Gravitational versus finite-temperature effects in SU(5) symmetry breaking

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(Received 27 June 1984)

The influence of classical gravitation in the symmetry breaking of the SU(5) model at early stages of the universe is considered. This is achieved by treating the gravitational field as a *c*-number external field in the path integral while the remaining particle fields are considered as quantum fields of the unified model. The symmetry breaking of the SU(5) model locally coupled to gravity is described through the effective potential of the quantum field theory, which is renormalized by making use of a zeta-function regularization, and its evolution in the course of the universal time of the treated standard cosmological model is presented. It is shown that the background gravitational field has the tendency to enhance the symmetry breaking that is postulated in the zero-temperature, flat—space-time quantum theory. Although this influence is overcome by the restoration due to the finite-temperature effects through two orders of magnitude in temperature above the mass scale of the SU(5) model, there exists a temperature for which the effective potential develops a strong symmetry breakdown due to the classical gravitational effects. This temperature is between one and two orders of magnitude below the Planck mass scale.

I. INTRODUCTION

In recent years great effort has been made in understanding the way in which symmetry breaking occurs in grand unified theories.[1–4] This effort has been mainly centered in the minimal SU(5) model[5] while the standard cosmological vision has been adopted.[6] The existence of a hot phase after the big bang allows for the possibility that, at an early stage in the evolution of the universe after the Planck time $\sim 10^{-43}$ sec, the temperature at which interactions are in equilibrium reaches the larger mass scale of the model $\sim 10^{15}$ GeV. Above this temperature all the analyses done show that the gauge symmetry remains intact, but at a later moment during the cooling of the universe symmetry breaking proceeds by $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ as an intermediate stage to the present-day observed symmetry $SU(3)_{\rm color} \times U(1)_{\rm em}$.

At this point, it must be remarked that the notion of thermodynamic equilibrium and the assumption that this equilibrium has been reached are essential. Although we are not dealing with this problem, we will suppose that equilibrium is reached at some time after the Planck time[7] and that processes taking place in a nonstatic space-time can be considered as statistical processes and studied by making use of a quantum statistical theory of fields.[8] In this framework, the two symmetry breakings $SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$ are observed as phase transitions in which the statistical mean value of the Higgs field plays the role of order parameter of the transition. As is well known, the central point of the discussion lies in the nature of this transition and, consequently, the validity of the perturbative methods employed in its study.[1,2] It has been shown that the phase transition may be first or second order depending on the choice of the parameters of the model, i.e., bare mass of the Higgs field, scalar, and gauge coupling constants. In the more attractive case of symmetry breaking due to radiative corrections, i.e., the Coleman-Weinberg mode,[9] one has to face the consequences of a large supercooling.[3,4] This is, perhaps, one of the most important recent problems in the study of cosmological phase transitions, in relation to the monopole problem[10] and the baryon-number generation.[11]

Although the statistical formulation of the problem is correct, the influence of gravity considered as a local interaction has been forgotten, at a stage in which classical gravitation can play a significant role. (There have been, however, some attempts to deal with the influence that the curvature may have in the symmetry breaking of naive scalar models placed in an expanding universe. See Refs. 14 and 23, in which symmetry breaking is studied within the framework of Robertson-Walker universes.) In fact, after the Planck time gravity decouples as a quantum interaction from the remaining fundamental forces, and it is not necessary to include quantum fluctuations in the study of the symmetry breaking of the SU(5) grand unified theory as far as the ratio between the mass scale σ and the Planck mass $G^{-1/2}$, $\sigma/G^{-1/2} \sim 10^{-4}$, makes their influence negligible. But after the Planck time we still need to preserve the general covariance of our theory as long as we are dealing with a quantum field theory in a curved space-time.[12] First, this implies that the gravitational field acts as a *c*-number external source in the equations of motion (or in the path integral) for particle quantum fields and, on the other hand, that this gravitational field must be solved from the Einstein equations when the source of these equations is the vacuum expectation value of the energy-momentum tensor of the quantum field theory under study.[12] In this paper we do not attempt to solve this problem (the back-reaction problem) and simply suppose that the cosmological solution of the Einstein equations takes the form of a cosmological standard metric.[6]

We are going to consider the simplest case, the $k=0$

Robertson-Walker model with flat three-spatial sections, and to concentrate on the influence that this classical gravitational field may have in the symmetry breaking of the grand unified theory. The influence of this c-number field may not be negligible as can be shown with the following reasoning. The formulation of the quantum field theory in curved space-time introduces geometrical objects in the equations of motion (or in the path integral) which are related to the curvature of the space-time (Christoffel symbols, the Ricci tensor). The mass scale which is related to these geometrical objects may be seen from those which have the dimension of inverse length (or squared inverse length) and that, placed in the quantum Lagrangian, are effective mass couplings. From inspection of $\Gamma^{\mu}_{\nu\tau}, R_{\mu\nu}$, this mass scale turns out to be $\dot{\chi}/\chi$, i.e., the inverse of the universal time of the model, a scale which runs from 10^{19} GeV at the Planck time to 10^{12} GeV at the scale of the transition temperature $T_c \sim 10^{15}$ GeV. This shows that during that interval of time the mass scale introduced by the classical gravitational field may be equal to or greater than the mass scale of the SU(5) grand unified theory, which makes us expect a significant influence of classical gravitation during this period of time. This applies to the $k=0$ Robertson-Walker metric as well as to the other two standard cosmological models, because the order of magnitude of the objects $\Gamma^{\mu}_{\nu\tau}, R_{\mu\nu}$ is the same for the three models, so that we can hope that the conclusions reached in this paper are not influenced by the choice of a specific model.

To summarize, we deal with symmetry breaking in the SU(5) model under the following assumptions: (1) thermodynamic equilibrium is reached at some time after the Planck time and a quantum statistical formulation of the theory is possible; (2) a complete description of the problem requires the general covariance of the quantum field theory, which makes the gravitational field appear in the path integral as a c-number field coupled to the different fields in the SU(5) model. The paper is distributed in sections in the following way. Section II is devoted to compute the one-loop-order contribution to the effective potential of the SU(5) model including the gravitational field in the above-mentioned form. We only take into account the scalar and vectorial sectors of the model and disregard the contribution of the spinors, which is greatly suppressed because of the weakness of the Yukawa couplings. In Sec. III the generating functional is redefined by means of a generalized zeta function and a zeta-function regularization is made in the usual form. Then, the effective potential is defined in terms of finite quantities and explicit results concerning the way in which classical gravitation modifies the shape of the effective potential are shown. Finally, in Sec. IV the finite-temperature contributions to the effective potential are computed with the same method of regularization. The presented results support the evidence that classical gravitation tends to favor the symmetry breaking of the model in the range of time in which the gravitational mass scale $\dot{\chi}/\chi$ is equal to or greater than the mass scale σ of the SU(5) model. In the framework of the given cosmological model, this kind of effect seems to be negligible when the temperature reaches the σ mass scale. There exists, however, a period of time, for temperatures still below the Planck mass, in which the finite-temperature restoration is overcome by a stronger symmetry-breaking effect and the symmetry seems to be broken because of the presence of the classical gravitational field.

The approach adopted throughout this article is that of the path-integral formulation of quantum field theory, and functional methods are used in the study of the symmetry breaking. Henceforth we will suppose that the quantum theory is defined through a functional generator $Z(J)$ whose successive derivatives

$$\frac{1}{i^n}\frac{1}{Z(J)}\left.\frac{\delta^n Z(J)}{\delta J(x_1)\cdots\delta J(x_n)}\right|_{J=0} = \langle T\psi(x_1)\cdots\psi(x_n)\rangle_\beta \quad (1)$$

have the meaning of statistical mean values of time-ordered products of operators

$$\langle T\psi(x_1)\cdots\psi(x_n)\rangle_\beta = \frac{\mathrm{Tr}e^{-\beta H}T[\psi(x_1)\cdots\psi(x_n)]}{\mathrm{Tr}e^{-\beta H}} . \quad (2)$$

This essentially static approach will allow us to study the symmetry breaking through the effective potential of Higgs fields of the theory. For an extensive discussion on the feasibility of this procedure in a nonstatic space-time one is led to Refs. 13 and 14, in the latter of which the effective potential for an interacting $\lambda\phi^4$ theory is computed within the framework of a Robertson-Walker universe. Throughout this paper we suppose that the equilibrium conditions mentioned there are fulfilled in the framework of our cosmological standard scenario[7] and that the study of the symmetry breaking may be accomplished through the computation of the effective potential of the theory, which is the major result of the next section.

II. THE EFFECTIVE POTENTIAL

The starting point is the assumption that the generating functional can be expressed in terms of a functional integral

$$Z(J) = \int \mathscr{D}\psi\,\mathscr{D}\tilde{g}\exp\left[i\int d^4x\sqrt{-\tilde{g}}\,[\mathscr{L}_0(\psi,\tilde{g}) + \mathscr{L}_G(\tilde{g})]\right] \quad (3)$$

of the matter fields ψ and the gravitational field $\tilde{g}$. We suppose, as usual, that after the Planck time quantum gravitational effects become negligible, as far as the presumable rate of the quantum gravitational interactions $\tau^{-1}_{\mathrm{grav}} \lesssim GT^3$ is much lower than the rate of expansion of the universe $\tau^{-1} \propto G^{1/2}T^2$ and quantum gravity decouples from the remaining interactions. Then, in order to preserve the general covariance of the functional integral we have to consider the $g_{\mu\nu}$ field as a classical field satisfying the Einstein field equations [equivalently, this can be seen as if we were taking the zeroth order in an expansion in the functional integral around a background field $\tilde{g}=\tilde{g}_0$ (Ref. 15). For the sake of convenience, we are going to take the simplest cosmological solution for $g_{\mu\nu}$, i.e., the $k=0$ Robertson-Walker metric, but, as it has been said in the Introduction, we hope the final conclusions to be independent of the choice of a specific standard cosmological model.

In the coordinate system in which $-\infty < x^i < +\infty$ the $k=0$ Robertson-Walker metric may be put in the form

$$(\tilde{g}_0)_{\mu\nu} = \text{diag}(1, -\mathcal{X}^2(t), -\mathcal{X}^2(t), -\mathcal{X}^2(t)) . \quad (4)$$

The corresponding Christoffel symbols are[6]

$$\Gamma^0_{ij} = \mathcal{X}\dot{\mathcal{X}}\delta_{ij} , \quad \Gamma^i_{0j} = \frac{\dot{\mathcal{X}}}{\mathcal{X}}\delta^i_j , \quad \Gamma^i_{jk} = 0 , \quad (5)$$

and the components of the Ricci tensor are

$$R_{00} = -3\frac{\ddot{\mathcal{X}}}{\mathcal{X}} , \quad R_{ij} = (2\dot{\mathcal{X}}^2 + \mathcal{X}\ddot{\mathcal{X}})\delta_{ij} . \quad (6)$$

In the case $k=0$, the equation governing $\mathcal{X}(t)$ reduces to

$$\dot{\mathcal{X}}^2 = \frac{8\pi G}{3}\rho\mathcal{X}^2 . \quad (7)$$

In the range of time under consideration the universe is dominated by radiation, $\rho \propto \mathcal{X}^{-4}$, which in turn implies that $\mathcal{X}(t)$ has a solution of the form $\mathcal{X} \propto t^{1/2}$. Thus, one obtains the known relation between time and temperature in the early universe,[16]

$$t = \left[\frac{45}{16\pi^3}\right]^{1/2} \left[\frac{1}{g_I}\right]^{1/2} \left[\frac{\hbar^3 c^5}{G}\right]^{1/2} \frac{1}{(k_B T)^2} = \frac{2.42\times 10^{-6}}{g_I{}^{1/2}} \frac{1}{T(\text{GeV})^2} . \quad (8)$$

Afterward this expression will be of major interest to obtain the length scale of the classical gravitational couplings.

In a formal sense, the $k=0$ Robertson-Walker metric presents two types of advantages. The first lies in the fact that, since the chosen metric is conformally flat, we do not need to remark further on the quantization of the theory, assuming that this can be accomplished defining a functional integral $Z_0(J)$. It is worthwhile commenting, though, that all quantities with vectorial character will have to be conveniently rescaled with $\mathcal{X}(t)$, in order for said quantities to acquire physical dimensions. Keeping this in mind, all the standard techniques can be employed, in particular the functional representation of the effective potential.[17] The second advantage is concerned with the fact that for the $k=0$ Robertson-Walker metric the curvature scalar $R \equiv R_{\mu\nu} g^{\mu\nu}$ vanishes, which makes no distinction between conformal and minimal scalar fields in the formulation of our model. Actually, this is strictly true when the vacuum energy of the quantum theory is identically zero, i.e., when one is away from any period of inflationary expansion.[18] The study of the influence that an $R\phi^2$ term may have in the symmetry breaking of the SU(5) model during a period of supercooling after the transition temperature $T_c \sim 10^{15}$ GeV has been carried out by Abbott.[3] In the present paper and as a first approach to the problem, we suppose that the range of time in which the influence of gravitation is considered lies away from any period of metastability of the universe, which makes our approach complementary to that adopted by Abbott in his paper.

