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# Characteristic size for the neutrino

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The possibility of attributing a characteristic size to the neutrino is discussed in the context of the Glashow-Salam-Weinberg model of the electroweak interactions. We define the electroweak radius by means of the $\nu_l l'$ scattering amplitude at the one-loop level. This amplitude can be written, in a certain kinematical domain, in terms of only two form factors, which multiply the electromagnetic current and the weak neutral current, respectively. Since the first one does not depend on the properties of the charged lepton $l'$, its derivative gives a suitable measure of the neutrino size. By construction, the electroweak radius so defined is a finite and gauge-independent quantity of the order $10^{-33}$ cm$^2$, and it gets contributions from the proper neutrino electromagnetic vertex, from the $\gamma$-$Z$ self-energy, and from part of the box diagrams. We also show explicitly the gauge dependence of the first two contributions.

## I. INTRODUCTION

To our knowledge there are three kinds of "size" that can be assigned to a particle: classical, charge radius, and structure. It is clear that the size is defined for each interaction; thus, for example, the charge radius is related to the interaction between the photon and the particle ($e^-,\nu$, etc.), whereas the structure size is defined by the mass scale $\Lambda$ which characterizes the strength of the interaction that bounds the constituents of the composite object. In this paper we will be concerned with the electromagnetic charge radius (henceforth abbreviated ECR), in particular that of the $\nu$ (Ref. 1), and its extension—the electroweak radius.

In order to clarify the motivation for this work, let us summarize the facts we know about the ECR of other particles. The ECR is defined in terms of the electromagnetic form factor (see next section)

$$\langle r_{\rm EM}{}^2\rangle = 6\frac{\partial^2 F(q^2)}{\partial q^2}\bigg|_{q^2=0} . \qquad (1)$$

Depending on the nature of the particle under consideration there will be several contributions to the ECR. In the case of strongly interacting particles (such as the $p$) we will have contributions from the weak [Fig. 1(a)], electromagnetic [Fig. 1(b)], and strong interactions [Fig. 1(c)]. The weak contribution is negligible compared to the strong one, whereas the electromagnetic is infrared divergent. This infrared-divergent term is combined with the bremsstrahlung (Fig. 2) and included in the radiative corrections of the full process used to measure the form factor. In the case of charged leptons we have weak and electromagnetic contributions, but in analogy with the proton case the electromagnetic term is absorbed in the radiative corrections, so we are left solely with the weak part. Therefore if we want to extract a number for the ECR of the charged lepton, we have to be able to separate the weak contribution from the much larger electromagnetic radiative corrections. Finally, for neutral leptons the ECR only gets a contribution from the weak interaction and therefore even if this is a tiny quantity, it has the advantage that its measurement will not be masked by the effects of other interactions.

Several authors have studied the ECR of the $\nu$ (Refs. 4—6). Particularly interesting are the following results. Using the Glashow-Salam-Weinberg model, Bardeen, Gastmans, and Lautrup[5] showed that, in the unitary gauge, the ECR of the $\nu$ is infinite and therefore it is not a physical quantity. Later, S. Y. Lee[6] suggested considering the $l'\nu_l$ scattering and defined the ECR of the $\nu$ including, besides the usual terms, diagrams in which the photon is replaced by a neutral gauge boson $Z$. In this way he obtained a finite, although, as we will see later, gauge-dependent quantity.

The main point of this paper is to introduce, through the elastic scattering $l'\nu_l$, an electroweak radius which is finite, gauge independent, and independent of the properties of the lepton $l'$ used to define it.

In Sec. II we calculate the ECR of the $\nu$ and show ex-

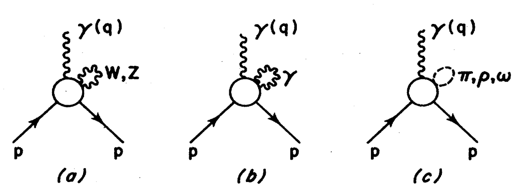


FIG. 1. Diagrams which contribute to the electromagnetic charge radius, in the case of a (a) weakly, (b) electromagnetically, (c) strongly interacting particle.

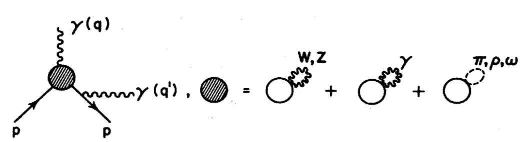


FIG. 2. Bremsstrahlung diagram.

