PHYSICAL REVIEW D VOLUME 31, NUMBER 6 15 MARCH 1985

Gravitation without black holes

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(Received 3 August 1983)

The Schwarzschild, Reissner-Nordström, and Kerr exterior solutions in general relativity are reconsidered adding to the vacuum a massless scalar field. The event horizons in the modified solutions all reduce to a point, thus preventing the formation of black holes.

I. INTRODUCTION

In the study of the global structure of spacetimes described by Einstein's theory of general relativity, an important role is played by Penrose's hypothesis of cosmic censorship.[1] In particular, all proofs of black-hole existence are based on this hypothesis. At present there is widespread confidence in the first-order stability of black holes, and the discussion of validity of the cosmic censorship hypothesis is focused on the appropriate equation of state to be used at extremely high density.[2]

Now, to our knowledge, a precise formulation of cosmic censorship has not yet appeared in the literature, its plausibility being supported on one hand by perturbative and computer calculations and on the other by the lack of any convincing counterexample disproving it. The main point under investigation is, therefore, whether one can obtain naked singularities by destroying, in a physically reasonable manner, the event horizons associated with the well-known black-hole solutions of the gravitational-collapse problem.

The efforts in this direction seem to indicate that the structure of the event horizons can be drastically changed if the gravitational field is coupled to a massless scalar field.[3] Recently the authors have shown that even a minimum amount of scalar charge in a body is sufficient to give to the Schwarzschild horizon the topology of a point.[4] The purpose of this paper is to generalize the above-mentioned result.

In Sec. II we summarize the effect of a scalar-field coupling for the Schwarzschild spacetime, and the same analysis is carried out in Secs. III and IV for the Reissner-Nordström and Kerr spacetimes, respectively, showing that in these cases too the event horizons shrink to a point. In all these cases the energy-momentum tensor describing the scalar field is chosen in its simplest form. In Sec. V we shall obtain and discuss some exact static solutions in the case of a conformally invariant massless scalar field coupled to gravity.

The concluding section contains some comments on the results obtained; in particular, we suggest that collapsing matter is always associated with a scalar field, thus providing a justification to the introduction of the scalar field itself. Finally, we mention that the stability of our solutions under time-dependent perturbations will not be discussed in this paper, even if there may exist some choices of the initial data such that the scalar charge be radiated away. This point is left to further investigations.

II. SCALAR-FIELD-MODIFIED SCHWARZSCHILD SPACETIME

We begin by examining the static spherically symmetric metric of gravitation coupled to a zero-rest-mass scalar field in the exterior region. The field equations of general relativity are

$$R_{ij}-\tfrac{1}{2}g_{ij}R=-8\pi T_{ij}\ , \tag{1}$$

where

$$T_{ij}=\frac{1}{4\pi}(\varphi_i\varphi_j-\tfrac{1}{2}g_{ij}\varphi^k\varphi_k) \tag{2}$$

is the energy-momentum tensor of the neutral massless scalar field and $\varphi_i\equiv\partial\varphi/\partial x^i$ $(i=0,1,2,3)$. Equations (1) and (2) are equivalent to

$$R_{ij}=-2\varphi_i\varphi_j \tag{3}$$

and moreover imply the d'Alembertian equation

$$\Box\varphi=0\ . \tag{4}$$

The required line element, which can be written in the form

$$ds^2=e^{\gamma(r)}dt^2-e^{\alpha(r)}dr^2-e^{\beta(r)}r^2(d\theta^2+\sin^2\theta\, d\varphi^2)\ , \tag{5}$$

was already found by the authors in Ref. 4.

In this paper it will be more convenient to use a coordinate system in which

$$\alpha(r)+\gamma(r)=0 \tag{6}$$

thus obtaining

$$ds^2=\left[1-\frac{2\eta}{r}\right]^{m/\eta}dt^2-\left[1-\frac{2\eta}{r}\right]^{-m/\eta}dr^2-\left[1-\frac{2\eta}{r}\right]^{1-m/\eta}r^2(d\theta^2+\sin^2\theta\, d\varphi^2)\ , \tag{7}$$

where

$$\eta=(m^2+\sigma^2)^{1/2}\ . \tag{8}$$

The real constant σ which appears in the time-independent scalar field

$$\varphi(r)=\frac{\sigma}{2\eta}\ln\left[1-\frac{2\eta}{r}\right] \tag{9}$$

can be interpreted as the scalar charge. The radial coordinate r varies in the range $2\eta<r<\infty$; correspondingly the standard radial coordinate

$$R=r\left[1-\frac{2\eta}{r}\right]^{(1-m/\eta)/2} \tag{10}$$

takes values between 0 and ∞.

