Consequences of the improved Kobayashi-Maskawa matrix for mixing and CP nonconservation in K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ systems

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In light of the new experimental results on B decays the parameters of the Kobayashi-Maskawa (KM) matrix are reanalyzed. The correlated constraints on the t-quark mass, the K^0-$\bar{K}^0$ matrix element, and $\cos\delta$ are presented. All uncertainties and ambiguities are fully exhibited. The consequences of the new knowledge of the KM matrix for mixing and CP nonconservation in B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ systems are deduced.

I. INTRODUCTION

The recent experimental measurements[1] of B lifetime and the tightened bounds[2] on $\Gamma(b\to u)/\Gamma(b\to c)$ have improved our knowledge of the Kobayashi-Maskawa[3] (KM) matrix considerably. Initially one used unitarity constraints[4] to place limits on the new elements of the mixing matrix. Then the demand that parameters of the K^0-$\bar{K}^0$ system be reproduced correctly was used[5] to provide much stronger constraints. However, this approach has been plagued with uncertainty about the hadronic-matrix-element evaluation.[6] As more data on the production and decays of heavy quarks become available, more direct and reliable information on the KM matrix elements could be extracted.[7] This program is now almost complete with the new data on B decays.

In this paper we summarize the knowledge of the KM matrix and its implications in light of these new data. The current knowledge of the quark mixing matrix is summarized, in general and then for three generations, and the bounds on the KM angles presented in the next section.

In Sec. III we calculate the parameters of the K^0-$\bar{K}^0$ mass matrix with the new KM matrix. We show, following the recent suggestion, bounds on m_t (for a given δ) to reproduce the observed value of ϵ. The uncertainty due to the unknown value of the matrix element, as well as the dependence on the precise value of the quark mass used, is spelled out clearly. We exhibit the values of the matrix elements needed to reproduce ϵ and $\delta m_{K_L\text{-}K_S}$ correctly.

In Sec. IV mixing and CP violation in B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ systems are calculated allowing the unknown matrix element to vary over a wide range. The resulting same-sign-dilepton rate and asymmetry in e^+-e^- production are exhibited.

In brief, our findings can be summarized as follows. In the KM matrix, $0.01\le s_2\le 0.14$ and $s_3\le 0.06$ for any value of δ. To reproduce ϵ, allowing for the uncertainty in B_ϵ (the matrix-element factor), m_t as low as 25 GeV is allowed for $c_\delta<0$. To reproduce $\delta m_{L\text{-}S}$ correctly, the (corresponding matrix-element factor) B_m has to be about 1.5. For $m_t<50$ GeV, mixing in B^0-$\bar{B}^0$ is very small, $r<0.1$ at $c_\delta\sim -1$ to $r<10^{-4}$ at $c_\delta\sim +1$. The mixing in B_s^0-$\bar{B}_s^0$ can be large, e.g., for $m_t\sim 30$ GeV, r can be 0.6. The charge asymmetries in both B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ can be as large as 10^{-2} but the observable effect ($\sim ar$) remains near 10^{-5} (B^0) and 10^{-4} (B_s^0).

II. KOBAYASHI-MASKAWA PARAMETERS

The parameters on which new experimental information has become available recently are $|U_{us}|$, $|U_{cb}|$, and $|U_{ub}|$. The recent analysis[8] by the CERN WA2 collaboration of hyperon semileptonic decays yields a new value for $|U_{us}|$ from their two-angle (θ_V and θ_A) fit: $|U_{us}|=0.231\pm0.003$. The lifetime of B mesons has been measured in two recent experiments. From the results,

$$\tau_B=(1.2^{+0.45}_{-0.436}\pm0.3)\times10^{-12}\ \text{s}$$

and

$$\tau_B=(1.8\pm0.6\pm0.4)\times10^{-12}\ \text{s}\ ,$$

we take τ_B to lie in the range $0.6\le\tau_B\le1.8$ (in units of 10^{-12} s). Assuming a spectator model for B decays, and the phase-space and QCD factors as calculated, e.g., by Cortez, Pham, and Tounsi,[9] we find

$$0.04\le|U_{cb}|\le0.08\ . \tag{2.1}$$

These bounds include the uncertainty from the quark mass being taken to be current or constituent, although the experience[10] with semileptonic D decays as well as the phase-space requirement supports the constituent mass for the heavy quarks. In deriving this bound on $|U_{cb}|$ we already ignored $|U_{ub}|$ in comparison to $|U_{cb}|$. This is supported by the recent bound on $\Gamma(b\to u)/\Gamma(b\to c)\le0.05$. With the phase-space correction this yields