As it has been said in the Introduction, we will suppose the remaining (quantum) interactions to be unified in the SU(5) model. We will assume that symmetry breaking takes place by mediation of a $\underline{24}$ representation of Higgs fields ϕ^a ignoring effects due to the existence of lower-dimensional representations. In this way we take

$$Z_0(J) = \int \mathscr{D}\psi \exp\left[i \int d^4x \sqrt{-g_0}\, \mathscr{L}_0(\psi, \tilde{g}_0)\right] , \quad (9)$$

$$\mathscr{L}_0 = -\tfrac{1}{4} V^a_{\mu\nu} V^{\mu\nu}_a + \text{Tr}(D_\mu\Phi D^\mu\Phi) - m^2 \text{Tr}\Phi^2 - V(\Phi) + \text{fermion terms} + \text{gauge-fixing terms} + \text{ghost terms} ,$$
$$V^a_{\mu\nu} \equiv \nabla_\mu V^a_\nu - \nabla_\nu V^a_\mu - g f^{abc} V_{\mu b} V_{\nu c} , \quad D_\mu\Phi \equiv \nabla_\mu\Phi - ig[T^a V^a_\mu, \Phi] , \quad \Phi \equiv \tfrac{1}{2} T^a \phi^a , \quad a = 1, \ldots, 24 . \quad (10)$$

The inclusion of gravitation as an external source takes place through covariant derivatives $\nabla_\mu V_\nu = \partial_\mu V_\nu - \Gamma^\tau_{\mu\nu} V_\tau$, which are needed to make our Lagrangian invariant under general coordinate transformations. As it has been pointed out above, the appearance of a term $R\,\text{Tr}\Phi^2$ is prevented by the vanishing of the curvature scalar in the $k=0$ cosmological standard model.

As it follows we will face the case in which the squared gauge coupling constant g^2 is bigger than any other coupling constant in the scalar or spinorial sectors, whereby we only need to compute the gauge corrections to the one-loop order to get a reasonable description of the problem. On the other hand, a squared bare mass $m^2 \geq 0$ is maintained in the Lagrangian in order to avoid infrared instabilities which may appear in the computation of the effective potential, though at the end we will be interested in the Coleman-Weinberg mode $m^2 = 0$. The effective potential to the one-loop order can be represented by[17]

$$V(\hat{\psi}) = V_0(\hat{\psi}) + i \ln \int \mathscr{D}\psi \exp\left[i \int d^4x \sqrt{-g_0}\, \hat{\mathscr{L}}_0(\hat{\psi}, \psi)\right] , \quad (11)$$

where $V_0(\hat{\psi})$ is the "tree" approximation and $\hat{\mathscr{L}}_0$ is the Lagrangian quadratic in the fields after the shift $\psi \rightarrow \psi + \hat{\psi}$. Since we are only interested in the gauge contributions to the effective potential we take

$$\mathscr{L}_0 = \text{Tr}(\nabla_\mu\Phi - ig[T^a V^a_\mu, \Phi])(\nabla^\mu\Phi - ig[T^a V^a_\mu, \Phi]) - m^2 \text{Tr}\Phi^2 - \tfrac{1}{4} V^a_{\mu\nu} V^{\mu\nu}_a - \frac{1}{2\alpha}(\nabla_\mu V^\mu_a)^2 \quad (12)$$

and make the shift $\Phi \rightarrow \Phi + \langle\Phi\rangle$ in order to break the symmetry to SU(3)×SU(2)×U(1),

$$\langle \Phi \rangle \equiv \theta\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2})\,.$$

Then

$$\hat{\mathscr{L}}_0=\tfrac{1}{2}g_{\mu\nu}\nabla^\mu\phi^a\nabla^\nu\phi^a-\frac{m^2}{2}\phi^a\phi^a+5g\theta\sum_{\substack{a=9,\ldots,14\\16,\ldots,21}} g_{\mu\nu}\nabla^\mu\phi^{(a*)}V^{\nu a}-\tfrac{1}{4}(\nabla_\mu V^a_\nu-\nabla_\nu V^a_\mu)(\nabla^\mu V^\nu_a-\nabla^\nu V^\mu_a)$$
$$-\frac{1}{2\alpha}(\nabla_\mu V^\mu_a)^2+\tfrac{25}{2}g^2\theta^2\sum_{\substack{a=9,\ldots,14\\16,\ldots,21}} g_{\mu\nu}V^\mu_a V^\nu_a\,, \tag{13}$$

where $(a_*)\equiv$ conjugate particle of a.

Disregarding a four-divergence and making use of $\nabla_\delta\nabla_\beta V^\alpha=\nabla_\beta\nabla_\delta V^\alpha-V^\epsilon R^{\ \alpha}_{\epsilon\beta\delta}$, this Lagrangian can be transformed into

$$\hat{\mathscr{L}}_0=-\tfrac{1}{2}g_{\mu\nu}\phi^a\nabla^\mu\nabla^\nu\phi^a-\frac{m^2}{2}\phi^a\phi^a+5g\theta\sum_{\substack{a=9,\ldots,14\\16,\ldots,21}} g_{\mu\nu}V^{\mu a}\nabla^\nu\phi^{(a*)}+\tfrac{1}{2}g_{\alpha\beta}V^\alpha_a g_{\mu\nu}\nabla^\mu\nabla^\nu V^\beta_a-\tfrac{1}{2}\left[1-\frac{1}{\alpha}\right]V^\mu_a\nabla_\mu\nabla_\nu V^\nu_a$$
$$-\tfrac{1}{2}R_{\mu\tau}V^\mu_a V^\tau_a+\tfrac{25}{2}g^2\theta^2\sum_{\substack{a=9,\ldots,14\\16,\ldots,21}} g_{\mu\nu}V^{\mu a}V^{\nu a}\,. \tag{14}$$

Making explicit use of the Christoffel symbols and rescaling to quantities with dimensions $\chi x^i\to x^i,\chi V^i\to V^i$, we get

$$\hat{\mathscr{L}}_0=-\tfrac{1}{2}\phi^a\left[\partial_\mu\partial^\mu+\frac{3\dot\chi}{\chi}\partial_0+m^2\right]\phi^a+5g\theta\sum_{\substack{a=9,\ldots,14\\16,\ldots,21}} V^{\mu a}M^{ab}\partial_\mu\phi^b+\tfrac{1}{2}V^a_\nu\partial_\mu\partial^\mu V^{\nu a}-\tfrac{1}{2}\left[1-\frac{1}{\alpha}\right]V^\mu_a\partial_\mu\partial_\nu V^\nu_a$$
$$+\frac{3}{2\alpha}\frac{\dot\chi}{\chi}V^0_a\partial_0 V^0_a-\frac{1}{2}\frac{\dot\chi}{\chi}V^0_a\partial_i V^i_a+\frac{1}{2}\left[4-\frac{3}{\alpha}\right]\frac{\dot\chi}{\chi}V^i_a\partial_i V^0_a-2\frac{\dot\chi}{\chi}V^i_a\partial_0 V^i_a-\frac{3}{\alpha}\left[\frac{\dot\chi}{\chi}\right]^2V^0_aV^0_a-\left[\frac{\dot\chi}{\chi}\right]^2V^i_aV^i_a$$
$$+\tfrac{25}{2}g^2\theta^2\sum_{\substack{a=9,\ldots,14\\16,\ldots,21}} V^{\mu a}V_{\mu a}\,, \tag{15}$$

where $M^{ab}=0$, $a,b=1,\ldots,8,15,22,\ldots,24$,

$$M^{ab}\equiv\begin{bmatrix}0 & \mathbb{1}_3 & 0 & 0\\ \mathbb{1}_3 & 0 & 0 & 0\\ 0 & 0 & 0 & \mathbb{1}_3\\ 0 & 0 & \mathbb{1}_3 & 0\end{bmatrix},\quad a,b=9,\ldots,14,16,\ldots,21\,.$$

From the way in which the classical field θ is coupled to the gauge boson field V^a we conclude that the SU(5) symmetry is actually broken to an SU(3)×SU(2)×U(1) gauge symmetry, and the only particle fields which get masses from a vacuum expectation value of the Higgs field are the $X^{\pm 4/3)},Y^{(\pm 1/3)}$ (corresponding to $V^a,a=9,\ldots,14,16,\ldots,21$). If we have a look at the inverse-length-dimensional couplings generated by classical gravitation we observe that the relevant scale is $\dot\chi/\chi$, as it was advanced in the Introduction. In fact, this is the natural scale which is introduced by $\Gamma^\mu_{\nu\tau}$ and $R_{\mu\nu}$ after the rescaling $\chi V^i\to V^i,\chi x^i\to x^i$, and has the character of a geometrical scale related to the curvature of the space-time under consideration. In this sense and as a consequence of treating the gravitational field as an unperturbed background field, the dynamics of the gravitational interaction shows up only in an indirect way in the equations of motion for the quantum fields, as it may be seen from the fact that, although being responsible for the expansion of the space-time, the gravitational constant G does not enter in the geometrical scale $\dot\chi/\chi$. One must have in mind that this is true to the order of approximation at which we work but not necessarily in a self-consistent formulation of the problem, which would include the semiclassical version of the Einstein equations and could make corrections arise to the $\chi(t)\propto t^{1/2}$ law.

For the $k=0$ Robertson-Walker model, $\dot\chi/\chi$ is the inverse of the universal time. As the only natural mass scale which appears in the vacuum effective potential for the SU(5) model is the vacuum expectation value of the Higgs field σ, this is the scale with which $\dot\chi/\chi$ must be compared in order to find out the period of time in which the effective potential may be different from its flat—space-time form. We are not going to allow, anyway, the $\dot\chi/\chi$ mass scale to reach the Planck mass, $G^{-1/2}$, as this is the threshold above which quantum gravitational effects can become important. But, as far as we take as a basic assumption the existence of a grand-unification mass scale different from the Planck mass, there still remains a range, $\sigma<\dot\chi/\chi<G^{-1/2}$, in which relevant effects due to the presence of the classical gravitational field may appear.

Now we return to a few technical details of the calculation. To get a well-defined functional integral analytical continuation to Euclidean space-time is done in the form

$$x_4 = ix_0\ ,\quad V_4 = iV_0\ ,$$
$$\frac{\dot{\chi}}{\chi} = \frac{d}{dx_0}\ln\chi = i\frac{d}{dx_4}\ln(-ix_4)^{1/2} = \frac{i}{2x_4} \equiv \frac{i}{2\tau}\ . \tag{16}$$

The Lagrangian is quadratic in the fields,

$$\hat{\mathscr{L}}_0 = \tfrac{1}{2}\phi^a(x)i(\mathscr{D}^{-1})^{ab}(x-y)\phi^b(y) + \phi^a(x)P^{ab}_\mu(x-y)V^{\mu b}(y) + \tfrac{1}{2}V^{\mu a}(x)(i\Delta^{-1}_{\mu\nu})^{ab}(x-y)V^{\nu b}(y)\ ,$$
$$i(\mathscr{D}^{-1})^{ab}(x-y) = \left[\Box + \frac{3}{2\tau}\partial_4 - m^2\right]\delta^{ab}\delta(x-y)\ ,$$
$$P^{ab}_\mu(x-y) = 5g\theta\left[-M^{ab}\frac{3}{2\tau}\delta_{4\mu} + M^{ab}\partial_\mu\right]\delta(x-y)\ ,$$
$$(i\Delta^{-1}_{\mu\nu})^{ab}(x-y) = \left[\Box\delta_{\mu\nu} - \left[1-\frac{1}{\alpha}\right]\partial_\mu\partial_\nu + \frac{3}{\alpha}\frac{1}{2\tau}\delta_{\mu 4}\partial_4\delta_{4\nu} - \frac{1}{2\tau}\delta_{\mu 4}\partial_i\delta_{i\nu} + \left[4-\frac{3}{\alpha}\right]\frac{1}{2\tau}\delta_{\mu i}\partial_i\delta_{4\nu}\right.$$
$$\left. + \frac{2}{\tau}\delta_{\mu i}\partial_4\delta_{i\nu} - \frac{3}{\alpha}\frac{1}{2\tau^2}\delta_{\mu 4}\delta_{4\nu} + \frac{1}{2\tau^2}\delta_{\mu i}\delta_{i\nu} - 25g^2\theta^2\delta_{\mu\nu}\right]\delta^{ab}\delta(x-y)\ ,$$
$$a,b = 9,\ldots,14,16,\ldots,21 \tag{17}$$