Let us note that at the value

$$R=\frac{(\eta+m)^2}{2m}\left[\frac{\eta-m}{\eta+m}\right]^{(1-m/\eta)} \tag{11}$$

the six-dimensional embedding changes from a pseudo-Euclidean $\bar{E}(2,4)$ to a pseudohyperbolic $\bar{E}(3,3)$ one.[5] The analysis of the asymptotically flat spacetime corresponding to the metric in Eq. (7) is conveniently carried out in the Newman-Penrose null-tetrad formalism.[6]

Let us decompose the metric (7) as

$$g^{ij}=l^i n^j+n^i l^j-m^i\bar{m}^j-\bar{m}^i m^j\ , \tag{12}$$

where l,n,m and $\bar{m}$ are all null vectors, with l and n real and m and $\bar{m}$ complex conjugates of each other, and $l^i n_i=-m^i\bar{m}_i=1$, all other products vanishing. We obtain, up to tetrad rotations,

$$\begin{aligned} l^j&=(e^{-\gamma(r)},1,0,0)\ ,\\ n^j&=\tfrac{1}{2}(1,-e^{\gamma(r)},0,0)\ ,\\ m^j&=\frac{1}{\sqrt{2}re^{\beta(2)/2}}\left[0,0,1,\frac{i}{\sin\theta}\right]\ ,\\ \bar{m}^j&=\frac{1}{\sqrt{2}re^{\beta(r)/2}}\left[0,0,1,\frac{-i}{\sin\theta}\right]\ . \end{aligned} \tag{13}$$

It is then straightforward to compute the spin coefficients and the Weyl scalars in the chosen frame.

The nonvanishing quantities are, respectively,

$$\begin{aligned} \rho&=-\frac{r-m-\eta}{r^2(1-2\eta/r)}\ ,\quad \mu=-\frac{r-m-\eta}{2r^2(1-2\eta/r)^{1-m/\eta}}\ ,\\ \gamma&=\frac{m}{2r^2(1-2\eta/r)^{1-m/\eta}}\ ,\\ \alpha&=-\beta=-\frac{\cot\theta}{2\sqrt{2}r(1-2\eta/r)^{(1-m/\eta)/2}}\ , \end{aligned} \tag{14}$$

and

$$\psi_2=-\frac{m(r-m-\eta)-\sigma^2/3}{r^4(1-2\eta/r)^{2-m/\eta}}\ . \tag{15}$$

Therefore the metric is of Petrov type D, and moreover there appears a singularity at $r=2\eta$ which corresponds, in standard coordinates, to $R=0$.

We investigate the structure of this singularity by examining the properties of the equipotential surfaces $g_{00}=\text{constant}$, $t=\text{constant}$, and of the closed curves on these surfaces as r approaches the value 2η. The area of the equipotential surfaces is

$$A=\int_0^{2\pi}\int_0^{\pi}\sqrt{g_{22}g_{33}}\,d\theta\,d\varphi=4\pi r^2\left[1-\frac{2\eta}{r}\right]^{1-m/\eta} \tag{16}$$

while the proper lengths are, respectively,

$$L_\varphi=\int_0^{2\pi}\sqrt{-g_{33}}\,d\varphi=2\pi r\left[1-\frac{2\eta}{r}\right]^{(1-m/\eta)/2} \tag{17}$$

for a closed azimuthal curve $\theta=\pi/2$ and

$$L_\theta=2\int_0^{\pi}\sqrt{-g_{22}}\,d\theta=2\pi r\left[1-\frac{2\eta}{r}\right]^{(1-m/\eta)/2} \tag{18}$$

for a polar curve $\varphi=\text{constant}$.

It is now evident that the singularity at $r=2\eta$ has the topology of a point and the event horizon has therefore shrinked to a point. Thus we cannot speak of a black hole in the usual sense, even if the red-shift still approaches infinity as the radius of the body tends to zero.

III. SCALAR FIELD MODIFIED REISSNER-NORDSTRÖM SPACETIME

We shall now determine the static spherically symmetric line element (5) with the coordinate condition (6), in the case when the interior material system of mass m carries both scalar and electric charge. Accordingly, the energy-momentum tensor in Einstein's equations is the sum of the scalar field tensor (2) and of the electromagnetic field tensor

$$T_{ij}=\frac{1}{4\pi}(-F_{ik}F_j{}^k+\tfrac{1}{4}g_{ij}F_{rs}F^{rs})\ . \tag{19}$$

Due to the spherical symmetry, the quadripotential vector A_i has only one nonvanishing component $A_0\equiv V(r)$ which, from Maxwell's equations in the vacuum, must satisfy

$$e^{\beta(r)}r^2\frac{d}{dr}V(r)=q\ . \tag{20}$$

Here the constant q is the total electric charge of the body which is the source of the field.

