Static properties of the nucleon octet in a relativistic potential model with center-of-mass correction

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The static properties, such as magnetic moment, charge radius, and axial-vector coupling constants, of the quark core of baryons in the nucleon octet have been studied in an independent-quark model based on the Dirac equation with equally mixed scalar-vector potential in harmonic form in the current quark mass limit. The results obtained with the corrections due to center-of-mass motion are in reasonable agreement with experimental values.

I. INTRODUCTION

In the study of the static properties of baryons much progress has been made by virtue of the dynamical theory of quarks; nevertheless, the level of accuracy is far from the experimental standards. As the quarks inside ordinary hadrons are believed to be light, a relativistic description seems to be indispensable. One such relativistic quark model describing the static properties of baryons is the MIT bag model.[1] The original bag-model calculations, however, give a proton magnetic moment $\mu_p \simeq 1.9\mu_N$ (μ_N=nuclear magneton), which is about 30% smaller than the experimental value. There have been many attempts to improve the predictions further. Donoghue and Johnson,[2] with recoil corrections, have improved the proton magnetic moment to $\mu_p \simeq 2.24\mu_N$. The pion-loop corrections in the cloudy-bag-model calculations[3] have yielded a still better result, $\mu_p \simeq 2.6\mu_N$. However, the virtue of these models is almost entirely due to the spherical-cavity approximation. Therefore, it is worthwhile to study the baryons with an alternative approach based on the independent-quark Dirac equation with some average confining potential replacing the spherical-bag boundary. Such schemes with equally mixed scalar and vector parts of the potential in harmonic,[4] linear,[4] and non-Coulombic power-law[5] form have been followed by many authors in the recent past. The average potential in the form of an equal admixture of scalar and vector parts not only simplifies calculations by converting the single-quark Dirac equation into an effective Schrödinger equation, but also produces no spin-orbit splitting as required by the experimental baryon spectrum. However, in a Lagrangian formulation of this scheme, the term corresponding to the scalar part of the potential in the Lagrangian density breaks chiral invariance, which can only be accounted for by an additional pionic component. Therefore, keeping in view the need to include effects of the pionic cloud supposed to surround the assembly of quarks in the baryon and also to incorporate the spurious-center-of-mass-motion corrections at appropriate stages, we prefer here to work with a harmonic form of the scalar-vector mixed potential for its tractability in these respects. For the moment we leave aside the pion cloud and study the contributions of the quark core alone to the static properties of a baryon, taking into account appropriate center-of-mass corrections.

Keeping the Lagrangian mass parameters of the quarks as the current quark masses within the limits of broken SU(3) ($m_u \simeq m_d \neq m_s$), we present in Sec. II a brief outline of the potential model and its solutions, leading to a complete description of the relativistic bound states of individually confined quarks of the baryon core. Then with the Dirac wave function for the ground state in hand, the core contribution to the static properties of the nucleon ($\frac{1}{2}^+$) octet in terms of the magnetic moments, charge radius, and axial-vector coupling constant g_A for β-decay processes are calculated in the usual manner. We also give an account of the prescription adopted here for the center-of-mass correction for the above quantities. In Sec. III, we estimate the potential parameter and the quark-mass parameters suitably in order to yield an appropriate ground-state energy for the nonstrange quarks which gives the average nucleon and $\Delta(1232)$ mass approximately, keeping in mind the corrections due to the center-of-mass motion and the pionic cloud. The static properties predicted after the center-of-mass corrections turn out to be in reasonable agreement with the experimental results.

II. THEORETICAL FRAMEWORK

In this section we briefly outline the framework of the model adopted here to study the core contributions to the static properties of the baryons and the prescriptions used to account for the corrections due to the center-of-mass motion.