 

plicitly the dependence of this quantity on the parameter $\xi$ that is used to fix the gauge of the electroweak group (in the linear $R_\xi$ gauge). In Sec. III, we define, using the results of Marciano and Sirlin,[7] the electroweak radius (EWR), we give a numerical estimate, and we show its relation to the effective Hamiltonian (that is, the $\nu l'$ scattering matrix element) in which the radiative corrections have been included. Finally, in Sec. IV we discuss our results. The calculations have been performed using the dimensional-regularization scheme and the following properties of the $\gamma$-matrix algebra:

$$\{\gamma_\mu,\gamma_5\}=0,\quad \{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu},\quad g^\mu{}_\mu=n\ .$$

## II. THE NEUTRINO CHARGE RADIUS IN THE LINEAR $R_\xi$ GAUGE

For a massless left-handed neutrino the matrix element of the electromagnetic current can be written in terms of a single form factor $F(q^2)$ as

$$M_\mu=ieF(q^2)\bar{u}\gamma_\mu(1-\gamma_5)u\ . \tag{2}$$

The one-loop contributions to the form factor can be divided into two classes, depending on whether they arise from (I) proper vertices [Figs. 3(a)—3(f)] or (II) $\gamma$-$Z$ self-energy diagrams [Figs. 4(a)—4(g)]. Within the proper-vertex contribution we still can distinguish those in which the photon is coupled to the intermediate lepton, $F_{If}$ [Figs. 3(a) and 3(b)], and those in which the photon is coupled to bosons, $F_{Ib}$ [Figs. 3(c)—3(f)]. On the other hand, the $\gamma Z$-mixing diagrams get contributions both from bosonic [Figs. 4(a)—4(f)] and fermionic loops [Fig. 4(g)].

It is well known that, in contrast to the unitary gauge, in order to get a vanishing $\nu$ charge in the Feynman-t' Hooft gauge it is necessary to introduce the $\gamma Z$ mixing diagrams.[8,9] On the other hand, the $\nu$ charge radius gets contributions from these diagrams in both gauges. In fact, as we will see later, this contribution contains poles due to ultraviolet divergences as well as singularities for some values of the $\xi$ parameter introduced through the use of the $R_\xi$ gauge. In particular, the $\nu$ charge radius is singular for $\xi=0$ which explains the result obtained by Bardeen, Gastmans, and Lautrup,[5] even if they did not work in the unitary gauge, defined as the limit $\xi\to 0$ of the amplitude once the parametric Feynman integrals have been performed.[9]

In the rest of this section, we will concentrate on the contribution of the different diagrams to the derivative of the form factor which is related to the ECR of the $\nu$ by Eq. (1). The contribution from the vertex diagrams where the photon is coupled to the internal lepton is finite and gauge independent and is given by

$$F'_{If}(0)=\frac{g^2}{96\pi^2M^2}\left[\frac{1}{6}+\ln\frac{M^2}{m^2}\right], \tag{3}$$

where $M$ and $m$ are the $W$ and charged-lepton masses, respectively. For the bosonic-vertex contribution $F_{Ib}$, we obtain

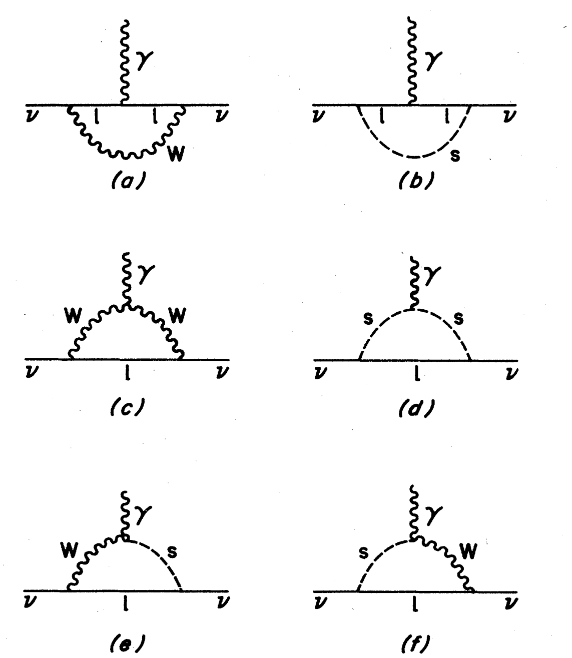


FIG. 3. Diagrams which contribute to the neutrino electromagnetic form factor at the lowest order.

