Valons in mesons

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(Received 23 August 1984)

We present a systematic study of the valon distribution functions and valon structure functions for pions and kaons. The implications of our results for low-p_T inclusive reactions are emphasized. The complications with the structure function of strange valons are discussed and suggestions are made for future experiments to extract it from the structure function of kaons at high Q^2.

I. INTRODUCTION

In many hadron-hadron collisions at high center-of-mass energies, most of the final-state hadrons are collimated within cylinders of fixed transverse momentum along the beam direction with an average transverse momentum of 0.2 GeV/c. The very short nature of the potential is responsible for the high-p_T reactions which do not contribute significantly to the total cross section. Consequently, soft hadronic reactions ($p_T \leq 1$ GeV/c) populate most of the final-state particles and most of the cross section. The relevance of the constituent picture of hadrons in the framework of QCD to soft hadronic reactions has been demonstrated through many experimental and theoretical investigations.[1] Since it is not yet possible to calculate the QCD predictions for these reactions, many of the theoretical studies are based on phenomenological models motivated by QCD.

Among several proposed models to explain the bulk of data available on soft hadron reactions, the so-called "valon-parton" model[2] has been successfully applied to many hadron-hadron and hadron-nucleus reactions.[3] Valons, as defined in this model, are conglomerations of valence quarks, sea quarks, and gluons. In a bound-state situation, valons saturate all of the momentum carried by a hadron, i.e., a proton contains three valons UUD, shown by capital letters to distinguish them from quarks, and a meson is a bound state of valon-antivalon, e.g., π^+ is $U\bar{D}$, etc.

In this study our aim is to study the properties of valon distribution and structure functions in strange and nonstrange mesons in the framework of the valon-proton model. Mesons present many attractive features for studying soft hadronic reactions because of their simple qq structure as opposed to the qqq structure of baryons. In the following, we first present some brief review of the essential formalism of the valon-parton model; then we describe our method for determination of valon distribution functions and structure functions in pions and kaons, respectively.

The static (bound-state) properties of valons are characterized by their wave functions in hadrons where they are considered as constituent quarks. On the other hand, they respond to a dynamical probe in a way specified by their structure function which represents the internal structure of cluster quarks.[2] The precise knowledge of both properties as mentioned above is very crucial in the determination of the subsequent behavior of quarks and produced hadrons in a scattering process. Of course, the ultimate solution should be presented by QCD to connect these perturbative and nonperturbative regions. Nevertheless, one may try to gain some understanding of the underlying mechanism just by resorting to phenomenological viewpoints in the context of the valon-parton model.

Let us consider the nucleon as our example. We denote by $G_{V/N}(y)$ the valon distribution function in the nucleon, where y is the fraction of momentum carried by valon V. Then, the average momentum y_V is

$$\bar{y}_V = \int_0^1 yG_{V/N}(y)dy$$

and (1)

$$\sum_V \bar{y}_V = 1 \ .$$

The nucleon structure function can be written as the convolution of the valon distribution and valon structure function,

$$F_N(x) = \sum_V \int_x^1 dy\, G_{V/N}(y)F_V(x/y) \ , \quad (2)$$

where $F_V(x)$ is the structure function of valon V. By this function we mean, when a valon undergoes a collision, it is no longer an indivisible pointlike object. The virtual cloud of partons are excited, however, the fast and slow partons behave differently. Now for this quark cluster, one can write

$$F_V(x) = \sum_q \int_x^1 dy\, G_{q/V}(y)F_q(x/y) \ , \quad (3)$$

where $G_{q/V}(z)$ is the distribution of quarks in the valon V, and $F_q(x)$ is the quark structure function. Assuming a pointlike structure for quarks (which is appropriate to the kind of reactions we are considering), then $F_q(x) = \delta(x-1)$ and Eqs. (2) and (3) give

$$F_V(x) = \sum_q xG_{q/V}(x) \ , \quad (4)$$

$$F_N(x) = \sum_{q/V} \int_x^1 dy\, G_{V/N}(y)x/yG_{q/V}(x/y) \ . \quad (5)$$

Denoting the quark distribution in the nucleon by $G_{q/N}(x)$, the sum in Eq. (5) separates into two parts: the valence quark distribution

$$xG_{v/N}(x)=\sum_V \int_x^1 dy\, G_{V/N}(y)x/yG_{v/V}(x/y)\ , \quad (6)$$

and sea quark distribution

$$xG_{s/N}(x)=\sum_V \int_x^1 dy\, G_{V/N}(y)x/yG_{S/V}(x/y)\ . \quad (7)$$

Therefore, the quark distributions have been expressed in terms of valon distributions in hadrons.