A. Potential model

We start with the assumption that the quarks in a baryon core move independently in an average flavor-independent potential taken in the form

$$U(r)=\tfrac{1}{2}(1+\gamma^0)ar^2, \quad a>0 \tag{2.1}$$

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and obey the Dirac equation derivable from a Lagrangian density

$$\mathscr{L}_q=\frac{i}{2}\bar{q}(x)\gamma^\mu\overleftrightarrow{\partial}_\mu q(x)-\bar{q}(x)[U(r)+m_q]q(x) . \quad (2.2)$$

Hence the Lagrangian mass parameter m_q for the quarks must be regarded as the current quark mass. Then the independent quark wave functions $\psi_q(\mathbf{r})$ satisfy the equation

$$[\gamma^0 E_q-\boldsymbol{\gamma}\cdot\mathbf{p}-m_q-U(r)]\psi_q(\mathbf{r})=0 . \quad (2.3)$$

If we now assume that all the three quarks in the baryon core corresponding to the nucleon octet are in their ground state with $J^P=\frac{1}{2}^+$ and $J_z=\frac{1}{2}$, then a solution to the independent-quark (normalized) wave function $\psi_q(\mathbf{r})$ can be written in the two-component form as

$$\psi_q(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} ig_q(r)/r \\ \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}f_q(r)/r\end{bmatrix}\chi_\uparrow , \quad (2.4)$$

where the reduced radial parts $f_q(r)$ and $g_q(r)$ can be found to satisfy the equations

$$f_q(r)=\frac{1}{\lambda_q}\left[\frac{d}{dr}-\frac{1}{r}\right]g_q(r) ,$$
$$\frac{d^2 g_q(r)}{dr^2}+\lambda_q(E_q-m_q-ar^2)g_q(r)=0 , \quad (2.5)$$

where $\lambda_q=(E_q+m_q)$. Now, substituting a length scale $r_{0q}=(a\lambda_q)^{-1/4}$, the energy-eigenvalue condition for the ground state is given as

$$\lambda_q r_{0q}{}^2(E_q-m_q)=\left[\frac{\lambda_q}{a}\right]^{1/2}(E_q-m_q)=3 , \quad (2.6)$$

which yields the ground-state ($1S\frac{1}{2}$) individual quark-binding energy E_q. The reduced radial parts of the upper- and lower-component solutions obtainable from Eqs. (2.5) and (2.6) are

$$g_q(r)=N_q(r/r_{0q})\exp(-r^2/2r_{0q}{}^2) ,$$
$$f_q(r)=-\frac{N_q}{\lambda_q r_{0q}}(r/r_{0q})^2 e^{-r^2/2r_{0q}{}^2} , \quad (2.7)$$

where N_q is the overall normalization factor satisfying the relation

$$\frac{N_q{}^2\sqrt{\pi}r_{0q}}{8\lambda_q}=\frac{1}{(3E_q+m_q)} . \quad (2.8)$$

B. Static properties of the S-wave baryon core

We can now present some consequences of the model in terms of derived expressions for the quark-core contributions to certain measurable quantities of the S-wave baryons in the nucleon octet which are obtained simply by appropriately adding the contributions of each individual quark.

With the ground-state quark wave function $\psi_q(\mathbf{r})$ known from (2.4) and (2.7), the confined quark magnetic moment μ_q can be computed in the usual manner[5] to give

$$\mu_q=\frac{4M_p e_q}{(3E_q+m_q)}\mu_N . \quad (2.9)$$

Here M_p is the proton mass and e_q is the electric charge of the quark in the unit of proton charge. Then, with the usual assumption of quark additivity, the quark-core contribution to the octet-baryon magnetic moments can be obtained from the well-known relations[5,6]

$$\mu_p=\tfrac{1}{3}(4\mu_u-\mu_d),\ \ \mu_n=\tfrac{1}{3}(4\mu_d-\mu_u),\ \ \mu_\Lambda=\mu_s ,$$
$$\mu_{\Sigma^+}=\tfrac{1}{3}(4\mu_u-\mu_s),\ \ \mu_{\Sigma^-}=\tfrac{1}{3}(4\mu_d-\mu_s),\ \ \mu_{\Sigma^0}=\tfrac{1}{3}(2\mu_u+2\mu_d-\mu_s) , \quad (2.10)$$
$$\mu_{(\Sigma^0,\Lambda)}=\frac{1}{\sqrt{3}}(\mu_d-\mu_u),\ \ \mu_{\Xi^-}=\tfrac{1}{3}(4\mu_s-\mu_d),\ \ \mu_{\Xi^0}=\tfrac{1}{3}(4\mu_s-\mu_u) .$$

Here $\mu_{u,d,s}$ are the magnetic moments of the core quarks u, d, and s, respectively.

