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Is there another set of left-handed W and Z bosons?

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Experimental consequences of the $Sp_L(6)\times U_Y(1)$ model proposed earlier are studied. It is found that the model suggests another set of light ($\sim$ a few TeV), left-handed, W and Z bosons. Rare decays of (K,B,μ,τ), resulting from the new gauge couplings, are predicted.

It has been shown[1] that the standard $SU(3)\times SU(2)\times U(1)$ model can be extended to the left-right-symmetric, anomaly-free, partial-unification group

$$SU(4)\times Sp_L(6)\times Sp_R(6) \quad , \tag{1}$$

under which the three generations of fermions transform as $[\mathbf{4},\mathbf{6},\mathbf{1}]\oplus[\bar{\mathbf{4}},\mathbf{1},\mathbf{6}]$ (Ref. 2). In this Rapid Communication, we wish to present a further study on the experimental signatures of this model. It is found that, although most of the new gauge bosons of (1) must be heavy ($\gtrsim 10^{2-3}$ TeV), a triplet ($W'^{\pm},Z'$) can be light ($\sim$ a few TeV). A number of processes [rare (K,B,μ,τ) decays, B^0-$\bar{B}^0$ mixing] is discussed so that these ideas may be checked.

Let us first recapitulate the main features of (1). In (1), (gauged) horizontal symmetry is not introduced as a direct product, but is unified with the electroweak interactions.[3] Moreover, this unification is accomplished without extra unknown fermions (except for three right-handed neutrinos). To the best of our knowledge, this is the only existing horizontal scheme which satisfies these two aspects simultaneously. As for the fermion mass spectrum, it was shown that several subgroups of (1) can yield light families naturally. Among these subgroups, the smallest is[4]

$$SU_C(3)\times Sp_L(6)\times U_Y(1) \quad . \tag{2}$$

The model (1) may be motivated from another perspective. It is well known that, when the electroweak interaction is turned off, there emerges a global

$$SU_L(6)\times SU_R(6) \tag{3}$$

symmetry for the quarks, which is believed to be dynamically and spontaneously broken by QCD.[5] Now, the usual electroweak theory gauges only a limited set of generators of (3). This is the case both in the standard model and in the left-right-symmetric model.[6] It is thus natural to wonder why those generators have to be singled out among (3). One possible scheme to explain this may be a composite model in which the generation structure disappears at a fundamental level, making (3) valid only at the quark level. In the absence of plausible composite models, it seems more natural to gauge (3) as much as possible while keeping the fermion species intact. However, not all the generators of (3) can be gauged because of the triangle anomaly. The maximal gauge group is thus $Sp_L(6)\times Sp_R(6)$ uniquely. By unifying leptons with quarks through Pati-Salam SU(4), we arrive at (1).

It is well known that one of the best ways to test extensions of the standard model is to look for processes involving flavor-changing neutral current (FCNC) and/or lepton-flavor-violating current (LFVC). They are ideal to check such a model as (1), because it contains a number of gauge fields which can induce FCNC/LFVC processes. Since strong suppressions are observed in them, most masses of such dangerous gauge fields should be chosen to be 10^{2-3} TeV. The only question is whether there can be exceptionally light gauge fields other than the standard W and Z^0. If there are such fields, then they may induce certain observable phenomena.

In order to answer this question, it is sufficient to consider a subgroup of (2), i.e.,

$$SU_1(2)\times SU_2(2)\times SU_3(2)\times U_Y(1) \quad . \tag{4}$$

Here, when particle mixings are ignored, the gauge fields of $SU_k(2)$ ($\mathbf{A}^{(k)}$) couple only to the left-handed fermions of the kth generation ($k=1,2,3$).[7] Note that all these three gauge couplings are identical since they are equal to the original $Sp_L(6)$ coupling. However, depending on the symmetry-breaking pattern, physical gauge bosons will become linear combinations of $\mathbf{A}^{(k)}$, and hence couple to all the generations. Let us study them by assuming that the symmetry (4) is broken according to the chain