$$|U_{ub}|/|U_{cb}|<0.14\ . \tag{2.2}$$

With the bound on $|U_{cb}|$ this yields for $|U_{ub}|$,

$$|U_{ub}|<0.0115\ . \tag{2.3}$$

Inserting this new information our current knowledge of the KM matrix can be summarized by

$$U_{\rm KM}=\begin{bmatrix} |U_{ud}|=0.9734\pm0.024 & |U_{us}|=0.231\pm0.003 & 0\le|U_{ub}|\le0.0115 \\ 0.2\le|U_{cd}|\le0.24 & 0.8\le|U_{cs}|\le0.98 & 0.04\le|U_{cb}|\le0.08 \\ 0\le|U_{td}|\le0.13 & 0\le|U_{ts}|\le0.56 & 0.82\le|U_{tb}|\le0.99 \end{bmatrix}, \tag{2.4}$$

where we have used unitarity to bound the t couplings. These values are quite general and are valid even if the number of generations is larger than 3.

In the case of only three generations, the KM matrix can be parametrized,

$$U_{\rm KM}=\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{bmatrix}, \tag{2.5}$$

where $c_i=\cos\theta_i$, $s_i=\sin\theta_i$; θ_i are chosen to be between 0 and $\pi/2$, whereas $0\le\delta\le2\pi$. The knowledge of U_{ud}, U_{us}, and the fact that s_3 is very small fixes s_1 to be about 0.23.

Using the constraints on $|U_{bc}|$ and $\Gamma(b\to u)/\Gamma(b\to c)$ and assuming a value of c_δ, it is possible to further reduce the region of allowed values of s_2 and s_3. These regions are plotted[11] in Figs. 1, 2, and 3 for $c_\delta=0$, π, $\pi/2$, respectively. The point ($s_2=s_3=0.037$) in the $\delta=0$ plot is the prediction of the permutation symmetry model. For any δ, the allowed ranges are: $0.01\le s_2\le0.14$ and $0\le s_3\le0.06$.

III. THE K^0-$\bar{K}^0$ SYSTEM

The evaluation of the $K^0\to\bar{K}^0$ matrix element is plagued by the unknown dispersive (long-distance) contribution. It was pointed out by Ginsparg *et al.*[12] that for the *CP*-violating piece this contribution is perhaps negligible and the short-distance calculation may then be used to constrain parameters reliably. The only remaining uncertainty is then the parameter B_ϵ relating the actual matrix element to the vacuum-saturation value. They used $B_\epsilon=0.33$ as determined from relating K^0-$\bar{K}^0$ to $\Delta T=\frac{3}{2}$ $K^+\to\pi^+$ matrix element.[13] We follow their procedure to find minimum allowed m_t by requiring ϵ_K to be reproduced correctly. However, we do not believe B_ϵ is rigorously 0.33 (it may be off by as much as a factor of 2) and we allow B_ϵ to take on larger values. The b-decay widths in the spectator approximation (with $c_1\approx c_2\approx c_3\approx1$) are given by

$$\Gamma(b\to c)=\frac{1}{\tau_\mu}\left[\frac{m_b}{m_\mu}\right]^5|U_{bc}|^2\alpha\,, \tag{3.1}$$

$$\Gamma(b\to u)=\frac{1}{\tau_\mu}\left[\frac{m_b}{m_\mu}\right]^5|U_{bu}|^2\beta\,, \tag{3.2}$$

where α and β are dynamical three-body phase-space factors incorporating all allowed decay modes. For fixed values of $m_b=4.6$, $m_c=1.4$ GeV these can be inverted to give

$$s_3{}^2=3.6\times10^{-2}R_{bu}\left[\frac{10^{-12}}{\tau_B}\right], \tag{3.3}$$

$$|U_{bc}|^2=(s_2{}^2+s_3{}^2+2s_2s_3c_\delta)$$
$$=4.1\times10^{-3}R_{bc}\left[\frac{10^{-12}}{\tau_B}\right]. \tag{3.4}$$

The relationships (3) and (4) place strict upper limits on the value of s_3 and select out a value of s_2 once a value of c_δ is chosen.

