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Chiral fermions beyond the standard model

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A scheme is discussed for constructing anomaly-free, charge-vectorial chiral sets of fermions which acquire masses by coupling to the Higgs doublet of the standard model.

I. INTRODUCTION

The standard $SU_c(3)\times SU_W(2)\times U_Y(1)$ model (SM) (Refs. 1 and 2) of electroweak interactions has so far passed every experimental test.[3] Maintaining this harmony with experiment imposes a strong restriction on any proposed extension of the SM. Many extended models which have been proposed, for example, the grand unified and/or supersymmetric models, have involved expanding the theoretical framework of the SM. In this paper we continue a program aimed at extending the SM without the introduction of larger groups or new theoretical ideas.[4] The objective of this program is to clarify and enumerate the physical parameters and experimental signatures of fermions which in addition to the familiar three families of quarks and leptons are allowed by the basic principles of the SM and by the restrictions imposed by present experiments.

Fermions in the SM form multiplets which we denote as $(r_c,2I^W+1)^Y_{L,R}$, where r_c is the color representation, I^W the weak-isospin representation, and Y the weak hypercharge. The electric charge of any member of the multiplet is $Q=I_3^W+\frac{1}{2}Y$. The subscript L (R) denotes a left- (right-) handed field. The hypercharge anomaly (the only anomaly which will concern us) of such a state is

$$A=(\pm)d_c(2I^W+1)Y^3 .$$

The plus (minus) sign applies to L (R), and d_c is the dimensionality of the color representation.

The relevant basic principles of the SM are the following.

(1) The electric and color currents are vectorlike.

(2) The breaking of $SU_W(2)\times U_Y(1)$ into $U_{EM}(1)$ is induced by the neutral component of an $I^W=\frac{1}{2}$, $Y=1$, color-singlet scalar field (the Higgs doublet) acquiring a vacuum expectation value.

(3) There is no Y anomaly.[5]

Condition 1 implies that for every left- (right-) handed field there is a corresponding right- (left-) handed field with the same charge and color quantum numbers. (There is thus no color and charge anomaly.) As will become clear from the discussion in Sec. II, this feature is necessary to allow a charge- and color-conserving mass for each colored or charged particle. Condition 2, which states that the $SU_W(2)\times U_Y(1)$ symmetry is broken in the $I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ channel, is satisfied to considerable accuracy for the usual quarks and leptons. We ignore the possibility of additional $\Delta I_W=1,\frac{3}{2},\ldots$ symmetry-breaking effects. Condition 3 is required for renormalizability.

Since the Y anomaly cancels among the ordinary quarks and leptons in the SM, the contribution to the anomaly coming from any additional particles must cancel among themselves. Such a cancellation would occur if the new particles were completely vectorlike in $SU_c(3)\times SU_W(2)\times U_Y(1)$. However they could also be vectorlike with respect to the color and electric charges but chiral (i.e., non-vectorlike) with respect to I^W and combined in such a way that the Y anomaly vanishes. In the first case $\Delta I^W=0$ mass terms could exist without violating the underlying $SU_c(3)\times SU_W(2)\times U_Y(1)$ symmetry.[6,7] In the second case such mass terms could not be present, and every charged or colored particle (but not necessarily charge and color neutral particles) would have to acquire a $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ mass as a consequence of the symmetry breaking.[4,6] This latter scenario is followed by the usual quarks and leptons in the SM.

In this paper we consider the construction of new chiral sets in the SM which follow the second scenario above. We will insist that with the possible exception of color and charge neutrals all particles can have $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ mass terms. In the construction of these sets we will not insist that the Y anomaly cancels, because given sets i with anomalies A_i we can always combine them into a larger anomaly-free set by taking n_i multiplets of set i such that $\sum n_i A_i=0$. (Recall that the sign of the anomaly changes when we exchange the labels L and R in set i.)

The remainder of this paper discusses the problem of constructing the chiral sets. In Sec. II the problem is formulated and simple examples of chiral sets are presented. In Sec. III a procedure is given for generating the general chiral sets. We have not succeeded in writing such sets in general closed form but instead state a systematic method of construction which also leads us to some conjectures on the general form. Section IV states some implications of this work. In an appendix we list the most useful ("low-lying") chiral sets.

II. STRUCTURE OF THE CHIRAL SETS

A chiral set is composed of left-handed and right-handed weak isospin multiplets, which combine in pairs to give color-singlet, $\Delta I^W = \frac{1}{2}$, $\Delta Y = \pm 1$ masses to all the particles in the set. We call a chiral set *irreducible* if, assuming all allowed mass terms are actually present, the multiplets cannot be separated into two or more subsets such that there are no mass couplings involving multiplets in different subsets. Only irreducible sets will be considered explicitly, since reducible sets are simply collections of particles obtained by combining those in two or more irreducible sets.