$$[SU_L(2)]^3\times U_Y(1) \underset{M''}{\rightarrow} SU_{12}(2)\times SU_3(2)\times U_Y(1) \underset{M'}{\rightarrow} SU_{123}(2)\times U_Y(1) \underset{M}{\rightarrow} U_{EM}(1) \quad , \tag{5}$$

where $SU_{12}(2)$ [$SU_{123}(2)$] is diagonal SU(2) of $SU_1(2)\times SU_2(2)$ [$SU_1(2)\times SU_2(2)\times SU_3(2)$]. The $SU_{123}(2)$ gauge fields can be expressed as

$$\mathbf{A}=\frac{1}{\sqrt{3}}(\mathbf{A}^{(1)}+\mathbf{A}^{(2)}+\mathbf{A}^{(3)}) \quad , \tag{6}$$

and will be identified as the standard $SU_L(2)$ gauge fields. It is clear from (6) that they have a universal coupling to all the generations. The other combinations of the $SU_{12}(2)\times SU_3(2)$ gauge fields, orthogonal to (6), are

$$\mathbf{A}'=\frac{1}{\sqrt{6}}(\mathbf{A}^{(1)}+\mathbf{A}^{(2)}-2\mathbf{A}^{(3)}) \quad , \tag{7}$$

which couple equally to the first and second generations, but differently to the third. The remaining combinations out of $[SU_L(2)]^3$ are

$$\mathbf{A}''=\frac{1}{\sqrt{2}}(\mathbf{A}^{(1)}-\mathbf{A}^{(2)}) \quad , \tag{8}$$

which exhibits no universality. As long as M, M', and M'' are chosen such that $M \ll M' \ll M''$, the mixings among $\mathbf{A}$, $\mathbf{A}'$, and $\mathbf{A}''$ remain small.

We will next study gauge couplings of fermions. To do this, we introduce diagonalization matrices U_u (U_d) of the left-handed $Q=+\frac{2}{3}$ $(-\frac{1}{3})$ quarks. The structures of the standard gauge couplings are well known. Namely, the W coupling contains the Kobayashi-Maskawa (KM) matrix

$$C^{ud} \equiv U_u^{-1} U_d \ , \tag{9}$$

and the Z coupling is diagonal since

$$C^{jj} \equiv U_j^{-1} U_j = 1, \quad j=u,d \ . \tag{10}$$

In our model, there also exist the $\mathbf{A}'$ and $\mathbf{A}''$ couplings, which contain

$$C'^{jk} = \frac{1}{\sqrt{2}} U_j^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} U_k \ ,$$

$$C''^{jk} = \sqrt{3/2}\, U_j^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} U_k \ , \tag{11}$$

respectively, where $j,k=u,d$. From (11), we can see that the $\mathbf{A}'$ and $\mathbf{A}''$ couplings are nondiagonal in general, giving rise to FCNC/LFVC. Unlike C, C' and C'' are nonunitary by themselves, but satisfy instead

$$(C'^{jk})^\dagger (C'^{jk}) + (C''^{jk})^\dagger (C''^{jk}) = 2 \ , \quad \text{(no sum in } j,k) \ . \tag{12}$$

This analysis can be carried out similarly in the lepton sector. There, however, one can choose $U_\nu = U_e$ so that the lepton family numbers may become approximate symmetries. In other words, $\mathbf{A}$ couples diagonally ($C^{ll}=1$) as in the standard model. The lepton flavor is violated, however, by the $\mathbf{A}'$ and $\mathbf{A}''$ couplings ($C'^{ll}, C''^{ll} \neq$ diagonal).

To study the FCNC/LFVC couplings of $\mathbf{A}'$ further, we will rewrite C' as

$$C'^{jk} = \frac{1}{\sqrt{2}} C^{jk} - \frac{3}{\sqrt{2}} U_j^{-1} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} U_k \ . \tag{13}$$

In particular, for an off-diagonal Z' coupling such as $C'^{dd}_{\alpha\beta}$, $\alpha \neq \beta$, (13) reads

$$C'^{dd}_{\alpha\beta} = -\frac{3}{\sqrt{2}} U^*_{d3\alpha} U_{d3\beta}, \quad \alpha \neq \beta, \quad \alpha,\beta = 1,2,3 \ . \tag{14}$$