and the functional integral in

$$V_1(\theta) = i\ln\int\mathscr{D}\psi\exp\left[i\int d^4x\sqrt{-g_0}\,\hat{\mathscr{L}}_0(\theta,\psi)\right]$$
$$= i\ln\int\mathscr{D}\phi\,\mathscr{D}V\exp\left[i\int d^4x\left(\tfrac{1}{2}\phi i\mathscr{D}^{-1}\phi + VP\phi + \tfrac{1}{2}Vi\Delta^{-1}V\right)\right] \tag{18}$$

is trivial to compute. If we make the integration in the order

$$V_1(\theta) = i\ln\frac{1}{\det(i\Delta^{-1})^{1/2}}\int\mathscr{D}\phi\exp\left[i\int d^4x\left(\tfrac{1}{2}\phi i\mathscr{D}^{-1}\phi + \tfrac{1}{2}\phi Pi\Delta P\phi\right)\right]$$
$$= i\ln\frac{1}{\det(i\Delta^{-1})^{1/2}}\,\frac{1}{\det(i\mathscr{D}^{-1}+Pi\Delta P)^{1/2}} \tag{19}$$

we get an effective potential to the one-loop order of the form (Appendix A)

$$V(\theta) = \tfrac{15}{2}m^2\theta^2 + 12\,\mathrm{Tr}\ln\left[k^2 + 25g^2\theta^2 - \frac{1}{2\tau^2}\right] + 6\,\mathrm{Tr}\ln(k^6 + a'k^4 + b'k^2 + c')\ , \tag{20}$$

where

$$a' \equiv 25g^2\theta^2 + m^2 + \frac{3}{4\tau^2}\ ,$$
$$b' \equiv m^2 50g^2\theta^2 - \frac{5}{4\tau^2}25g^2\theta^2 + \frac{3}{4\tau^2}m^2 - \frac{3}{4\tau^4} + \frac{1}{4\tau^2}k_4{}^2\ , \tag{21}$$
$$c' \equiv 625g^4\theta^4\left[m^2 - \frac{9}{4\tau^2}\right] + m^2\frac{1}{\tau^2}25g^2\theta^2 + \frac{9}{8\tau^4}25g^2\theta^2 + \frac{1}{2\tau^2}k_4{}^2 25g^2\theta^2 - m^2\frac{3}{4\tau^4} + m^2\frac{1}{4\tau^2}k_4{}^2\ ,$$

the traces and logarithms being defined in the functional sense.

This is the formal expression for the effective potential that we will use henceforth to study the symmetry breaking of the SU(5) model during the evolution of the treated cosmological model. The trace for each one of the operators has to be defined in accordance with the boundary conditions imposed by the choice of the vacuum state of the theory. In our case we are going to suppose, as an *a priori* simplification, that the thermodynamic equilibrium is maintained during the cosmological expansion for temperatures under the Planck mass, whereby we are going to take the statistical mixture of the states of a canonical ensemble at temperature T as a natural choice for the vacuum state of the quantum model. This implies, in turn, that the trace has to be defined over an ensemble of discrete frequencies $\omega_n = 2\pi nT$, that is the standard way by which finite-temperature effects are introduced. The outstanding feature of this effective potential is the presence of the parameter τ related to the background gravitational field. This mass scale τ is connected with the temperature within the framework of the standard model by the relation between universal time and temperature in early universe (8), and this fact forces us to study the evolution of the effective potential as a function of only a free mass parameter, that we will choose as the temperature. We will keep in mind, however, that this is an evolution in the course of the cosmological expansion of the universe, in which the background gravitational field may play a significant role.

The next section deals with the regularization of the formal expression (20) at zero temperature, in order to control its ultraviolet behavior. As is well known, the finite-temperature contributions do not spoil the procedure of renormalization at zero temperature, and this fact allows us to postpone the inclusion of the finite-temperature effects until the final discussion of the results.

III. RENORMALIZATION

As a method of renormalization we will choose zeta-function regularization,[15,19] following the approach adopted by Hawking to renormalize path integrals of Gaussian integrand.[15] As will be seen, this method demands a rather more tedious calculation than dimensional regularization but is deprived of such ambiguities as counterterms depending on the physical parameter τ^{-1}.

Our purpose is to give a finite expression for a functional integral of the form

$$Z=\int \mathscr{D}\phi \exp\left[-\tfrac{1}{2}\int d^4x\sqrt{-g_0}\,\phi A\phi\right], \qquad (22)$$

A being a real, positive-definite operator. If we call λ_n the eigenvalues of A and ϕ_n their respective eigenfunctions, then

$$A\phi_n=\lambda_n\phi_n ,$$

$\{\phi_n\}$ being an orthonormal complete set in the space of ϕ fields

$$\int d^4x\sqrt{-g_0}\,\phi_n\phi_m=\delta_{nm} , \quad \phi=\sum_n a_n\phi_n .$$

By defining the functional measure in the form $\phi=\prod_n \mu\, da_n$, we get an expression for the functional integral,

$$Z=\int \mathscr{D}\phi \exp\left[-\tfrac{1}{2}\int d^4x\sqrt{-g_0}\,\phi A\phi\right] =\prod_n \tfrac{1}{2}\mu\pi^{1/2}\lambda_n{}^{-1/2} . \qquad (23)$$

In our case, this expression has a purely formal sense as far as the determinant $\prod_n \lambda_n$ of the operators which enter in our functional integral diverges, and some kind of regularization procedure has to be devised to give a finite expression to Z. The zeta-function regularization method consists in defining a generalized zeta function of the form

$$\zeta(s)=\sum_n \frac{1}{\lambda_n{}^s} \qquad (24)$$

and through this $\zeta(s)$,

$$\ln Z=\ln\det\left[\frac{A}{\bar\mu^2}\right]^{-1/2}=\tfrac{1}{2}\zeta'(0)+\tfrac{1}{2}\ln\bar\mu^2\zeta(0) \qquad (25)$$

which is now a finite expression since the zeta function $\zeta(s)$ and its derivative $\zeta'(s)$ have a regular limit $s\to 0$.

Let us now show that for the particular form of our operators, defined through their representations in momentum space as

$$D_1\equiv k^2+25g^2\theta^2-\frac{1}{2\tau^2} , \qquad (26)$$

$$D_2\equiv k^6+\left[25g^2\theta^2+\frac{3}{4\tau^2}+m^2\right]k^4+\frac{1}{4\tau^2}k_4{}^2k^2+\left[\frac{3}{4\tau^2}m^2+50g^2\theta^2m^2-25g^2\theta^2\frac{5}{4\tau^2}-\frac{3}{4\tau^4}\right]k^2+\frac{1}{4\tau^2}m^2k_4{}^2$$
$$+25g^2\theta^2\frac{1}{2\tau^2}k_4{}^2+625g^4\theta^4\left[m^2-\frac{9}{4\tau^2}\right]+25g^2\theta^2\frac{1}{\tau^2}m^2-m^2\frac{3}{4\tau^4}+25g^2\theta^2\frac{9}{8\tau^4} , \qquad (27)$$

a zeta-function regularization may be accomplished to give a finite expression to the formal one:

$$V_1(\theta)=12\ln\det\left[\frac{D_1}{\mu^2}\right]+6\ln\det\left[\frac{D_2}{\mu^6}\right] . \qquad (28)$$

Using zeta-function regularization we define

$$\ln\det\left[\frac{D_1}{\mu^2}\right]\equiv-\zeta_1'(0)-\ln\mu^2\zeta_1(0) , \quad \zeta_1(s)=\int_{-\infty}^{+\infty}\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+25g^2\theta^2-1/2\tau^2)^s} \qquad (29)$$

and

$$\ln\det\left[\frac{D_2}{\mu^6}\right]\equiv-\zeta_2'(0)-\ln\mu^6\zeta_2(0) , \quad \zeta_2(s)=\int_{-\infty}^{+\infty}\frac{d^4k}{(2\pi)^4}\frac{1}{(k^6+a'k^4+b'k^2+c')^s} , \qquad (30)$$

where

$$a' \equiv 25g^2\theta^2 + \frac{3}{4\tau^2} + m^2 ,$$

$$b' \equiv 50g^2\theta^2 m^2 - 25g^2\theta^2 \frac{5}{4\tau^2} m^2 + \frac{3}{4\tau^2} m^2 - \frac{3}{4\tau^4} + \frac{1}{4\tau^2} k_4^2 ,$$

$$c' \equiv 625g^4\theta^4 \left[m^2 - \frac{9}{4\tau^2} \right] + 25g^2\theta^2 m^2 \frac{1}{\tau^2} + 25g^2\theta^2 \frac{9}{8\tau^4} - m^2 \frac{3}{4\tau^4} + \frac{1}{4\tau^2} m^2 k_4^2 + 25g^2\theta^2 \frac{1}{2\tau^2} k_4^2 .$$

In these expressions μ is a parameter with dimensions of mass which has to be thought of as a standard renormalization mass scale, adjusted so that our theory is in agreement with some experimental parameter. In our case this simply means that μ has to be fixed in order for the flat space-time effective potential to have its minimum at a value of θ in accordance with the expected grand-unification mass scale

$$\left. \frac{dV(\theta)}{d\theta} \right|_{\substack{\theta=\sigma \\ \dot{\chi}/\chi \to 0}} = 0 . \tag{31}$$

In what follows we explicitly show that $\ln \det(D_1/\mu^2)$ is a finite quantity in terms of the zeta function $\zeta_1(s)$. As we know, the generalized zeta function $\zeta_1(s)$ admits a regular limit $s \to 0$, but a convenient transformation has to be made in its integral representation in order to take this limit. This last is defined for $s > 2$, but after an integration by parts

$$\begin{aligned} \zeta_1(s) &= \int_{-\infty}^{+\infty} \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + 25g^2\theta^2 - 1/2\tau^2)^s} = \frac{1}{(4\pi)^2} \int_0^\infty dx \frac{x}{(x + 25g^2\theta^2 - 1/2\tau^2)^s} \\ &= \frac{1}{(4\pi)^2} \left[\frac{x^2}{(x + 25g^2\theta^2 - 1/2\tau^2)^s} \right]_0^\infty - \frac{1}{(4\pi)^2} \int_0^\infty \frac{x\,dx}{(x + 25g^2\theta^2 - 1/2\tau^2)^s} + \frac{1}{(4\pi)^2} s \int_0^\infty \frac{x^2 dx}{(x + 25g^2\theta^2 - 1/2\tau^2)^{s+1}} \end{aligned} \tag{32}$$

we get an expression

$$(2-s)\zeta_1(s) = -s \left[25g^2\theta^2 - \frac{1}{2\tau^2} \right] \frac{1}{(4\pi)^2} \int_0^\infty dx \frac{x}{(x + 25g^2\theta^2 - 1/2\tau^2)^{s+1}} \tag{33}$$

which can be analytically extended for $s > 1$. By making reiterated use of this procedure we get at the end an expression in which the limit $s \to 0$ can be taken. We have

$$(2-s)(1-s)\zeta_1(s) = (s+2)(s+1) \left[25g^2\theta^2 - \frac{1}{2\tau^2} \right]^3 \frac{1}{(4\pi)^2} \int_0^\infty dx \frac{x}{(x + 25g^2\theta^2 - 1/2\tau^2)^{s+3}} , \tag{34}$$

$$\zeta_1(0) = \left[25g^2\theta^2 - \frac{1}{2\tau^2} \right]^3 \frac{1}{(4\pi)^2} \int_0^\infty dx \frac{x}{(x + 25g^2\theta^2 - 1/2\tau^2)^3} , \tag{35}$$

$$\begin{aligned} \zeta_1'(0) &= 3 \left[25g^2\theta^2 - \frac{1}{2\tau^2} \right]^3 \frac{1}{(4\pi)^2} \int_0^\infty dx \frac{x}{(x + 25g^2\theta^2 - 1/2\tau^2)^3} \\ &\quad - \left[25g^2\theta^2 - \frac{1}{2\tau^2} \right]^3 \frac{1}{(4\pi)^2} \int_0^\infty dx \frac{x \ln(x + 25g^2\theta^2 - 1/2\tau^2)}{(x + 25g^2\theta^2 - 1/2\tau^2)^3} , \end{aligned} \tag{36}$$

and finally

$$\ln \det \left[\frac{D_1}{\mu^2} \right] = \frac{1}{(4\pi)^2} \left[\frac{(25g^2\theta^2 - 1/2\tau^2)^2}{2} \ln \frac{25g^2\theta^2 - 1/2\tau^2}{\mu^2} - \frac{3}{4} \left[25g^2\theta^2 - \frac{1}{2\tau^2} \right]^2 \right] . \tag{37}$$

This expression is the same that would have been obtained by the method of dimensional regularization, which shows that this and zeta-function regularization are equivalent in the case of renormalization of an operator quadratic in k. The nonvanishing imaginary part that at first sight arises for θ values $25g^2\theta^2 < 1/2\tau^2$ has no physical meaning and, in fact, does not survive in the final expression for the effective potential. We have to remember that the τ parameter, as it stands in the previous formulas, is not a physical quantity, and that only if the transformation $\tau \to i\tau$ is inverted can it be identified with the rate of expansion of the cosmological model. We will delay this rotation until we have formed the complete expression of the renormalized effective potential at the end of this section.