If a multiplet within a chiral set is represented by a set of fields ψ_i, the charge-conjugate fields ψ_i^c, as well as the ψ_i, are available for the construction of mass terms. It is thus a matter of convention whether the set is considered to contain the multiplet associated with ψ_i or the charge-conjugate multiplet associated with ψ_i^c. But the charge conjugate of a left- (right-) handed multiplet is a right- (left-) handed multiplet, and thus the chiralities of the individual multiplets within a chiral set are ambiguous without further definition, which we discuss below.

All mass terms have either the Dirac or Majorana form. Thus, since they must also be color singlets, any two multiplets in an irreducible chiral set must transform according to the same or according to conjugate representations of the color group. Thus, by considering the charge conjugate of multiplets as necessary, it is possible to require that all multiplets in an irreducible chiral set transform according to the same color representation. If the representation is not self-conjugate (1,8,27, . . .), this condition determines the chirality of each multiplet (to within a charge conjugation of the whole set). It also implies that all mass terms are of the Dirac form. Further, since a $\Delta I^W = \frac{1}{2}$ mass term is obtained by combining a multiplet of isospin I^W with another multiplet of isospin $I^W \pm \frac{1}{2}$, the multiplets of a given chirality have multiplicities which are either all even or all odd. For irreducible chiral sets which transform according to self-conjugate representations of the color group, this last feature can be incorporated simply by considering the charge conjugate of any multiplet for which it is not *a priori* satisfied. All mass terms are then of the Dirac form independently of the color representation.

It is convenient to represent a chiral set by dots and crosses located at the vertices of a two-dimensional square lattice. The dots and crosses represent left- and right-handed particles, respectively, and a multiplet of multiplicity M $(=2I^W+1)$ is indicated by a vertical column of M dots or crosses located at adjacent vertices. The hypercharges of the multiplets are indicated by the overall vertical positions of the multiplets in such a way that all dots and crosses in the same horizontal row correspond to the same electric charge. No absolute scale for the hypercharge is necessary. That is, a set obtained from another by adding a constant y to the hypercharges of all the multiplets does not correspond to a different representation. Also, two representations which differ only in the interchange of dots and crosses are not to be distinguished, since one is simply the charge conjugate of the other.

A mass term is constructed by coupling a column with M dots with a column of $M \pm 1$ crosses. Since the mass term carries hypercharge $Y = \pm 1$, the smaller multiplet, "lines up" horizontally with the larger multiplet either at the top (highest charge) or the bottom (lowest charge). Thus all members of the smaller multiplet are matched in charge with members of the larger multiplet, with one member of the larger multiplet unmatched. As an example, Figs. 1(a) and 1(b) show the two possible configurations wherein a $\Delta I^W = \frac{1}{2}$, $\Delta Y = \pm 1$ mass term can be constructed with a left-handed quadruplet and a right-handed triplet. No such mass term can be constructed with the multiplets in Fig. 1(c), since their hypercharges differ by more than unity.

The multiplets in Figs. 1(a) and 1(b) form a chiral set when the unmatched member is a charge and color neutral without mass, and we can now give in closed form the chiral sets involving neutrals. Let n be the multiplicity of the larger multiplet. Then the chiral set is

$$(1,n)_L^{\pm(n-1)} + (1,n-1)_R^{\pm n} , \tag{2.1}$$

with anomaly

$$A = \mp n(n-1)(2n-1) . \tag{2.2}$$

The upper (lower) sign corresponds to the neutral state at the bottom (top) of the left-handed multiplet.

Henceforth we will require that the multiplets in a chiral set can combine to generate $\Delta I^W = \frac{1}{2}$, $\Delta Y = \pm 1$ masses for all the states. Then the multiplets in Fig. 1(a), do not by themselves comprise a chiral set. Suppose another right-handed triplet is included with the two multiplets in Fig. 1(a). If its hypercharge is one unit less than the hypercharge of the quadruplet in Figs. 1(a) (two units less than the hypercharge of the triplet), the three multi-

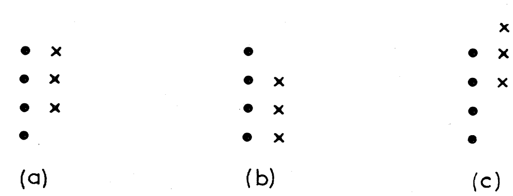

FIG. 1. Three configurations involving a left-handed quadruplet and a right-handed triplet. The two pairs of multiplets in (a) and (b) can combine to form a $\Delta I^W = \frac{1}{2}$, $\Delta Y = \pm 1$ mass term. No such mass term can be constructed with the pair of multiplets represented in (c), since their hypercharges differ by more than unity.