To estimate (14), we will make one assumption: It is known empirically that the KM matrix C^{ud} $(=U_u^{-1}U_d)$ has a hierarchy structure.[8,9] Unless there is an unnatural cancellation between U_u and U_d, it seems reasonable to assume the U_u and U_d themselves have similar structures. We will hence assume the parametrization[10]

$$U_d = \begin{pmatrix} 1-\frac{1}{2}\lambda_d^2 & \lambda_d & \lambda_d^3 A_d \rho_d \\ -\lambda_d & 1-\frac{1}{2}\lambda_d^2 & \lambda_d^2 A_d \\ \lambda_d^3 A_d (1-\rho_d) & -\lambda_d^2 A_d & 1 \end{pmatrix} , \tag{15}$$

and use $U_d \sim C^{ud}$, when appropriate, to make order-of-magnitude estimates. From this, we find approximately that $|C'^{dd}_{12}| \sim 4.9\times 10^{-4}$, $|C'^{dd}_{13}| \sim 1.1\times 10^{-2}$, and $|C'^{dd}_{23}| \sim 0.093$. It is therefore concluded that, although there is no Glashow-Iliopoulos-Maiani mechanism for the $\mathbf{A}'$ couplings, FCNC/LFVC suppressions still occur because of the empirical structure of the KM matrix.[11] Furthermore, this suppression is rather pronounced between the first and the second generations, but less so for processes involving the b quark.

When the scale M' is reduced, $\mathbf{A}$ and $\mathbf{A}'$ start to mix with each other. As a result, physical W and Z will contain W' and Z' with fractions, $\xi_{WW'}$ and $\xi_{ZZ'}$ $[\sim (M/M')]$. Then, certain FCNC/LFVC processes will be induced by the exchange diagram (Fig. 1). For example, $K_L \to \mu\bar{\mu}$ is calculated to be[12]

$$\frac{\Gamma(K_L \to \mu\bar{\mu})^{\text{exch}}}{\Gamma(K \to \mu\nu)} = |\xi_{ZZ'} C'^{dd}_{12}|^2 \left(\frac{\cos\theta_W}{\sqrt{2}|C^{ud}_{12}|} \right)^2 \ . \tag{16}$$

Now, the standard box-diagram contribution $[\lesssim 10^{-10}$ (Ref. 13)] is known to be too small to account for the short-distance part $[(3.2\pm 2.4)\times 10^{-9}$ (Ref. 14)] of $B(K_L \to \mu\bar{\mu})$. We thus propose to remedy this by demanding that the Z' contribution should saturate this part. Namely, we will set

$$B(K_L \to \mu\bar{\mu})^{\text{exch}} = 5.6\times 10^{-9} \ , \tag{17a}$$

or

$$|\xi_{ZZ'} C'^{dd}_{12}| = 1.7\times 10^{-5} \ . \tag{17b}$$

The result (17b) suggests that M' is of order of a few TeV.[15]

Among LFVC processes, the most stringent bound is set for $\mu \to ee\bar{e}$ $[B(\mu \to 3e) < 2.4\times 10^{-12}$ (Ref. 16)]. Since

$$B(\mu \to ee\bar{e}) = |\xi_{ZZ'} C'^{ee}_{12}|^2 \cos^2\theta_W \left[\frac{(1-2\sin^2\theta_W)^2}{2} + \sin^4\theta_W \right] \ , \tag{18}$$

we find $|C'^{ee}_{12}/C'^{dd}_{12}|^2 < 0.052$. This ratio can be small, even if $U_e \sim U_d$, because of the following reason. Let us parametrize U_e as (15) with the replacements $\lambda_d \to \lambda_e$, etc. Then, we see that the ratio contains $(\lambda_e/\lambda_d)^{10}$. Thus, one can obtain $|C'^{ee}_{12}/C'^{dd}_{12}|^2 < 0.052$ by choosing, e.g.,

$$A_e/A_d \sim 1, \quad \rho_e/\rho_d \sim 1, \quad \lambda_e/\lambda_d \lesssim 0.74 \ . \tag{19}$$