The relevant Einstein equations are

$$\begin{aligned} -e^{-2\gamma}R_{00}&=\tfrac{1}{2}\gamma_{11}+\tfrac{1}{2}\gamma_1{}^2+\tfrac{1}{2}\beta_1\gamma_1+\frac{\gamma_1}{r}=\frac{q^2}{r^4}e^{-(2\beta+\gamma)}\ ,\\ R_{11}&=\beta_{11}+\tfrac{1}{2}\beta_1{}^2+\tfrac{1}{2}\gamma_{11}+\tfrac{1}{2}\gamma_1{}^2+\tfrac{1}{2}\beta_1\gamma_1+\frac{\gamma_1}{r}+\frac{2}{r}\beta_1\\ &=\frac{q^2}{r^4}e^{-(2\beta+\gamma)}-\frac{2\sigma^2}{r^4}e^{-2(\beta+\gamma)}\ ,\\ R_{22}&=-1+\frac{r^2}{2}e^{(\beta+\gamma)}\left[\beta_{11}+\beta_1{}^2+\beta_1\gamma_1+\frac{2}{r}\gamma_1+\frac{4}{r}\beta_1+\frac{2}{r^2}\right]\\ &=-\frac{q^2}{r^2}e^{-\beta}\ . \end{aligned} \tag{21}$$

After some algebraic manipulations, one obtains the system

$$\gamma_{11}+\gamma_1{}^2+\beta_1\gamma_1-\frac{2}{r}\gamma_1=\frac{2q^2}{r^4}e^{-(2\beta+\gamma)} ,$$
$$(\beta_{11}+\gamma_{11})+(\beta_1+\gamma_1)^2+\frac{4}{r}(\beta_1+\gamma_1)=\frac{2}{r}(e^{-(\beta+\gamma)}-1) , \tag{22}$$

which can be integrated with the conditions

$$\beta\rightarrow 0,\ \ \gamma\rightarrow 0 ,$$
$$r^2\beta_1\rightarrow 2(\eta-m),\ \ r^2\gamma_1\rightarrow 2m,\ \ \text{as}\ \ r\rightarrow\infty . \tag{23}$$

These are appropriate requirements to recover the previously known solutions when σ and/or q vanish.

Performing the integration, in the case $q^2<m^2$, we get

$$e^{\beta(r)}=\left[1-\frac{2\eta}{r}+\frac{q^2}{r^2}\right]^{1-[(m^2-q^2)/(\eta^2-q^2)]^{1/2}}\left[\frac{1}{2}\left[1+\frac{m}{\sqrt{m^2-q^2}}\right]\left[1-\frac{\eta-\sqrt{\eta^2-q^2}}{r}\right]^{[(m^2-q^2)/(\eta^2-q^2)]^{1/2}}\right.$$
$$\left.+\frac{1}{2}\left[1-\frac{m}{\sqrt{m^2-q^2}}\right]\left[1-\frac{\eta+\sqrt{\eta^2-q^2}}{r}\right]^{[(m^2-q^2)/(\eta^2-q^2)]^{1/2}}\right] ,$$
$$e^{\gamma(r)}=\left[1-\frac{2\eta}{r}+\frac{q^2}{r^2}\right]e^{-\beta(r)} , \tag{24}$$

where the radial coordinate r now satisfies $\eta+\sqrt{\eta^2-q^2}<r<\infty$ and the corresponding standard radial coordinate $R=re^{\beta(r)/2}$ varies in the range $0<R<\infty$.

Proceeding as illustrated in the preceding section, we obtain that the nonvanishing spin coefficients and Weyl scalars are, respectively,

$$\rho=-\frac{1}{R}\frac{d}{dr}R,\ \ \mu=-\frac{(r^2-2\eta r+q^2)}{2R^3}\frac{d}{dr}R ,$$
$$\gamma=\mu+\frac{r-\eta}{2R^2},\ \ \alpha=-\beta=-\frac{\cot\theta}{2\sqrt{2}R} , \tag{25}$$

and

$$\psi_2=-\frac{1}{R^2}\left\{(r^2-2\eta r+q^2)\left[\frac{1}{2R}\frac{d^2}{dr^2}R-\left[\frac{1}{R}\frac{dR}{dr}\right]^2\right]\right.$$
$$\left.+(r-\eta)\left[\frac{1}{R}\frac{dR}{dr}\right]-\frac{\sigma^2}{3(r^2-2\eta r+q^2)}\right\} . \tag{26}$$

Here also the metric is of Petrov type D and there appears a singularity at $r=\eta+(\eta^2-q^2)^{1/2}\equiv r_+$, which corresponds, in standard coordinates, to $R=0$.