Now interpreting the weak β decays of the octet baryons $B\rightarrow B'+e^-+\bar{\nu}_e$ as quark β decays like $q_j\rightarrow u+e^-+\bar{\nu}_e$ occurring inside the baryon core (where q_j could be d or s quarks), one can obtain, in a similar manner as bag-model calculations,[7] the axial-vector coupling constant g_A as

$$g_A(B)=g_A^{\mathrm{SU}(6)}(B)\int_0^\infty dr[g_u(r)g_j(r)-\tfrac{1}{3}f_u(r)f_j(r)]$$
$$=g_A^{\mathrm{SU}(6)}(B)I_{uj} . \quad (2.11)$$

Here $g_A^{\mathrm{SU}(6)}$ stands for the matrix element $\langle B'\uparrow|\sigma_{uj}^{(3)}\tau/2|B\uparrow\rangle$. The integral expression in (2.11) can be obtained as

$$I_{uj}=\left[\frac{\pi}{2}\right]^{1/2}\frac{N_u N_j r_{0u}{}^2 r_{0j}{}^2}{(r_{0u}{}^2+r_{0j}{}^2)^{3/2}}\left[1-\frac{(r_{0j}/r_{0u})^4}{\lambda_u{}^2(r_{0u}{}^2+r_{0j}{}^2)}\right] , \quad (2.12)$$

which reduces for neutron β decay to

$$I_{uu}=\frac{N_u{}^2\sqrt{\pi}r_{0u}}{4}\left[1-\frac{1}{2\lambda_u{}^2 r_{0u}{}^2}\right] . \quad (2.13)$$

Then, with $g_A^{\mathrm{SU}(6)}=\frac{5}{3}$, the axial-vector coupling constant for neutron β decay is obtained as

$$g_A(n)=\frac{5}{9}\frac{(5E_u+7m_u)}{(3E_u+m_u)} . \quad (2.14)$$

Similarly $g_A(B)$ values corresponding to all the members of the nucleon octet can be obtained with appropriate values of $g_A^{SU(6)}$ and I_{uj}. Finally, the mean-square charge radius of the quark core of the proton can be obtained from the expression

$$\langle r^2\rangle_p = \langle p\uparrow | \sum_q e_q \int d^3\mathbf{r}\, r^2 \psi_q^\dagger(\mathbf{r})\psi_q(\mathbf{r}) | p\uparrow\rangle = \sum_q e_q \langle r^2\rangle_q = \langle r^2\rangle_u , \quad (2.15)$$

where $\langle r^2\rangle_u$ is the individual quark contribution obtained as

$$\langle r^2\rangle_u = \frac{3}{2}\frac{(11E_u + m_u)}{(3E_u+m_u)(E_u{}^2 - m_u{}^2)} . \quad (2.16)$$

C. Center-of-mass correction

Clearly our shell-type relativistic independent-quark model is not translationally invariant. Therefore the independent motion of quarks inside the baryon core does not lead to a state of definite total momentum as it should to represent the physical state. The problem appears in the same way in nuclear physics in the case of ^{3}He and also in the bag model and therefore has to be resolved accordingly.[8,9] Particularly we adopt here the prescription followed by Wong[9] and other workers[10] for such purposes. We decompose the static three-quark baryon-core state with the core center at $\mathbf{x}$ into components $\varphi(\mathbf{p})$ of plane-wave momentum eigenstates

