Anomalous magnetic moment of light quarks and dynamical symmetry breaking

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(Received 12 July 1984)

It is shown that in theories in which chiral symmetry breaks dynamically, quarks can have a rather large anomalous magnetic moment. This has been first shown, as an example, in a modified Nambu—Jona-Lasinio model. Next, using light-quark dynamical masses in QCD, derived and used by various authors, the light-quark anomalous magnetic moment has been calculated. This has been done in the one-gluon-exchange approximation using a nonsingular or singular form of the gluon propagator in a consistent way. It has been found that not all forms of quark dynamical masses give sensible results. Finally, some of the phenomenological consequences of the presence of such a term have also been worked out.

I. INTRODUCTION

There has been recently a revival in interest in calculating the magnetic moment of old ordinary baryons in terms of confined constituent quarks.[1] One way to make quantitative improvement in this direction would be to improve the calculations of the anomalous magnetic moment (AMM) of these constituent quarks. Also the knowledge of the AMM of quarks (at high energy) has implications in deciding whether they have any composite structure.[2] That the chiral symmetry in QCD is dynamically broken has been shown analytically[3] as well as by lattice calculations.[4] The resultant dynamical quark mass dominates low-energy physics. This has been shown in quite a few works.[5] In view of this last fact, it will be interesting to study the effect of the introduction of dynamical quark mass on the AMM of quarks. We will restrict our attention to the light quarks u and d, which we can assume, to a good approximation, to have vanishing current quark masses.

In QED, the AMM term has the form

$$\frac{i}{2m}\frac{\alpha}{2\pi}\sigma_{\mu\nu}q^{\nu} .$$

In QCD, where chiral symmetry has been broken dynamically, one naively expects the same form except for the change $m \rightarrow m_{\text{dyn}}$ and $\alpha \rightarrow \alpha_s C_2$. Taking m_{dyn} to be proportional to α_s (Ref. 6), one observes that the AMM term will be of $O((\alpha_s)^0)$. That is, in such a theory there appears to be room for a rather large AMM term.

First of all, to have a feeling for how the things are working, we shall consider a simple model example. Thus Sec. II deals with a modified form of the Nambu—Jona-Lasinio model.[7] The modification consists in the introduction of a gauge term into the original Lagrangian. The AMM corresponding to this gauge coupling is calculated. Dynamical mass generation is assumed to occur as in the original model. In Sec. III, we take up the QCD case. Here we employ various forms of "running" dynamical mass which have been calculated and used earlier by different authors to calculate low-energy quantities in hadronic physics. We shall see that not all of these forms are suitable for getting a sensible result for AMM. We find that the gluon propagator obtained by Baker, Ball, and Zachariasen[8] and the resultant quark propagator obtained by Ball and Zachariasen[9] can be used consistently for this calculation. In Sec. IV, we derive some phenomenological consequences of the presence of such a term. Section V is devoted to the discussion of our conclusions.

II. MODIFIED NAMBU—JONA-LASINIO MODEL

As an example, we shall consider the following Lagrangian:

$$\mathscr{L}=\bar{\psi}(i\partial\!\!\!/-eA\!\!\!/)\psi-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2 +g[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2] \qquad (2.1)$$

which is invariant under the ordinary local phase transformation

$$\psi(x)\rightarrow\exp[i\theta(x)]\psi(x),\quad A_\mu\rightarrow A_\mu-\partial_\mu\theta(x)/e ,$$

as well as the chiral (global) transformation

$$\psi(x)\rightarrow e^{i\gamma_5\theta}\psi(x),\quad \bar{\psi}(x)\rightarrow\bar{\psi}(x)e^{i\gamma_5\theta} .$$

Gauge terms will be considered as small perturbations on the original field theory. After the chiral symmetry has been broken dynamically, the fermion develops mass $\Sigma(p)=m$ given by[7] (neglecting the effect of gauge interaction)

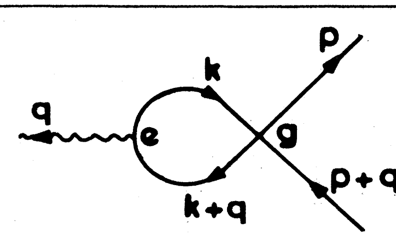

FIG. 1. The lowest-order diagram which contributes to the anomalous magnetic moment of the quark in the Nambu—Jona-Lasinio model.

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$$\frac{2\pi^2}{g\Lambda^2}=1-\frac{m^2}{\Lambda^2}\ln\left[\frac{\Lambda^2}{m^2}+1\right], \tag{2.2}$$

where Λ is the ultraviolet cutoff and $0<2\pi^2/g\Lambda^2<1$ for getting real Λ/m. Next, we shall use the resultant fermion propagator

$$S(p)=\frac{1}{\not p-\Sigma(p)} \tag{2.3}$$

to calculate the lowest-order diagram given in Fig. 1. This will give the AMM term

$$\Gamma_\mu(p,b+q)-\gamma_\mu=2ig\int(dk)\frac{i}{\not k-\Sigma(k)}\gamma_\mu\frac{i}{\not k+\not q-\Sigma(k+q)}$$

$$=-\frac{i\sigma_{\mu\nu}q^\nu}{2m}\frac{m^2g}{4\pi^2}\left[\ln\left[\frac{\Lambda^2}{m^2}+1\right]-\frac{\Lambda^2}{\Lambda^2+m^2}\right]$$

$$=\frac{i\sigma_{\mu\nu}q^\nu}{2m}\frac{1/2}{2\pi^2/g\Lambda^2}\left[\frac{2\pi^2}{g\Lambda^2}-\frac{\Lambda^2}{\Lambda^2+m^2}\right] \tag{2.4a}$$

$$\simeq-\frac{i\sigma_{\mu\nu}q^\nu}{2m}\frac{m^2}{2\Lambda^2}\ln\frac{\Lambda^2}{m^2}\quad(\text{for}\,\Lambda^2\gg m^2), \tag{2.4b}$$

where $(dk)=d^4k/(2\pi)^4$. Given the equation (2.4a) and the constraint $0<2\pi^2/g\Lambda^2<1$, one can see that the AMM can be large.

III. QCD WITH CHIRAL QUARKS

A. The model

Following Delbourgo and Scadron,[10] we shall assume that in QCD with light quarks (u and d), chiral flavor symmetry is realized in the Nambu-Goldstone[7] mode. QED will also be assumed to be operative, but its effect on the mass splitting of u and d quarks will be ignored. In literature various forms of running quark masses have been derived and used for calculating low-energy hadronic quantities. They can be classified in two main categories: (i) solutions based on the asymptotic behavior of QCD, and (ii) solutions based on the infrared behavior of QCD. We shall use them in the quark propagator in the following Schwinger-Dyson equation satisfied by the quark—electromagnetic-current vertex function:

$$\Gamma_\mu(p,p+q)=Z_2\gamma_\mu-\int(dk)[S(k)\Gamma_\mu(k,k+q)S(k+q)]\times K(p,q,k), \tag{3.1}$$

where K denotes the full, exact, renormalized quark-antiquark Bethe-Salpeter scattering kernel, which contains two distinct pieces: $K=K_{\rm P}+K_{\rm NP}$, where $K_{\rm P}$ admits the familiar skeleton expansion

$$K_{\rm P}(p,q,k)=ig^2\Gamma_\mu^\alpha(p,k)D_{\alpha\beta}^{\mu\nu}(p-k)\Gamma_\nu^\beta(k+q,p+q)+\cdots \tag{3.2}$$

in terms of the quark-gluon vertex function. QED contributions to $K_{\rm P}$ will be ignored. $K_{\rm NP}$ denotes the nonperturbative instanton contributions and vanishes to any finite order in perturbation.[11]