The finite expression for $\ln \det(D_2/\mu^6)$ is obtained in Appendix B in terms of the three roots β, δ, r (real or complex) of the polynomial $k^6 + a'k^4 + b'k^2 + c'$. It is worthwhile remarking that this result for $\ln \det(D_2/\mu^6)$ not only is not the same that would have been obtained by dimensional regularization, but even differs from the one which would have been obtained if more than one zeta function, associated to different factors entering D_2, had been employed in its definition. Nevertheless, it seems more natural to carry out the calculation as it has been done here with only a zeta function, as long as there is only one mass scale μ associated to the physical dimension of the ϕ field.

The final expression for the renormalized zero-temperature part of the effective potential is

$$\begin{aligned} V(\theta)=&\tfrac{15}{2}m^2\theta^2+\frac{12}{(4\pi)^2}\left[\frac{(25g^2\theta^2-1/2\tau^2)^2}{2}\ln\frac{25g^2\theta^2-1/2\tau^2}{\mu^2}-\frac{3}{4}\left[25g^2\theta^2-\frac{1}{2\tau^2}\right]^2\right]\\ &+\frac{6}{(4\pi)^2}\left[\tfrac{1}{6}(\beta^2+\delta^2+r^2)\left[\ln\frac{-\beta}{\mu^2}+\ln\frac{-\delta}{\mu^2}+\ln\frac{-r}{\mu^2}\right]-\tfrac{3}{4}(\beta^2+\delta^2+r^2)-\beta\delta-\delta r-\beta r-(\tfrac{1}{3}\beta^3+\tfrac{1}{3}\delta^3)J(1,1,0)\right.\\ &-(\tfrac{1}{3}\beta^3+\tfrac{1}{3}r^3)J(1,0,1)-(\tfrac{1}{3}\delta^3+\tfrac{1}{3}r^3)J(0,1,1)-(\tfrac{1}{6}\beta^4-\tfrac{1}{2}\beta^2\delta^2)J(2,1,0)-(\tfrac{1}{6}\delta^4-\tfrac{1}{2}\delta^2\beta^2)J(1,2,0)\\ &-(\tfrac{1}{6}r^4-\tfrac{1}{2}\beta^2r^2)J(1,0,2)-(\tfrac{1}{6}r^4-\tfrac{1}{2}\delta^2r^2)J(0,1,2)\\ &\left.-(\tfrac{1}{6}\beta^4-\tfrac{1}{2}\beta^2r^2)J(2,0,1)-(\tfrac{1}{6}\delta^4-\tfrac{1}{2}\delta^2r^2)J(0,2,1)\right],\\ J(a,b,c)\equiv&\int_0^\infty dx\frac{1}{(x-\beta)^a(x-\delta)^b(x-r)^c}\,. \end{aligned} \tag{38}$$

This expression contains the parameters g^2, m^2, μ^2, which naturally appear in the description of the quantum effective potential, and a parameter τ which is related to the classical background gravitational field. By inverting the Wick rotation $\tau\rightarrow i\tau$, we can identify this parameter with the universal time of the $k=0$ Robertson-Walker model, while $1/2\tau$ becomes the rate of expansion of this cosmological model. Once this transformation has been done, the effective potential turns out to be

$$\begin{aligned} V(\theta)=&\tfrac{15}{2}m^2\theta^2+\frac{12}{(4\pi)^2}\left[\frac{(25g^2\theta^2+1/2\tau^2)^2}{2}\ln\frac{25g^2\theta^2+1/2\tau^2}{\mu^2}-\frac{3}{4}\left[25g^2\theta^2+\frac{1}{2\tau^2}\right]^2\right]+6\ln\det\left[\frac{D_2}{\mu^6}\right],\\ D_2\equiv&k^6+a'k^4+b'k^2+c',\\ a'\equiv&25g^2\theta^2-\frac{13}{16\tau^2}+m^2,\\ b'\equiv&50g^2\theta^2m^2+25g^2\theta^2\frac{9}{8\tau^2}-\frac{13}{16\tau^2}m^2-\frac{3}{4\tau^4},\\ c'\equiv&625g^4\theta^4\left[m^2+\frac{9}{4\tau^2}\right]-25g^2\theta^2m^2\frac{1}{\tau^2}+25g^2\theta^2\frac{9}{8\tau^4}-m^2\frac{3}{4\tau^4}, \end{aligned} \tag{39}$$

which is a real definite quantity as long as the J integrals are defined through their principal values. In this sense, the zero-temperature effective potential does not introduce any kind of singularity even when one of the three roots β,δ,r, gets a negative value.

On dealing with the definition of the renormalized parameters of the quantum model, we are going to take the flat—space-time limit $\dot{X}/X\rightarrow 0$, in which these quantities can be related to observable magnitudes, as it is usually done in flat—space-time quantum field theory. For the purpose of circumventing the arbitrariness on the value of the renormalized mass m_R of the Higgs field

$$\left.\frac{d^2V(\theta)}{d\theta^2}\right|_{\substack{\theta=\sigma\\ \dot{X}/X\rightarrow 0}}=m_R{}^2 \tag{40}$$

we are going to set the bare mass in the Lagrangian equal to zero, i.e., we are going to deal in what follows with the effective potential in the Coleman-Weinberg mode $m=0$.

The remaining free mass parameter, the renormalization mass scale μ, must be adjusted in order to fix the minimum σ of the effective potential

$$\left.\frac{dV(\theta)}{d\theta}\right|_{\substack{\theta=\sigma\\ \dot{X}/X\rightarrow 0}}=0 \tag{41}$$

at the point which gives the desired values for the renormalized masses of the $X^{(\pm 4/3)}, Y^{(\pm 1/3)}$. It is to be noticed that the limit $\dot{X}/X\rightarrow 0$, $m^2\rightarrow 0$, is a regular limit in the above expression for the effective potential. It can be explicitly taken in the form $\delta\rightarrow 0, r\rightarrow 0, \beta\rightarrow 25g^2\theta^2$. The resulting expression

$$V(\theta)\underset{\substack{\dot{X}/X\rightarrow 0\\ m^2\rightarrow 0}}{\longrightarrow}\frac{1}{(4\pi)^2}\left[9(25g^2\theta^2)\ln\frac{25g^2\theta^2}{\mu^2}-\frac{23}{2}(25g^2\theta^2)^2\right] \tag{42}$$

agrees with the naive flat—space-time form of the effective potential that is found in other schemes of renormalization. After imposing the condition $V'(\sigma)=0$, we get the relation $\mu^2=25g^2\sigma^2e^{-7/9}$. Finally, the remaining free parameter, the gauge coupling constant g, is not affected by the renormalization in the scalar sector of the model, which allows us to identify it with the renormalized gauge coupling constant which shows up in the unification of strong, weak, and electromagnetic interactions at the grand-unification scale.

At this point we have an effective potential which is defined in terms of renormalized parameters g and σ. In the following we are going to assign the standard values $g^2/4\pi=\frac{1}{42}$, $\sigma=10^{15}$ GeV, which are assumed in the flat—space-time quantum field theory. This is a fair procedure only if the classical gravitational background field does not affect the description of the quantum model between the grand-unification scale and the low-energy scales in the present world. In other words stated, one has to be sure that the running of the different renormalized coupling constants is not affected by the background field in such a way that it might modify the standard grand-unification scale. The results that we are going to obtain for the effective potential support, *a posteriori*, the standard procedure by which the scale σ is obtained, since it can be shown that classical gravity concentrates its influence in the period of time between the Planck scale and some scale above the grand-unification one. The representation of the θ function (39) when $(1/\tau^2)$ runs from $0.12\times10^{-4}\sigma^2$ to $0.49\times10^{7}\sigma^2$ is shown in Figs. 1–4

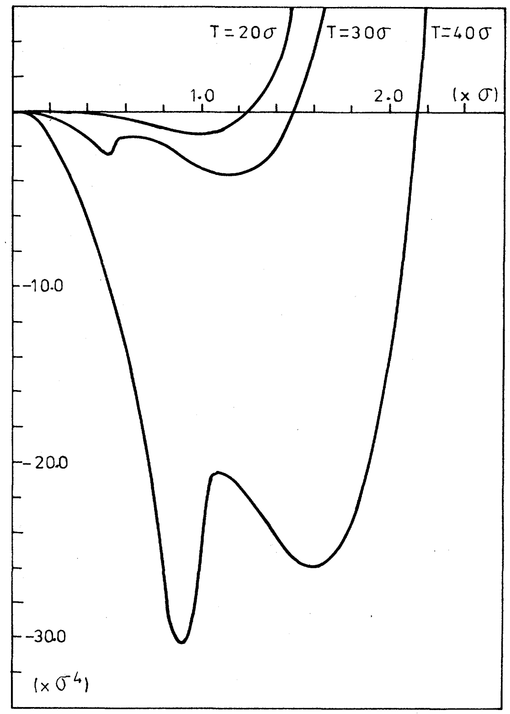

FIG. 2. The zero-temperature part of the effective potential for temperatures $T=20\sigma$, $T=30\sigma$, and $T=40\sigma$ in the cosmological model.

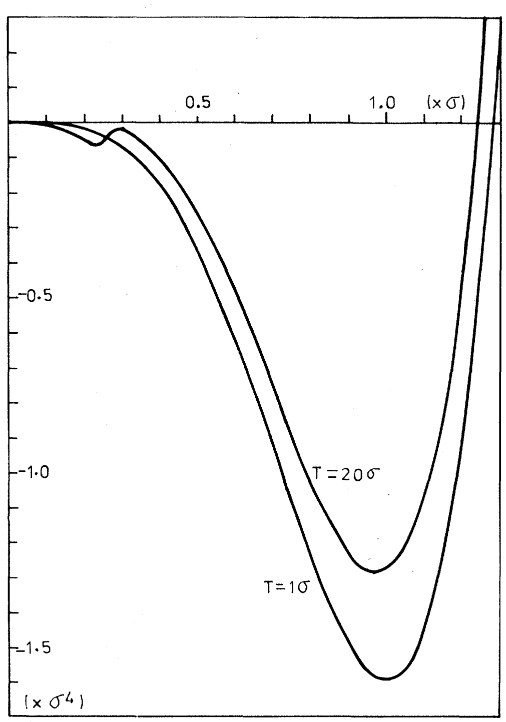

FIG. 1. The zero-temperature part of the effective potential for temperatures $T=1\sigma$ and $T=20\sigma$ in the cosmological model.

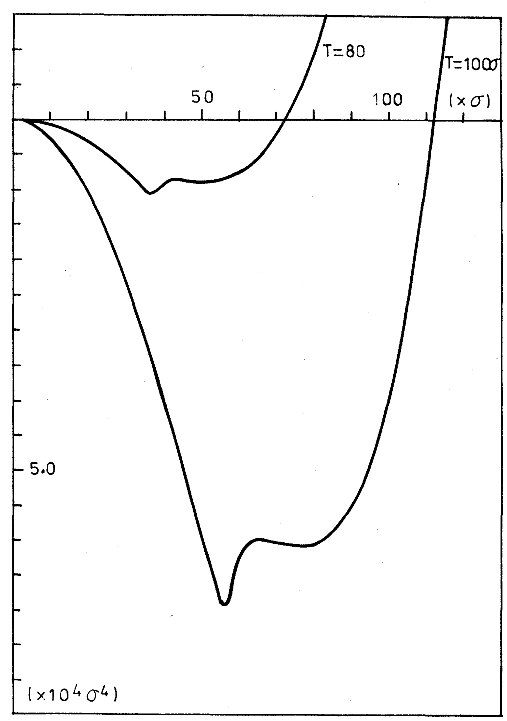

FIG. 3. The zero-temperature part of the effective potential for temperatures $T=80\sigma$ and $T=100\sigma$ in the cosmological model.