II. VALON DISTRIBUTIONS IN MESONS

The valon distributions in mesons follow a sum rule similar to Eq. (1) for nucleons,

$$\sum_V \int_0^1 yG_{V/\pi}(y)dy=1\ . \quad (8)$$

Furthermore, for pions, one expects a symmetric distribution, i.e.,

$$G_{V/\pi}(y)=G_{D/\pi}(y)\ , \quad (9)$$

while for kaons the SU(3) breaking has to be considered. Let us parametrize the double-valon distribution in the pions and kaons as

$$G_{V_1V_2/\pi}(y_1,y_2)=\alpha_\pi(y_1y_2)^\alpha\delta(y_1+y_2-1)\ , \quad (10)$$

$$G_{V_1V_2/K}(y_1,y_2)=\alpha_K(y_1)^\alpha(y_2)^\beta\delta(y_1+y_2-1)\ , \quad (11)$$

where the δ functions have been introduced for momentum conservation. Integrating Eqs. (10) and (11), the single-valon distribution in the pions and kaons becomes

$$G_{V/\pi}(y)=y^\alpha(1-y)^\alpha/B(\alpha+1,\alpha+1)\ , \quad (12)$$

$$G_{V/K}(y)=y^\alpha(1-y)^\beta/B(\alpha+1,\beta+1)\ , \quad (13)$$

$$G_{S/k}(y)=y^\beta(1-y)^\alpha/B(\alpha+1,\beta+1)\ , \quad (14)$$

where B is the Euler beta function. We first consider the case of pions.

Pions

The method employed in extracting valon distribution functions in nucleons is described in detail in Ref. 3. We are going to use those results plus the recently available comprehensive fits[4] to quark distributions in nucleons and pions at $Q^2\geq 4$ GeV^2, to calculate the distribution of valons in the pions. Actually, the connection is quite simple if one knows the structure function of nonstrange valons, say, U valons. We assume this function, $G_{u/U}(z)$, to be independent of its host particle, i.e., the internal structure of a particular valon in the proton or pion is the same (it probably differs between strange and nonstrange valons, and we will discuss this point later in the next section). Consequently, we proceed by its determination using the results of Refs. 3 and 4 for valon and quark distributions, respectively. According to Eq. (6) the connection between, say, d quark and D valon is

$$xd_v(x,Q^2=4)=\int_x^1 G_{D/N}(y)x/yG_{d/D}(x/y)dy\ . \quad (15)$$

The valence d-quark distribution is given by[4]

$$xd_v(x,Q^2=4)=2.8x^{0.763}(1-x)^4\ , \quad (16)$$

and the D-valon distribution is[3]

$$G_{D/N}(y)=17y^{0.88}(1-y)^3\ . \quad (17)$$

We use the following parametrization for $G_{d/D}(z)$, for which the motivation is discussed in detail in Ref. 3:

$$zG_{d/D}(z)=Az^ae^{bz}\ . \quad (18)$$

A is the normalization factor and is given by

$$A=\left[\int_0^1 z^{a-1}e^{bz}dz\right]^{-1}\ . \quad (19)$$

Substituting Eqs. (16)—(19) in Eq. (15), we find the values of a and b to be 0.8 and -0.4, respectively. The normalization factor A in (19) for the given values of a and b is $A\simeq 1$, consequently from Eq. (18) we obtain

$$zG_{d/D}(z)=z^{0.8}e^{-0.4z}\ . \quad (20)$$

Equation (20) is precisely what we need to determine the valon distribution function in the pions, i.e.,