We choose a slightly different parametrization of ϵ, namely,

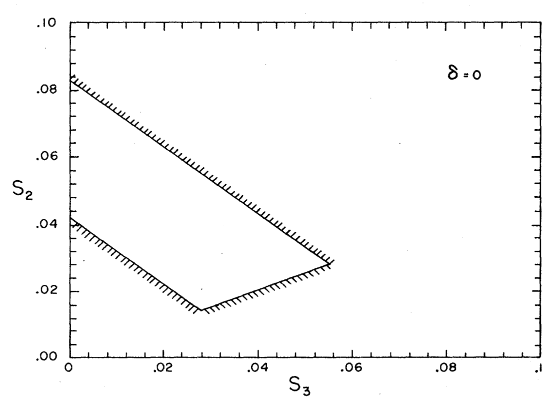

FIG. 1. The allowed region in the s_2-s_3 plane for $\delta=0$. The point corresponds to the prediction of Y. Yamanaka, H. Sugawara, and S. Pakvasa [Phys. Rev. D **25**, 1895 (1982); **29**, 2135(E) (1984)].

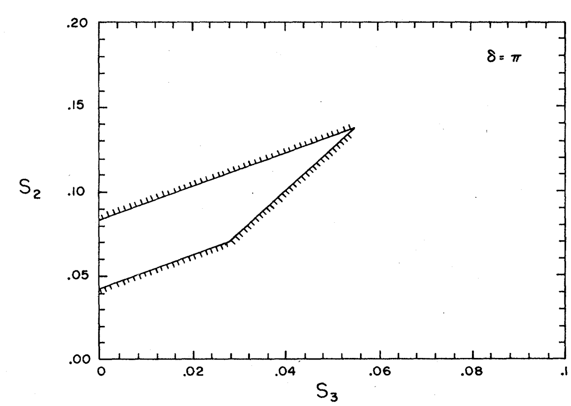

FIG. 2. The allowed region in the s_2-s_3 plane for $\delta=\pi$.

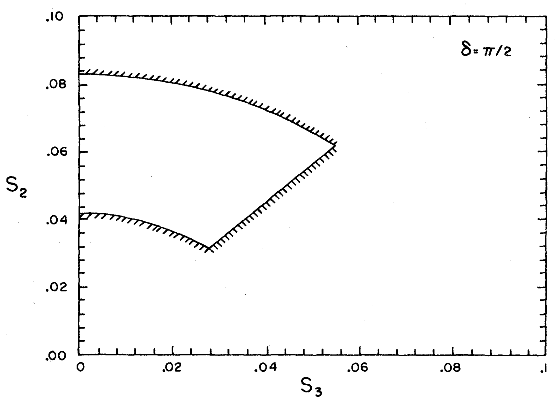

FIG. 3. The allowed region in the s_2-s_3 plane for $\delta=\pi/2$.

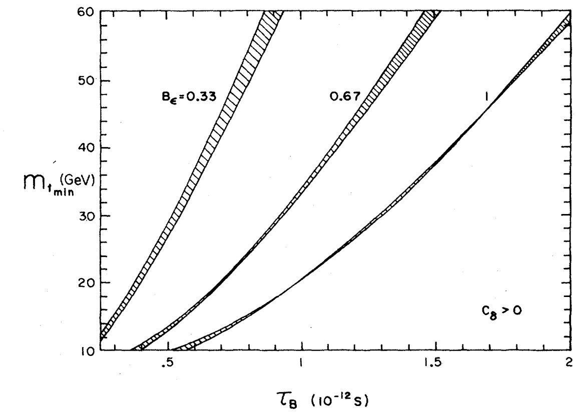

FIG. 4. Minimum value of m_t required to reproduce ϵ_K plotted against τ_B for several values of $B_\epsilon(\cos\delta>0)$. The band corresponds to the change as input quark masses are varied.

$$\epsilon=-\frac{B_\epsilon f_K{}^2 m_K G_F{}^2 m_c{}^2 s_1{}^2}{6\sqrt{2}\,\pi^2\delta m}c_2s_2s_3s_\delta\left[\eta_1(-c_1c_2{}^2c_3+s_2s_3c_2c_\delta)+\eta_2\left[\frac{m_t}{m_c}\right]^2(c_1s_2{}^2c_3+s_2s_3c_2c_\delta)\right.$$
$$\left.+\eta_3\ln\left[\frac{m_t}{m_c}\right]^2(c_1c_2{}^2c_3-c_1s_2{}^2c_3-2s_2c_2s_3c_\delta)\right]e^{i\pi/4}\,, \tag{3.5}$$

where B_ϵ represents the uncertainty in the relationship of the physical value of ϵ to the value predicted in the vacuum insertion approximation, $f_K=0.1722$ GeV is the K decay constant, and η_1, η_2, η_3 are the strong-interaction corrections. Using the experimental values $|\epsilon|=2.27\times10^{-3}$, $\delta m_{L\text{-}S}=m_{K_S}-m_{K_L}=-3.5\times10^{-15}$ GeV, and $s_1=0.229$, we examined the value of m_t as c_δ was varied from -1 to $+1$ and selected the minimum m_t allowed for each value of τ_B and B_ϵ.