It is easily verified that the area of the equipotential surfaces and the proper lengths of the closed curves L_φ and L_θ on it, all tend to zero as one approaches the singularity at $r=r_+$. This implies that at $r=r_+$ we have the topology of a point and the black-hole behavior of the standard solution has again been eliminated by the introduction of the scalar field.

IV. SCALAR FIELD MODIFIED KERR SPACETIME

We examine how the Kerr solution is modified if the rotating body acquires a scalar charge. In the Boyer-Lindquist coordinates,[7] a general stationary axisymmetric metric can be written as

$$ds^2=e^{2\nu}dt^2-e^{2\psi}(d\varphi-\omega\, dt)^2-e^{2\lambda}dr^2-e^{2\mu}d\theta^2 , \tag{27}$$

where φ denotes the azimuthal angle in the equatorial plane and r and θ are the two remaining spatial coordinates. The quantities ν, ψ, ω, λ, and μ in Eq. (27) are functions of r and θ only.

The relevant equations are

$$[e^{\mu-\lambda}(e^{\psi+\nu})_r]_r+[e^{\lambda-\mu}(e^{\psi+\nu})_\theta]_\theta=0 ,$$
$$[e^{\psi+\nu+\mu-\lambda}(\psi-\nu)_r]_r+[e^{\psi+\nu+\lambda-\mu}(\psi-\nu)_\theta]_\theta=-e^{3\psi-\nu}[e^{\mu-\lambda}(\omega_r)^2+e^{\lambda-\mu}(\omega_\theta)^2] ,$$
$$[e^{3\psi-\nu+\mu-\lambda}\omega_r]_r+[e^{3\psi-\nu+\lambda-\mu}\omega_\theta]_\theta=0 ,$$
$$[\nu_{r\theta}+\nu_r\nu_\theta+\psi_{r\theta}+\psi_n\psi_\theta-\mu_r(\psi+\nu)_\theta-\lambda_\theta(\psi+\nu)_r]-\tfrac{1}{2}e^{2(\psi-\nu)}\omega_r\omega_\theta=-2\varphi_r\varphi_\theta , \tag{28}$$
$$[\nu_r(\psi+\mu)_r+\psi_r\mu_r]+e^{2(\lambda-\mu)}[\nu_{\theta\theta}+\nu_\theta(\nu-\mu)_\theta+\psi_{\theta\theta}+\psi_\theta(\psi+\nu-\mu)_\theta]+\tfrac{1}{4}e^{2(\psi-\nu)}[(\omega_r)^2-e^{2(\lambda-\mu)}(\omega_\theta)^2]=\varphi_r{}^2-e^{2(\lambda-\mu)}\varphi_\theta{}^2 ,$$
$$[e^{\psi+\nu+\mu-\lambda}\varphi_r]_r+[e^{\psi+\nu+\lambda-\mu}\varphi_\theta]_\theta=0 ,$$

where the "comma" notation for partial derivatives has been adopted. These are obtained by using the components of the Einstein and Ricci tensors appropriate to the chosen form of the metric as written down by Chandrasekhar and Friedman.[8] We notice that, in the absence of the scalar field, Eqs. (28) are the basis for a direct and simple derivation of Kerr solution.[9] In the case at hand it seems more convenient to try whether the Newman-Janis construction[10] retains its validity in the presence of a scalar field and still provides a transformation from the Schwarzschild to the Kerr solutions. Some evidence that such a method could be effective comes from its success in deriving a Kerr-type metric when applied to Brans-Dicke theory.[11]

Turning to our problem, let us consider the radiation form of the modified Schwarzschild line element (7):

$$ds^2=e^{\gamma(r)}du^2+2\,du\,dr-e^{\beta(r)}r^2(d\theta^2+\sin^2\theta\,d\varphi^2)\ , \qquad (29)$$

where the time coordinate u is chosen such that

$$du=dt-e^{-\gamma(r)}dr \qquad (30)$$

and

$$e^{\beta(r)}=\left[1-\frac{2\eta}{r}\right]^{1-m/\eta}\ ,\qquad e^{\gamma(r)}=\left[1-\frac{2\eta}{r}\right]^{m/\eta}\ . \qquad (31)$$