We can now deduce the effects of Z' or W' on other FCNC/LFVC processes. (See Tables I and II.) Among the rare decays (Table I), we find the relations

$$\frac{\Gamma(K_L \to \mu\mu)^{\text{exch}}}{\Gamma(K \to \mu\nu)} : \frac{\Gamma(K \to \pi\nu\bar{\nu})^{\text{exch}}}{\Gamma(K \to \pi^0 e\nu)} = 1:3 \ , \tag{20a}$$

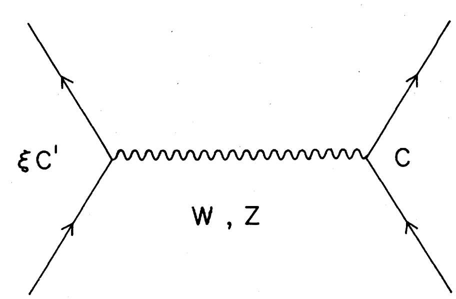

FIG. 1. A typical exchange diagram inducing FCNC/LFVC processes. It contains the standard W (Z) exchange, but with a single W-W' (Z-Z') mixing.

TABLE I. Branching ratios of several FCNC/LFVC decays. The standard-model results are obtained as usual for the given C and m_t ($30 \lesssim m_t \lesssim 50$ GeV). (See also Ref. 13.) The results of the present model are calculated from diagrams containing a single Z-Z' (W-W') mixing. Some results are obtained based on assumptions as indicated. Others are derived from them by Eq. (20). The results denoted by (a), (b) are also used to derive other results, as indicated. Experimental data are taken from Ref. 17 except that $B(B\rightarrow l\bar{l}X)$ and $B(\mu\rightarrow ee\bar{e})$ are given in Refs. 18 and 16, respectively.

	Calculated result: Standard model	This model: Results	This model: Assumptions used	Experimental data
$K_L\rightarrow\mu\bar{\mu}$	$\sim 1\times10^{-10}$	5.6×10^{-9}	(input)	$(9.1\pm1.9)\times10^{-9}$
$K\rightarrow\pi\nu\bar{\nu}$	$(2\text{–}7)\times10^{-11}$	3.1×10^{-10}		$<1.4\times10^{-7}$
$B\rightarrow l\bar{l}X$	$\lesssim 3\times10^{-7}$	0.031%	$U_d\sim C^{ud}$	$<0.31\%$
$\mu\rightarrow ee\bar{e}$	0	$<2.4\times10^{-12}$ (a)	(input)	$<2.4\times10^{-12}$
$\mu\rightarrow e\gamma$	0	$<0.89\times10^{-13}$	$\xi_{WW'}=\xi_{ZZ'}$, (a)	$<1.7\times10^{-10}$
$\tau\rightarrow\mu\mu\bar{\mu}$	0	$<1.0\times10^{-7}$ (b)	Eq. (19), (a)	$<4.9\times10^{-4}$
$\tau\rightarrow\mu e\bar{e}$	0	$<6.3\times10^{-8}$	(b)	$<4.4\times10^{-4}$
$\tau\rightarrow\mu\pi^0$	0	$<1.3\times10^{-7}$	(b)	$<8.2\times10^{-4}$
$\tau\rightarrow\mu\rho^0$	0	$<0.91\times10^{-7}$	(b)	$<4.4\times10^{-4}$

$$\frac{\Gamma(\tau\rightarrow\mu\mu\bar{\mu})}{\Gamma(\tau\rightarrow\mu\nu\bar{\nu})}:\frac{\Gamma(\tau\rightarrow\mu e\bar{e})}{\Gamma(\tau\rightarrow\mu\nu\bar{\nu})}:\frac{\Gamma(\tau\rightarrow\mu\pi^0)}{\Gamma(\tau\rightarrow\nu\pi)}:\frac{\Gamma(\tau\rightarrow\mu\rho^0)}{\Gamma(\tau\rightarrow\nu\rho)}$$

$$=[(1-2\sin^2\theta_W)^2+2\sin^4\theta_W]:[(1-2\sin^2\theta_W)^2/2+2\sin^4\theta_W]:1:(1-2\sin^2\theta_W)^2\ , \tag{20b}$$

$$\frac{\Gamma(\mu\rightarrow e\gamma)}{\Gamma(\mu\rightarrow ee\bar{e})}=\frac{\Gamma(\tau\rightarrow\mu\gamma)}{\Gamma(\tau\rightarrow\mu\mu\mu)}\ . \tag{20c}$$