In Figs. 4 and 5, we have plotted the results of this procedure for $c_\delta>0$ and $c_\delta<0$, respectively, for three values of B_ϵ. As can be seen in these diagrams, lower values of B_ϵ require significantly higher values of $m_{t\min}$ in all cases. The upper line of each pair represents a maximal choice for the quark masses ($m_b=5.2$, $m_c=1.86$), while the lower line is for the values $m_b=4.6$, $m_c=1.4$ GeV. We have used the value $m_w=80$ GeV in all calculations. Comparing the two figures, we see the choice of quark masses has little effect for $c_\delta>0$. However, for $c_\delta<0$ we find that the higher light-quark masses increase the minimum m_t.

All these considerations are critically dependent on the value of the parameter B_ϵ; this effect is illustrated graphically in Fig. 6 for an arbitrarily fixed value of $c_\delta=-0.4$. The larger values of B_ϵ lower the allowed m_t significantly. In Fig. 7, for each value of m_t and τ_β we vary c_δ over the full range $[-1,+1]$ and thus find the minimum value which B_ϵ can assume and still satisfy Eq. (5) for ϵ.

The expression for δm from the box diagram is

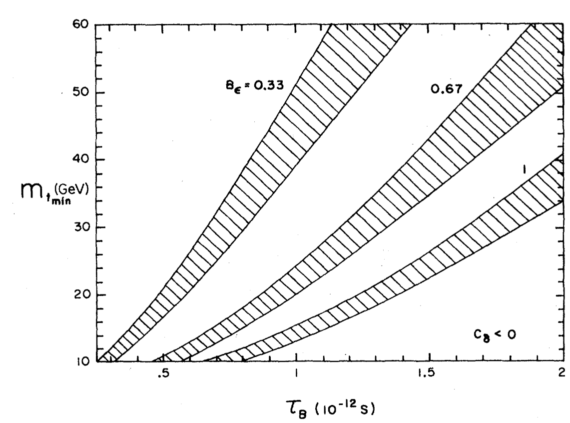

FIG. 5. Minimum value of m_t required to reproduce ϵ_K plotted against τ_B for several values of B_ϵ ($\cos\delta<0$). The upper edges of the bands correspond to $m_b\sim5.2$ GeV and the lower edges to $m_b\sim4.6$ GeV, respectively.

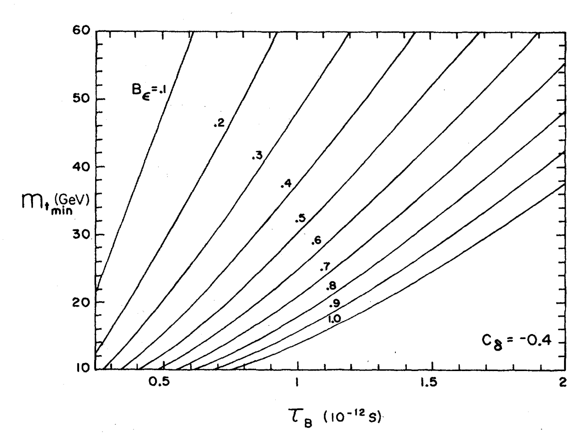

FIG. 6. The variation of $m_{t\min}$ with B_ϵ is shown for a fixed $\cos\delta=-0.4$.

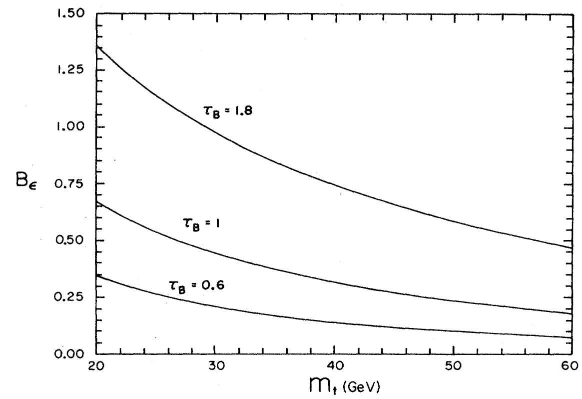

FIG. 7. The value of B_ϵ needed to reproduce ϵ_K is plotted against m_t for several values of τ_B.