plets are represented as in Fig. 2(a). With these three multiplets one can construct $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ masses for all the particles except for a linear combination of the two crosses in the second row and, separately, in the third row (the linear combinations orthogonal to whatever combinations couple to the quadruplet). To also give masses to these two linear combinations without introducing new massless particles it is necessary to add a left-handed doublet with the same hypercharge as the quadruplet. The resulting four multiplets are shown in Fig. 2(b). The sum of all possible $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ Dirac mass terms constructed with these four multiplets induces a mass for each of the six left-handed and six right-handed particles. The configuration in Fig. 2(b) thus corresponds to a chiral set as defined above.

Chiral sets may be classified by the number p_L (p_R) of left- (right-) handed multiplets in the chiral set. We may restrict ourselves to sets for which $p_L \le p_R$ since sets where $p_R \ge p_L$ are equivalent to these via charge conjugation. It is then convenient to define $p \equiv p_L$. The chiral set in Fig. 2(b) has $p=p_L=p_R=2$. The unique chiral set with $p=1$ is shown in Fig. 3. It can be written as

$$(r_c,2)_L^Y+(r_c,1)_R^{Y+1}+(r_c,1)_R^{Y-1}\ .$$

Two chiral sets with $p=2$ are shown in Fig. 4. In the second set, Fig. 4(b), the multiplicities contain an arbitrary constant. The freedom to add a constant to the multiplicities of all the multiplets by changing the value of M is characteristic of sets where $p_L=p_R$. It does not occur when $p_L \neq p_R$. Three chiral sets which are simply combinations of two sets with $p=1$ are shown in Fig. 5. The sets represented in Figs. 5(a) and 5(b) are irreducible, whereas the one in Fig. 5(c) is reducible.

For arbitrary p we say that a chiral set is *simple* if it does not result from a combination of sets with smaller values of p. A simple chiral set is necessarily irreducible, although as the examples in Figs. 5(a) and 5(b) illustrate, an irreducible set need not be simple. In the following we will be interested in constructing simple chiral sets, since for any p the chiral sets which are not simple are already implicit in the structure of the sets with smaller values of p. The only simple chiral sets for $p=2$ are those represented in Fig. 4.

Consider the chiral set configurations for $p=3$ shown in Fig. 6. The two configurations in Figs. 6(a) and 6(b) are simple, but the configuration in Fig. 6(c) results from combining the sets represented in Fig. 2(b) and in Fig. 3. However, from the configuration in Fig. 6(c) one can generate a simple chiral set by "hinging" the doublet of dots in the second column and the singlet in the third column

FIG. 2. Additional multiplets included with the pair in Fig. 1(a). (b) represents a chiral set allowing $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ masses for all the states.

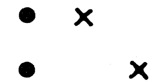

FIG. 3. The unique chiral set with $p=1$.

of crosses about the second row from the top. This hinging is indicated by the arrows in Fig. 7(a), and the resulting new configuration is shown in Fig. 7(b). Similarly, if the doublet of dots and the singlet cross in Fig. 6(b) are hinged about the third row from the top of the figure, a simple chiral-set configuration results, which, however, is the same configuration as would be obtained by turning the configuration in Fig. 6(b) upside down. This inversion of a chiral-set configuration, which can also be viewed as a hinging of the entire configuration about any row, always leads to a single chiral set with the same simplicity and reducibility. It is thus more economical not to consider a configuration and its inverse as distinct. This point of view will be adopted in the following.

The chiral-set configurations in Figs. 6(a) and 6(b) and in Fig. 7(b) represent the only simple chiral sets with $p=3$. The configuration in Fig. 7(b), however, results from hinging the one in Fig. 6(c). Thus the configurations in Fig. 6 may be considered to be the basic configurations for $p=3$ in the sense that any configuration corresponding to a simple chiral set is either included among them or can be obtained from one of them by hinging. These three basic configurations have the form of a "pyramid" arising from the fact that the maximum charges of the dot and cross multiplets decrease by unity as one moves column by column away from the multiplets with the largest maximum charge located at the center of the configuration. The three pyramids in Fig. 6 are irreducible.