B. Solutions based on asymptotic properties

Lane,[12] in his pioneering work, had shown that in an asymptotically free theory (such as QCD) the kernel is well approximated by the ladder kernel for large momenta, where the instantons are unimportant. It has been argued on experimental grounds[11] that the following asymptotic solution emerges (in Landau gauge):

$$\Sigma(p^2)\underset{p^2\to\infty}{\sim}=\frac{4m_D{}^3}{p^2} \tag{3.3}$$

up to a logarithm which will be ignored. This solution has been successfully used by Pagels and Stokar[5] to calculate the pion decay constant, electromagnetic form factor of the pion, and quark electromagnetic self-energy in QCD. Writing[5]

$$S^{-1}(p)=\not p-\Sigma(p^2) \tag{3.4}$$

with $\Sigma(p^2)$ given by Eq. (3.3), a consistent way would be to calculate the lowest-order vertex diagram given in Fig. 2. It turns out that the Pauli form factor $F_2(0)$ defined by $(m_D\to m)$

$$\Gamma_\mu(p+q,p)=F_1(q^2)\gamma_\mu+\frac{i}{2m}\sigma_{\mu\nu}q^\nu F_2(q^2) \tag{3.5}$$

becomes complex when (3.3) is used (but with the replacement $\Sigma\to m$ in the denominator). Evidently the form (3.3) which mimics a massless scalar propagator is not suitable for the low-energy region. As a remedy, following Cornwall,[5] we shall parametrize $\Sigma(p^2)$ as

$$\Sigma(p^2)=\frac{M\Lambda^2}{\Lambda^2-p^2}. \tag{3.6}$$

This gives the expression

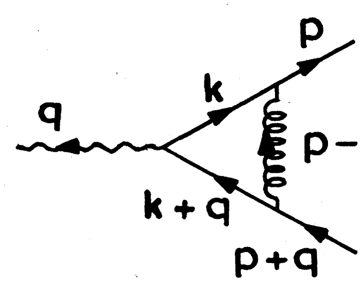

FIG. 2. Correction to the quark-photon vertex by single-gluon exchange.

$$F_2(q^2)=-\frac{g^2C_2m}{4\pi^2}\int_0^1 dx\int_0^1 dy\left[\frac{-m(1-x)}{m^2-q^2y(1-y)}+\frac{2m(1-x)}{m^2x+(\Lambda^2-m^2)(1-y)-q^2xy(1-y)}\right], \tag{3.7}$$

which for $q^2=0$ reduces to

$$F_2(0)=\frac{g^2C_2}{4\pi}\frac{1}{2\pi}+\frac{g^2C_2}{4\pi}\frac{m^2}{\pi(\Lambda^2-m^2)}\left[\ln\frac{m^2}{\Lambda^2-m^2}-\frac{\Lambda^4}{m^4}\ln\frac{\Lambda^2}{\Lambda^2-m^2}+\frac{\Lambda^2}{m^2}-1\right] \tag{3.8a}$$

$$=\frac{g^2C_2}{4\pi^2}f(\Lambda^2/m^2), \tag{3.8b}$$

where

$$m=\Sigma(p^2)\,|_{p=m}=\frac{M\Lambda^2}{\Lambda^2-m^2},$$

provided that it can be solved for m for a given M and Λ. The first term on the right-hand side of (3.8a) is the standard result whereas the second term becomes complex for $m>\Lambda$.

The plot of f vs Λ^2/m^2 in Fig. 3 shows that it starts with a negative value and at $\Lambda^2/m^2\simeq 7.8$ it becomes zero. As Λ^2/m^2 becomes infinitely large, it tends to $\frac{1}{2}$, the standard result. Cornwall[5] has given an estimate of $M\simeq 300$ MeV and $\Lambda\simeq 600$ MeV. But with this set of values, a solution to the equation

$$m=\frac{M\Lambda^2}{\Lambda^2-m^2}$$

does not exist. As an example, the following set of values satisfy this equation: $M\simeq 300$ MeV, $\Lambda\simeq 900$ MeV, and $m\simeq 360$ MeV; and for this set $f(\Lambda^2/m^2)\simeq -0.11$.

C. Solutions based on infrared properties of QCD

Several authors[8] have pointed out that the effective gluon propagator could be written just as in QED with the replacement (modulo gauge terms)

$$-g_{\mu\nu}\frac{e^2}{k^2}\to -g_{\mu\nu}\frac{\bar{g}^2(k^2)}{k^2}C_2,$$

where $\bar{g}^2(k^2)\sim 1/k^2$ as $k^2\to 0$ (which, in a rough sense, corresponds to a linearly rising potential at large distances in configuration space). Some authors have also calculated the form of the quark propagator in chirally symmetric QCD using this kind of effective gluon propagator and found solutions which break chiral symmetry (dynamically)[3,9] and which are entire functions with neither poles nor branch points.[9] In the following we shall use the singular effective gluon propagator, and the chiral-symmetry-breaking quark propagator which has been used along with it or, consistently, which has been derived using such a singular gluon propagator. Furthermore, following Acharya and Narayana-Swamy,[9] we shall assume for small momenta, where $K_{\rm NP}$ is expected to dominate over $K_{\rm P}$, that the leading (most singular) term in K has the ladder form with a singular effective gluon propagator, $D_{\mu\nu}(k)\sim k^{-4}$ (Fig. 2). "This approximation to the dynamics may be regarded as an effective strong-coupling approximation." Works of Cornwall[5] and Richardson[8] support this hypothesis.

If we use expression (3.6) for $\Sigma(p^2)$ (and the mass-shell condition) along with the singular part of the gluon propagator

$$D_{\mu\nu}(k)\sim g_{\mu\nu}\frac{m'^2}{k^4}\to g_{\mu\nu}\frac{m'^2}{(k^2-\mu^2+i\epsilon)^2},$$

which has been used by Cornwall[5] for calculating f_π, etc., then the Pauli form factor becomes linearly divergent as $\mu\to 0$. Cornwall has argued that μ should be kept finite and equal to a typical hadronic mass scale. On the other hand, if, following Cornwall, quarks are assumed as confined particles having no mass shell, then the Pauli term does not arise.

Acharya and Narayana-Swamy[9] have also examined the feasibility of dynamically broken chiral symmetry in QCD with zero-bare-mass quarks interacting via single gluon exchange when the gluon propagator has the infrared behavior: $D_{\mu\nu}\sim k^{-4}$. They have found that the chiral symmetry can be, although not necessarily, realized in the Nambu-Goldstone mode with

$$S^{-1}(p)=-m(p^2)=\text{constant}. \tag{3.9}$$

As can be easily checked this result is too strong to give the Pauli form factor.