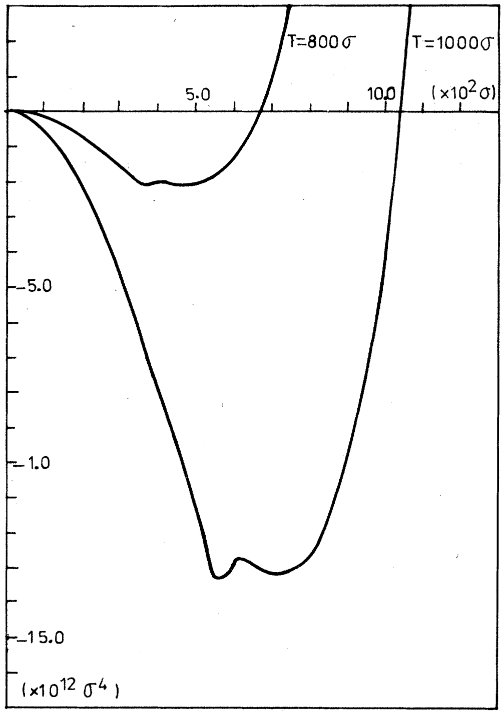

FIG. 4. The zero-temperature part of the effective potential for temperatures $T=800\sigma$ and $T=1000\sigma$ in the cosmological model.

which display the evolution of the zero-temperature part of the effective potential for respective temperatures in the cosmological model from $T=1\sigma$ to $T=800\sigma$. Although this is the evolution of the part of the effective potential that does not take into account the finite temperature contributions, it has two features that are worth thinking about before the computation of the finite-temperature effects. First, it can be seen from Fig. 1 that the gravitational effects that we have introduced in the grand unified model do not appreciably modify the effective potential for temperatures of the cosmological model below $T=1\sigma$, as long as for this temperature the vacuum contribution to the effective potential is superposed to that for $T=0$, i.e., $\dot{\chi}/\chi=0$. In this way, even though we have not computed the finite-temperature effects that presumably lead to the restoration of the gauge symmetry, we are in conditions to assure that the background gravitational field effects do not affect the mechanism of the transition from the symmetric vacuum state $\theta=0$ to the asymmetric one $\theta\neq0$, if this takes place for a temperature below $T=1\sigma$. On the other hand, the presence of the background gravitational field leads to a stronger symmetry breakdown than that present in the flat—space-time grand unified model. As can be seen in Figs. 2—4, this is reflected in the shift of the θ value of the minimum for the zero-temperature effective potential, and in the lowering of its effective potential value for several orders of magnitude. Having in mind that the finite-temperature contributions that have to be added to the vacuum effective potential are going to favor the restoration of the gauge symmetry for temperatures above $T=1\sigma$, we may expect a confrontation between these finite-temperature effects and the classical gravitational effects which, as we have shown in the previous figures, work in the opposite direction to the restoration of the gauge symmetry.

In the following section we accomplish the computation of the finite-temperature contributions to the effective potential that, as has been pointed out, do not need further renormalization. We match these two kinds of effects in the final expression for the effective potential at the one-loop level, and, lying in the range of temperatures between $T=1\sigma$ and $T=800\sigma$, try to show what picture arises for the evolution of the effective potential when the restoration due to the finite-temperature effects and the enhancing of the symmetry breaking due to the classical gravitational field are taken into account.

IV. FINITE-TEMPERATURE EFFECTS

In this section we deal with the computation of the finite-temperature contributions to the effective potential of the model. In the preceding section we have made use of specific boundary conditions in the definition of $\ln\det(D_1/\mu^2)$ and $\ln\det(D_2/\mu^6)$, which have allowed us to proceed to the renormalization of these determinants. Returning to the physical aspect of the problem, we have to remember that these boundary conditions are, in fact, determined by the hypothesis of thermodynamic equilibrium at temperature T of the system of elementary particles—a hypothesis that we have adopted as a first approximation to the problem from the start.

As is well known, the hypothesis of thermodynamic equilibrium implies the periodicity of the configurations for the bosonic fields in the Euclidean variable x_4, with period $\beta\equiv1/T$ (Ref. 20). This, in turn, provides the boundary conditions needed for the calculation of $\ln\det(D_1/\mu^2)$ and $\ln\det(D_2/\mu^6)$, under the form that the determinants of these operators must include only the product over discrete frequencies $k_4\equiv\omega_n=2\pi n/\beta$. Alternatively, the definition of the zeta functions $\zeta_1(s),\zeta_2(s)$, turns out to be

$$\zeta_1(s)=\frac{1}{(2\pi)^3}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int d^3k\frac{1}{(\omega_n{}^2+\vec{k}^2+25g^2\theta^2+1/2\tau^2)^s} \tag{43}$$

and

$$\zeta_2(s)=\frac{1}{(2\pi)^3}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int d^3k\frac{1}{(\omega_n{}^2+\vec{k}^2+c^2)^s(\omega_n{}^2+\vec{k}^2+d^2)^s(\omega_n{}^2+\vec{k}^2+\bar{d}^2)^s}\ , \tag{44}$$

$$(x+c^2)(x+d^2)(x+\bar{d}^2)\equiv x^3+a'x^2+b'x+c'\ .$$

Our first purpose is to relate these $\zeta_1(s)$ and $\zeta_2(s)$ with the zero-temperature zeta functions of the preceding section, that we will call in what follows $\zeta_{1\,\text{vac}}(s),\zeta_{2\,\text{vac}}(s)$.

As an illustration, we deal with $\zeta_1(s)$. After the set of transformations

$$\begin{aligned}\zeta_1(s)&=\frac{1}{(2\pi)^3}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int d^3k\frac{1}{(\omega_n{}^2+\vec{k}^2+a^2)^s}=\frac{1}{\beta}\frac{1}{(2\pi)^3}\int d^3k\frac{1}{\Gamma(s)}\sum_{n=-\infty}^{+\infty}\int_0^\infty dx\,x^{s-1}e^{-(\omega_n{}^2+\vec{k}^2+a^2)x}\\&=\frac{1}{\beta}\frac{1}{2\pi^2}\frac{1}{\Gamma(s)}\int_0^\infty dx\,x^{s-1}\sum_{n=-\infty}^{+\infty}\int_0^\infty d\hat{k}\,\hat{k}^2e^{-(\hat{k}^2+\omega_n{}^2+a^2)x}\\&=\frac{1}{\beta}\frac{1}{2\pi^2}\frac{1}{\Gamma(s)}\sum_{n=-\infty}^{+\infty}\frac{\pi^{1/2}}{4}\int_0^\infty dx\,x^{s-5/2}e^{-(\omega_n{}^2+a^2)x}\\&=\frac{1}{\beta}\frac{1}{8\pi^{3/2}}\frac{\Gamma(s-3/2)}{\Gamma(s)}\sum_{n=-\infty}^{+\infty}\frac{1}{(\omega_n{}^2+a^2)^{s-3/2}}\,,\end{aligned}\tag{45}$$

we are able to express $\zeta_1(s)$ in terms of one series alone. By means of a Sommerfeld-Watson transformation we can rewrite

$$\sum_{n=-\infty}^{+\infty}\frac{1}{(\omega_n{}^2+a^2)^{s-3/2}}=\frac{1}{2\pi i}\oint_{C_0}\frac{i\beta e^{iz\beta}/(e^{iz\beta}-1)}{(z^2+a^2)^{s-3/2}}dz\,,\tag{46}$$

where C_0 (Fig. 5) is a contour that only contains the poles $\omega_n=2\pi n/\beta$ in the real axis. If we restrict the s variable in the form $\frac{5}{2}>s>2$, the C_0 contour can be deformed to follow the branch cuts drawn from $z=\pm ia$ (Fig. 6). The integral becomes now

$$\sum_{n=-\infty}^{+\infty}\frac{1}{(\omega_n{}^2+a^2)^{s-3/2}}=\frac{\beta}{2\pi}\left[2\cos\pi s\int_a^\infty dt\frac{e^{t\beta/2}+e^{-t\beta/2}}{e^{t\beta/2}-e^{-t\beta/2}}\frac{1}{|t-a|^{s-3/2}|t+a|^{s-3/2}}\right]\tag{47}$$

and the expression for $\zeta_1(s)$:

$$\zeta_1(s)=\frac{1}{8\pi^{5/2}}\frac{\Gamma(s-\frac{3}{2})}{\Gamma(s)}\cos\pi s\int_a^\infty dt\frac{e^{t\beta/2}+e^{-t\beta/2}}{e^{t\beta/2}-e^{-t\beta/2}}\frac{1}{|t-a|^{s-3/2}|t+a|^{s-3/2}}\,.\tag{48}$$

After decomposing it into a zero-temperature part and a finite-temperature contribution,

$$\begin{aligned}\zeta_1(s)&=\frac{1}{8\pi^{5/2}}\frac{\Gamma(s-\frac{3}{2})}{\Gamma(s)}\cos\pi s\int_a^\infty\frac{1}{(t^2-a^2)^{s-3/2}}dt+\frac{1}{4\pi^{5/2}}\frac{\Gamma(s-\frac{3}{2})}{\Gamma(s)}\cos\pi s\int_a^\infty\frac{e^{-t\beta}}{1-e^{-t\beta}}\frac{1}{(t^2-a^2)^{s-3/2}}\\&\equiv\zeta_{1\,\text{vac}}(s)+\zeta_{1T}(s)\end{aligned}\tag{49}$$

we recognize all the divergences concentrated in the part that survives in the limit $T\to 0$, from what follows that only the vacuum contribution has to be conveniently regularized. By the same procedure of the preceding section, we can extend this contribution, after an integration by parts, to an expression which has a regular limit $s\to 0$. The two first orders of the zeta function are

$$\zeta_1(s)=\frac{1}{8\pi^2}\left[\frac{a^4}{4}+s\left[-\frac{a^4}{4}\ln a^2+\frac{3a^4}{8}\right]\right]+\frac{1}{3\pi^2}s\int_a^\infty dt\frac{e^{-t\beta}}{1-e^{-t\beta}}(t^2-a^2)^{3/2}+O(s^2)\tag{50}$$

from which we can obtain

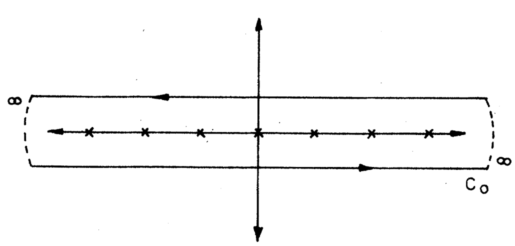

FIG. 5. The C_0 contour contains all the poles $\omega_n=2\pi n/\beta$ in the z plane.

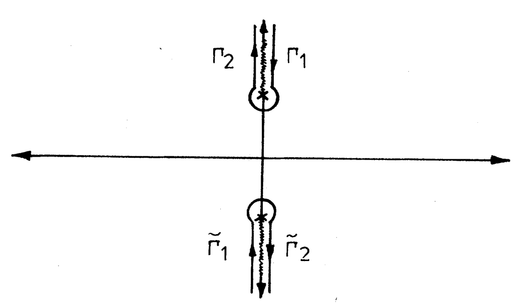

FIG. 6. The z integration is done along the two branch cuts shown in the figure.

$$\zeta_1(0)=\frac{1}{32\pi^2}a^4\,, \tag{51}$$

$$\zeta_1'(0)=-\frac{1}{32\pi}a^4\ln a^2+\frac{3}{64\pi^2}a^4+\frac{1}{3\pi^2}\int_a^\infty dt\frac{e^{-t\beta}}{1-e^{-t\beta}}(t^2-a^2)^{3/2}\,. \tag{52}$$

Both expressions agree in the limit $T\rightarrow 0$ with those obtained in the preceding section for the zero-temperature part of the effective potential. The renormalized expression

$$\begin{aligned}\ln\det\left[\frac{D_1}{\mu^2}\right]=-\zeta_1'(0)-\ln\mu^2\zeta_1(0)&=\frac{1}{32\pi^2}\left[a^4\ln\frac{a^2}{\mu^2}-\frac{3}{2}a^4\right]+\frac{1}{3\pi^2}\int_a^\infty dt\frac{e^{-t\beta}}{1-e^{-t\beta}}(t^2-a^2)^{3/2}\\&=\frac{1}{32\pi^2}\left[a^4\ln\frac{a^2}{\mu^2}-\frac{3}{2}a^4\right]+\frac{1}{\pi^2}T^4\int_0^\infty dx\,x^2\ln(1-e^{[x^2+(a\beta)^2]^{1/2}})\end{aligned} \tag{53}$$

also reproduces the standard form for the trace of the logarithm obtained by other methods of renormalization.