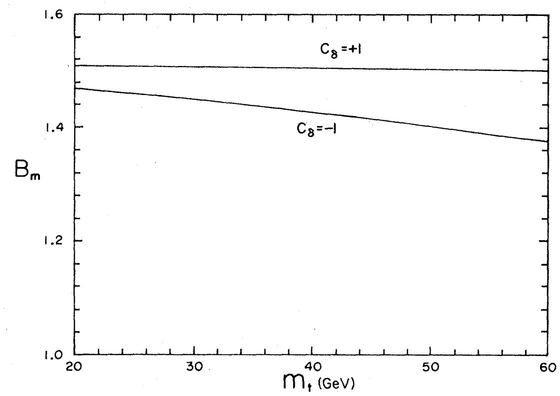

FIG. 8. The value of B_m needed to reproduce $\delta m_{K_L\text{-}K_S}$ is plotted against m_t. The band shows the variation as $\cos\delta$ is changed from $+1$ to -1.

$$\delta m = -\frac{B_m f_K{}^2 m_K G_F{}^2 m_c{}^2}{6\pi^2}\Bigg[\eta_1 s_1{}^2 c_2{}^2(c_1{}^2c_2{}^2c_3{}^2+s_2{}^2s_3{}^2c_{2\delta}-2c_1c_2c_3s_2s_3c_\delta) + \eta_2\left[\frac{m_t}{m_c}\right]^2 s_1{}^2 s_2{}^2(c_1{}^2s_2{}^2c_3{}^2+c_2{}^2s_3{}^2c_{2\delta}+2c_1s_2c_3c_2s_3c_\delta) + 2\eta_3 \ln\left[\frac{m_t}{m_c}\right]^2 s_1{}^2c_2s_2(c_1{}^2c_2c_3{}^2s_2-s_2{}^2s_3c_1c_3c_\delta+c_1c_2{}^2c_3s_3c_\delta-s_2s_3{}^2c_2c_{2\delta})\Bigg], \quad (3.6)$$

where for reasonable values of m_t, the dominant contribution is expected to be that of the charmed quark (the η_1 term). In order to estimate B_m, we fix τ_B and then examine the variation of B_m as we change c_δ from -1 to $+1$. The results of this procedure are illustrated in Fig. 8 for $\tau_B = 1\times10^{-12}$ s. The c-quark dominance tightly constrains B_m for all values of m_t. We can say with some confidence that $B_m \approx 1.5$ which gives some measure of the dispersive long-range contribution to δm. Assuming that ϵ has negligible dispersive contribution and parametrizing the dispersive contribution to δm in the manner of Wolfenstein[6] we have

$$\delta m_{\text{expt}} = B_m \delta m_{\text{box}} = B_\epsilon \delta m_{\text{box}} + D\,\delta m_{\text{expt}}\,; \quad (3.7)$$

hence

$$B_m = \frac{B_\epsilon}{1-D}\,, \quad (3.8)$$

and from the fact that $B_m \approx 1.5$, $D \approx 0.5$ to 0.8 for $B_\epsilon \approx \frac{2}{3}$ to $\frac{1}{3}$ (if $D>0$).[14]

IV. MIXING AND CP VIOLATION IN B^0-$\bar{B}^0$ AND B_s^0-$\bar{B}_s^0$ SYSTEMS

The experimental signal for mixing in the B^0-$\bar{B}^0$ (or B_s^0-$\bar{B}_s^0$) system is same-sign dileptons (ee, $\mu\mu$, or $e\mu$) in, e.g., $e^+ + e^- \to B^0\bar{B}^0 \to llX$. The size of the signal as parametrized by $r = (N^{--}+N^{++})/N^{+-}$, where N^{+-}, N^{++}, and N^{--} are the number of events with leptonic charges $(+\,-)$, $(+\,+)$, and $(-\,-)$, respectively. Possible CP violation in mixing is measured by the charge asymmetry

$$a = (N^{--}-N^{++})/(N^{--}+N^{++})\,.$$

The parameters r and a are given by

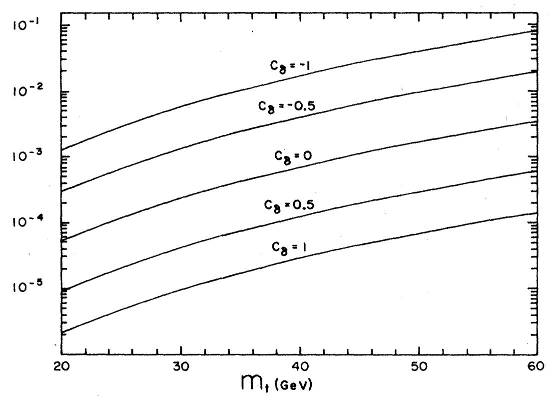

FIG. 9. r_B as a function of m_t for several values of cosδ. $f_B{}^2B_B = 0.01$ GeV^2.