$$|3q,\mathbf{x}\rangle = \int \frac{d^3\mathbf{P}}{W(P)} e^{i\mathbf{P}\cdot\mathbf{x}} \varphi(\mathbf{P}) | B(\mathbf{P})\rangle , \quad (2.17)$$

where the momentum eigenstates $|B(\mathbf{P})\rangle$ of the baryon core B are normalized usually as

$$\langle B(\mathbf{P}') | B(\mathbf{P})\rangle = (2\pi)^3 \delta(\mathbf{P}-\mathbf{P}') W(P) , \quad (2.18)$$

with

$$W(P) = (M_B{}^2 + \mathbf{P}^2)^{1/2}/M_B .$$

The momentum-profile function $\varphi(\mathbf{P})$ can be obtained from (2.17) and (2.18) as

$$\varphi^2(\mathbf{P}) = \frac{W(P)}{(2\pi)^3}\tilde{I}(\mathbf{P}) , \quad (2.19)$$

where

$$\tilde{I}(\mathbf{P}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{r}\, e^{-i\mathbf{P}\cdot\mathbf{r}} \langle 3q,0 | 3q,\mathbf{r}\rangle \quad (2.20)$$

is the Fourier transform of the Hill-Wheeler overlap function.[9] This result along with (2.17) can be used to calculate expectation values of any $F(\mathbf{P})$ as

$$\langle 3q,0 | F(\mathbf{P}) | 3q,0\rangle = \int d^3\mathbf{p}\, \tilde{I}(\mathbf{P}) F(\mathbf{P}) . \quad (2.21)$$

Then it is a simple matter to evaluate various c.m. corrections, once the Hill-Wheeler function $\tilde{I}(\mathbf{P})$ is calculated which, with Hill-Wheeler overlap

$$I(\mathbf{r}) = [(1 - cr^2/r_{0q}{}^2)\exp(-r^2/4r_{0q}{}^2)]^3 \quad (2.22)$$

for three $1S_{1/2}$ quarks in this model, comes out as

$$\tilde{I}(P) = \left[\frac{r_{0q}}{3\pi}\right]^{3/2} \exp\left[-r_{0q}{}^2\frac{P^2}{3}\right] \sum_{n=0}^{3} c_n (P^2 r_{0q}{}^2)^n ,$$

where

$$c = \tfrac{1}{6}(E_q - m_q)/(3E_q + m_q) , \qquad c_0 = (1 - 6c + 20c^2 - \tfrac{280}{9}c^3) , \quad (2.23a)$$

$$c_1 = \left[\frac{4c}{3} - \frac{80c^2}{9} + \frac{560c^3}{27}\right] , \qquad c_2 = \frac{16}{27}\left[c^2 - \frac{14c^3}{3}\right] \text{ and } c_3 = (\tfrac{8}{27})^2 c^3 . \quad (2.23b)$$

This overlap function permits ready estimates of center-of-mass effects in terms of the expectation values $\langle M_B{}^2/E_B{}^2\rangle$, $\langle M_B/E_B\rangle$, $\langle \mathbf{P}^2\rangle$, and $\langle R^2\rangle$, where

$$R = \sum_q E_q r_q \Big/ \sum_q E_q ,$$

evaluated according to (2.21). In fact we get

$$\langle \mathbf{P}^2\rangle = \sum_q \langle \mathbf{p}^2\rangle_q \quad (2.24)$$

where $\langle \mathbf{p}^2\rangle_q$ is the average value of the square of the individual quark momentum taken over the $1S$ single-quark states and is obtained as

$$\langle \mathbf{p}^2\rangle_q = \frac{(11E_q + m_q)}{6(3E_q + m_q)}(E_q{}^2 - m_q{}^2) . \quad (2.25)$$

Then with (2.24) and (2.25), one easily gets

$$\langle M_B{}^2/E_B{}^2\rangle = \left[1 - \sum_q \langle \mathbf{p}^2\rangle_q / E_B{}^2\right] = \delta_B{}^2 . \quad (2.26)$$