The metric (29) can be written in term of a null tetrad

$$l^j=(0,1,0,0)\ ,\quad n^j=\left[1,-\frac{1}{2}e^{\gamma(r)},0,0\right]\ ,\quad m^j=\frac{1}{\sqrt{2}re^{\beta(r)/2}}\left[0,0,1,\frac{i}{\sin\theta}\right]\ ,\quad \overline{m}^j=\frac{1}{\sqrt{2}re^{\beta(r)/2}}\left[0,0,1,\frac{-i}{\sin\theta}\right]\ , \qquad (32)$$

where l and n are repeated principal null vectors of the Weyl tensor [with $x^j=(u,r,\theta,\varphi)$]. Then the complex coordinate transformation

$$u'=u-ia\cos\theta\ ,\quad r'=r+ia\cos\theta\ ,\quad \theta'=\theta\ ,\quad \varphi'=\varphi \qquad (33)$$

is performed, and the new coordinates are restricted to real values. Of course, because of the Newman-Janis construction, what is finally obtained is not a genuine coordinate transformation.

Dropping the prime, the resulting tetrad is

$$l^j=(0,1,0,0)\ ,\quad n^j=(1,-\tfrac{1}{2}e^{\gamma(r,\theta)},0,0)\ ,$$
$$m^j=\frac{1}{\sqrt{2}(r+ia\cos\theta)e^{\beta(r,\theta)/2}}\left[ia\sin\theta,-ia\sin\theta,1,\frac{i}{\sin\theta}\right]\ ,\quad \overline{m}^i=\frac{1}{\sqrt{2}(r-ia\cos\theta)e^{\beta(r,\theta)/2}}\left[-ia\sin\theta,ia\sin\theta,1,\frac{-i}{\sin\theta}\right]\ , \qquad (34)$$

where

$$e^{\beta(r,\theta)}=\left[1-\frac{2\eta r}{r^2+a^2\cos^2\theta}\right]^{1-m/\eta}\ ,\qquad e^{\gamma(r,\theta)}=\left[1-\frac{2\eta r}{r^2+a^2\cos^2\theta}\right]^{m/\eta}\ . \qquad (35)$$

By means of the coordinate transformation

$$du=d\overline{t}-\left[\frac{e^{\beta(r,\theta)}(r^2+a^2\cos^2\theta)+a^2\sin^2\theta}{r^2-2\eta r+a^2}\right]dr\ ,\qquad d\varphi=d\overline{\varphi}-\left[\frac{a}{r^2-2\eta r+a^2}\right]dr \qquad (36)$$

the corresponding line element is, dropping the overbar,

$$ds^2=e^{\gamma(r,\theta)}dt^2-\frac{1}{e^{\gamma(r,\theta)}+\dfrac{a^2\sin^2\theta}{e^{\beta(r,\theta)}(r^2+a^2\cos^2\theta)}}dr^2+2(1-e^{\gamma(r,\theta)})a\sin^2\theta\,dt\,d\varphi$$
$$-e^{\beta(r,\theta)}(r^2+a^2\cos^2\theta)\left\{d\theta^2+\left[1+\frac{(2-e^{\gamma(r,\theta)})a^2\sin^2\theta}{e^{\beta(r,\theta)}(r^2+a^2\cos^2\theta)}\right]\sin^2\theta\,d\varphi^2\right\}\ . \qquad (37)$$

The line element (37) has now the form required by expression (27), so by comparison we obtain

$$e^{2\nu}=\frac{R^2\Delta}{\Sigma^2},\quad e^{2\psi}=\frac{\Sigma^2}{R^2}\sin^2\theta,\quad e^{2\lambda}=\frac{R^2}{\Delta},\quad e^{2\mu}=R^2\ ,$$
$$\omega=\frac{a(R^2+a^2\sin^2\theta-\Delta)}{\Sigma^2}\ , \tag{38}$$

where

$$R^2=(r^2+a^2\cos^2\theta)\left[1-\frac{2\eta r}{r^2+a^2\cos^2\theta}\right]^{1-m/\eta}\ ,$$
$$\Delta=r^2-2\eta r+a^2\ , \tag{39}$$
$$\Sigma^2=(R^2+a^2\sin^2\theta)^2-a^2\sin^2\theta\Delta\ ,$$

and the radial coordinate r varies in the θ dependent range

$$\eta+(\eta^2-a^2\cos^2\theta)^{1/2}<r<\infty$$

which corresponds to $0<R<\infty$. It is easily verified that, in the limit of vanishing scalar charge, one obtains the Kerr solution.