$$xu_{v/\pi}(x,Q^2=4)=\int_x^1 dy\, G_{V/\pi}(y)x/yG_{u/V}(x/y)\ , \quad (21)$$

where $G_{u/V}(z)$ is given (according to our assumption) by

$$zG_{u/U}(z)=zG_{d/D}(z)=z^{0.8}e^{-0.4z}\ . \quad (22)$$

Furthermore, from Ref. 4, the valence u-quark distribution in the pion is

$$xu_{v/\pi}(x,Q^2=4)=0.52x^{0.4}(1-x)^{0.7}\ , \quad (23)$$

consequently, using Eqs. (22) and (23) along with Eq. (12) we obtain the distribution of valons in the pion to be

$$G_{U/\pi}(y)=G_{D/\pi}(y)=0.66y^{-0.22}(1-y)^{-0.22}\ . \quad (24)$$

For a large range of y, this equation is consistent with the simple result of Ref. 2, which is $G_{U/\pi}(y)=1$. It is interesting to note that if one uses the same line of arguments as in Ref. 2, the result of Eq. (24) will emerge approximately. The idea is that in order to obtain a $(1-x)^{0.7}$ behavior for the valence u quark near $x=1$, the power α in Eq. (12) must be -0.3, because $G_{u/U}(x)$ is finite near $x=1$. This will give

$$G_{U/\pi}\propto y^{-0.3}(1-y)^{-0.3}\ ,$$

which is similar to the result in Eq. (24). In Fig. 1, the valon distributions in the pion as given by Eq. (24) and Ref. 2 are plotted. The authors of Ref. 5 predicted similar results using the nonrelativistic quark model.

Kaons

For kaons the situation is somewhat different. The main differences are (i) SU(3) breaking among the constituents of kaons, and (ii) lack of knowledge with regard to quark distributions in kaons at different values of Q^2. However, the following technique can be employed to get the distribution of nonstrange valence quarks, from which

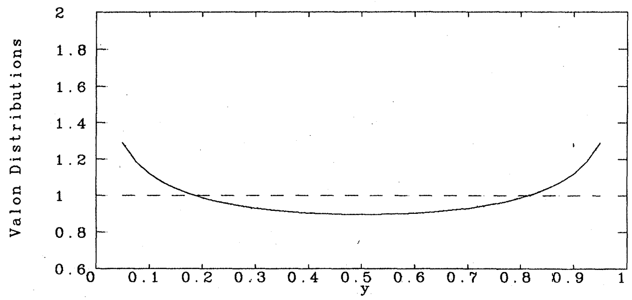

FIG. 1. Valon distribution function in pion given by Eq. (24), solid curve, and Ref. 2, dashed curve.

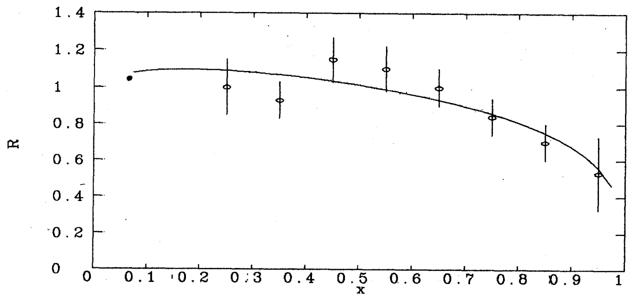

FIG. 2. Data for the ratio $R=u_{v/\pi}(x,Q^2=20)/u_{v/K}(x,Q^2=20)$ from the CERN NA3 experiment. The solid curve is our fit to this data.

we shall attempt to calculate the valon distribution. The details of this method are given in Ref. 5 and here we present a brief account of it.

Let us define the moments of nonsinglet (in this case valence quarks) distribution in hadron h as

$$M^n_{\mathrm{NS}/h}(Q^2)=\int_0^1 x^{n-1}q(x,Q^2)dx\ , \tag{25}$$

where $q(x,Q^2)$ is the quark (valence quark) distribution in hadron h. Then to all orders of perturbative QCD, but to leading-twist order, one can write[5]

$$\frac{M^n_{\mathrm{NS}/p}(Q_1{}^2)}{M^n_{\mathrm{NS}/p}(Q_2{}^2)}=\frac{M^n_{\mathrm{NS}/\pi}(Q_1{}^2)}{M^n_{\mathrm{NS}/\pi}(Q_2{}^2)}=\frac{M^n_{\mathrm{NS}/K}(Q_1{}^2)}{M^n_{\mathrm{NS}/K}(Q_2{}^2)}\ , \tag{26}$$

which implies that the ratio of moments at different values of Q^2 is particle independent. As long as the quark distributions do not contain any corrections of order $(1/Q^2)^n$, one can choose the values of Q^2 in (26) arbitrarily. Solving Eq. (26) for $M^n_{\mathrm{NS}/K}(Q^2)$, we get

$$M^n_{\mathrm{NS}/K}(Q_1{}^2)=\frac{M^n_{\mathrm{NS}/K}(Q_2{}^2)}{M^n_{\mathrm{NS}/\pi}(Q_2{}^2)}M^n_{\mathrm{NS}/\pi}(Q_1{}^2)\ . \tag{27}$$

