Path-integral solution for a two-dimensional model with axial-vector-current–pseudoscalar derivative interaction

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(Received 4 September 1984)

We study a two-dimensional quantum field model with axial-vector-current–pseudoscalar derivative interaction using path-integral methods. We construct an effective Lagrangian by performing a chiral change in the fermionic variables leading to an exact solution of the model.

Analysis of quantum field models in two space-time dimensions has proved to be a useful theoretical laboratory to understand phenomena like dynamical mass generation, topological excitations, and confinement; all features expected to be present in a realistic four-dimensional theory of strong interactions. Recently, a powerful nonperturbative technique has been used to analyze several two-dimensional fermionic models in the (Euclidean) path-integral approach. This technique is based on a chiral change of variables in the functional fermionic measure.[1-4] It is the purpose of this Brief Report to solve exactly another fermionic model by means of this nonperturbative technique. The model to be studied describes the interaction of a massive pseudoscalar field with massless fermion fields in terms of a derivative coupling and was analyzed previously in Ref. 5 by using the operator approach.

We start our study by considering the Euclidean Lagrangian of the model:

$$\mathscr{L}_1(\psi,\bar\psi,\phi)(x)=[-i\bar\psi\gamma_\mu\partial_\mu\psi+\tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi)+\tfrac{1}{2}m^2\phi^2+g\bar\psi\gamma^5\gamma_\mu\psi\partial_\mu\phi](x)\ , \tag{1}$$

where $\psi=(\psi_1,\psi_2)$ denotes a massless fermion, ϕ a massive pseudoscalar field, and g the coupling constant. The Lagrangian (1) is invariant under the global Abelian and Abelian-chiral groups

$$\psi\to e^{i\alpha}\psi,\quad \psi\to e^{i\gamma_5\beta}\psi,\quad (\alpha,\beta)\in\mathbb{R}\ ,$$

with the formally Noether conserved currents

$$\partial_\mu(\bar\psi\gamma^5\gamma_\mu\psi)=0,\quad \partial_\mu(\bar\psi\gamma^\mu\psi)=0\ .$$

The Hermitian γ matrices we are using satisfy the (Euclidean) relations

$$\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu},\quad \gamma_\mu\gamma_5=i\epsilon_{\mu\nu}\gamma_\nu,\quad \gamma_5=i\gamma_0\gamma_1\ ,$$

$$\epsilon_{01}=-\epsilon_{10}=1\quad (\mu,\nu=0,1)\ .$$

In the framework of path intergals, the generating functional of the Green's functions of the quantum field theory associated with the Lagrangian (1) is given by

$$Z[J,n,\bar n]=\int D[\phi]D[\psi]D[\bar\psi]\exp\left[-\int d^2x[\mathscr{L}_1(\psi,\bar\psi,\phi)(x)+\phi J+\bar n\psi+\bar\psi n](x)\right]\ . \tag{2}$$

In order to perform a chiral change of variable in (2) it appears to be convenient to write the interaction Lagrangian in a form closely parallel to the usual fermion-vector coupling in gauge theories by making use of the indentity

$$\exp\left[-\int d^2x\, g(\psi\gamma^5\gamma_\mu\psi\partial_\mu\phi)(x)\right]=\int D[A_\mu]\delta(A_\mu+i\epsilon_{\mu\alpha}\partial_\alpha\phi)\exp\left[\int d^2x\, g(\bar\psi\gamma_\mu A_\mu\psi)(x)\right]\ , \tag{3}$$

where $A_\mu(x)$ is an auxiliary Abelian vector field. Substituting (3) into (2) we obtain a more suitable form for $Z[J,n,\bar n]$,

$$Z[J,n,\bar n]=\int D[\phi]D[\psi]D[\bar\psi]D[A_\mu]\delta(A_\mu+i\epsilon_{\mu\alpha}\partial_\alpha\phi)\exp\left[-\int d^2x[\mathscr{L}_2(\psi,\bar\psi,\phi,A_\mu)+J\phi+\bar n\psi+\bar\psi n]\right]\ , \tag{4}$$

with

$$\mathscr{L}_2(\psi,\bar\psi,\phi,A_\mu)=-i\bar\psi\gamma_\mu(\partial_\mu-igA_\mu)\psi+\tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi)+\tfrac{1}{2}m^2\phi^2\ . \tag{5}$$

Now we can proceed as in the case of gauge theories[2,3] in order to decouple the fermion fields from the auxiliary vector field $A_\mu(x)$ by making the chiral change of variables

$$\begin{aligned}\psi(x)&=e^{i\gamma_5\beta(x)}\chi(x)\ ,\\ \bar\psi(x)&=\bar\chi(x)e^{i\gamma_5\beta(x)}\ ,\\ A_\mu(x)&=-\frac{i}{g}(\epsilon_{\mu\nu}\partial_\nu\beta)(x)\ .\end{aligned} \tag{6}$$