The relations (20) are independent of ξ and C', and hence characteristic of the present model. Particularly, (20a) yields $B(K\rightarrow\pi\nu\bar{\nu})^{\text{exch}}=3.1\times10^{-10}$, which overwhelms the standard result [$\lesssim(2\text{–}7)\times10^{-11}$ (Ref. 13)], although there may be considerable interference. Besides (20), we used, if necessary, specific assumptions such as $U_d\sim C^{ud}$ and (19). When estimating the neutral pseudoscalar mixings (Table II), we further assumed that the vacuum saturates the intermediate state, and that $f_P{}^2m_P=\text{const}$. Owing to these additional assumptions, the numbers given in Table II are more uncertain than those of Table I.

Several remarks are in order. The rate for $K\rightarrow\pi\nu\bar{\nu}$ is within reach of future experiments.[19] Also, $B\rightarrow l\bar{l}X$ may soon become observable. The $K\rightarrow\pi\nu\bar{\nu}$ mode is particularly interesting in view of checking the model via (20a). However, the interference with the standard box diagram may modify it substantially. The rare τ decays are also important to test this model by (20b) although they require improvements of the existing bounds by roughly three orders of magnitude. As for the pseudoscalar mixings (Table II), it seems clear, despite the ambiguities, that the effect is too small to affect $K^0\leftrightarrow\bar{K}^0$ or $B^0\leftrightarrow\bar{B}^0$, but large enough to mix $(\bar{b}s)$ and $(\bar{s}b)$ significantly.

We would like to add a few comments. (i) It is somewhat hard to observe W'/Z' directly, not only because they are heavy, but also because they decay predominantly into the

TABLE II. Neutral pseudoscalar mixings. Here $\text{Re}T(K^0\text{-}\bar{K}^0)$, for example, is the real part of the K^0-$\bar{K}^0$ transition amplitude. Vacuum saturation, $\xi_{ZZ'}=m_Z/m_{Z'}$, and other assumptions as indicated are used. The two classes (A) and (B) correspond to the two parametrizations of the KM matrix.

	Standard model (box diagram)	This model (Z,Z' exchange)	Extra assumptions
$\dfrac{\text{Re}T(K^0\text{-}\bar{K}^0)}{2\Gamma(K_S)}$	0.6 (A),(B)	7.4×10^{-4}	
$\dfrac{\text{Re}T(B^0\text{-}\bar{B}^0)}{2\Gamma(B^0)}$	$(1\text{–}8)\times10^{-3}$ (A) $(1\text{–}6)\times10^{-2}$ (B)	6×10^{-3} (A),(B)	$U_d\sim C^{ud}$ $f^2m=\text{const}$
$\dfrac{\text{Re}T((\bar{b}s)\text{-}(\bar{s}b))}{2\Gamma(B^0)}$	0.1–0.5 (A),(B)	0.4 (A) 0.05 (B)	$U_d\sim C^{ud}$ $f^2m\sim\text{const}$ $\Gamma(\bar{b}s)=\Gamma(B^0)$

third generation ($B=\frac{2}{3}$). It is, however, not necessary to know precise mass values of W'/Z' in order to estimate their effects on FCNC/LFVC processes, as long as they are not too heavy. (ii) Observations of the FCNC/LFVC processes will supply evidence for extended models such as ours. Our specific model will be favored if the B/τ processes are relatively enhanced. (iii) The ultimate test of our model is given by Eq. (20). Such tests, however, are much harder experimentally. (iv) Some future experiments are devoted to observing $K_L \rightarrow \mu e$ and $K \rightarrow \pi\mu e$ (Ref. 19). They, as well as $\tau \rightarrow \mu K_S$ and $\tau^- \rightarrow \mu^+ e^- e^-$, are excluded in this analysis because the gauge-field contributions to them are extremely small ($\lesssim 10^{-17}$) (Ref. 20). (v) Since we considered (2) or (4) here, ν were assumed to be massless. In more extended models such as (1), ν will be massive naturally, and hence exhibit a variety of interesting LFVC phenomena such as ν oscillation.