It is consistent with our experience in constructing more general chiral sets that the configuration for any simple chiral set is either included among the irreducible pyramids or can be obtained from them by hinging. For example, consider the five irreducible pyramids with $p=4$ shown in Fig. 8. Except for the pyramid in Fig. 8(e), they represent simple chiral sets. The three simple chiral-set configurations in Figs. 9(d)—9(f) result from configurations in Fig. 8 by hinging as shown in Figs. 9(a)—9(c). There are other possible hingings of the configurations in Fig. 8, for example, the hinging of the configuration in Fig. 8(c) shown in Fig. 10, but they either lead to an unacceptable set where not every particle acquires a $\Delta I^W=\frac{1}{2}$,

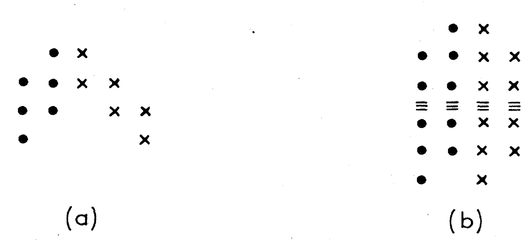

FIG. 4. The only simple chiral sets with $p=2$. The horizontal lines in (b) indicate the possibility of extending the entire figure vertically with dots and crosses. This notation is used in subsequent figures as well.

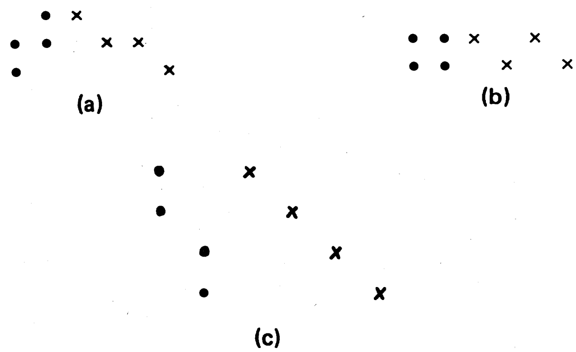

FIG. 5. Three examples of nonsimple chiral sets with $p=2$.

$\Delta Y=\pm 1$ mass, or they lead to configurations which are simply inverted versions of the ones given. Although there are irreducible pyramids with $p=4$ other than those shown in Fig. 8, they are not simple nor can they be hinged to form simple configurations. All the simple chiral sets with $p=4$ are represented by the configurations in Fig. 8 and in Figs. 9(d)–9(f).

Up till now we have considered only examples with low values of p. However, some simple observations may be made about generalizations of the sets we have considered as well as about systematics with p. For example, the chiral sets of Figs. 3, 4(a), 6(a), and 8(a), in which $(n-1)$ left-handed n-plets combine with n right-handed $(n-1)$-plets generalize in a clear way. Their anomaly is

$$A=d_c n(n-1)(2n-1)Y\ .$$

Comparing to Eq. (2.2) for the chiral set involving a neutral, we are tempted to combine these to make an anomaly-free chiral set. This is the choice made by nature in the SM, with $n=2$.

For a generalization on p, we first note that p_L and p_R cannot both be odd. This rule follows because the multiplicities of one chirality are all even, and the multiplicities of the other chirality are all odd. Thus, if p_L and p_R were both odd, the number of particles of one chirality would be even and the number of the other chirality would be odd (the sum of an odd number of odd numbers is an odd number). But then not every particle could acquire a Dirac mass. As a corollary, we note that when there are p multiplets of each chirality then p must be even.

Finally, we observe that in all the examples of simple chiral sets discussed above p_R equals either p (and only when p is even) or $p+1$ (when p is either even or odd). We conjecture that this is a general feature of all simple chiral sets.

In Sec. III we give a systematic scheme for constructing all the irreducible pyramids corresponding to arbitrary p.

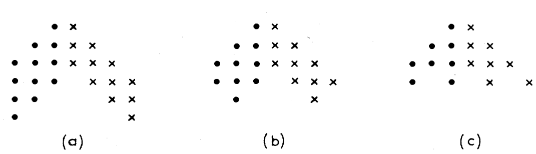

FIG. 6. Three chiral sets with $p=3$. The two sets in (a) and (b) are simple, but the one in (c) is not simple.

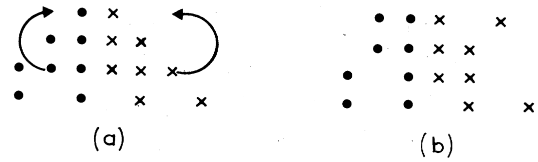

FIG. 7. A hinging of the pyramid in Fig. 6(c) which leads to a simple chiral set.

We conjecture that the configurations for all simple chiral sets can be obtained by hinging these irreducible pyramids. However, we have not proven this conjecture nor, for that matter, have we been able to construct a systematic scheme to generate all the possible hingings of these pyramids which lead to simple configurations.

Since the chiral sets most likely to be useful are the ones with small values of p, we list in the Appendix all chiral sets and their Y anomalies through $p=p_L=4$. We do not separately list sets which differ from those given by exchanging left and right chiralities.

