , additional interactions coupling the fermions to the technifermions are required. This is most easily seen by considering the fermion-fermion-Goldstone-boson vertex for which an \mathcal{L}_{fT}^i must

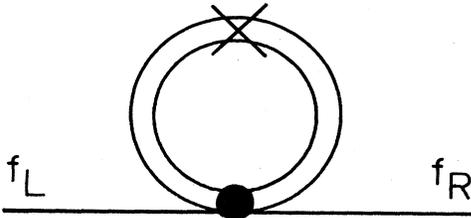


FIG. 6. Mass-generating diagram for ordinary fermions (single lines) using the four-fermion interaction \mathcal{L}_m . The technifermion (double line) condensate is marked with a cross.

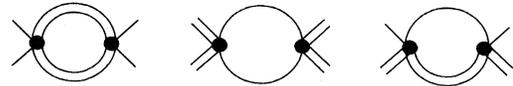


FIG. 7. Four-fermion interactions induced by iterations of \mathcal{L}_m .

also be used. If the G_i are of the same order as G and G' in \mathcal{L}_m then the \mathcal{L}_4 operators will be generated with strength suppressed by two powers of a Yukawa coupling.

We turn next to the operators \mathcal{L}_{fT}^i . Upon Fierz rearrangement, they produce \mathcal{L}_m [Eq. (22)] along with other interactions. Since these couplings involve technifermions, they contribute to ordinary-particle physics only when the technifermions are integrated out. An example of this, of course, is the mass formula [Eq. (23)]. There are, however, many other possibilities. The basic gauge interactions [Eqs. (18) and (19)] will be renormalized by a technifermion loop (see Fig. 8), and many of the dimension-four operators \mathcal{L}_4^i [Eqs. (20)] will be similarly generated. Mass insertions on the technifermion lines will cut off the loop integrals at momentum $k \approx \Lambda_{TC}$.

The low-energy \mathcal{L}_4^i operators were listed and their dependence on the Yukawa couplings was estimated, in Sec. III, using the nonlinear σ model. The result was the quadratic dependence on Yukawa couplings of Eqs. (21). Now, however, the technicolor dynamics determines a different strength for these couplings—it leads to a qualitatively new result. Contributions of the sort shown in Fig. 8 can be estimated and clearly give a linear connection between the various δ_i 's in Eq. (20) and the constants G_5 – G_{12} . One finds $\delta_i \approx G\Lambda_{TC}^2/4\pi^2$ [for example, both \mathcal{L}_{fT}^1 and \mathcal{L}_{fT}^5 contribute to δ_1 giving $\delta_1 \approx (G_5 + G_9)\Lambda_{TC}^2/4\pi^2$]. If G_i are of order G and G' in \mathcal{L}_m then the corresponding δ 's are of the same order as the Yukawa couplings (y and y_3) of the appropriate fermion. For the known fermions the strength of these new operators is therefore small.

To summarize up to this point, the \mathcal{L}_{ff}^i interactions have a (mild) tree-level effect on the low-energy physics while the \mathcal{L}_{fT}^i interactions affect the low-energy physics only after integrating out the technifermions. Radiative contributions from the \mathcal{L}_{fT}^i interactions dominate over those of \mathcal{L}_{ff}^i interactions since the latter are suppressed by powers of ordinary Yukawa couplings.

The further integration over ordinary fermion loops will yield the operator \mathcal{L}_2 and consequently a contribution to the ρ parameter. Figure 9, for example, shows how \mathcal{L}_2 can be generated at the two loops from the \mathcal{L}_{fT}^i interactions. The operator \mathcal{L}_2 is also generated at this two-loop level from the \mathcal{L}_{TT}^i interactions, as shown in Fig. 10. The interactions which give the largest contribution to $\Delta\rho$ are \mathcal{L}_{fT}^4 and \mathcal{L}_{TT}^4 .

To estimate the contribution of \mathcal{L}_{fT}^4 and \mathcal{L}_{TT}^4 to $\Delta\rho$ imagine turning off the U(1) gauge coupling g_1 . In this limit $\theta_W = 0$ and ρ is M_W^2/M_Z^2 . Restoring g_1 will give higher-order gauge corrections to the dominant contributions.

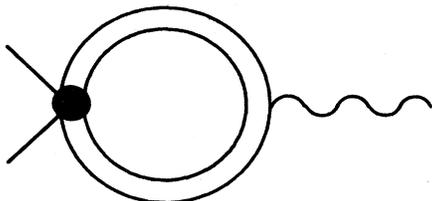


FIG. 8. An example of a technicolor contribution to \mathcal{L}_4^i .

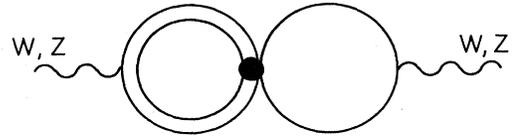


FIG. 9. Two-loop contribution to the gauge-boson self-energy diagram from interaction \mathcal{L}_{fT}^i .