Our task is now to check if the functions (38) together with a suitable scalar field $\varphi(r,\theta)$ satisfy Eqs. (28). It soon became very apparent to us that the required calculations were quite involved and easily affected by errors, so that we preferred to test our results on a computer. In this way we have verified that the functions (38) are indeed solutions of Eqs. (28) adopting for the scalar field the θ independent ansatz

$$\varphi(r)=\frac{\sigma}{2\sqrt{\eta^2-a^2}}\ln\left[1-\frac{\eta+\sqrt{\eta^2-a^2}}{r}\right]. \tag{40}$$

Of course, this ansatz satisfies the d'Alembertian Eq. (4) which, written explicitly, corresponds to the last of Eqs. (28). The fact that in our solution the scalar field is only r dependent is perhaps not surprising if we consider that the source of the Kerr field can be a rotating spherical body and that a θ dependence might only arise from a departure from sphericity. We remark that in this case we should also take into account, for example, the scalar field modifications to the $\delta\neq 1$ Tomimatsu-Sato families of solutions,[12] which however will not be pursued here.

It is also interesting to notice that the asymptotic behaviors

$$e^{2\nu}\sim 1-\frac{2m}{r},\quad e^{2\psi}\sim\left[1-\frac{2(\eta-m)}{r}\right]r^2\sin^2\theta\ ,$$
$$e^{2\lambda}\sim 1+\frac{2m}{r},\quad e^{2\mu}\sim\left[1-\frac{2(\eta-m)}{r}\right]r^2\ , \tag{41}$$
$$\omega\sim\left[1-\frac{4(\eta-m)}{r}\right]\frac{2am}{r^3}$$

of the various metric coefficients, clearly shows that the modified Kerr metric approaches the modified Schwarzschild metric as $r\to\infty$. Moreover, interpreting ω as the "dragging of the inertial frame," we are allowed to conclude that the parameter a can again be identified with the angular momentum per unit mass.

The Kinnersley frame,[13] which provides a suitable basis for the description of the Kerr geometry, now becomes

$$l^j=\frac{1}{\Delta}(R^2+a^2\sin^2\theta,\Delta,0,a)\ ,$$
$$n^j=\frac{1}{2R^2}(R^2+a^2\sin^2\theta,-\Delta,0,a)\ ,$$
$$m^j=\frac{1}{\sqrt{2}R_+}\left[ia\sin\theta,0,1,\frac{i}{\sin\theta}\right]\ , \tag{42}$$
$$\bar{m}^j=\frac{1}{\sqrt{2}R_-}\left[-ia\sin\theta,0,1,\frac{-i}{\sin\theta}\right]\ ,$$

where

$$R_\pm=(r\pm ia\cos\theta)\left[1-\frac{2\eta r}{r^2+a^2\cos^2\theta}\right]^{(1-m/\eta)/2}. \tag{43}$$

The spin coefficients in this frame are

$$\kappa=\frac{1}{\sqrt{2}R_+\Delta}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ ,$$
$$\rho=-\frac{1}{2R^2}\left[\frac{\partial}{\partial r}R^2+2ia\cos\theta\right]\ ,$$
$$\sigma=\frac{ia\sin\theta}{2R_+{}^2\Delta}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ ,$$
$$\mu=-\frac{\Delta}{4R^4}\left[\frac{\partial}{\partial r}R^2+2ia\cos\theta\right]\ ,$$
$$\lambda=\frac{ia\sin\theta}{2R^2R_-{}^2}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ ,$$
$$\tau=\frac{ia\sin\theta}{2\sqrt{2}R^2R_+}\left[\frac{\partial}{\partial r}R^2+2ia\cos\theta\right]\ ,$$
$$\nu=-\frac{\Delta}{4\sqrt{2}R^4R_-}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ , \tag{44}$$
$$\pi=\frac{ia\sin\theta}{2\sqrt{2}R^2R_-}\left[\frac{\partial}{\partial r}R^2+2ia\cos\theta\right]\ ,$$
$$\epsilon=-\frac{ia\sin\theta}{4R^2\Delta}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ ,$$
$$\gamma=\mu+\frac{r-\eta}{2R^2}-\frac{ia\sin\theta}{8R^4}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ ,$$
$$\beta=\frac{\cot\theta}{2\sqrt{2}R_+}-\frac{ia\sin\theta}{4\sqrt{2}R_+}\frac{\partial}{\partial r}\ln\frac{R^2}{r^2+a^2\cos^2\theta}+\frac{1}{2\sqrt{2}R^2R_+}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ ,$$
$$\alpha=\pi-\bar{\beta}+\frac{1}{2\sqrt{2}R^2R_-}\frac{\partial}{\partial\theta}(R^2+a^2\sin^2\theta)\ .$$