Now taking $\langle M_B/E_B\rangle$ as roughly equal to $\delta_B = \langle M_B{}^2/E_B{}^2\rangle^{1/2}$ and $\langle R^2\rangle$ for the proton as $\frac{1}{3}\langle r_u{}^2\rangle$, one can compute the corrected[10] static properties in the following manner:

$$\mu_B' = \left[3\mu_B + \frac{QM_p}{M_B}(1-\delta_B)\right]\Big/(1+\delta_B+\delta_B{}^2) ,$$

$$g_A'(B) = 3g_A(B)/(1+2\delta_B) , \quad (2.27)$$

$$\langle r^2\rangle_p' = \langle r^2\rangle_p/(1+\tfrac{1}{2}\delta_B{}^2) .$$

Here Q is the total charge of the baryon, the unprimed quantities μ_B, g_A, and $\langle r^2\rangle_p$ are the uncorrected expressions given through Eqs. (2.10), (2.11), and (2.15), respectively, and the primed quantities are the corresponding corrected ones.

III. RESULTS AND CONCLUSION

The static quantities to be calculated are the rms charge radius of the proton, axial-vector coupling constants, and the magnetic moments. The expressions for these quantities, as derived in Sec. II, are found to depend on the Lagrangian mass parameters m_q and the single-quark energy eigenvalue E_q. Although the parameters a and m_q are

TABLE I. Magnetic moments of the nucleon octet with and without center-of-mass correction in comparison with experiment (all numbers are in nuclear magnetons).

Baryons	Magnetic Moment μ_B Uncorrected	Corrected	Experiment	Reference
p	2.302	2.6067	2.7928	11
n	-1.5346	-1.7113	-1.9130	11
Λ	-0.5581	-0.6165	-0.613 ± 0.004	11
Σ^+	2.2321	2.494	$2.33\ \pm0.13$	11
Σ^0	0.6975	0.7705	$0.46\ \pm0.28$	
Σ^-	-0.837	-0.9528	$-1.10\ \pm0.03$	13
			$-0.89\ \pm0.14$	14
Ξ^0	-1.2556	-1.3759	$-1.25\ \pm0.014$	11
Ξ^-	-0.4884	-0.5585	$-0.69\ \pm0.04$	11
(Σ^0,Λ)	-1.329	-1.482	$-1.82^{+0.18}_{-0.25}$	12

a priori unconstrained, we have to make a suitable choice by reasonable assumptions.

The Lagrangian mass parameters of the quarks are small and may well be chosen according to the prediction of current algebra, which would restrict m_q for the nonstrange quark not to exceed some 10 or 20 MeV ($m_d > m_u$). However, since no physical quantity considered here depends appreciably on these parameters as long as they are small, we keep them fixed as $m_u \simeq m_d = 10$ MeV. Then the potential parameter a has to be adjusted properly to yield the appropriate single-quark ground-state energy E_u. Normally $E_u = E_d$ is expected to be of the order of $\frac{1}{3}\overline{M}_N$, where

$$\overline{M}_N = \left[\frac{4M_N + 16M_\Delta}{20}\right] = 1173 \text{ MeV}$$

is the spin-isospin average mass of the nucleon and $\Delta(1232)$. However if one admits that spurious-center-of-mass-motion corrections, and other possible corrections such as pion-cloud effects not considered in this model, are to be accounted for at appropriate stages, then E_u must be somewhat larger than $\frac{1}{3}\overline{M}_N$. Since we are not particularly interested here in the detailed baryonic mass spectrum, we only choose a and hence E_u appropriately to obtain an order-of-magnitude prediction for the quark-core contributions to bare-nucleon properties before any possible corrections are applied. We find that with

$$a = 2.273 \text{ fm}^{-3},\quad m_u = m_d = 10 \text{ MeV}\ , \tag{3.1}$$

the energy-eigenvalue condition (2.6) yields $E_u = E_d = 540$ MeV, which results in the bare-nucleon properties as

$$\mu_p = 2.3\mu_N,\quad \langle r^2\rangle_p{}^{1/2} = 0.85 \text{ fm}\ ,\qquad g_A = 0.944\ . \tag{3.2}$$

Now with the assumed flavor independence of the potential, the parameter a obtained in (3.1), along with $m_s = 252$ MeV chosen well within the limits of current-algebra predictions, gives rise to a unique positive real root $E_s = 663$ MeV from Eq. (2.6). This yields a reasonable value of $\mu_\Lambda = -0.56\mu_N$.