\mathcal{L}_{fT}^4 contributes to the gauge-boson two-point function through the graph shown in Fig. 9. To conserve helicity (with $g_1 = 0$ there are only left-handed gauge interactions) there must be two mass insertions on the ordinary-fermion line. The two τ_3 matrices explicit in \mathcal{L}_{fT}^4 allow the generation of the structure \mathcal{L}_2 . The contribution to $\Delta\rho$ may be estimated by a two-step procedure. First, integrate out the technifermion fields—this gives an isospin-breaking effective coupling (\mathcal{L}_4^6) of the gauge fields to right-handed fermions with strength $\delta_6 \approx G_8\Lambda_{TC}^2/4\pi^2$. The final integration over the ordinary-fermion loop (with the mass insertions) will yield a $\Delta\rho$ of order $\delta_6 m^2/4\pi^2 f^2$. Since δ is expected to be similar in magnitude to $y = m/f$ this contribution to $\Delta\rho$ is smaller than the standard model result of Eq. (4).

Consider now \mathcal{L}_{TT}^4 . It is a product of two isospin-violating currents and directly yields \mathcal{L}_2 and consequently a contribution to $\Delta\rho$. For \mathcal{L}_{TT}^4 , as opposed to \mathcal{L}_{fT}^4 , there are only technifermions in the loops. Technifermions develop a dynamical mass at the scale Λ_{TC} where the technicolor interactions become strong. The necessary mass insertions on the technifermion lines thus cut off the loop integrals at momenta of order Λ_{TC} . An estimate analogous to that above gives

$$\Delta\rho \approx \frac{y_{TT}}{4\pi^2}, \tag{25}$$

where $y_{TT} = G_{14}\Lambda_{TC}^2/4\pi^2$. A color factor of 3 should be included for the techniquark contribution. Rewritten in terms of mass splittings this gives

$$\Delta\rho \approx \left[\frac{G_{14}}{G'} \right] \frac{\Delta m}{4\pi^2 f}. \tag{26}$$

For G_{14} of order G' this gives

$$\Delta\rho \approx \frac{1}{4\pi^2} \frac{\Delta m}{f}. \tag{27}$$

Note that this is linear, as compared to quadratic, in the mass splitting. Although the numerical factors in the expression Eq. (27) are only rough estimates, it is interesting to compare the size of $\Delta\rho$ with the result of Eq. (5b). The two results are comparable when Δm is of order f , giving a $\Delta\rho$ of a few percent. This is barely less than the current

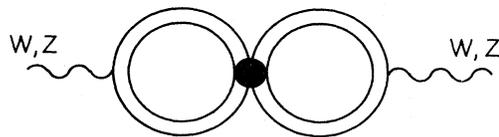


FIG. 10. Gauge-boson self-energy diagram from interaction \mathcal{L}_{TT}^i .

experimental bound. If this bound can be reduced, the linear expression [Eq. (27)] will provide the strongest constraint on the mass splitting Δm (Ref. 17).

V. CONCLUSION

The consequences of strong isospin breaking, inherent in the mass matrix of the known fermions, have been explored in strongly interacting Higgs theories and, in particular, in dynamical (technicolorlike) theories. The isospin breaking, both in the fermion mass matrix and in the U(1) gauge coupling, makes its most important impact on the gauge-boson mass matrix, shifting the ρ parameter from its tree-level value of unity. The one-loop effect of the U(1) coupling is an $O(\alpha/\pi)$ correction to $\Delta\rho \equiv \rho - 1$. In the limit of a heavy, strongly interacting Higgs-boson sector, this correction grows like $\ln(M_H)$. An analysis of corrections of higher order in the Higgs-boson self-coupling shows that, in this limit, the coefficient of the $O(\alpha/\pi)$ term is nonperturbative but probably of order unity, a value consistent with the experimental upper limit on $\Delta\rho$. This result is known as Veltman's screening theorem.

The one-loop correction to $\Delta\rho$ arising from the fermion mass matrix, that is, the Yukawa couplings in the standard model, had previously been computed by several authors. The result [Eq. (4)], that $\Delta\rho$ grows quadratically with the intradoublet mass splittings, was summarized in Sec. II. The computation at this level is completely insensitive to the Higgs-boson mass. However, higher-order

corrections can depend on M_H .

To study this dependence in the case of a strongly interacting Higgs sector and to set the stage for the analysis of technicolor theories, the nonlinear σ model coupled to fermions was next analyzed.

A generalized screening theorem was found. Amplitudes quadratically dependent on fermion mass splitting are never simultaneously quadratically dependent on the Higgs-boson mass. Such a quadratic sensitivity to the Higgs-boson mass is always screened by further powers of the Yukawa or gauge couplings. This is shown to be a consequence of the structure of isospin-symmetry breaking and the underlying renormalizability of the linear model. This means that the one-loop result [Eq. (4)] can be reliably used even in the presence of a strongly interacting Higgs sector.

Finally a specific dynamical model for electroweak-symmetry breaking was discussed. Here technicolor gauge interactions determine the strength of the amplitudes induced. New sources of isospin-symmetry breaking are found associated with four (techni)fermion interactions which are natural partners of the effective mass-generating interactions. For fermion mass splitting less than the electroweak-symmetry-breaking scale these are the most sensitive manifestations of isospin-symmetry breaking in the theory. The improved measurements of gauge-boson masses expected in the next decade should thus yield constraints on the mechanism of electroweak-symmetry breaking. These will be of paramount importance if a light Higgs boson is not directly observed.

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