The components of the Weyl tensor can be calculated, by (44), from the Ricci identities; once the Weyl scalars are obtained, it is possible to arrive at the Petrov classification. Since in this paper we are primarily interested in the nature of singularities, we shall confine ourselves to the analysis of the curvature scalar

$$g^{ij}R_{ij}=\frac{2\sigma^2}{(r^2-2\eta r+a^2)(r^2+a^2\cos^2\theta)\left[1-\dfrac{2\eta r}{r^2+a^2\cos^2\theta}\right]^{1-m/\eta}} . \tag{45}$$

One sees immediately that there appears a singularity at

$$r=\eta+(\eta^2-a^2\cos^2\theta)^{1/2}=r_+(\theta)$$

i.e., at $R=0$.

Here again the area A of the equipotential surfaces and the proper lengths of the closed curves l_φ and L_θ all tend to zero as one approaches the coordinate location of the singularity at $r=r_+(\theta)$. Therefore the singularity at $r=r_+(\theta)$ has the topology of a point, and consequently also the Kerr ergosurfaces and event horizons are reduced to a point by the presence of a scalar field.

V. STATIC SPHERICALLY SYMMETRIC SOLUTIONS WITH A CONFRMALLY INVARIANT SCALAR FIELD

In the previous section we have described the scalar field by employing the particularly simple energy-momentum tensor shown in Eq. (2). More generally, we can consider the stress tensor[14]

$$T_{ij}=\frac{1}{4\pi}[(1-2\xi)\varphi_i\varphi_j+(2\xi-\tfrac{1}{2})g_{ij}\varphi^k\varphi_k-2\xi\varphi\nabla_i\nabla_j\varphi$$
$$+\tfrac{1}{2}\xi\varphi_{ij}\varphi\Box\varphi-\xi(G_{ij}+\tfrac{3}{2}\xi Rg_{ij})\varphi^2$$
$$+\tfrac{1}{2}(1-3\xi)\mu^2g_{ij}\varphi^2] , \tag{46}$$

where G_{ij} is Einstein's tensor, R is the scalar curvature, ξ is an adjustable parameter, and μ is the mass of the free scalar field φ which, in curved spacetime, satisfies the wave equation

$$(\Box+\xi R+\mu^2)\varphi=0 . \tag{47}$$

In the massless case, which we shall consider, and with the choice $\xi=\frac{1}{6}$ (four dimensions) T_{ij} is manifestly traceless and Einstein's equations imply $R=0$. As a consequence the field equations (1) can now be written as

$$3\left[1-\frac{\varphi^2}{3}\right]R_{ij}=-4\varphi_i\varphi_j+g_{ij}\varphi^k\varphi_k+2\varphi\nabla_i\nabla_j\varphi \tag{48}$$

and the wave equation (47) reduces to the d'Alembertian equation (4).

Our task is that of obtaining, as in Sec. II, the static spherically symmetric line element [Eq. (5)] in the exterior region. The explicit solutions to this problem which appeared in the literature cover only particular cases.[15]

We shall impose the coordinate condition

$$\beta-\frac{\alpha-\gamma}{2}=f(r) , \tag{49}$$

where $f(r)$ is a function at our disposal to select a suitable coordinate system. If we choose

$$f(r)=\ln\left\{\left[1-\frac{2\eta}{r}\right]\cosh^2\left[\frac{\sigma}{2\sqrt{3}\eta}\ln\left[1-\frac{2\eta}{r}\right]\right]\right\} , \tag{50}$$

the required solutions are given by

$$ds^2=\cosh^2\left[\frac{\sigma}{2\sqrt{3}\eta}\ln\left[1-\frac{2\eta}{r}\right]\right]$$
$$\times\left\{\left[1-\frac{2\eta}{r}\right]^{m/\eta}dt^2-\left[1-\frac{2\eta}{r}\right]^{-m/\eta}dr^2\right.$$
$$\left.-\left[1-\frac{2\eta}{r}\right]^{1-m/\eta}r^2(d\theta^2+\sin^2\theta\,d\varphi^2)\right\} \tag{51}$$

and

$$\varphi(r)=\sqrt{3}\tanh\left[\frac{\sigma}{2\sqrt{3}\eta}\ln\left[1-\frac{2\eta}{r}\right]\right] . \tag{52}$$

It is easily seen, by comparing the line elements (7) and (51), that the spacetimes obtained by Einstein's equations with the choices $\xi=0$ and $\xi=\frac{1}{6}$, respectively in the stress tensor (46), are conformally related at $\mu=0$.