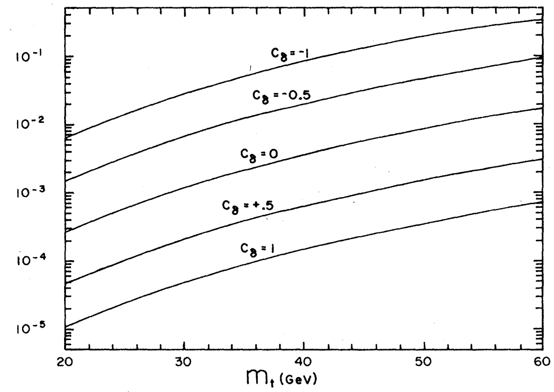

FIG. 10. r_B as a function of m_t for several values of cosδ. $f_B{}^2B_B = 0.025$ GeV^2.

$$r = \frac{2\Delta}{1+\Delta^2}$$

and (4.1)

$$a = \frac{4\,\mathrm{Re}\epsilon_B(1+|\epsilon_B|^2)}{(1+|\epsilon_B|^2)^2+4(\mathrm{Re}\epsilon_B)^2} .$$

In Eq. (4.1) Δ and ϵ_B in turn are

$$\Delta = \frac{(\delta m/\Gamma)^2+(\delta\Gamma/\Gamma)^2/4}{2+(\delta m/\Gamma)^2-(\delta\Gamma/\Gamma)^2/4}$$

and (4.2)

$$\epsilon_B = \frac{-\mathrm{Re}M_{12}+\frac{1}{2}i\,\mathrm{Re}\Gamma_{12}+(M_{12}-\frac{1}{2}i\Gamma_{12})(M_{12}^*-\frac{1}{2}i\Gamma_{12}^*)}{i\,\mathrm{Im}M_{12}+\frac{1}{2}\mathrm{Im}\Gamma_{12}} .$$

For comparison, we recall that for the K_L^0-K_S^0 system $r \cong 1$ and $a \approx 4\times10^{-3}$.

These quantities for the B^0-$\bar{B}^0$ (and B_s^0-$\bar{B}_s^0$) system have been calculated before.[15] Here we calculate them once more with the KM parameters as determined from τ_B as well as constrained to reproduce ϵ_K. We also allow f_B to vary over a wide range of values.

To a good approximation Γ_{12} and m_{12} are given by the absorptive and dispersive parts of the box diagram which for $m_t < 60$ GeV yields

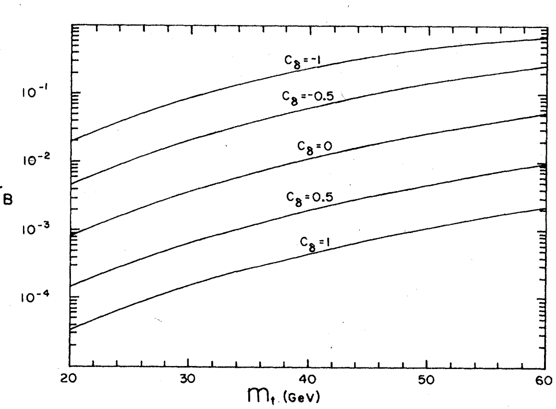

FIG 11. r_B as a function of m_t for several values of cosδ. $f_B{}^2B_B = 0.04$ GeV^2.

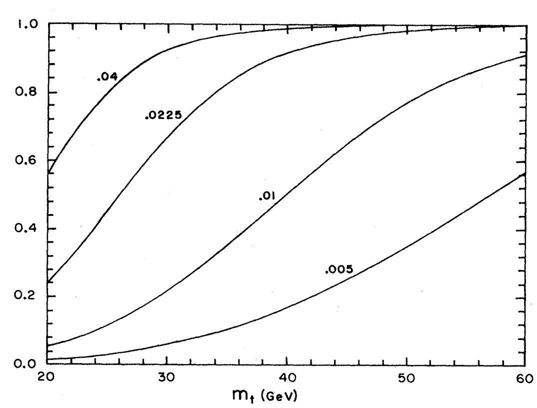

FIG. 12. r_{B_s} as a function of m_t for several values of $f_B{}^2B_B$ in GeV^2.