Let us choose $Q_1{}^2=4$ GeV^2 and $Q_2{}^2=20$ GeV^2. Then Eq. (27) gives the moments of nonsinglet quark distributions [in this case $u_{v/K}(x,Q^2)$] at $Q_1{}^2=4$ GeV^2 if we know its moments at $Q_2{}^2=20$ GeV^2. The rest of the quantities in Eq. (27) are given using the Q^2-dependent quark distributions of Ref. 4. Therefore, we proceed to determine $M^n_{uv/K}(Q_2{}^2=20\ \mathrm{GeV}^2)$.

The valence u-quark distribution at $Q^2=20$ GeV^2 can be calculated using the respective quark distribution in the pions which is given by[4]

$$xu_{v/\pi}(x,Q^2=20)=0.514x^{0.381}(1-x)^{0.895}\ , \tag{28}$$

and the result of the CERN NA3 experiment[6] shown in Fig. 2. A fit to the data of Fig. 2 results in the following distribution for u_v quark in kaons,

$$xu_{v/K}(x,Q^2=20)=0.656x^{0.438}(1-x)^{1.169}\ . \tag{29}$$

We now return to Eq. (27), where all of the quantities on the right-hand side are given through Eqs. (23), (25), (28), and (29). Finally, the inverse transformation of the moments of Eq. (27) gives the valence u-quark distribution at $Q^2=4$ as

$$xu_{v/K}(x,Q^2=4)=0.680x^{0.466}(1-x)^{0.968}\ . \tag{30}$$

The difference between Eqs. (29) and (30) near $x=1$ is mainly due to gluon radiation which causes the quark distributions to become steeper as Q^2 goes higher.

The procedure for the determination of valon distributions is similar to what we used for pions, i.e., first we make the connection through Eq. (6),

$$xu_{v/K}(x,Q^2=4)=\int_x^1 G_{U/K}(y)\frac{x}{y}G_{u/U}(x/y)dy \tag{31}$$

with

$$zG_{u/U}(z)=z^{0.8}e^{-0.4z}\ .$$

Then, using Eqs. (13) and (30), we obtain

$$G_{U/K}(y)=0.5y^{-0.4}(1-y)^{-0.2}\ . \tag{32}$$

This equation also implies [see Eq. (14)]

$$G_{S/K}(y)=0.5y^{-0.2}(1-y)^{-0.4}\ . \tag{33}$$

Figure 3 shows plots of valon distributions in the kaons given by Eqs. (32) and (33).

The average momentum fraction, carried by U valons and S valons are

$$\bar{x}_U=0.43\ ,\qquad \bar{x}_S=0.57\ , \tag{34}$$

which indicates the ratio of momentum fractions carried by respective valons to be

$$\frac{\bar{x}_S}{\bar{x}_U}=\frac{4}{3}\simeq 1.33\ . \tag{35}$$

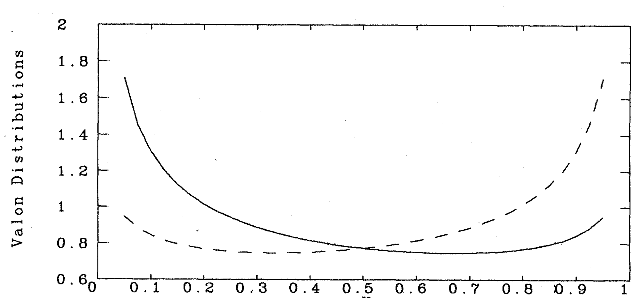

FIG. 3. The strange (solid curve) and nonstrange (dashed curve) valon distribution in kaons.