The form of the gluon propagator introduced by Baker, Ball, and Zachariasen[8] and the resultant chiral-symmetry-breaking quark propagator derived by Ball and Zachariasen[9] (BZ) is the one which we have found to be the most suitable combination for this calculation. Baker, Ball, and Zachariasen have found that in the infrared region in axial gauges the singular part of the gluon propagator has the same spin structure as that of the free propagator:

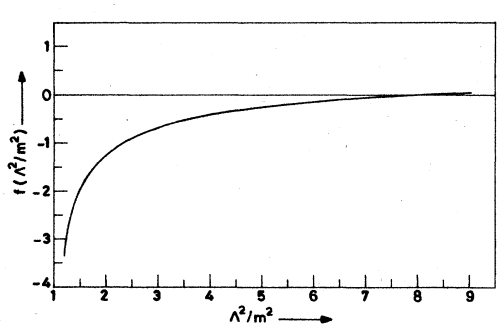

FIG. 3. Plot of $f(\Lambda^2/m^2)$ vs Λ^2/m^2. $f=0$ for $\Lambda^2/m^2\simeq 7.8$, and for large Λ^2/m^2, f asymptotically approaches the standard result.

$$D^{(s)}_{\mu\nu}(q)=-\frac{Z(M)AM^2}{q^4}\left[\delta_{\mu\nu}-\frac{q_\mu n_\nu+q_\nu n_\mu}{q\cdot n}+\frac{q_\mu q_\nu n^2}{(q\cdot n)^2}\right]. \qquad (3.10)$$

The angular average of the tensor part of (3.10) vanishes.[8] This kills any possible infrared divergence.

In a chirally symmetric QCD, using the longitudinal part of the quark-gluon vertex, BZ have found that the introduction of the singular gluon propagator (3.10) can break chiral symmetry so that the resultant quark propagator will have the form

$$S(p)=\not{p}F(p^2)+G(p^2)+(\not{p}H+I)\not{n}\ . \qquad (3.11)$$

We have taken this opportunity to work out the derivation of F and G, and found that the correct expressions for F and G are slightly different from the ones found by BZ:

$$F(p^2)=-\frac{1}{\beta M^2}\Psi(1;1;-p^2/\beta M^2)\ , \qquad (3.12a)$$

$$G(p^2)=C_1\Phi(2;\tfrac{3}{2};-p^2/\beta M^2)\ , \qquad (3.12b)$$

where we have followed the notations of BZ. According to BZ, functions H and I may be dropped in case one is working in the gauge $n\cdot p=0$. Henceforward we shall assume this special choice of gauge for convenience, wherein we may set $H=I=0$. To find out C_1, instead of following BZ, we shall follow the normalization procedure introduced by Cornwall[5] (remembering that our Green's functions are Euclidean):

$$C_1^{-1}=S^{-1}(0)\simeq 300\ \text{MeV}\ . \qquad (3.13)$$

If we take the asymptotic behavior of (3.11) and compare it with the usual form of the Euclidean fermion propagator for asymptotically large momenta, then we find that $m(p^2)\propto 1/p^2$, which may be compared to (3.3). Thus we see that the quark propagator defined by Eq. (3.11) and (3.12) has correct momentum dependence even for asymptotically large momenta.

Again we shall evaluate the ladder graph given by Fig. 2, but here the quark-photon vertex will be the complete one:

$$\Gamma_\mu(p,p+q)=Z_2\gamma_\mu+g_0{}^2C_2\int(dk)\gamma_\lambda S(k)\Gamma_\mu(k,k+q)S(k+q)\gamma_\sigma D_{\lambda\sigma}(p-k)\ . \qquad (3.14)$$

Following Ref. 9, we shall approximate the full quark-photon vertex by the longitudinal one. Since the quark-photon vertex and the quark-gluon vertex obey similar Ward-Takahashi identities, following Acharya and Narayana-Swamy,[9] identical longitudinal decompositions (except for the difference that the quark-gluon vertex will have a color matrix) can be written for both vertices, at least for small momentum transfer. Thus, we shall write[9]

$$S(k)\Gamma^{(L)}_\mu(k,k')S(k')=-\left[\tfrac{1}{2}(F+F')\gamma_\mu+\tfrac{1}{2}(F'-F)\frac{2\not{k}\gamma_\mu\not{k}'+(k^2+k'^2)\gamma_\mu}{k'^2-k^2}+(G'-G)\frac{\gamma_\mu\not{k}'+\not{k}\gamma_\mu}{k'^2-k^2}\right]+\not{n}\text{ terms} \qquad (3.15)$$

for the quark-photon vertex as well, for small $q=k'-k$. Here $F=F(k)$ and $F'=F(k')$, etc. It is to be noted that if the mass-shell condition is not used (which is the case for the confined quarks we are dealing with), then the contribution to the Pauli form factor can come only from the last term in the square brackets of Eq. (3.15). Thus we are interested in calculating the integral

$$I_\mu=\int\frac{d^4k}{(k-p)^4}\frac{G(k'^2)-G(k^2)}{k'^2-k^2}\gamma_\lambda(\gamma_\mu\not{k}'+\not{k}\gamma_\mu)\gamma_\sigma\left[\delta_{\lambda\sigma}-\frac{(k-p)_\lambda n_\sigma+(k-p)_\sigma n_\lambda}{(k-p)\cdot n}+\frac{(k-p)_\lambda(k-p)_\sigma n^2}{[(k-p)\cdot n]^2}\right]. \qquad (3.16)$$

Angular integration in (3.16) was performed, for the sake of simplicity, by assuming q to be in the same direction as p (for details see Appendix). For a general direction of q the result of integration is (all Euclidean γ matrices are anti-Hermitian):

$$\begin{aligned}I_\mu=q_\nu[\gamma_\nu,\gamma_\mu]&\left[\pi^2\left[1+\frac{p\cdot q}{q^2}\right]\left[2-\frac{p\cdot q+q^2}{(p+q)^2}\right]\int_{p^2}^{(p+q)^2}dk^2\frac{G(k^2)}{(k^2+p^2+p\cdot q)^2}\right.\\&\left.-\pi^2\int_0^{q^2/4}dk^2G(k^2)\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\right]\\&-\frac{q_\mu}{q^2}2\pi^2\left[\frac{1}{(p+q)^2}\int_0^{(p+q)^2}dk^2G(k^2)-\frac{1}{p^2}\int_0^{p^2}dk^2G(k^2)\right]+n_\mu\text{ terms}\ ,\end{aligned} \qquad (3.17)$$

where the constant vector n has been chosen to be perpendicular to the hyperplane defined by p, q, and $(i\sigma_{\mu\nu}q_\nu)$. We have not been able to locate any formula in the literature which can be used to perform various integrations, which appear in (3.17), analytically, if the transcendental form given by Eq. (3.12b) is used for $G(k^2)$. However, the integrations can be done in certain limiting cases:

(i) When p^2, $q^2\ll\beta M^2$ so that the series expansion for Φ function is valid and it can be approximated by the first few terms in the series:

$$I_\mu \simeq q_\nu[\gamma_\nu,\gamma_\mu]C_1 \left\{ \pi^2 \left[1+\frac{p\cdot q}{q^2}\right]\left[2-\frac{p\cdot q+q^2}{(p+q)^2}\right] \right.$$

$$\times \left[\left[1+\frac{4}{3}\frac{p^2+p\cdot q}{\beta M^2}+\frac{4}{5}\frac{(p^2+p\cdot q)^2}{\beta^2 M^4}\right]\left[\frac{1}{2p^2+p\cdot q}-\frac{1}{(p+q)\cdot(2p+q)}\right]\right.$$

$$+\frac{4}{5}\frac{(q^2+2p\cdot q)}{\beta^2 M^4}-\left[\frac{4}{3\beta M^2}+\frac{8}{5}\frac{(p^2+p\cdot q)}{\beta^2 M^4}\right]\ln\frac{(p+q)\cdot(2p+q)}{p\cdot(2p+q)}\Bigg]$$

$$-\pi^2\left[\frac{4}{q^2 r}-\frac{4}{3\beta M^2}\left[-2+(r+1)^{1/2}\ln\frac{(r+1)^{1/2}+1}{(r+1)^{1/2}-1}\right]\right.$$

$$\left.\left.+\frac{1}{5}\frac{q^2}{\beta^2 M^4}\left[\tfrac{4}{3}+4r-2r(r+1)^{1/2}\ln\frac{(r+1)^{1/2}+1}{(r+1)^{1/2}-1}\right]\right]\right\}+q_\mu \text{ terms}+n_\mu \text{ terms}\,, \qquad (3.18a)$$

where

$$r=\frac{4p\cdot(p+q)}{q^2}\,.$$

Now, on taking the $p\to 0$ limit,

$$I_\mu \to -q_\nu[\gamma_\nu,\gamma_\mu]C_1\frac{4\pi^2}{3\beta M^2}\left[1-\frac{2q^2}{5\beta M^2}\right]$$

$$+q_\mu \text{ terms}+n_\mu \text{ terms}\,, \qquad (3.18b)$$

$$\xrightarrow[q^2\to 0]{} -q_\nu[\gamma_\nu,\gamma_\mu]C_1\frac{4\pi^2}{3\beta M^2}+q_\mu \text{ terms}+n_\mu \text{ terms}\,. \qquad (3.18c)$$

This gives us

$$\Gamma_\mu(0,q)=-i\sigma_{\mu\nu}q_\nu\frac{C_1}{2}\frac{4}{3}+\gamma_\mu \text{ terms}+\cdots\,, \qquad (3.19)$$

where dots indicate other possible Lorentz structures.

(ii) On the other hand, in the limit $q\to 0$ (but with finite p)

$$I_\mu=-q_\nu[\gamma_\nu,\gamma_\mu]\frac{\pi^2}{p^2}\left[\frac{G(0)}{2}-G(p^2)\right]$$

$$+q_\mu \text{ terms}+n_\mu \text{ terms}. \qquad (3.20)$$

This gives

$$\Gamma_\mu(p,p+q)$$

$$=-i\sigma_{\mu\nu}q_\nu\frac{\beta(M^2)M^2}{2p^2}C_1[\tfrac{1}{2}-\Phi(2;\tfrac{3}{2};-p^2/\beta M^2)$$

$$+\gamma_\mu \text{ terms}+\cdots]\,. \qquad (3.21a)$$

If we substitute numerical values for the quantities appearing in Eq. (3.21a), namely, $\beta(M^2)M^2\simeq(8/3\pi)\times(0.16\ \text{GeV})^2$ (Ref. 13) and $p^2\simeq(0.3\ \text{GeV})^2$ (Refs. 5 and 6), then we get

$$\Gamma_\mu(p,p+q)\simeq-\frac{i\sigma_{\mu\nu}q_\nu}{2\times 300\ \text{MeV}}\times\tfrac{1}{9}+\gamma_\mu \text{ terms}+\cdots\,. \qquad (3.21b)$$

Similarly,

$$\Gamma_\mu(p+q,p)\simeq\frac{i\sigma_{\mu\nu}q_\nu}{2\times 300\ \text{MeV}}\times\tfrac{1}{9}+\gamma_\mu \text{ terms}+\cdots\,. \qquad (3.21c)$$

In the Minkowskian metric, this can be written as

$$\Gamma_\mu(p+q,p)\,|_{\text{Mink}}\simeq-\frac{i\sigma_{\mu\nu}q^\nu}{2\times 300\ \text{MeV}}\times\tfrac{1}{9}$$

$$+\gamma_\mu \text{ terms}+\cdots\,, \qquad (3.21d)$$

where we have followed the common practice[6] in assuming the same form for $m_{\text{dyn}}(p^2)$ in the spacelike region ($p^2<0$) as in the timelike region.

Since the longitudinal parts of the quark-gluon and quark-photon vertices are the same [Eq. (3.15)], their transverse parts are not expected to be much different from each other. Thus one expects that the counterpart of Eq. (3.21d) for the quark-gluon case will be

$$\Gamma_\mu(p+q,p) \simeq -\frac{i\sigma_{\mu\nu}q^\nu}{2\times 300\ \text{MeV}} \times \frac{C}{9} + \gamma_\mu \text{ terms} + \cdots , \qquad (3.21\text{e})$$

where C is a number, expected to be ~ 1, which takes into account the non-Abelian character of the gluon. There will be a similar change in Eq. (3.19).

(iii) When $q^2 \gg p^2 \sim \beta M^2$, since momentum transfer is no longer small, relation (3.15) can be applied only for the quark-gluon vertex. We are assuming that when Eqs. (3.15), (3.16), and (3.17) are used in Eq. (3.14), the result so obtained can be extrapolated for large q. In that case

$$\Gamma_\mu(p,p+q) \simeq -i\sigma_{\mu\nu}q_\nu \frac{\beta^2(M^2)M^4 C_1 C'}{4p^2q^2} + \gamma_\mu \text{ terms} + \cdots , \qquad (3.22\text{a})$$

$$\Gamma_\mu(p,p+q)\,|_{\text{Mink}} \simeq -i\sigma_{\mu\nu}q^\nu \frac{\beta^2 M^4 C_1 C'}{4p^2q^2} + \gamma_\mu \text{ terms} + \cdots , \qquad (3.22\text{b})$$

$$\Gamma_\mu(p+q,p) \simeq i\sigma_{\mu\nu}q_\nu \frac{\beta^2(M^2)M^4 C_1 C'}{4p^2q^2} + \gamma_\mu \text{ terms} + \cdots , \qquad (3.22\text{c})$$

$$\Gamma_\mu(p+q,p)\,|_{\text{Mink}} \simeq i\sigma_{\mu\nu}q^\nu \frac{\beta^2(M^2)M^4 C_1 C'}{4p^2q^2} + \gamma_\mu \text{ terms} + \cdots , \qquad (3.22\text{d})$$

where C' is another number, which it is hoped is ~ 1, and appears for the same reason as C in Eq. (3.21e).

IV. EXPERIMENTAL CONSEQUENCES OF THE ANOMALOUS-MAGNETIC-MOMENT TERM

In this section we shall explore some of the experimental consequences of the presence of the Pauli terms given by Eqs. (3.21) and (3.22).