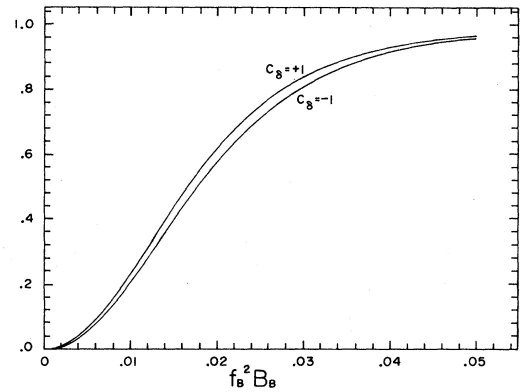

FIG. 13. r_{B_s} as a function of $f_B{}^2B_B$ in GeV^2 as $\cos\delta$ is varied from $+1$ to -1.

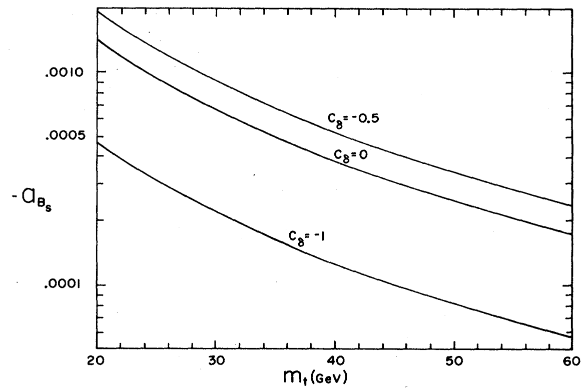

FIG. 15. a_{B_s} as a function of m_t for several values of $\cos\delta$.

$$\Gamma_{12}=-\frac{G_F{}^2f_B{}^2B_Bm_B}{8\pi}\left[\tfrac{1}{3}(2f_+{}^2+f_-{}^2)(U_{td}U_{tb}^*)^2m_B{}^2+\tfrac{8}{3}(\tfrac{3}{2}f_+{}^2-f_+f_-+\tfrac{1}{2}f_-)^2(U_{td}U_{tb}^*U_{cd}U_{cb}^*)^2m_c{}^2\right],$$

$$m_{12}=\frac{\eta G_F{}^2f_B{}^2B_Bm_B}{12\pi^2}\left[(U_{td}U_{tb}^*)^2\left\{m_t{}^2+\tfrac{1}{3}m_b{}^2+\tfrac{3}{4}m_b{}^2\ln\frac{m_t{}^2}{m_b{}^2}\right\}\right], \tag{4.3}$$

δm and $\delta\Gamma$ are given by $\delta m=2\,\mathrm{Re}\,m_{12}$, and $\delta\Gamma=2\,\mathrm{Re}\Gamma_{12}$. Leading-logarithmic QCD corrections are incorporated into deviation from 1 of $(\frac{2}{3}f_++\frac{1}{3}f_-)$, (f_+-f_-), and η. We have taken $2f_+{}^2+f_-{}^2$ to be 3.23, $(2f_+-f_-)^2=0.18$, and η, which depends weakly on m_t, is around 0.8. These values correspond to a $\Lambda_{\overline{\mathrm{MS}}}$ ($\overline{\mathrm{MS}}$ denotes modified minimal-subtraction scheme) of about 100 MeV.

We find that $\mathrm{Im}\epsilon_B$ is large (near 1) over a large range of parameters. Of course in the asymmetry a in B^0-$\bar{B}^0$ mixing this only has the effect of making the effect slightly smaller (by a factor of 2). The only possible way to exploit a large $\mathrm{Im}\epsilon_B$ is through observing *CP* violation in exclusive decay modes.[16]

In Fig. 9, we show the mixing r_B as a function of m_t as $\cos\delta$ is varied from $+1$ to -1 corresponding to $f_B{}^2B_B=0.01\ \mathrm{GeV}^2$. In Figs. 10 and 11 r_B is given for $f_B{}^2B_B=0.025$ and $0.04\ \mathrm{GeV}^2$, respectively. In Fig. 12, we show r_{B_s} as a function of m_t as $f_B{}^2B_B$ is varied from 0.04 GeV^2 to 0.005 GeV^2. Figure 13 shows that r_{B_s} has very little dependence on $\cos\delta$ provided the KM angles are constrained by τ_B as well as ϵ_K. The charge asymmetry exhibits very little variation as $f_B{}^2B_B$ is varied. In Fig. 14 (15) a_B (a_{B_s}) is shown against m_t with the variation with $\cos\delta$ exhibited. Here $f_B{}^2B_B$ was taken to be 0.0225 GeV^2. Finally, in Figs. 16 and 17 the total charge asymmetry $l=ar=(N^{--}-N^{++})/N^{+-}$ is exhibited for B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$, respectively.[17]