If this ratio is approximately equal to the ratio of masses of respective valons, then we must have

$$\frac{M_S}{M_U} \simeq 1.33 . \tag{36}$$

In the literature different values for the ratio (36) have been assumed which have a range between 1 and 2.

III. INCLUSIVE REACTIONS

An interesting area of the application of the results presented so far is the inclusive meson production in meson-proton collisions. The invariant cross section for such reactions (after integrating over p_T) can be written as[7]

$$\frac{x}{\sigma_{\text{in}}}\frac{d\sigma}{dx} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} F_{q\bar{q}}(x_1,x_2) R(x_1,x_2;x) , \tag{37}$$

where $F_{q\bar{q}}$ is the joint quark distribution and R is the recombination function which gives the probability of quark $q(x_1)$ and antiquark $\bar{q}(x_1)$ to recombine and produce a meson at $x = x_1 + x_2$. σ_{in} is the total inelastic cross section for the reaction under consideration. The valon-parton model gives prescriptions to calculate $F_{q\bar{q}}$ and R, however, in this paper, we like to use a simpler version of the model for the purpose of illustration. According to the simple recombination model,[7] the spectrum of the final-state particle in a single-particle inclusive reaction is similar to the distribution of the common valence quark between the beam and final-state particle. That means, at small p_T, in a reaction like $K^- p \to \pi^- + X$, we have

$$\left.\frac{x}{\sigma_{\text{in}}}\frac{d\sigma}{dx}\right|_{K^- \to \pi^-} \sim x u_{v/K}(x) \underset{x\to 1}{\sim} (1-x)^{\sim 1} . \tag{38}$$

The data[8] indicate a slightly steeper distribution than (38), i.e., $(1-x)^{\sim 1.4}$. This is actually related to the fact that the simple recombination model does not introduce any correlation between q and $\bar{q}$ in $F_{q\bar{q}}$, which in general results in a steeper distribution. Furthermore, the data include resonance decay of the type $K^- \to \rho \to \pi^+\pi^-$. The decay products usually populate the low-x region, resulting in an effectively steeper x distribution. Since the simple recombination model as presented here does not account for such complications, the distribution should be slightly flatter as indicated by Eq. (38).

Another interesting example is the inclusive production of fast ϕ's in K^+p reactions. The data[9] is taken at 70 GeV/c and the invariant cross section for $x \geq 0.5$ is

$$\left[\frac{d\sigma}{dx}\right]_{K^+ \to \phi} \propto x(1-x) . \tag{39}$$

In this case, the common quark between the beam (K^+) and final-state particle (ϕ) is the s quark. Thus, according to the simple recombination model, the distribution of s quarks in the kaons must be flatter than u_v quarks near $x=1$. This has been reflected in the valon distributions given by Eqs. (32) and (33). In other words, if $u_{v/K}(x) \sim (1-x)^1$ as shown in (30), then $s_{v/K}(x)$ should be $\sim (1-x)^{0.8}$ by virtue of (32) and (33), which is reflected by data point of Ref. 9.

As we mentioned above, the valon-parton model presents a well prescribed method to calculate joint quark distributions. However, a detailed calculation for reactions like $K^- p \to \bar{K}^0 + X$, or $K^+ p \to \phi + X$ requires the structure function of strange valons, i.e., $G_{s/S}(z)$, to determine the evolution of quarks from strange valons. The authors of Ref. 10 used low-p_T data to determine that function. It is interesting to investigate for some other methods independent of data at low p_T. Precise measurements of the kaon structure function at high Q^2 will be very interesting. The method we already described for the determination of the u_v-quark distribution in the kaon can be easily applied to calculate the s_v-quark distribution which is related to S valons through Eq. (6). Therefore, one can extract the structure of strange valons using that relationship.

IV. SUMMARY

In this paper we studied some of the methods for the determination of valon distribution functions and valon structure functions in pions and kaons. Our final results are given by Eqs. (22), (24), and (32). We indicated the complications with the structure function of the strange valons, and the need for future experiments to determine this function from hard-scattering data. The implications of our results for the single-particle inclusive reactions were studied in the context of the simple recombination model, and it was shown, semiqualitatively, that they agree with existing data.

ACKNOWLEDGMENTS

The author wishes to thank Professor M. Suzuki for reading the manuscript. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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