A. Baryon magnetic moment

If we make the usual assumption that the baryon magnetic moments arise solely from the constituent-quark moments, then following Barik and Das,[1] and references given therein, we can obtain expressions for the magnetic moments of the proton and neutron in terms of the magnetic moments of the corresponding constituent quarks in the following manner:

$$\mu_B = \sum_q \langle B\uparrow | \mu_q \sigma_z^q | B\uparrow \rangle , \qquad (4.1)$$

where $|B\uparrow\rangle$ stands for the state vectors of the baryon in question and in the present case, it represents the regular SU(6) state vectors. The well-known relations between the baryon magnetic moments and the corresponding constituent-quark moments are

$$\mu_p = \tfrac{1}{3}(4\mu_u - \mu_d), \quad \mu_n = \tfrac{1}{3}(4\mu_d - \mu_u) . \qquad (4.2)$$

If we take the simple expression for the quark (Dirac) magnetic moment in units of the nuclear magneton as

$$\mu_q^D = \frac{M_p}{m_q} e_q , \qquad (4.3)$$

where M_p is the proton mass, m_q is the effective mass[5] of the quark defined by Eq. (3.13), and e_q is the electric charge of the quark in the unit of the proton charge. Thus from Eqs. (4.3) and (3.21d) we have (in nuclear magnetons)

$$\mu_q = \frac{M_p}{m_q} e_q (1 - \tfrac{1}{9}) , \qquad (4.4\text{a})$$

$$\mu_u \simeq 1.86, \quad \mu_d \simeq -0.93 . \qquad (4.4\text{b})$$

This result, when substituted in Eq. (4.2), gives

$$\mu_p = 2.79, \quad \mu_n = -1.86 \qquad (4.5)$$

which can be compared to the experimental numbers 2.79 and -1.91, respectively. Numerical results obtained in (4.5) might be accidental (particularly in view of the fact that the value chosen for m_q was a bit uncertain), but what is remarkable is the fact that the dynamical mass, given by Eq. (3.13) (which was so chosen in a different context, namely, in the calculation of the pion decay constant[5]), together with AMM given by (3.21d) (where again the same dynamical mass was used) can yield a number which is so close to the experimental one.

B. Spin-dependent potential energy between quarks

Here we shall calculate spin-dependent potential energy between a heavy quark and a light antiquark. The light antiquark may be regarded as moving within the strong field provided by the heavy quark. Our argument below will not be as airtight as we might wish. For the sake of simplicity, we shall depart from our earlier convention and assume that the quarks may be treated as on mass shell within the framework of the confining potential.[14] A simple calculation of the spin-dependent part of the potential with one-gluon exchange in the nonrelativistic limit shows[15] that

$$\tilde{V}_s(q) = -\tfrac{4}{3} Z(M) \frac{AM^2}{\vec{q}^{\,4}} g_0^2 \Bigg[i\vec{\sigma}_1 \cdot \frac{\vec{p}_1 \times \vec{p}\,'_1}{4m_1^2}(1+2K_1) + \frac{i\vec{\sigma}_1 \cdot \vec{p}_1 \times \vec{p}\,'_1}{2m_1 m_2}(1+K_1) + \frac{i\sigma_2 \cdot \vec{p}_1 \times \vec{p}\,'_1}{4m_2^{\,2}}(1+2K_2) + \frac{i\vec{\sigma}_2 \cdot \vec{p}_1 \times \vec{p}\,'_1}{2m_1 m_2}(1+K_2) - \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) - \vec{q}^{\,2} \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4m_1 m_2}(1+K_1)(1+K_2) \Bigg] , \qquad (4.6\text{a})$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are Pauli matrices which act on the particle 1 and 2 spin wave functions, respectively; K_1 and K_2 are the (constant) chromomagnetic Pauli form factors and q is the momentum transfer, $q=p_1'-p_1$. Following the usual convention[8] that the $1/q^4$ type of potential corresponds to linearly rising potential in the configuration space, we can Fourier-transform Eq. (4.6a) as

$$V_s(r)=\frac{1}{r}\frac{dV(r)}{dr}\left[\frac{\vec{\sigma}_1\cdot\vec{\mathrm{L}}}{4m_1^2}(1+2K_1)+\frac{\vec{\sigma}_1\cdot\vec{\mathrm{L}}}{2m_1m_2}(1+K_1)+\frac{\vec{\sigma}_2\cdot\vec{\mathrm{L}}}{4m_2^2}(1+2K_2)+\frac{\vec{\sigma}_2\cdot\vec{\mathrm{L}}}{2m_1m_2}(1+K_2)\right]$$
$$+\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\nabla}^2-(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})}{4m_1m_2}V(r)(1+K_1)(1+K_2)\;, \tag{4.6b}$$

where

$$V(r)=\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\widetilde{V}(q)\;,$$
$$\widetilde{V}(q)=\tfrac{4}{3}\frac{Z(M)AM^2g_0^2}{\vec{q}^{\,4}}\;. \tag{4.6c}$$

If we apply Eq. (4.6b) to the ground-state ($L=0$) mesons and remember that

$$\tfrac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\nabla}^2-(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})$$

does not contribute for a spherically symmetric potential, then

$$m(B^*)-m(B)=\tfrac{1}{3}\frac{1}{m_bm}\vec{\nabla}^2V(r)(1+K)(1+K_b)\;, \tag{4.7a}$$

$$m(D^*)-m(D)=\tfrac{1}{3}\frac{1}{m_cm}\vec{\nabla}^2V(r)(1+K)(1+K_c)\;. \tag{4.7b}$$

The only conclusion one can draw from Eq. (4.7a) and (4.7b) is that

$$m(B^*)-m(B)=\frac{(1+K_b)}{(1+K_c)}\frac{m_c}{m_b}[m(D^*)-m(D)]\;. \tag{4.7c}$$

b and c quarks have large current-quark masses, and hence K_b and K_c are presumably small; hence (4.7c) is essentially the result obtained by Eichten and Feinberg.[16] This result agrees well with experimental numbers.[16] However, Eq. (4.6b) has a lesser number of free parameters than Eq. (6.2) of Eichten and Feinberg.

C. Reaction cross section of $p\bar{p}\rightarrow$hadrons

In this subsection, we shall calculate essentially the reaction cross section of $q\bar{q}\rightarrow q\bar{q}$ through one-gluon exchange (Fig. 4). For quark-gluon vertices we shall use (3.22) and see the effect of the Pauli term over and above the bare vertex. Since we are looking here at the processes involving large momentum transfer, we shall use the conventional form of the gluon propagator $D_{\mu\nu}(q)\sim q^{-2}$. External quark and antiquark lines are assumed to be on the mass shell. If θ is the scattering angle and $(p+k)^2=(2E)^2$, then the differential cross section for this process in the center-of-mass frame is given by

$$\left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{c.m.}}=\frac{g^4}{128}\frac{2/9}{\pi^2E^2}\left[\frac{1+\cos^4\Theta/2}{\sin^4\Theta/2}-\frac{2\cos^4\Theta/2}{\sin^2\Theta/2}+\frac{1}{2}(1+\cos^2\Theta)\right]$$
$$+\left[\frac{g^4}{128\pi^2E^2}\right]\frac{2}{9}\frac{\beta^2(M^2)M^4C_1}{mE^2}\left[-\frac{1+\cos^4\Theta/2}{2\sin^6\Theta/2}+\frac{\cos^4\Theta/2}{2\sin^4\Theta/2}-\frac{\cos^4\Theta/2}{2\sin^2\Theta/2}+\frac{1}{2}\right]$$
$$+\frac{g^4}{128\pi^2E^2}\frac{C'\beta^2(M^2)M^4C_1}{mE^2}\frac{2}{9}\left[\frac{1+\cos\Theta}{2\sin^6\Theta/2}+\frac{6+\sin^2\Theta+2\cos\Theta}{16\sin^4\Theta/2}+\frac{\sin^2\Theta+2\cos\Theta}{8\sin^2\Theta/2}\right]\;, \tag{4.8}$$

where the first two terms are the contributions coming from the bare vertex part and the third term is the contribution from the Pauli term. It is clear that for high energies, the contribution of the third term can be significant only for very small values of Θ. To get an estimate of the angle at which the contribution of the third term is of the same order or one order of magnitude smaller than that of the first term, we have

$$\frac{C'\beta^2M^4C_1}{mE^2}\frac{1}{2\sin^2\Theta/2}\sim 1\text{ or }10^{-1}\;. \tag{4.9a}$$