We note the relation $\beta=g\Phi$ between the chiral phase β and the pseudoscalar field Φ due to the constraint $\delta(A_\mu+i\epsilon_{\mu\alpha}\partial_\alpha\phi)$ in (4). After the change (6), the Lagrangian (5) takes a form where now the fermion fields $\chi(x)$, $\bar\chi(x)$ are free and decoupled from the auxiliary vector field $A_\mu(x)$,

$$\mathscr{L}_3(\chi,\bar\chi,\phi)=-i\bar\chi\gamma_\mu\partial_\mu\chi+\tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi)+\tfrac{1}{2}m^2\phi^2\ . \tag{7}$$

On the other hand, the fermionic measure $D\psi D\bar{\psi}$, defined in terms of the normalized eigenvectors of the Hermitian operator $-i\gamma_\mu(\partial_\mu - igA_\mu)$, is not invariant under the chiral change and gives the Jacobian[2,6]

$$\exp\left[-\frac{g^2}{2\pi}\int d^2x(A_\mu A_\mu)(x)\right] = \mathrm{Det}[-i\gamma_\mu(\partial_\mu - igA_\mu)] \ . \tag{8}$$

In terms of the new fermionic fields $\chi(x)$, $\bar{\chi}(x)$ and taking into account the Jacobian (8), the generating functional reads

$$Z[J,n,\bar{n}] = \int D[\chi]D[\bar{\chi}]D[\phi]\exp\Big(-\int d^2x[-i\bar{\chi}\gamma_\mu\partial_\mu\chi + \tfrac{1}{2}(1-g^2/\pi)(\partial_\mu\phi)(\partial_\mu\phi) + \tfrac{1}{2}m^2\phi^2 + J\phi + \bar{n}e^{ig\gamma_5\phi}\chi + \bar{\chi}e^{ig\gamma_5\phi}\eta](x)\Big) \ . \tag{9}$$

We shall now use the effective generating functional (9) to compute exactly the (bare) Green's functions of the model. The two-point Green's function of the pseudoscalar field $\phi(x)$ is straightforwardly obtained,

$$\langle(\phi(x)\phi(y))\rangle = \frac{1}{2\pi}K_0\left[\frac{m}{(1-g^2/\pi)^{1/2}}|x-y|\right] , \tag{10}$$

where $K_0(w)$ denotes the Hankel function of an imaginary argument of order zero. The two-point fermion Green's function is easily computed by noting that the fermion fields $\chi(x)$, $\bar{\chi}(x)$ are free and decoupled in (9). The result reads

$$\langle(\psi_\alpha(x)\bar{\psi}_\beta(y))\rangle = \frac{1}{2\pi}(\gamma_\mu)_{\alpha\beta}\frac{(x_\mu - y_\mu)}{|x-y|^2}\exp\left[\frac{g^2}{2\pi(1-g^2/\pi)}K_0\left[\frac{m}{(1-g^2/\pi)^{1/2}}|x-y|\right]\right] , \tag{11}$$

where we have used the dimensional regularization scheme to assign the value 1 to the "tadpole" contribution that appears in (11), i.e.,

$$\exp\left[\frac{g^2}{1-g^2/\pi^2}K_0(0)\right] = 1 \ .$$

It is also interesting to compute correlation functions involving fermions $\psi(x)$, $\bar{\psi}(x)$ and the pseudoscalar field $\phi(x)$. For instance we get the following expression for the vertex $\Gamma_{\alpha\beta}(x,y,z) = \langle(\psi_\alpha(x)\bar{\psi}_\beta(y)\phi(z))\rangle$:

$$\Gamma_{\alpha\beta}(x,y,z) = \frac{1}{2\pi}(\gamma_\mu\gamma_5)_{\alpha\beta}\frac{(x_\mu - y_\mu)}{|x-y|^2}\frac{ig}{2\pi(1-g^2/\pi)^{1/2}}\left[K_0\left[\frac{m}{(1-g^2/\pi)^{1/2}}|x-z|\right] - K_0\left[\frac{m}{(1-g^2/\pi)^{1/2}}|y-z|\right]\right] \times\exp\left[\frac{g^2}{2\pi(1-g^2/\pi)}K_0\left[\frac{m}{(1-g^2/\pi)^{1/2}}|x-y|\right]\right] . \tag{12}$$