In conclusion, the known generation structure seems to suggest a natural extension of the standard model, as given by (1), (2), and (5). It is argued that a second SU(2)$'$, besides the standard $SU_L(2)$, can have a scale as low as a few TeV. The model suggests a natural correlation between FCNC/LFVC suppressions and the known structure of the KM matrix. According to this scenario, the FCNC/LFVC suppression is very strong between the first and the second generations, but less so for the third. As a result a number of rare FCNC/LFVC processes is predicted.

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[1]T. K. Kuo and N. Nakagawa, Phys. Lett. **138B**, 135 (1984); **140B**, 63 (1984); Phys. Rev. D **30**, 2011 (1984); Nucl. Phys. **B250**, 641 (1985).

[2]Here, Pati-Salam SU(4) unifies leptons with quarks. Thus, the representation [**4**, **6**, **1**] contains $(\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau)_L$ and $(u,c,t,d,s,b)_L$, while [$\bar{\mathbf{4}}$, **1**, **6**] contains the corresponding charge-conjugated right-handed fermions.

[3]Such horizontal symmetries are chiral, as compared to the usual vectorlike schemes.

[4]In the model (2), even the right-handed neutrinos can be eliminated if necessary.

[5]We take the usual standpoint that the axial-vector baryon number is broken explicitly by the QCD instanton. See G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).

[6]J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975); G. Senjanovic and R. N. Mohapatra, *ibid.* **12**, 1502 (1975).

[7]The possibility of having several W's and Z's, as well as gauge groups like (4), was first analyzed by V. Barger, E. Ma, and K. Whisnant [Phys. Rev. Lett. **46**, 1501 (1981)] and X.-Y. Li and E. Ma [*ibid.* **47**, 1788 (1981)]. Our results are similar to theirs in some aspects, although the two approaches differ. We would like to thank Professor E. Ma for a discussion and for calling our attention to these references.

[8]L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[9]The recent data on B decay suggest that $|C_{12}^{ud}|=0.23$, $|C_{23}^{ud}|=0.044$, and $|C_{13}^{ud}|=0.0051$. See P. Langacker, talk given at the 22nd International Conference on High-Energy Physics, Leipzig, 1984 (unpublished). Owing to the phase ambiguity, however, there exist two classes [(A) or (B)] of solutions for $(|C_{31}^{ud}|, |C_{32}^{ud}|)$ [(0.0052, 0.044) or (0.015, 0.042)]. We choose (A) for definiteness.

[10]In this Rapid Communication, imaginary parts (and hence *CP* violation) are ignored.

[11]See also Li and Ma, Ref. 7.

[12]A kinematical correction is ignored ($0.993 \sim 1$).

[13]F. J. Gilman and J. S. Hagelin, Phys. Lett. **133B**, 443 (1983).

[14]R. E. Shrock and M. B. Voloshin, Phys. Lett. **87B**, 375 (1979).

[15]If $U_d \sim C^{ud}$, then $|C_{12}'^{dd}| \sim 4.9\times 10^{-4}$, and hence $|\xi_{ZZ'}| \sim 0.035$, or $M' \sim 3$ TeV.

[16]Here, we have used the preliminary result given by the SINDRUM collaboration, as reported at 10th International Conference on Particles and Nuclei, Heidelberg, 1984 (unpublished). We thank Dr. H. K. Walter for communicating their result to us.

[17]Particle Data Group, C. G. Wohl *et al.*, Rev. Mod. Phys. **56**, S1 (1984).

[18]CLEO Collaboration, P. Avery *et al.*, Phys. Rev. Lett. **53**, 1309 (1984).

[19]L. S. Littenberg, BNL Report No. BNL-35086 (unpublished).

[20]The heavy gauge fields, or the leptoquark scalar field introduced earlier (Ref. 1) may take over for such processes, depending on their masses.