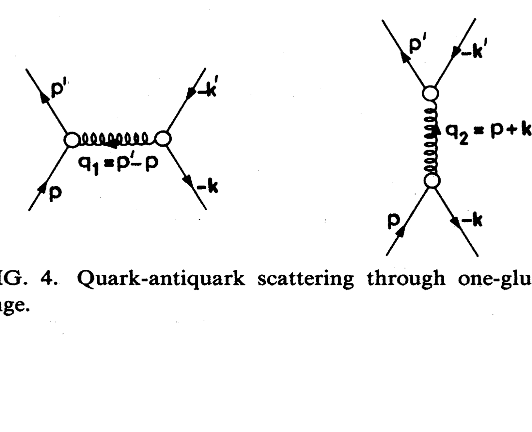

FIG. 4. Quark-antiquark scattering through one-gluon exchange.

Even for $m \simeq 10$ MeV (a number which is usually assigned at high energies) and $E \simeq 50$ GeV, this gives

$$\Theta \sim \sqrt{10} \times 10^{-3} \text{ or } 10^{-2} . \tag{4.9b}$$

Even if we use the full Dirac form factor (instead of just 1), the crude estimate (4.9b) is unlikely to change. Thus it will be difficult to detect the effect of the Pauli term this way.

V. SUMMARY AND CONCLUSION

We have seen that the anomalous magnetic moment (of the fermions) can be significant in theories where mass generation occurs through dynamical symmetry breaking. To calculate the AMM of light quarks we essentially used the one-gluon-exchange approximation and employed solutions of both kinds: solutions based on the asymptotic and infrared behavior of QCD derived and used by various authors. We found that all solutions cannot be used for this calculation. Solutions for the gluon propagator derived by Baker, Ball, and Zachariasen[8] and the resultant chiral-symmetry-breaking and confining solution for the quark propagator derived by Ball and Zachariasen,[9] in particular, form a good combination. The result thus arrived at was used to check the experimental consequences of the presence of the AMM (both electromagnetic as well as chromomagnetic) term. It gave very good results for p and n magnetic moments when regular SU(6) state vectors for baryons were used. In fact, use of Eq. (4.2) alone gives the correct ratio of magnetic moments of proton and neutron; using dynamical mass (here the effective quark mass) given by Eq. (3.13) and the resulting AMM term given by Eq. (3.21d), one obtains correct values for the individual baryons. In the case of the spin-dependent-potential calculation, we obtained a result which agreed with an earlier calculation and, moreover, the potential had a lesser number of free parameters; while in the case of the differential-cross-section calculation of $q\bar{q} \to q\bar{q}$, we found that the effect of the presence of the Pauli term was insignificant (at high energy) at any angle $\theta \gtrsim 10^{-2}$.

ACKNOWLEDGMENT

The author is grateful to Professor H. S. Mani, who suggested the problem, guided the work throughout, and checked some of the important steps. The author thanks Professor R. Ramachandran also for reading the manuscript.

APPENDIX

We have to perform the following integration:

$$\int \frac{d^4k}{(k-p)^4} \frac{G(k'^2)-G(k^2)}{k'^2-k^2} \gamma_\lambda(\gamma_\mu q - 2k_\mu)\gamma_\sigma \left[\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n\cdot(k-p)} + \frac{(k-p)_\lambda (k-p)_\sigma n^2}{[n\cdot(k-p)]^2} \right] ,$$
$$k' = k+q . \tag{A1}$$

First consider

$$I_{\lambda\sigma} = \int \frac{d\Omega_k}{(k-p)^4(q^2+2k\cdot q)} \left[\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n\cdot(k-p)} + \frac{(k-p)_\lambda (k-p)_\sigma n^2}{[n\cdot(k-p)]^2} \right] . \tag{A2}$$

Choose the gauge $n\cdot p = n\cdot q = 0$ and assume that $p_\mu = \kappa q_\mu$. Then

$$I_1 = \int \frac{d\Omega_k}{(k-p)^4(q^2+2k\cdot q)}$$
$$= \frac{\kappa}{k^2+p^2+p\cdot q} \left[\frac{2\pi^2}{k_>{}^2 |k^2-p^2|} + \frac{1}{k^2+p^2+p\cdot q} \left[\frac{2\pi^2}{k_>{}^2} + \frac{S(k,q)}{\kappa} \right] \right] , \tag{A3}$$

where we have used $[k_>{}^2 = \max(k^2,p^2)]$

$$\int \frac{d\Omega_k}{(k-p)^2} = \frac{2\pi^2}{k_>{}^2}, \quad \int \frac{d\Omega_k}{(k-p)^4} = \frac{2\pi^2}{k_>{}^2 |k^2-p^2|} ,$$

$$\int \frac{d\Omega_k}{q^2+2k\cdot q} \to \mathrm{P} \int \frac{d\Omega_k}{q^2+2k\cdot q} = \frac{\pi^2}{k^2} \left[1 - \left[1 - \frac{4k^2}{q^2} \right]^{1/2} \theta \left[\frac{q^2}{4} - k^2 \right] \right] \equiv S(k,q) ,$$

where in the last integral (which is not defined for $2k > q$) we have taken the principal value because in the context of the whole integral (A1), it is the principal value which is relevant.

For those integrals in which $(n\cdot k)$ or its power occurs in the denominator, we shall use the standard principal-value prescription,[17] which in simple cases reads

$$(k\cdot n)^{-1}\to\tfrac{1}{2}\{[(k\cdot n)+i\epsilon']^{-1}-[-(k\cdot n)+i\epsilon']^{-1}\}$$

$$\Longrightarrow \int_0^{\pi}\frac{\sin\theta\, d\theta}{\cos\theta}=0\ , \tag{A4a}$$

$$(k\cdot n)^{-2}\to\tfrac{1}{2}\{[(k\cdot n)+i\epsilon']^{-2}+[-(k\cdot n)+i\epsilon']^{-2}\}$$

$$\Longrightarrow \int_0^{\pi}\frac{\sin\theta\, d\theta}{\cos^2\theta}=-2\ . \tag{A4b}$$