The zeta function associated to the second operator

$$\zeta_2(s)=\frac{1}{(2\pi)^3}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int d^3k\frac{1}{(\omega_n{}^2+\vec{k}^2+c^2)^s(\omega_n{}^2+\vec{k}^2+d^2)^s(\omega_n{}^2+\vec{k}^2+\bar{d}^2)^s} \tag{54}$$

admits a similar additive decomposition into a zero-temperature part and a finite-temperature contribution. The easiest way to show it begins with making a Sommerfeld-Watson transformation

$$\zeta_2(s)=\frac{1}{(2\pi)^3}\frac{1}{\beta}\int d^3k\frac{1}{2\pi i}\oint_{C_0}\frac{i\beta[e^{iz\beta}/(e^{iz\beta}-1)]dz}{(z^2+\vec{k}^2+c^2)^s(z^2+\vec{k}^2+d^2)^s(z^2+\vec{k}^2+\bar{d}^2)^s}\,, \tag{55}$$

where C_0 is the same contour as in the above. In the present situation, there are six branch cuts in the complex z plane, which are characterized through the definitions

$$\begin{aligned}&z^2+\vec{k}^2+c^2\equiv(z-i\gamma)(z+i\gamma)\,,\quad z^2+\vec{k}^2+d^2\equiv(z-\omega-i\delta)(z+\omega+i\delta)\,,\\&z^2+\vec{k}^2+\bar{d}^2\equiv(z-\omega+i\delta)(z+\omega-i\delta)\,.\end{aligned} \tag{56}$$

In this case, a restriction must be done in the s variable $1>s>\frac{2}{3}$ in order to deform the contour of integration along the branch cuts (Fig. 7). When this is accomplished, the integral representation of the zeta function $\zeta_2(s)$ takes the form

$$\begin{aligned}\zeta_2(s)=\frac{1}{(2\pi)^4}\int d^3k\Bigg[&2\sin\pi s\int_\gamma^\infty dt\frac{1}{|t^2-\gamma^2|^s[((t+\delta)^2+\omega^2)((t-\delta)^2+\omega^2)]^s}\\&+4\sin\pi s\int_\gamma^\infty dt\frac{e^{-t\beta}(1-e^{-t\beta})}{|t^2-\gamma^2|^s[((t+\delta)^2+\omega^2)((t-\delta)^2+\omega^2)]^s}\\&+2\sin\pi s\int_\delta^\infty dt\frac{1}{|t^2-\delta^2|^s[(\omega-it+i\gamma)(\omega-it-i\gamma)(2\omega-it-i\delta)(2\omega-it+i\delta)]^s}\\&+2\sin\pi s\int_\delta^\infty dt\frac{1}{|t^2-\delta^2|^s[(\omega+it-i\gamma)(\omega+it+i\gamma)(2\omega+it+i\delta)(2\omega+it-i\delta)]^s}\\&+4\sin\pi s\int_\delta^\infty dt\frac{(e^{i\omega\beta}e^{t\beta}-1)^{-1}}{|t^2-\delta^2|^s[(\omega-it+i\gamma)(\omega-it-i\gamma)(2\omega-it-i\delta)(2\omega-it+i\delta)]^s}\\&+4\sin\pi s\int_\delta^\infty dt\frac{(e^{-i\omega\beta}e^{t\beta}-1)^{-1}}{|t^2-\delta^2|^s[(\omega+it-i\gamma)(\omega+it+i\gamma)(2\omega+it+i\delta)(2\omega+it-i\delta)]^s}\Bigg]\\\equiv\zeta_{2\,\mathrm{vac}}(s)+\zeta_{2T}(s)\,.&\end{aligned} \tag{57}$$

For the definition of the renormalized expression $\ln\det(D_2/\mu^6)$ we need to know the first two orders in s in the expansion of $\zeta_2(s)$. The only contribution to the zeta function which has to be conveniently regularized is $\zeta_{2\,\mathrm{vac}}(s)$, whose first two orders in s have already been calculated in the preceding section. At this point we only need to take care of the finite-temperature contribution

$$\zeta_{2T}(s)=\frac{1}{\pi^2}s\int_0^\infty d\hat{k}\,\hat{k}^2\int_\gamma^\infty dt\frac{e^{-t\beta}}{1-e^{-t\beta}}+\frac{1}{\pi^2}s\int_0^\infty d\hat{k}\,\hat{k}^2\int_\delta^\infty dt\left[\frac{1}{e^{-i\omega\beta}e^{t\beta}-1}+\frac{1}{e^{i\omega\beta}e^{t\beta}-1}\right]+O(s^2)$$

$$=-s\frac{1}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-e^{-\gamma'})-s\frac{1}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-2\cos\omega' e^{-\delta'}+e^{-2\delta'})+O(s^2)\,,$$

$$\gamma'\equiv\left[\hat{k}^2+\left[\frac{c}{T}\right]^2\right]^{1/2},\quad \omega'+i\delta'\equiv\left[\hat{k}^2+\left[\frac{d}{T}\right]^2\right]^{1/2}. \tag{58}$$

From this expression one can easily read

$$\zeta_{2T}(0)=0\,, \tag{59}$$

$$\zeta'_{2T}(0)=-\frac{1}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-e^{-\gamma'})-\frac{1}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-2\cos\omega' e^{-\delta'}+e^{-2\delta'})\,, \tag{60}$$

and the renormalized $\ln\det(D_2/\mu^6)$ with the finite-temperature contributions explicitly shown takes the form

$$\ln\det\left[\frac{D_2}{\mu^6}\right]=-\zeta'_{2\,\mathrm{vac}}(0)-\ln\mu^6\zeta_{2\,\mathrm{vac}}(0)-\zeta'_{2T}(0)-\ln\mu^6\zeta_{2T}(0)$$

$$=\ln\det\left[\frac{D_2}{\mu^6}\right]\bigg|_{T=0}+\frac{1}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-e^{-\gamma'})+\frac{1}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-2\cos\omega' e^{-\delta'}+e^{-2\delta'})\,, \tag{61}$$

where $\ln\det(D_2/\mu^6)\,|_{T=0}$ stands for the renormalized expression (B8) from Appendix B.

At this point we possess an expression for the effective potential to the one-loop order which takes into account the effects due to the presence of the classical gravitation field as well as the finite-temperature effects

$$V(\theta)=12\ln\det\left[\frac{D_1}{\mu^2}\right]\bigg|_{T=0}+6\ln\det\left[\frac{D_2}{\mu^6}\right]\bigg|_{T=0}+\frac{12}{\pi^2}T^4\int_0^\infty dx\,x^2\ln(1-e^{-[x^2+(25g^2\theta^2+1/2\tau^2)/T^2]^{1/2}})$$

$$+\frac{6}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-e^{-\gamma'})+\frac{6}{\pi^2}T^4\int_0^\infty d\hat{k}\,\hat{k}^2\ln(1-2\cos\omega' e^{-\delta'}+e^{-2\delta'})\,, \tag{62}$$

where $\ln\det(D_1/\mu^2)\,|_{T=0}$ and $\ln\det(D_2/\mu^6)\,|_{T=0}$ stand, respectively, for the formulas (37) and (B8). We have learned from the representation of the zero-temperature part $V(\theta)\,|_{T=0}$ that the classical gravitational effects give rise to a stronger symmetry breaking than that present in the flat—space-time quantum field theory, whereby we can infer that in the above expression for the effective potential the tendency to restore the symmetry by the finite-temperature effects is going to contend with the enhancing of the symmetry breaking that the classical gravitational field generates. These two opposite effects have a strength that monotonically increases for the universal time reaching the Planck time scale or, equivalently, for the range of temperature from 10^{15} GeV to 10^{19} GeV. But, as long as the effective potential is insensitive to the background field effects at the mass scale of the grand unified model $T=1\sigma$, the balance in the competition between the two effects, if attained, can only take place at a higher temperature.

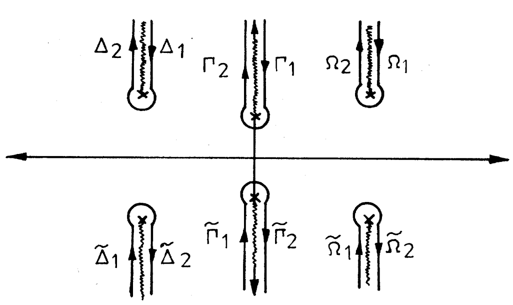

FIG. 7. The z integration is done along the six branch cuts shown in the figure.

In what follows we present the graphic representation of the $V(\theta)$ function (62) in order to show the evolution of the effective potential as the temperature of the cosmological scenario changes. For the sake of convenience, we are going to study this evolution in the direction of increasing temperatures, i.e., the direction opposite to the evolution of the universal time of the model, but this choice has no significance as long as we are not interested in the phase transitions that can take place in the quantum statistical model. In the above expression, the parameters of the quantum model are set equal to their flat—space-time values, as conceded in the preceding section, $g^2/4\pi=\frac{1}{42}$, $\sigma=10^{15}$ GeV, while the two remaining mass parameters are constrained by the relation between temperature and universal time $(\dot{X}/X)\propto G^{1/2}T^2$. In Figs. 8—11, the evolution of the effective potential is given in terms of the temperature of the cosmological model from $T=1\sigma$ to $T=800\sigma$. As could be expected, for a temperature $T=1\sigma$, the symmetry is restored due to the finite-temperature effects, and from inspection of Figs. 2,

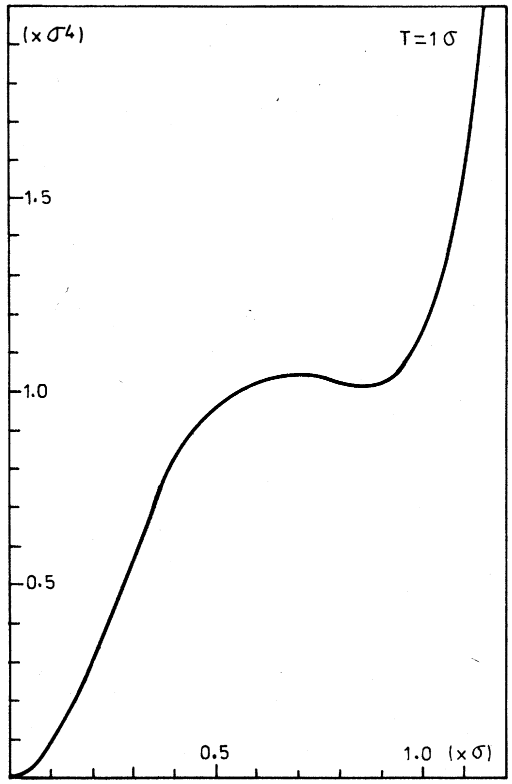

FIG. 8. The finite-temperature effective potential at $T=1\sigma$.

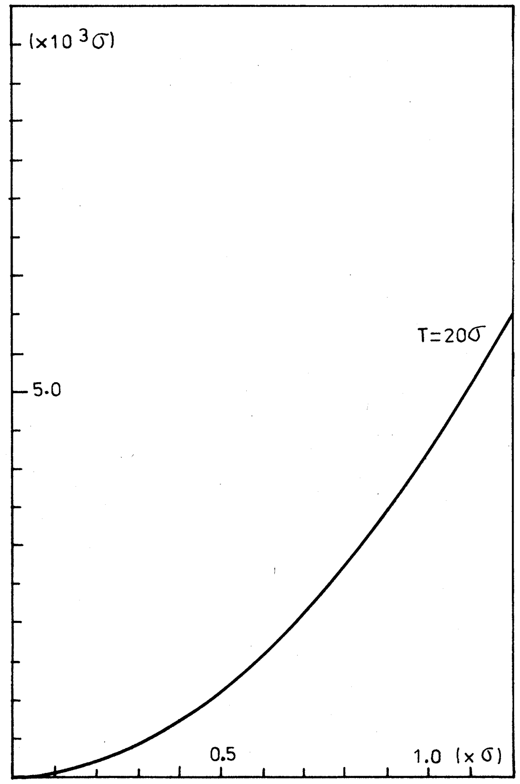

FIG. 9. The finite-temperature effective potential at $T=20\sigma$.

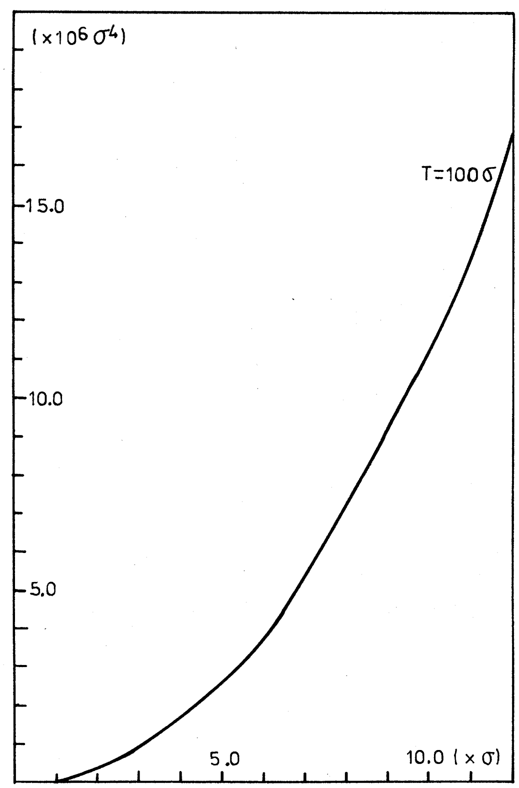

FIG. 10. The finite-temperature effective potential at $T=100\sigma$.