III. CONSTRUCTION OF IRREDUCIBLE PYRAMIDS

For arbitrary p the irreducible pyramids have p multiplets of one chirality and, if p is even, either p or $p+1$ multiplets of the other chirality or, if p is odd, $p+1$ multiplets of the other chirality. That is, if there were more than $p+1$ multiplets of the other chirality at least one of them would have a maximum charge that was more than one unit less than all the maximum charges of the p multiplets with the former chirality (since the maximum charges of the multiplets decrease by unity as one moves out column by column from the pair of dot and cross multiplets at the center of the pyramid). As is clear from the discussion in Sec. II in connection with Fig. 1, such multiplets could not couple with any of the p multiplets to form a $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ mass term.

The irreducible pyramids in Figs. 4, 6, and 8 have the feature that the multiplicities of the multiplets remain the same or decrease by two as one moves column by column away from the multiplets with the largest maximum

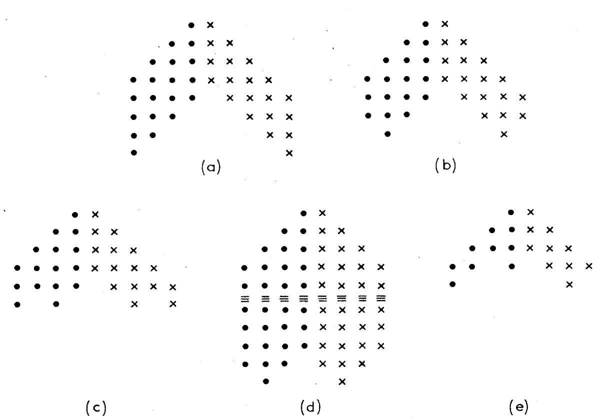

FIG. 8. Five irreducible pyramids with $p=4$. The example in (e) can be generalized by adding an arbitrary non-negative integer n to the multiplicities of all the multiplets.

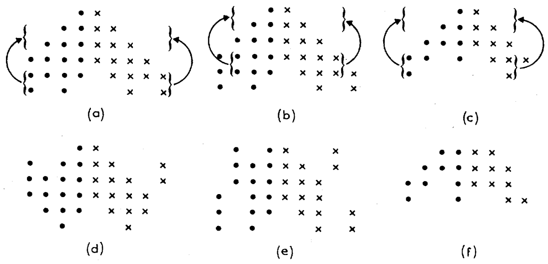

FIG. 9. Three simple chiral sets as they result from hinging the pyramids in Figs. 8(c) and 8(e).

charge at the center of the pyramid. This feature is characteristic of all irreducible pyramids. For example, consider the pyramid in Fig. 11, in which the multiplicity four of the third column of dots is two larger than the multiplicity two of the second column of dots. It is evident (see discussion in connection with Fig. 1) that no $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ mass term can be constructed between dot (cross) multiplets in the third and fourth columns with cross (dot) multiplets in the first and second columns (counting out from the center of the pyramid). The pyramid in Fig. 11 is thus reducible; any pyramid not satisfying the above rule must also be reducible.

Consider an irreducible pyramid with p multiplets of one chirality and $p+1$ multiplets of the other chirality. The pyramid of this kind with the largest number of dots and crosses is composed of p multiplets with multiplicity $p+1$ together with $p=1$ multiplets of multiplicity p. Such a pyramid for $p=6$ is shown in Fig. 12(a). If one deletes the two lowest dots and the two lowest crosses in the two outside columns of this pyramid, one obtains the irreducible pyramid in Fig. 12(b). Similarly, if the lowest two dots in the fifth column and the lowest two crosses in the sixth column are deleted from the configuration in Fig. 12(b), the pyramid in Fig. 12(c) results. Continuing in this way removing the bottom two dots and crosses in four more pairs of columns yields the irreducible pyramids in Figs. 12(d)—12(g). In Figs. 12(f) and 12(g) the subpyramids constructed with the first two pairs of columns of dots and crosses, and with the remaining columns of dots and crosses, are individually chiral set configurations. These pyramids are therefore not simple. However, there are allowed $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ mass terms which are not already allowed by the subpyramids (for example, the multiplet in the second column of dots can couple with the multiplet in the third column of crosses), and hence the pyramids in Figs. 12(f) and 12(g) are irreducible.

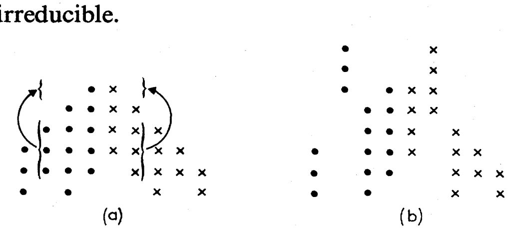

FIG. 10. A hinging of the pyramid in Fig. 8 which leads to a set where not all the particles can acquire $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ masses.