To analyze the properties of the line element so obtained, let us consider the standard radial coordinate R defined as

$$R(r)=r\left[1-\frac{2\eta}{r}\right]^{(1-m/\eta)/2}\cosh\left[\frac{\sigma}{2\sqrt{3}\eta}\ln\left[1-\frac{2\eta}{r}\right]\right] . \tag{53}$$

In general it is not possible to recover from Eq. (53) the inverse function $r=r(R)$ explicitly; however, some remarks on the behaviors of the radial coordinates r and R can be made. Let us point out that even if asymptotically they nearly coincide, great differences arise when r approaches the value $r=2\eta$ from above. The investigation near this point has been carried out numerically and the results are the following: if $\sigma^2>3m^2$ the function $R(r)$ decreases monotonically with r and takes the value $R=0$ when $R=2\eta$. On the other hand, when $\sigma^2\leq 3m^2$, $R(r)$ begins to decrease with r, reaches a minimum (of the order η) when r is quite close to 2η, and finally, as r approaches further the value 2η, R steeply tends to infinity. Thus the function $R(r)$ shows that there can be a singularity at $r=2\eta$ with the topology of a point only if $\sigma^2>3m^2$.

We remark that this condition does not seem very realistic; for instance, in this case the wavelengths of the radiation emitted by the collapsing body should be blueshifted to zero as $r\to 2\eta$. Furthermore, a computation of the energy of the scalar field in the exterior region, that is, from the body radius r_0 to infinity, by the Tolman expression

$$E=-\frac{1}{4\pi}\int \sqrt{-g}R_0{}^0 d^3x \tag{54}$$

yields

$$E=\frac{m}{2}\left\{\frac{\sigma}{\sqrt{3}m}-\tanh\left[\frac{\sigma}{2\sqrt{3}\eta}\ln\left[1-\frac{2\eta}{r_0}\right]\right]\right\}\times\sinh\left[\frac{\sigma}{\sqrt{3}\eta}\ln\left[1-\frac{2\eta}{r_0}\right]\right] \tag{55}$$

which is a positive quantity for $\sigma^2>3m^2$ becoming infinitely large as $r_0\to 2\eta$. In the case $\sigma^2\leq 3m^2$ and considering the values of $r>2\eta$ for which $R_{\min}\leq R(r)<\infty$ we can say that the minimum attainable surface has not the topology of a point; but it does not appear as a null surface either and, in this limiting situation the emitted radiation would be greatly but not infinitely red-shifted.

We are well aware that the illustrated results are not exhaustive and that more investigations are necessary; nevertheless, our analysis seems to indicate that the introduction of a scalar field with a traceless energy-momentum tensor leads to a model with very unusual physical properties.

VI. CONCLUSIONS

We have shown that the introduction of a scalar field with minimal coupling to gravity prevents the formation of the event horizons which are present in all the well-known exact solutions of the exterior problem which appeared in the literature. A possible physical effect of the scalar field stems from its masslessness, for the resulting long-range interaction may affect the numerical computation relative to the classical experimental tests of gravitation. Within our scheme we have checked that the only modification concerns the precession of perihelia, but the resulting correction turns out to be undetectable by the present apparatus at least when $\sigma^2\ll m^2$.

The meaning of the scalar charge is clearly an open question. It may well be that a satisfactory explanation should be sought for not on classical but on quantum grounds. For example, the capture of particles (or antiparticles) which are created in pairs in the curved vacuum surrounding the body during the gravitational collapse. This mechanism looks very similar to the one proposed by Hawking to describe black-hole evaporation.[16] Along these lines is the work of Roberts[17] who suggests that, as a consequence of the uncertainty of local energy arising from the curvature due to matter fields, the total stress energy tensor must contain a term which may be assigned to a massless scalar field. Let us finally remark that, apart from the difficulties it generates, the introduction of a massless scalar field into Einstein's equations leads to interesting results for the global structure of spacetime and future work in this direction will clarify many points which are now obscure.

ACKNOWLEDGMENTS

We are very grateful to professor N. Rosen for his encouragement, and to Professor A. Blasi for helpful discussions

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