3, 9, and 10, one can conclude that these effects are more efficient in their role for the symmetry restoration than the classical gravitational effects in the opposite direction. Figure 11 shows, however, that there must be a point between $T=100\sigma$ and $T=800\sigma$ in which the two tendencies balance, giving rise to a situation for higher temperatures in which the effective potential is dominated by the symmetry-breaking effects associated with the presence of the background gravitational field. We do not follow the description of the effective potential for temperatures higher than $T=800\sigma$, for which the mass scale $\dot{X}/X=1.1\times10^{18}$ GeV is only one order of magnitude below the Planck scale, since there must exist a point as the $\dot{X}/X$ parameter rises to the Planck mass in which the fluctuations of the quantum gravitational field become important and his description as a classical background field does not make sense. However, although we do not know in what direction these fluctuations operate, we can assert the existence of a period of time after the Planck time in which the configuration $\theta=0$ is not a vacuum configuration of the grand unified model, not even a metastable vacuum configuration. This period of time corresponds with a temperature two orders of magnitude below the Planck mass, and the symmetry breaking that characterizes it appears to be completely washed out for temperatures of the order of the grand-unification mass scale $\sigma=10^{15}$ GeV. Although all these considerations suggest the existence of a transition from a broken phase $\theta\neq0$ to the restored one $\theta=0$ by finite-temperature ef-

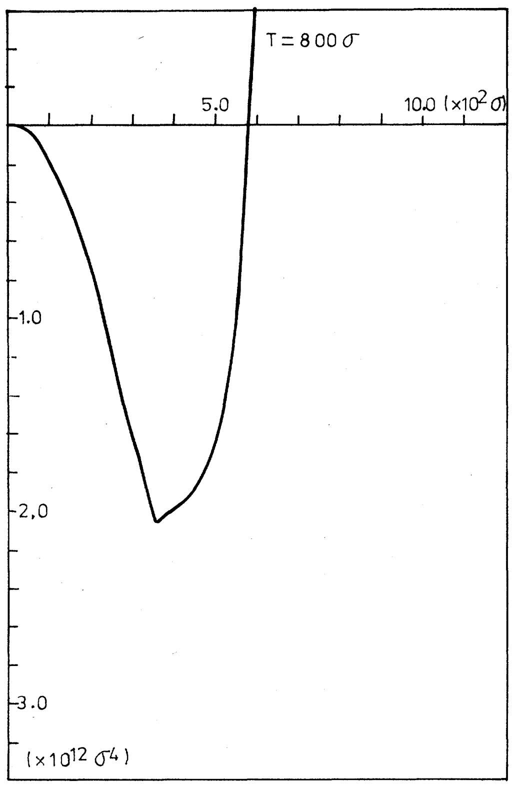

FIG. 11. The finite-temperature effective potential at $T=800\sigma$. The strong symmetry breakdown shown in the figure develops through classical gravitational effects for an expansion rate below the Planck mass, $\dot{\chi}/\chi = 1.1\times 10^{18}$ GeV.

fects, we are not going, for the time being, into the consequences of such a phase transition nor the existence of a primordial period of time with superheavy massive particles.

V. CONCLUSION

The main part of this paper has been devoted to working out the features that a classical background gravitational field can induce in the symmetry breaking of the SU(5) model. The importance of such an influence is revealed by the naive dimensional argument of Sec. II, that shows up $\dot{\chi}/\chi$ as the mass scale introduced by the classical gravitational couplings in the quantum model. The preceding section has ended with the explicit computation of these gravitational effects by representing the effective potential as a function of the temperature in the standard cosmological scenario. When the $k=0$ Robertson-Walker model is adopted as the metric of the space-time, the background gravitational field has the tendency to enhance the symmetry breaking that is postulated in the zero-temperature, flat—space-time quantum theory. Although this influence is overcome by the restoration due to the finite-temperature effects through two orders of magnitude in temperature above the mass scale of the SU(5) model, there exists a temperature for which the effective potential develops a strong symmetry breakdown due to the classical gravitational effects. This temperature is between one and two orders of magnitude below the Planck mass scale.

Although the features that are present in the evolution of the effective potential are rather surprising, the existence of such a strong symmetry breakdown can be explained once it is known that the background gravitational field tends to enhance the symmetry breaking of the model. The mass scales associated to the two opposite effects, $\dot{\chi}/\chi$ and T, tend to reach the Planck mass scale at the same time, and this explains why in the evolution of the effective potential the restoration due to the finite-temperature effects is balanced by the classical gravitational effects for sufficiently high temperature. From these considerations, it is also obvious that the results that are obtained for the effective potential strongly depend on the relation between universal time and temperature in the early universe, and that the strong symmetry breakdown which is present in the above picture for a temperature two orders of magnitude below the Planck mass could exist for a wider range of temperatures if the cosmological scenario were comparatively colder. This kind of argumentation implies, however, rejection of the standard scale factor $\chi(t)\propto t^{1/2}$, and we are not going to look at this possibility for the moment. Another more realistic situation in which the role of the background gravitational field is stressed comes out from the consideration of a grand-unification mass scale greater than 10^{15} GeV. Keeping in mind that the scale of the transition approximately coincides with the mass scale of the grand unified model, a greater σ mass scale would raise the transition temperature to a region in which the influence of the classical gravitational field could modify the mechanism of the transition.

The purpose of the present paper has been to show up the influence that gravity can have in the symmetry breaking of the SU(5) model at early stages of the universe. In this sense, the study has been centered in this grand unified model but, since we have not made use of any specific feature of the same, we can hope that the influence of gravity, considered at a classical level, would play the same role that has been shown in the above in other grand unified models. Finally, it has to be remarked that the approach adopted in this paper has been that of studying the influence of the classical gravitational field locally coupled to the grand unified model disregarding all kinds of topological effects. These have been the object of attention of some papers[21,22] and, although they seem to support the conclusion that their influence tends to restore the symmetry, an analysis on grand unified theories is needed. As long as the local approach is consistent with our choice of the $k=0$ Robertson-Walker model, which has a trivial spatial topology, this kind of effect could have significance under a different choice of cosmological model, in which case its influence would have to be set up.

ACKNOWLEDGMENT

I want to thank E. Alvarez for encouragement and useful comments on this work.

APPENDIX A: CALCULATION OF THE UNRENORMALIZED $V_1(\theta)$ PART

In this appendix we give the details of the computation of the order-one loop corrections to the effective potential

$$V_1(\theta)=i\ln\int \mathscr{D}\phi\,\mathscr{D}V\exp\left[i\int d^4x[\tfrac{1}{2}\phi^a(i\mathscr{D}^{-1})^{ab}\phi^b+V^{\mu a}P_\mu^{ab}\phi^b+\tfrac{1}{2}V^{\mu a}(i\Delta^{-1})_{\mu\nu}^{ab}V^{\nu b}]\right], \tag{A1}$$

where, in momentum space,

$$\begin{aligned}(i\mathscr{D}^{-1})^{ab}(k)&=\left[-k^2-\frac{3i}{2\tau}k_4-m^2\right]\delta^{ab},\quad P_\mu^{ab}(k)=5g\theta\left[-M^{ab}\frac{3}{2\tau}\delta_{4\mu}-iM^{ab}k_\mu\right],\\(i\Delta^{-1})_{\mu\nu}^{ab}(k)&=\left[-\delta_{\mu\nu}k^2+\left[1-\frac{1}{\alpha}\right]k_\mu k_\nu+\frac{i}{2\tau}\delta_{\mu4}\delta_{i\nu}k_i-\left[4-\frac{3}{\alpha}\right]\frac{i}{2\tau}\delta_{\mu i}\delta_{4\nu}k_i-\frac{3}{\alpha}\frac{i}{2\tau}\delta_{\mu4}\delta_{4\nu}k_4\right.\\&\qquad\left.-\frac{2i}{\tau}\delta_{\mu i}\delta_{i\nu}k_4-\frac{3}{\alpha}\frac{1}{2\tau^2}\delta_{\mu4}\delta_{4\nu}+\frac{1}{2\tau^2}\delta_{\mu i}\delta_{i\nu}-25g^2\theta^2\delta_{\mu\nu}\right]\delta^{ab}\quad a,b=9,\ldots,14,16,\ldots,21\,.\end{aligned} \tag{A2}$$

The easiest way to compute the functional integral in $V_1(\theta)$ is to make the integrations in the following order:

$$\begin{aligned}V_1(\theta)&=i\ln\int\mathscr{D}\phi\,\mathscr{D}V\exp\left[i\int d^4x(\tfrac{1}{2}\phi i\mathscr{D}^{-1}\phi+VP\phi+\tfrac{1}{2}Vi\Delta^{-1}V)\right]\\&=i\ln\frac{1}{\det(i\Delta^{-1})^{1/2}}\int\mathscr{D}\phi\exp\left[i\int d^4x(\tfrac{1}{2}\phi i\mathscr{D}^{-1}\phi+\tfrac{1}{2}\phi Pi\Delta P\phi)\right]\\&=i\ln\frac{1}{\det(i\Delta^{-1})^{1/2}}\frac{1}{\det(i\mathscr{D}^{-1}+Pi\Delta P)^{1/2}}\,.\end{aligned} \tag{A3}$$

The main problem now is the computation of the determinant and inverse of $i\Delta^{-1}$, because it cannot be decomposed in diagonal and transverse projectors in momentum space. In the particular case $\alpha=1, i\Delta^{-1}$ takes the form

$$i\Delta_{\mu\nu}{}^{-1}=\begin{bmatrix}\bar\alpha & & 0 & -x\\ & \bar\alpha & & -y\\ 0 & & \bar\alpha & -z\\ x & y & z & \bar\beta\end{bmatrix}, \tag{A4}$$

where

$$\bar\alpha=-k^2-\frac{2i}{\tau}k_4+\frac{1}{2\tau^2}-25g^2\theta^2\,,\quad \bar\beta=-k^2-\frac{3i}{2\tau}k_4-\frac{3}{2\tau^2}-25g^2\theta^2\,,$$

$$x=\frac{i}{2\tau}k_1\,,\quad y=\frac{i}{2\tau}k_2\,,\quad z=\frac{i}{2\tau}k_3\,.$$

In this gauge we have

$$\det(i\Delta_{\mu\nu}{}^{-1})=\bar\alpha^2\left[\bar\alpha\bar\beta-\frac{i}{4\tau^2}\vec{k}^2\right],\quad Pi\Delta P=-\delta^{ab}\frac{25g^2\theta^2}{\det(i\Delta^{-1})}\bar\alpha^2\left[\bar\alpha k_4{}^2+\bar\beta\vec{k}^2+\frac{3}{2\tau^2}\vec{k}^2+\bar\alpha\frac{9}{4\tau^2}\right],$$

$$a,b=9,\ldots,14,16,\ldots,21 \tag{A5}$$

and the effective potential to the one-loop order is

$$\begin{aligned}V_1(\theta)&=i\ln\frac{1}{\det(i\Delta^{-1})^{1/2}}+i\ln\frac{1}{\det(i\mathscr{D}^{-1}+Pi\Delta P)}\\&=\tfrac{1}{2}\operatorname{Tr}\ln\left[\bar\alpha^2\left[\bar\alpha\bar\beta-\frac{1}{4\tau^2}\vec{k}^2\right]\right]^{12}+\tfrac{1}{2}\operatorname{Tr}\ln\left[k^2+\frac{3i}{2\tau}k_4+m^2\right]^{12}\\&\quad+\tfrac{1}{2}\operatorname{Tr}\ln\left[k^2+\frac{3i}{2\tau}k_4+m^2+\frac{25g^2\theta^2}{\bar\alpha\bar\beta-(1/4\tau^2)\vec{k}^2}\left[\bar\alpha k_4{}^2+\bar\beta\vec{k}^2+\frac{3}{2\tau^2}\vec{k}^2+\bar\alpha\frac{9}{4\tau^2}\right]\right]^{12}\\&=\tfrac{12}{2}\operatorname{Tr}\ln\left[k^2+\frac{2i}{\tau}k_4-\frac{1}{2\tau^2}+25g^2\theta^2\right]+\tfrac{12}{2}\operatorname{Tr}\ln\left[k^2+\frac{3i}{2\tau}k_4+m^2\right]\\&\quad+\tfrac{12}{2}\operatorname{Tr}\ln\left[\left[k^2+\frac{3i}{2\tau}k_4+m^2\right]\left[\bar\alpha\bar\beta-\frac{1}{4\tau^2}\vec{k}^2\right]+25g^2\theta^2\left[\bar\alpha k_4{}^2+\bar\beta\vec{k}^2+\frac{3}{2\tau^2}\vec{k}^2+\bar\alpha\frac{9}{4\tau^2}\right]\right].\end{aligned} \tag{A6}$$

The second term being independent of θ, the relevant contribution of $V_1(\theta)$ to the effective potential proceeds from the first and third terms of the last expression. On the other hand, the $(i/\tau)k_4$ terms in the inverse propagators can be put aside by introducing fictitious particles $F^{\mu a}$ propagating the ∂_4 part of the boson gauge interactions, i.e., by introducing terms in the functional integral

$$\tfrac{1}{2}F^{\mu a}(\partial_4)^{-1}F^a_\mu - F^{\mu a}(a_{\mu\nu})^{1/2}V^{\nu a} \tag{A7}$$

with $a_{\mu\nu}=\text{diag}(3/2\tau,2/\tau,2/\tau,2/\tau)$. If the order of integration in the functional integral is the same as in the above, we get contributions of F particles

$$\int \mathscr{D}F \exp\left[\int d^4x\left(-\tfrac{1}{2}F\partial^{-1}F+\tfrac{1}{2}Fa^{1/2}\Delta a^{1/2}F\right)\right] \tag{A8}$$

to the effective potential. We will not evaluate this contribution, assuming that it is a purely gravitational effect and that the main features are contained in the terms that recover the flat—space-time effective potential in the limit $\tau\rightarrow\infty$.