Thus,

$$\int\frac{d\Omega_k}{(k-p)^4(q^2+2k\cdot q)(n\cdot k)}=0\ ,$$
$$\int\frac{d\Omega_k k_\lambda}{(k-p)^4(q^2+2k\cdot q)(n\cdot k)}=I_1\frac{n_\lambda}{n^2}\ , \tag{A5}$$

$$I_2=\int\frac{d\Omega_k n^2}{(k-p)^4(q^2+2k\cdot q)(n\cdot k)^2}$$
$$=\frac{-4\pi^2\kappa}{k^2+p^2+p\cdot q}\left[\frac{k^2+p^2}{k^2\,|\,(k^2-p^2)^3\,|}+\frac{1}{k^2+p^2+p\cdot q}\left[\frac{1}{k^2\,|\,k^2-p^2\,|}+\frac{\theta(q^2/4-k^2)}{\kappa k^2(q^4-4k^2q^2)^{1/2}}\right]\right]\ , \tag{A6}$$

where we have used

$$\int\frac{d\Omega_k n^2}{(k-p)^4(n\cdot k)^2}=-\frac{4\pi^2(k^2+p^2)}{k^2\,|\,(k^2-p^2)^3\,|}\ ,$$
$$\int\frac{d\Omega_k n^2}{(k-p)^2(n\cdot k)^2}=\frac{-4\pi^2}{k^2\,|\,k^2-p^2\,|}\ ,$$
$$\int\frac{d\Omega_k n^2}{(q^2+2k\cdot q)(n\cdot k)^2}\to \mathrm{P}\int\frac{d\Omega_k n^2}{(q^2+2k\cdot q)(n\cdot k)^2}=-\frac{4\pi^2\theta(q^2/4-k^2)}{k^2(q^4-4k^2q^2)^{1/2}}$$

(where in the last integral we have again taken the principal value for the same reason as explained above),

$$\int\frac{d\Omega_k k_\lambda}{(k-p)^4(q^2+2k\cdot q)(n\cdot k)^2}=-\frac{2\pi^2(k^2+p^2)q_\lambda}{n^2q^2k^2\,|\,(k^2-p^2)^3\,|}-\frac{I_2q_\lambda}{2n^2}\ , \tag{A7}$$

$$\int\frac{d\Omega_k k_\lambda k_\sigma}{(k-p)^4(q^2+2k\cdot q)(n\cdot k)^2}=Aq_\lambda q_\sigma+Bn_\lambda n_\sigma+C\delta_{\lambda\sigma}\ , \tag{A8}$$

where

$$A=\frac{1}{2q^2}\left[\frac{I_1}{n^2}-\frac{k^2}{n^2}I_2+3\frac{I_3}{p^2}\right]\ ,$$
$$B=\frac{1}{2n^2}\left[3\frac{I_1}{n^2}-\frac{k^2}{n^2}I_2+\frac{I_3}{p^2}\right]\ , \tag{A8a}$$
$$C=-\tfrac{1}{2}\left[\frac{I_1}{n^2}-\frac{k^2}{n^2}I_2+\frac{I_3}{p^2}\right]\ ;$$

$$I_3=\tfrac{1}{4}\int\frac{d\Omega_k}{(q^2+2k\cdot q)(n\cdot k)^2}\left[1-\frac{2(k^2+p^2)}{(k-p)^2}+\frac{(k^2+p^2)^2}{(k-p)^4}\right]$$
$$=-\frac{\pi^2\theta(q^2/4-k^2)}{n^2k^2(q^4-4k^2q^2)^{1/2}}+\frac{p^2+k^2}{2n^2}\frac{4\pi^2\kappa}{k^2+p^2+p\cdot q}\left[\frac{1}{k^2\,|\,k^2-p^2\,|}+\frac{\theta(q^2/4-k^2)}{\kappa k^2(q^4-4k^2q^2)^{1/2}}\right]+\frac{(k^2+p^2)^2}{4n^2}I_2\ . \tag{A8b}$$

When Eqs. (A3) and (A5)—(A8b) are substituted in Eq. (A2), we get

$$I_{\lambda\sigma}=\frac{q_\lambda q_\sigma}{q^2}\frac{1}{k^2}\left[-\frac{\pi^2\kappa(8p^2+5q\cdot p)}{2p^2(k^2+p^2+p\cdot q)^2}\theta(p^2-k^2)+\frac{\pi^2\kappa(8p^2+3p\cdot q)}{2p^2(k^2+p^2+p\cdot q)^2}\theta(k^2-p^2)+\frac{\pi^2}{2}\frac{1}{(k^2+p^2+p\cdot q)^2}\right.$$
$$\left.-2\pi^2\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\theta\left[\frac{q^2}{4}-k^2\right]\right]\ . \tag{A2$'$}$$

Next, we come to

$$I_\mu=\int \frac{d\Omega_k k_\mu}{(k-p)^4(q^2+2k\cdot q)}\left[2+\frac{n^2(k-p)^2}{(n\cdot k)^2}\right]=A_1 q_\mu/q^2+B_1 n_\mu/n^2\,, \tag{A9}$$

where

$$A_1=\frac{2\pi^2}{k^2}\left[-\frac{\theta(p^2-k^2)}{p^2}+\frac{q^2}{4\pi^2}\frac{\pi^2\kappa(8p^2+4p\cdot q)}{2p^2(k^2+p^2+p\cdot q)^2}\theta(p^2-k^2)-\frac{q^2}{4\pi^2}\frac{\pi^2\kappa(8p^2+4p\cdot q)}{2p^2(k^2+p^2+p\cdot q)^2}\theta(k^2-p^2)\right.$$
$$\left.+\frac{q^2}{2}\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\theta(q^2/4-k^2)\right]. \tag{A9a}$$

B_1 is not of interest to us. From Eqs. (A2), (A2′), (A9), and (A9a), we get

$$\int \frac{d^4k}{(k-p)^4}\frac{G(k^2)}{k'^2-k^2}\gamma_\lambda(\gamma_\mu q-2k_\mu)\gamma_\sigma\left[\delta_{\lambda\sigma}-\frac{(k-p)_\lambda n_\sigma+(k-p)_\sigma n_\lambda}{n\cdot(k-p)}+\frac{(k-p)_\lambda(k-p)_\sigma n^2}{[n\cdot(k-p)]^2}\right]$$
$$=-\frac{q\gamma_\mu}{2}\left[-\int_0^{p^2}dk^2\frac{\pi^2\kappa(8p^2+4p\cdot q)G(k^2)}{2p^2(k^2+p^2+p\cdot q)^2}+\int_{p^2}^{\infty}dk^2\frac{\pi^2\kappa(8p^2+4p\cdot q)G(k^2)}{2p^2(k^2+p^2+p\cdot q)^2}\right.$$
$$\left.-2\pi^2\int_0^{q^2/4}dk^2\left[\frac{(q^4-4k^2q^2)^{1/2}G(k^2)}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2G(k^2)}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\right]$$
$$-2\pi^2\frac{q_\mu}{q^2}\left[\int_0^{p^2}dk^2G(k^2)\left[-\frac{1}{p^2}+\frac{q^2}{4\pi^2}\frac{\pi^2\kappa(8p^2+4p\cdot q)}{2p^2(k^2+p^2+p\cdot q)^2}\right]-\int_{p^2}^{\infty}dk^2\frac{q^2\pi^2\kappa(8p^2+4p\cdot q)G(k^2)}{4\pi^2 2p^2(k^2+p^2+p\cdot q)^2}\right.$$
$$\left.+\int_0^{q^2/4}dk^2\left[\frac{(q^4-4k^2q^2)^{1/2}G(k^2)}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2G(k^2)}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\frac{q^2}{2}\right]+n_\mu\text{ terms}\,. \tag{A10}$$