Now with the parameters a and m_q along with the corresponding binding energy E_q being known, all the relevant quantities leading to the predictions of the static-core properties before c.m. correction can be calculated. First of all we obtain from (2.9) the bound-quark magnetic moments as $\mu_u = -2\mu_d = 1.5346\mu_N$ and $\mu_s = -0.558\mu_N$, which through expressions (2.10) provide the uncorrected magnetic moments of the quark core for the baryons in the nucleon octet. The magnetic moments calculated in this way are presented in Table I. Then, computing the expression I_{uj} in Eq. (2.12) as $I_{uu} = 0.5665$ and $I_{us} = 0.654$, we can obtain the axial-vector coupling constant g_A for the β-decay processes corresponding to all the members of the nucleon octet, with appropriate

TABLE II. Axial-vector coupling constant g_A in the β-decay processes of $\frac{1}{2}^+$ baryons.

Decay mode	$g_A = g_A^{SU(6)} I_{uj}$ Uncorrected	Corrected	Experimental data	Reference
$n \to pe^-\bar{\nu}$	$\frac{5}{3}I_{uu} = 0.944$	1.02	1.255 ± 0.06	11
$\Lambda \to ne^-\bar{\nu}$	$I_{us} = 0.6537$	0.702	0.690 ± 0.034	11
$\Sigma^- \to ne^-\bar{\nu}$	$-\frac{1}{3}I_{us} = -0.2179$	-0.234	$\pm0.385\pm0.06$	15
$\Xi^- \to \Lambda e^-\bar{\nu}$	$\frac{2}{9}I_{us} = 0.1453$	0.155	0.248 ± 0.05	11
$\Xi^- \to \Sigma^0 e^-\bar{\nu}$	$\frac{5}{3}I_{us} = 1.0895$	1.1624		
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}$	$\frac{5}{3}I_{us} = 1.0895$	1.1624		

values[10] of $g_A^{SU(6)}$. The results obtained here without c.m. corrections are presented in Table II. Finally, we may mention again that the rms charge radius of the proton comes out at this stage of the calculation to be 0.85 fm.

For introducing center-of-mass corrections, we estimate the factor δ_B from expressions (2.25) and (2.26), which certainly depend on the flavor combinations of the quark core in baryons. Accordingly we find $\delta_B = 0.893$, 0.902, and 0.91, respectively, for the baryon cores with three nonstrange quarks, two nonstrange quarks with one strange quark, and one nonstrange quark with two strange quarks. Then it is straightforward to obtain the corrected values of the magnetic moments, axial-vector coupling constants, and the proton rms charge radius from expressions (2.27). These quantities, so obtained after the center-of-mass corrections, are presented in the appropriate tables (I and II) in comparison with the corresponding uncorrected values as well as the experimental ones.

We observe that our results for the magnetic moments and axial constants compare reasonably well with the existing experimental data. However, the proton charge radius $\langle r^2 \rangle_p^{1/2}$, which was 0.85 fm before the center-of-mass correction, becomes 0.72 fm, as against the experimental value of 0.88 fm. Any departures of our calculated values in the (u,d) sector from the experimental data can hopefully be accounted for by the pionic contributions, which will be taken up in a subsequent work. But in the strange-quark sector we expect the pionic contribution to be less significant in view of its being heavier than the pion. In any case, our overall predictions for the static properties of the baryons in the nucleon octet show a significant improvement over the results obtained earlier in a similar model[4,5] without c.m. correction, which differs from ours in the choice of the potential form as well as the parameters. In view of the simplicity of the model, the results obtained are quite encouraging.

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