Bearing these considerations in mind, we get at the end a $V_1(\theta)$ part of the effective potential of the form

$$V_1(\theta)=12\,\mathrm{Tr}\ln\left[k^2+25g^2\theta^2-\frac{1}{2\tau^2}\right]+6\,\mathrm{Tr}\ln(k^6+a'k^4+b'k^2+c')\,, \tag{A9}$$

where

$$a'\equiv 25g^2\theta^2+m^2+\frac{3}{4\tau^2}\,,\quad b'\equiv 50g^2\theta^2m^2-25g^2\theta^2\frac{5}{4\tau^2}+\frac{3}{4\tau^2}m^2-\frac{3}{4\tau^4}+\frac{1}{4\tau^2}k_4{}^2\,,$$

$$c'\equiv 625g^4\theta^4\left[m^2-\frac{9}{4\tau^2}\right]+25g^2\theta^2\frac{1}{\tau^2}m^2+25g^2\theta^2\frac{9}{8\tau^4}-m^2\frac{3}{4\tau^4}+\frac{1}{4\tau^2}m^2k_4{}^2+25g^2\theta^2\frac{1}{2\tau^2}k_4{}^2\,.$$

APPENDIX B: RENORMALIZATION OF $\ln\det(D_2/\mu^6)$

We begin with the definition

$$\ln\det\left[\frac{D_2}{\mu^6}\right]=-\zeta_2'(0)-\ln\mu^6\zeta_2(0)\,, \tag{B1}$$

where

$$\zeta_2(s)=\int_{-\infty}^{+\infty}\frac{d^4k}{(2\pi)^4}\frac{1}{(k^6+a'k^4+b'k^2+c')^s} \tag{B2}$$

and the coefficients a',b',c', are defined as in Appendix A.

First, let us note that, if we want to carry out an analytic calculation of $\zeta_2'(0)$ and $\zeta_2(0)$, we must replace $k_4{}^2$ by $\frac{1}{4}k^2$ in the anisotropic terms of a',b',c'. This is a procedure which is obviously exact only up to the first order in an expansion of $\zeta_2(s)$ in the parameter $1/\tau^2$. However, it can be shown that, in fact, it does not appreciably influence the final results. Then we can replace

$$\begin{aligned}&a'\equiv 25g^2\theta^2+m^2+\frac{13}{16\tau^2}\,,\quad b'\equiv 50g^2\theta^2m^2-25g^2\theta^2\frac{9}{8\tau^2}+\frac{13}{16\tau^2}m^2-\frac{3}{4\tau^4}\,,\\&c'\equiv 625g^4\theta^4\left[m^2-\frac{9}{4\tau^2}\right]+25g^2\theta^2\frac{1}{\tau^2}m^2+25g^2\theta^2\frac{9}{8\tau^4}-m^2\frac{3}{4\tau^4}\,.\end{aligned} \tag{B3}$$

If we call β,δ,r, the three roots (real or complex) of the polynomial $x^3+a'x^2+b'x+c'$,

$$x^3+a'x^2+b'x+c'=(x-\beta)(x-\delta)(x-r) \tag{B4}$$

and apply the technique of integration by parts to get a well-behaved extension of $\zeta_2(s)$ in the limit $s\rightarrow 0$, we obtain at the end

$$-3(2-3s)(1-3s)\zeta_2(s)=\frac{1}{(4\pi)^2}\left[(s+2)(s+1)\beta^3\int_0^\infty dx\frac{x}{(x-\beta)^{s+3}(x-\delta)^s(x-r)^s}+\beta\leftrightarrow\delta+\beta\leftrightarrow r\right]$$
$$+\frac{1}{(4\pi)^2}\left[3(s+1)s\beta^2\delta\int_0^\infty dx\frac{x}{(x-\beta)^{s+2}(x-\delta)^{s+1}(x-r)^s}+\beta\leftrightarrow\delta\right]$$
$$+\frac{1}{(4\pi)^2}\left[3(s+1)s\delta^2 r\int_0^\infty dx\frac{x}{(x-\beta)^{s}(x-\delta)^{s+2}(x-r)^{s+1}}+\delta\leftrightarrow r\right]$$
$$+\frac{1}{(4\pi)^2}\left[3(s+1)s\beta^2 r\int_0^\infty dx\frac{x}{(x-\beta)^{s+2}(x-\delta)^{s}(x-r)^{s+1}}+\beta\leftrightarrow r\right]$$
$$+\frac{1}{(4\pi)^2}6\beta\delta r\int_0^\infty dx\frac{x}{(x-\beta)^{s+1}(x-\delta)^{s+1}(x-r)^{s+1}}\;. \qquad \text{(B5)}$$

This expression for $\zeta_2(s)$ has only poles at $s=\frac{1}{3},s=\frac{2}{3}$. With the help of the definition

$$J(a,b,c)\equiv\frac{1}{(4\pi)^2}\int_0^\infty dx\frac{x}{(x-\beta)^a(x-\delta)^b(x-r)^c}\;,$$

we have

$$\zeta_2(0)=-\tfrac{1}{3}[\beta^3 J(3,0,0)+\delta^3 J(0,3,0)+r^3 J(0,0,3)]\;, \qquad \text{(B6)}$$

$$\zeta_2'(0)=\frac{1}{3}\frac{1}{(4\pi)^2}\left[\beta^3\int_0^\infty dx[\ln(x-\beta)+\ln(x-\delta)+\ln(x-r)]\frac{x}{(x-\beta)^3}+\beta\leftrightarrow\delta+\beta\leftrightarrow r\right]$$
$$-2\beta^3 J(3,0,0)-2\delta^3 J(0,3,0)-2r^3 J(0,0,3)-\tfrac{1}{2}\beta^2\delta J(2,1,0)-\tfrac{1}{2}\beta\delta^2 J(1,2,0)$$
$$-\tfrac{1}{2}\delta^2 r J(0,2,1)-\tfrac{1}{2}\delta r^2 J(0,1,2)-\tfrac{1}{2}\beta r^2 J(1,0,2)-\tfrac{1}{2}\beta^2 r J(2,0,1)\;. \qquad \text{(B7)}$$

Until now we have not mentioned whether all three roots β,δ,r, are real or not, but, as far as it is concerned with the obtaining of well-defined expressions for $\zeta_2(0)$ and $\zeta_2'(0)$, we only need that the J integrals stand for their principal values. As for the range of large θ two of the roots (for example, β and δ) become complex, we also have to choose the sheet $-\pi<\arg\beta,\arg\delta<+\pi$, so that each one of the integrals becomes finite in the limit $\mathrm{Im}\beta,\mathrm{Im}\delta\to 0$. The renormalized expression for $\ln\det(D_2/\mu^6)$ in terms of β,δ,r, reads

$$\ln\det\left[\frac{D_2}{\mu^6}\right]=\frac{1}{(4\pi)^2}\left[\frac{1}{6}(\beta^2+\delta^2+r^2)\left[\ln\frac{-\beta}{\mu^2}+\ln\frac{-\delta}{\mu^2}+\ln\frac{-r}{\mu^2}\right]-\frac{3}{4}(\beta^2+\delta^2+r^2)-\beta\delta-\delta r-\beta r\right]$$
$$-(\tfrac{1}{3}\beta^3+\tfrac{1}{3}\delta^3)I(1,1,0)-(\tfrac{1}{3}\beta^3+\tfrac{1}{3}r^3)I(1,0,1)-(\tfrac{1}{3}\delta^3+\tfrac{1}{3}r^3)I(0,1,1)$$
$$-(\tfrac{1}{6}\beta^4-\tfrac{1}{2}\beta^2\delta^2)I(2,1,0)-(\tfrac{1}{6}\delta^4-\tfrac{1}{2}\delta^2\beta^2)I(1,2,0)-(\tfrac{1}{6}r^4-\tfrac{1}{2}\beta^2 r^2)I(1,0,2)$$
$$-(\tfrac{1}{6}r^4-\tfrac{1}{2}\delta^2 r^2)I(0,1,2)-(\tfrac{1}{6}\beta^4-\tfrac{1}{2}\beta^2 r^2)I(2,0,1)-(\tfrac{1}{6}\delta^4-\tfrac{1}{2}\delta^2 r^2)I(0,2,1)\;, \qquad \text{(B8)}$$

$$I(a,b,c)\equiv\frac{1}{(4\pi)^2}\int_0^\infty dx\frac{1}{(x-\beta)^a(x-\delta)^b(x-r)^c}\;.$$

Finally, we have to remark that this renormalized $\ln\det(D_2/\mu^6)$ contains the standard form of the logarithm in the flat—space-time limit $\beta\to 0,\delta\to 0,r\to 25g^2\theta^2$,

$$\ln\det\left[\frac{D_2}{\mu^6}\right]\to\frac{1}{(4\pi)^2}\left[\frac{(25g^2\theta^2)^2}{2}\ln\frac{25g^2\theta^2}{\mu^2}-\frac{5}{12}(25g^2\theta^2)^2\right]. \qquad \text{(B9)}$$

It can be shown that all divergences in (B8) cancel out to give the previous formula, which we took into account in the computation of the limit expression (42).

APPENDIX C: NOTATION

In this appendix we detail the notation used in this paper. The metric tensor of the Riemannian space-time is denoted by $\tilde{g}$ and has signature -2. Greek indices are used as general coordinate indices $\mu,\nu,\ldots=0,\ldots,3$, whereas latin indices $i,j,\ldots=1,\ldots,3$ are used as coordinate indices of a spatial section of the space-time. $\Gamma^{\mu}_{\nu\tau}$

denote the Christoffel symbols, $R^{\mu}_{\epsilon\nu\tau}$ the curvature tensor, $R_{\mu\nu}=R^{\tau}_{\mu\tau\nu}$ the Ricci tensor, and $R=g^{\mu\nu}R_{\mu\nu}$ the scalar curvature. The volume element of the space-time is chosen as $(-g_0)^{1/2}=(-\det\tilde{g}_0)^{1/2}$.

Latin indices $a,b,c,\ldots=1,\ldots,24$ are used as SU(5) indices in a space with metric δ_{ab}. T_a denote the generators in the fundamental representation, normalized by $\mathrm{Tr}(T^aT^b)=2\delta^{ab}$:

$$T^a=\begin{bmatrix}\lambda^a & 0 \\ 0 & \begin{matrix}0&0\\0&0\end{matrix}\end{bmatrix}\quad a=1,\ldots,8,\ \lambda^a=\text{Gell-Mann matrices}$$

$$T^9=\begin{bmatrix} & & & 1 & 0\\ & 0 & & 0 & 0\\ & & & 0 & 0\\ 1 & 0 & 0 & & \\ 0 & 0 & 0 & & 0\end{bmatrix},$$

$$T^{10}=\begin{bmatrix} & & & -i & 0\\ & 0 & & 0 & 0\\ & & & 0 & 0\\ i & 0 & 0 & & \\ 0 & 0 & 0 & & 0\end{bmatrix},\ldots,$$

$$T^{15}=\frac{1}{\sqrt{6}}\begin{bmatrix}1 & & & & 0\\ & 1 & & & \\ & & 1 & & \\ & & & -3 & \\ 0 & & & & 0\end{bmatrix},$$

$$T^{16}=\begin{bmatrix} & & & 0 & 1\\ & 0 & & 0 & 0\\ & & & 0 & 0\\ 0 & 0 & 0 & & \\ 1 & 0 & 0 & & 0\end{bmatrix},$$

$$T^{17}=\begin{bmatrix} & & & 0 & -i\\ & 0 & & 0 & 0\\ & & & 0 & 0\\ 0 & 0 & 0 & & \\ i & 0 & 0 & & 0\end{bmatrix},\ldots,$$

and $T^{24}=(1/\sqrt{10})\,\mathrm{diag}(1,1,1,1,-4)$. f_{abc} denote the structure constants of the SU(5) group, $[T_a,T_b]=if_{abc}T_c$.

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