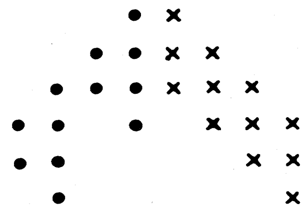

FIG. 11. An example of a reducible pyramid.

New irreducible pyramids can be generated from those in Figs. 12(c)—12(g) by deleting the bottom two dots and crosses in the pairs of outside columns. For example, the irreducible pyramid in Fig. 12(h) is generated in this way from the one in Fig. 12(e). Further, the irreducible

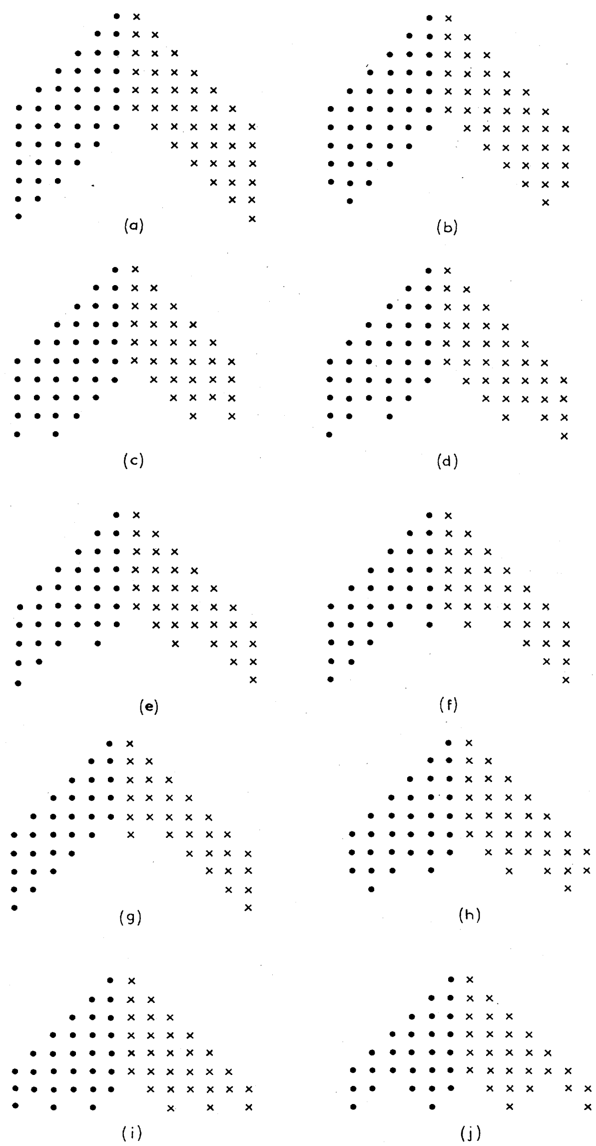

FIG. 12. Irreducible pyramids with $p=6$. As illustrated in (b)—(j), and as discussed in the text, all irreducible pyramids with $p=6$ can be obtained by deleting pairs of dots and crosses from the pyramid in (a).

pyramids in Figs. 12(i) and 12(j) are generated from the one in Fig. 12(h) by successively deleting pairs of dots and crosses in the second column, and then in the second and third columns, from the outside of the pyramid in Fig. 12(h).

Other irreducible pyramids corresponding to $p=6$ are generated by deleting pairs of dots and crosses from the irreducible pyramids in Fig. 12. Some of these may result from deleting the last two crosses in the outside column. The two pyramids in Figs. 13(a) and 13(b) result in this way from deleting the outside pairs of crosses in Figs. 12(i) and 12(j). These pyramids have six columns of crosses as well as six columns of dots. Whenever the numbers of left-handed and right-handed multiplets are the same, the pyramid can be generalized by adding any integer n to the multiplicities of all the multiplets. Thus, for example, the two pyramids in Figs. 13(a) and 13(b) can be generalized to those in Figs. 13(c) and 13(d).

Beginning with the irreducible pyramid having $p+1$ multiplets of multiplicity p with one chirality and p multiplets of multiplicity $p+1$ with the other chirality one can generate all the irreducible pyramids corresponding to the given value of p by successively deleting pairs of dots and crosses as in the examples discussed above. If pairs of dots and crosses are deleted from the bottoms of two columns, the dots and crosses must be in the same two rows, and the multiplets in the two columns must be capable of combining to form a $\Delta I^W=\frac{1}{2}$, $\Delta Y=\pm 1$ mass term. Further, if the resulting pyramid is to be irreducible, it must satisfy the rule discussed earlier in this section regarding the way the multiplicities can vary as one moves out column by column from the center of the pyramid. This last condition implies that a deletion of dots and crosses can only occur from a pair of columns which extend one row below the columns to which they are immediately adjacent. Whenever p is even the procedure of deleting pairs of dots and crosses will lead to pyramids with p multiplets of either chirality. As illustrated in Fig. 13 for $p=6$, such pyramids can be generalized by adding an integer n to the multiplicities of all the multiplets.