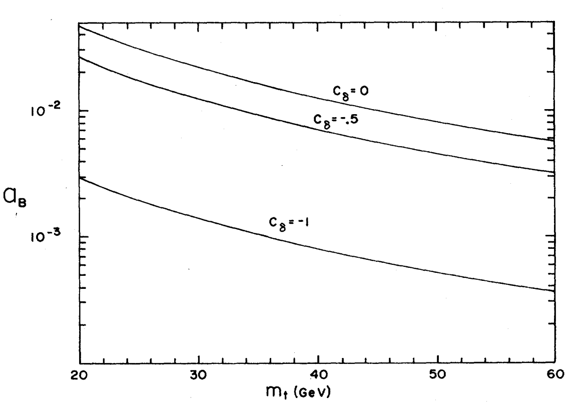

FIG. 14. a_B as a function of m_t for several values of $\cos\delta$.

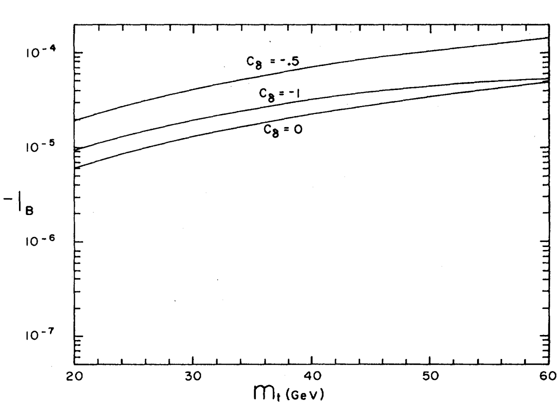

FIG. 16. l_B as a function of m_t for several values of $\cos\delta$.

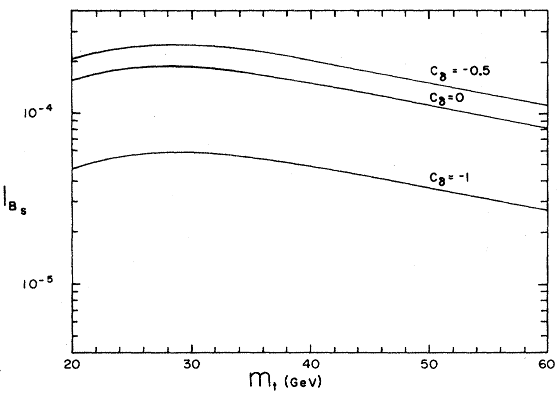

FIG. 17. l_{B_s} as a function of m_t for several values of cosδ.

V. CONCLUSION

To repeat, our main results are as follows. The new results on τ_B and $\Gamma(b\to u)/\Gamma(b\to c)$ restrict the KM angles considerably. In particular, s_2 and s_3 must satisfy $0.01 \le s_2 \le 0.14$ and $0 \le s_3 \le 0.06$.

With these small angles, the t quark (for a reasonable mass) contributes rather little to $\delta m_{K_L - K_S}$ and the c quark (in the manner of Gaillard-Lee) dominates. Then the effective value of the matrix element (including corrections due to long-distance contributions) relative to the vacuum situation has to be about $B_m \cong 1.5$. The similar factor B_ϵ for the CP-violating matrix element (i.e., ϵ_K) is in general different. For a given value of B_ϵ, a lower bound on m_t can be deduced. We find that for B_ϵ between $\frac{1}{3}$ and 1 m_t can be as low as 25 GeV. The lower values of m_t favor $\cos\delta < 0$. We estimate mixing and CP violation in B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ systems allowing large latitude in the unknown matrix elements. We find generally rather small mixing (r) in the B^0-$\bar{B}^0$ system but fairly large mixing in the B_s^0-$\bar{B}_s^0$ system. CP-violating asymmetries (a) are found to be rather small and the "observable" effect (ar) to be smaller than 10^{-4}. Our results for B_m and mixing (r) in B^0 and B_s^0 systems are valid even if CP violation is not described by the phase in the KM matrix and are to that extent more reliable.

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