Next, consider the following integration:

$$\int \frac{d^4k}{(k-p)^4}\frac{G(k'^2)}{k'^2-k^2}\gamma_\lambda(\gamma_\mu q-2k_\mu)\gamma_\sigma\left[\delta_{\lambda\sigma}-\frac{(k-p)_\lambda n_\sigma+(k-p)_\sigma n_\lambda}{n\cdot(k-p)}+\frac{(k-p)_\lambda(k-p)_\sigma n^2}{[n\cdot(k-p)]^2}\right]$$
$$=\int \frac{d^4k}{(k-p')^4}\frac{G(k^2)}{2k\cdot q-q^2}\gamma_\lambda(\gamma_\mu q-2k_\mu+2q_\mu)\gamma_\sigma\left[\delta_{\lambda\sigma}-\frac{(k-p')_\lambda n_\sigma+(k-p')_\sigma n_\lambda}{n\cdot k}+\frac{(k-p')_\lambda(k-p')_\sigma n^2}{(n\cdot k)^2}\right]$$
$$+\delta\text{-function part (if any)}\,, \tag{A11}$$

where $p'=p+q$. Also define $p'_\mu=\kappa' q_\mu$. Then,

$$I'_{\lambda\sigma}=-\int \frac{d\Omega_k}{(k-p')^4[q^2+2k\cdot(-q)]}\left[\delta_{\lambda\sigma}-\frac{(k-p')_\lambda n_\sigma+(k-p')_\sigma n_\lambda}{n\cdot k}+\frac{(k-p')_\lambda(k-p')_\sigma n^2}{(n\cdot k)^2}\right]$$
$$=-I_{\lambda\sigma}(p\to p',\ q\cdot p\to -q\cdot p',\ \kappa\to-\kappa')$$
$$=-\frac{q_\lambda q_\sigma}{q^2}\frac{\pi^2}{k^2}\left[\frac{\kappa'(8p'^2-5p'\cdot q)}{2p'^2(k^2+p'^2-p'\cdot q)^2}\theta(p'^2-k^2)-\frac{\kappa'(8p'^2-3p'\cdot q)}{2p'^2(k^2+p'^2-p'\cdot q)^2}\theta(k^2-p'^2)+\frac{1}{2}\frac{1}{(k^2+p'^2-p'\cdot q)^2}\right.$$
$$\left.-2\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p'^2-p'\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p'^2-p'\cdot q)}\right]\theta(q^2/4-k^2)\right]. \tag{A12}$$

Here, $k^2+p'^2-p'\cdot q=k^2+p^2+p\cdot q$. It can be easily checked that

$$\int I_{\lambda\sigma}k^2dk^2=\int I'_{\lambda\sigma}k^2dk^2=0\,.$$

Hence by shifting the variable, as in Eq. (A11), the apparent (infrared-) divergent nature of the integral does not introduce any δ function.

In a similar way,

$$I'_\mu=\int\frac{d\Omega_k k_\mu}{(k-p')^4(2k\cdot q-q^2)}\left[2+\frac{(k-p')^2n^2}{(n\cdot k)^2}\right]=A'_1\frac{q_\mu}{q^2}+B'_1\frac{n_\mu}{n^2}\ , \tag{A13}$$

where

$$\begin{aligned}A'_1&=A_1(p\to p',\ q\cdot p\to -q\cdot p',\ \kappa\to-\kappa')\\&=\frac{2\pi^2}{k^2}\left[-\frac{\theta(p'^2-k^2)}{p'^2}-\frac{q^2}{4\pi^2}\frac{\pi^2\kappa'(8p'^2-4p'\cdot q)}{2p'^2(k^2+p'^2-p'\cdot q)^2}\theta(p'^2-k^2)+\frac{q^2}{4\pi^2}\frac{\pi^2\kappa'(8p'^2-4p'\cdot q)}{2p'^2(k^2+p'^2-p'\cdot q)^2}\theta(k^2-p'^2)\right.\\&\left.\quad+\frac{q^2}{2}\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p'^2-p'\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p'^2-p'\cdot q)}\right]\theta(q^2/4-k^2)\right]\ .\end{aligned}\tag{A13a}$$

Substituting Eqs. (A12)—(A13a) in Eq. (A11), we get

$$\begin{aligned}&\int\frac{d^4k}{(k-p)^4}\frac{G(k'^2)}{k'^2-k^2}\gamma_\lambda(\gamma_\mu q\!\!\!/-2k_\mu)\gamma_\sigma\left[\delta_{\lambda\sigma}-\frac{(k-p)_\lambda n_\sigma+(k-p)_\sigma n_\lambda}{n\cdot(k-p)}+\frac{(k-p)_\lambda(k-p)_\sigma n^2}{[n\cdot(k-p)]^2}\right]\\&=\frac{q\!\!\!/\gamma_\mu+2q_\mu}{2}\left[\int_0^{p'^2}dk^2G(k^2)\frac{\pi^2\kappa'(8p'^2-4p'\cdot q)}{2p'^2(k^2+p^2+p\cdot q)^2}-\int_{p'^2}^{\infty}dk^2G(k^2)\frac{\pi^2\kappa'(8p'^2-4p'\cdot q)}{2p'^2(k^2+p^2+p\cdot q)^2}\right.\\&\qquad\left.-2\pi^2\int_0^{q^2/4}dk^2G(k^2)\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\right]\\&\quad-\frac{q_\mu}{2}\left[\int_0^{p'^2}dk^2G(k^2)\frac{\pi^2\kappa'(8p'^2-4p'\cdot q)}{2p'^2(k^2+p^2+p\cdot q)^2}-\int_{p'^2}^{\infty}dk^2G(k^2)\frac{\pi^2\kappa'(8p'^2-4p'\cdot q)}{2p'^2(k^2+p^2+p\cdot q)^2}\right.\\&\qquad\left.-2\pi^2\int_0^{q^2/4}dk^2\left[\frac{(q^4-4k^2q^2)^{1/2}G(k^2)}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2G(k^2)}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\right]\\&\quad-\frac{q_\mu}{q^2}\frac{2\pi^2}{p'^2}\int_0^{p'^2}dk^2G(k^2)+n_\mu\ \text{terms}\ .\end{aligned}\tag{A14}$$

Substituting Eqs. (A10) and (A14) in Eq. (A1), we get

$$\begin{aligned}&\int\frac{d^4k}{(k-p)^4}\frac{G(k'^2)-G(k^2)}{k'^2-k^2}\gamma_\lambda(\gamma_\mu q\!\!\!/-2k_\mu)\gamma_\sigma\left[\delta_{\lambda\sigma}-\frac{(k-p)_\lambda n_\sigma+(k-p)_\sigma n_\lambda}{n\cdot(k-p)}+\frac{(k-p)_\lambda(k-p)_\sigma n^2}{[n\cdot(k-p)]^2}\right]\\&=q_\nu[\gamma_\nu,\gamma_\mu]\left[\pi^2\kappa'\int_{p^2}^{p'^2}dk^2\frac{G(k^2)(2p'^2-p'\cdot q)}{p'^2(k^2+p^2+p\cdot q)^2}\right.\\&\qquad\left.-\pi^2\int_0^{q^2/4}dk^2G(k^2)\left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+p\cdot q)^2}+\frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p\cdot q)}\right]\right]\\&\quad-\frac{q_\mu}{q^2}2\pi^2\left[\frac{1}{p'^2}\int_0^{p'^2}dk^2G(k^2)-\frac{1}{p^2}\int_0^{p^2}dk^2G(k^2)\right]+n_\mu\ \text{terms}\ .\end{aligned}\tag{A15}$$

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