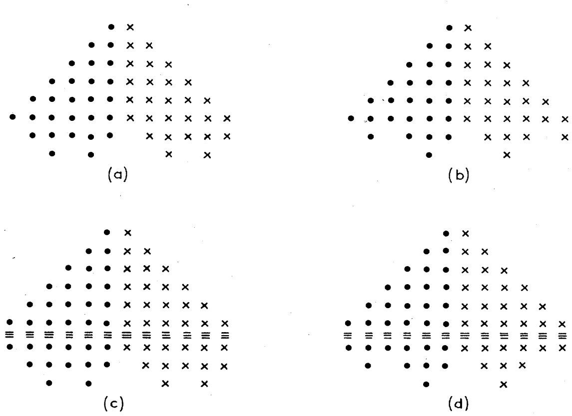

FIG. 13. Generalizing two irreducible pyramids with $p_L=p_R=6$ by adding an arbitrary integer n to the multiplicities of all the multiplets.

IV. CONCLUSIONS

We have described a method for the construction of chiral sets of fermions consistent with the principles of the SM. If such sets exist with a $\Delta I^W=\frac{1}{2}$ mass scale much higher than that of the usual fermions of the SM, then they might not have been detected up to now because of decoupling phenomena. Nevertheless their existence would have consequences both for rare processes at low energies, through (small) mixing with the usual fermions, and for high-energy processes through direct production and decay. Detailed calculations of mixing with the usual fermions would reveal such consequences in terms of the new mass scale; in particular, because we have done nothing in our approach to alter the gauge and Higgs sector of the SM, couplings to and through Z's and W's are predictable. Work along these lines is in progress.

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APPENDIX

We list here all allowed simple sets through $p=4$. Figures are indicated in each case, and we work for a given r_c. The charge spread ΔQ is defined as the maximum charge of the set minus the minimum charge. The multiplicity M is the multiplicity of each charge starting with the maximum charge. A is the anomaly.

1. Fig. 3 ,

$$(2)_L^Y+(1)_R^{Y+1}+(1)_R^{Y-1} ,$$
$$\Delta Q=1 ,$$
$$A=-6d_c y ,$$
$$M=[1,1] .$$

2. Fig. 4(a) ,

$$(3)_L^{Y+1}+(3)_L^{Y-1}+(2)_R^{Y+2}+(2)_R^{Y}+(2)_R^{Y-2} ,$$
$$\Delta Q=3 ,$$
$$A=-30d_c Y ,$$
$$M=[1,2,2,1] .$$

3. Fig. 4(b) ,

$$(n)_L^{Y+1}+(n)_L^{Y-1}+(n+1)_R^{Y}+(n-1)_R^{Y} ,$$
$$\Delta Q=n ,$$
$$A=6d_c nY ,$$
$$M=[1,2,2,\ldots,2,2,1] .$$

4. Fig. 6(a),

$$(4)_L^{Y+2}+(4)_L^{Y}+(4)_L^{Y-2}+(3)_R^{Y+3}+(3)_R^{Y+1}+(3)_R^{Y-1}+(3)_R^{Y-3},$$

$\Delta Q=5$,

$A=-84d_c Y$,

$M=[1,2,3,3,2,1]$.

5. Fig. 6(b),

$$(4)_L^{Y+1}+(4)_L^{Y-1}+(2)_L^{Y-1}+(3)_R^{Y+2}+(3)_R^{Y}+(3)_R^{Y-2}+(1)_R^{Y-2},$$

$\Delta Q=4$,

$A=-6d_c(9Y-1)$,

$M=[1,2,3,3,1]$.

6. Fig. 6(b) inverted,

$$(4)_L^{Y+1}+(4)_L^{Y-1}+(2)_L^{Y+1}+(3)_R^{Y+2}+(3)_R^{Y}+(3)_R^{Y-2}+(1)_R^{Y+2},$$

$\Delta Q=4$,

$A=-6d_c(9Y+1)$,

$M=[1,3,3,2,1]$.

7. Fig. 7(b),

$$(4)_L^{Y}+(2)_L^{Y+2}+(2)_L^{Y-2}+(3)_R^{Y+1}+(3)_R^{Y-1}+(1)_R^{Y+3}+(1)_R^{Y-3},$$

$\Delta Q=3$,

$A=-24d_c Y$,

$M=[2,2,2,2]$.