It is instructive to point out that in a perturbative analysis of the model, the previous correlation functions correspond to the full sum of nonrenormalized Feynman diagrams involving all possible radiative corrections. The perturbative renormalization analysis can be implemented by applying the asymptotic Lehmann-Symanzik-Zimmermann conditions (or equivalently, the Dyson prescription) in the (bare) propagators and the vertex function of the model.[5,7] This analysis results in that the pseudoscalar field ϕ gets (finite) wave-function and mass renormalizations, $\phi^{(R)} = (Z_\phi)^{-1}\phi$ and $m_{(R)}{}^2 = m^2(Z_\phi)^2$, respectively, with

$$Z_\phi = \frac{1}{(1-g^2/\pi)^{1/2}}$$

and the coupling constant gets a multiplicative renormalization $g_{(R)} = gZ_\phi$. For instance, the renormalized two-point fermion Green's function is given by

$$\langle(\psi_\alpha(x)\bar{\psi}_\beta(y))\rangle^{(R)} = \frac{1}{2\pi}(\gamma_\mu)_{\alpha\beta}\frac{(x_\mu - y_\mu)}{|x-y|^2}\times\exp\left[\frac{g_{(R)}{}^2}{2\pi}K_0(m_{(R)}|x-y|)\right] , \tag{13}$$

which has the following short- and long-distance behavior:

$$\langle(\psi_\alpha(x)\bar{\psi}_\beta(y))\rangle^{(R)}_{|x-y|\to 0} \cong (\gamma_\mu)_{\alpha\beta}|x-y|^{-(1+g_{(R)}{}^2/2\pi)} , \tag{14}$$

$$\langle(\psi_\alpha(x)\bar{\psi}_\beta(y))\rangle^{(R)}_{|x-y|\to\infty} \cong \frac{1}{2\pi}(\gamma_\mu)_{\alpha\beta}\frac{(x_\mu - y_\mu)}{|x-y|^2} \ . \tag{15}$$

The ultraviolet behavior (14) implies that the fermion field carries an anomalous dimension $\gamma_\psi = g_{(R)}{}^2/4\pi$. We remark that the model displays the appearance of the axial anomaly as a consequence of the presence of the massive field ϕ.[1,5] For instance, in terms of the bare field ϕ, we have

$$\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = \frac{g}{\pi}\Box\phi = \frac{g}{\pi}\frac{m^2}{1-g^2/\pi}\phi \ . \tag{16}$$

Next, we shall consider the case of massive fermions in the model

$$\mathscr{L}(\psi,\bar{\psi},\phi) = [-i\bar{\psi}\gamma_\mu\partial_\mu\psi + \mu\bar{\psi}\psi + \tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) + \tfrac{1}{2}m^2\phi^2 + g\bar{\psi}\gamma^5\gamma_\mu\psi\partial_\mu\phi] \ . \tag{17}$$

In order to get an effective action as in (9), we make again the chiral change (6) where it becomes important to remark that the fermion measure $D\psi D\bar{\psi}$ is to be defined by the eigenvectors of the massless Hermitian Dirac operator $\not{D}(A_\mu) = -i\gamma_\mu(\partial_\mu - igA_\mu)$ and thus yields the same Jacobian (8). This fact is related to the fermion mass independence of the anomalous part of the divergence of chiral current $\partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi)$ in fermion models interacting with gauge fields. Proceeding as above we obtain the new effective (bare) Lagrangian

$$\begin{aligned}\mathscr{L}(\chi,\bar{\chi},\phi)(x) = [&-i\bar{\chi}\gamma_\mu\partial_\mu\chi + \mu\bar{\chi}e^{2ig\gamma_5\phi}\chi \\ &+ \tfrac{1}{2}(1-g^2/\pi)(\partial_\mu\phi)(\partial_\mu\phi) \\ &+ \tfrac{1}{2}m^2\phi^2](x) \ , \end{aligned} \tag{18}$$

where now the fermion fields $\chi(x)$, $\bar{\chi}(x)$ possess an interacting term $\mu\bar{\chi}[\cos(2g\phi)+i\gamma_5\sin(2g\phi)]\chi$ which contributes to the phenomenon of formation of fractionally fermionic solitons.[8]

As we have shown, chiral changes in path integrals provide a quick, mathematically and conceptually simple way to solve and analyze the model studied in this Brief Report.

Note added. We would like to make some clarifying remarks on the analysis implemented in this Brief Report. First, in order to implement the regularization rule in Eqs. (3)–(11) we consider the associated perturbative power series in the coupling constant g dimensionally regularized which automatically takes into account the Wick normal-order operation in the correlation functions Eqs. (9)–(11). Second, we observe that the model should be redefined in the region $g^2 > \pi$ since its associated Gell-Mann–Low function has a nontrivial zero for $g^2 = \pi$ (the model contains a tachyonic excitation).[5]

We are grateful to C. G. Bollini, J. J. Giambiagi, and E. C. Marino for comments. This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

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