8. Fig. 8(a),

$$(5)_L^{Y+3}+(5)_L^{Y+1}+(5)_L^{Y-1}+(5)_L^{Y-3}+(4)_R^{Y+4}+(4)_R^{Y+2}+(4)_R^{Y}+(4)_R^{Y-2}+(4)_R^{Y-4},$$

$\Delta Q=7$,

$A=-180d_c Y$,

$M=[1,2,3,4,4,3,2,1]$.

9. Fig. 8(b),

$$(5)_L^{Y+2}+(5)_L^{Y}+(5)_L^{Y-2}+(3)_L^{Y-2}+(4)_R^{Y+3}+(4)_R^{Y+1}+(4)_R^{Y-3}+(2)_R^{Y-3},$$

$\Delta Q=6$,

$A=-6d_c(23Y-5)$,

$M=[1,2,3,4,4,3,1]$.

10. Fig. 8(b) inverted,

$$(5)_L^{Y+2}+(5)_L^{Y}+(5)_L^{Y-2}+(3)_L^{Y+2}+(4)_R^{Y+3}+(4)_R^{Y+1}+(4)_R^{Y-1}+(4)_R^{Y-3}+(2)_R^{Y+3},$$

$\Delta Q=6$,

$A=-6d_c(23Y+5)$,

$M=[1,3,4,4,3,2,1]$.

11. Fig. 8(c),

$$(5)_L^{Y+1}+(5)_L^{Y-1}+(3)_L^{Y-3}+(3)_L^{Y-1}+(4)_R^{Y+2}+(4)_R^{Y}+(4)_R^{Y-2}+(2)_R^{Y-4}+(2)_R^{Y-2},$$

$\Delta Q=5$,

$A=-6d_c(16Y-10)$,

$M=[1,2,3,4,4,2]$.

12. Fig. 8(c) inverted,

$$(5)_L^{Y+1}+(5)_L^{Y-1}+(3)_L^{Y+3}+(3)_L^{Y+1}+(4)_R^{Y+2}+(4)_R^{Y-2}+(2)_R^{Y+4}+(2)_R^{Y+2},$$

$\Delta Q=5$,

$A=-6d_c(16Y+10)$,

$M=[2,4,4,3,2,1]$.

13. Fig. 8(d),

$$(n)_L^{Y+2}+(n)_L^{Y}+(n)_L^{Y-2}+(n-2)_L^{Y-2}+(n+1)_R^{Y+1}+(n+1)_R^{Y-1}+(n-1)_R^{Y-1}+(n-3)_R^{Y-1},$$

$\Delta Q=n+1$,

$A=6d_c[(4n-3)Y-n+2]$,

$M=[1,2,3,4,4,\ldots,4,3,1]$.

14. Fig. 8(d) inverted,

$$(n)_L^{Y+2}+(n)_L^{Y}+(n)_L^{Y-2}+(n-2)_L^{Y+2}+(n+1)_R^{Y+1}+(n+1)_R^{Y-1}+(n-1)_R^{Y+1}+(n-3)_R^{Y+1},$$

$\Delta Q=n+1$,

$A=6d_c[(4n-3)Y+n-2]$,

$M=[1,3,4,4,\ldots,4,3,2,1]$.

15. Fig. 9(d),

$$(5)_L^{Y+1}+(5)_L^{Y-1}+(3)_L^{Y+1}+(3)_L^{Y-1}+(4)_R^{Y+2}+(4)_R^{Y}+(4)_R^{Y-2}+(2)_R^{Y+2}+(2)_R^{Y-2},$$

$\Delta Q=5$,

$A=-96d_c Y$,

$M=[1,3,4,4,3,1]$.

16. Fig. 9(e) ,

$$(5)_L^{Y+1}+(5)_L^{Y-1}+(3)_L^{Y-3}+(3)_L^{Y+3}+(4)_R^{Y+2}+(4)_R^{Y}+(4)_R^{Y-2}+(2)_R^{Y-4}+(2)_R^{Y+4} ,$$

$\Delta Q=5$,

$A=-96d_c Y$,

$M=[2,3,3,3,3,2]$.

17. Fig. 9(f) ,

$$(4)_L^{Y+2}+(2)_L^{Y+4}+(2)_L^{Y+2}+(2)_L^{Y}+(3)_R^{Y+3}+(3)_R^{Y+3}+(3)_R^{Y+1}+(1)_R^{Y-1} ,$$

$\Delta Q=3$,

$A=-12d_c(Y-1)$,

$M=[2,3,3,2]$.

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