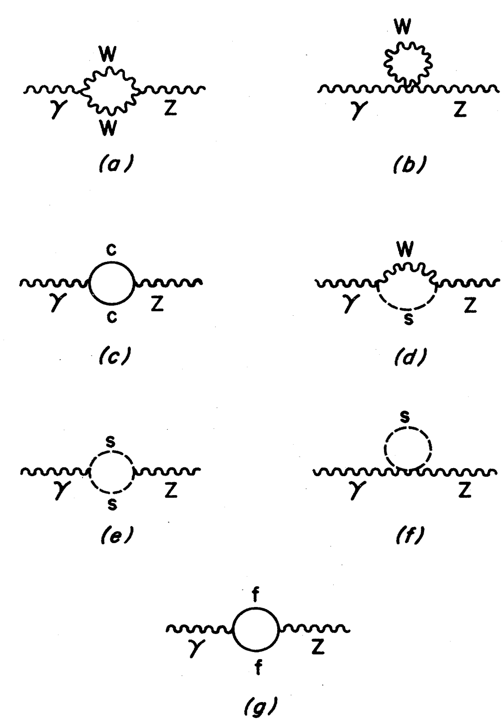


FIG. 4. Diagrams which contribute to the $\gamma$-$Z$ self-energy.

$$F'_{Ib}(0)=\frac{g^2}{128\pi^2M^2}\left(\tfrac{10}{9}+I\right),$$

$$I=\int_0^1 dx\left[\frac{\ln\xi}{\xi}+2\left[\frac{1-x}{\xi}-x^2+2x-2\right]\ln\left[x+\frac{1-x}{\xi}\right]-\frac{4x(x-1)^2\left[1-\frac{1}{\xi}\right]}{x+\frac{1-x}{\xi}}\right]$$
$$+\frac{m^2}{M^2}\int_0^1 dx\int_0^x dy\left[\ln\frac{A^2}{B\left[\frac{x}{\xi}+\frac{m^2}{M^2}(1-x)\right]}-4\left[\frac{1}{A}-\frac{1}{B}\right]\right], \tag{4a}$$

where

$$A=x+y\left[\frac{1}{\xi}-1\right]+\frac{m^2}{M^2}(1-x),$$
$$B=x+\frac{m^2}{M^2}(1-x), \tag{4b}$$

and $\xi$ is the gauge-fixing parameter in terms of which the $W$ propagator takes the form

$$\Delta_{\mu\nu}=-\left[g_{\mu\nu}-\delta\frac{q_\mu q_\nu}{q^2-M^2/\xi}\right]\frac{1}{q^2-M^2},$$

where $\delta=1-1/\xi$.

For $\xi=1$ this contribution reduces to

$$F'_{Ib}(0,\xi=1)=\frac{5g^2}{576\pi^2M^2}.$$

Therefore in the 't Hooft–Feynman gauge the contribution of the proper-vertex diagrams [Figs. 3(a)—3(f)] to the derivative of the form factor is

$$F'_{I}(0)=\frac{g^2}{96\pi^2M^2}\left[1+\ln\frac{M^2}{m_l^2}\right], \tag{5}$$

which is in agreement with previous calculations.[10]

Now, let us consider the $\gamma Z$ mixing diagrams. The contribution of the $\gamma$-$Z$ self-energy

$$\pi^{\gamma Z}_{\mu\nu}=A(q^2)g_{\mu\nu}+q_\mu q_\nu B(q^2) \tag{6}$$

to the derivative of the form factor is

$$F'_{II}(0)=\frac{g\cos\theta_W}{4M^2}[M^{-2}A(0)\cos^2\theta_W+A'(0)]. \tag{7}$$

The bosonic contribution to $A(q^2)$ is [Figs. 4(a)—4(f)]

$$A(q^2)_b=ig^2\tan\theta_W\frac{M^2}{8\pi^2}\left[\frac{1}{2-n/2}(1-\tfrac{1}{4}\delta)-\tfrac{1}{8}\delta+\tfrac{3}{4}+\frac{1}{4\delta}(1-\delta)(3-\delta)\ln(1-\delta)\right]$$
$$+ig^2\tan\theta_W\frac{q^2}{96\pi^2}\frac{1}{2-n/2}+ig^2\sin2\theta_W\frac{q^2}{32\pi^2}\frac{3+\delta}{2-n/2}$$
$$+ig^2\tan\theta_W\frac{q^2}{16\pi^2}\left[\left[\frac{5}{3\delta^3}-\frac{3}{2\delta^2}\right]\ln(1-\delta)-\tfrac{1}{6}\ln(1-\delta)+\frac{5}{3\delta^2}-\frac{2}{3\delta}+\frac{5}{36}\right]$$
$$+ig^2\sin2\theta_W\frac{q^2}{32\pi^2}\left[\left[-\frac{3}{\delta}+4-\delta\right]\ln(1-\delta)-\frac{7}{3}+\frac{\delta}{2}\right]+O(q^4), \tag{8}$$

which agrees with the result of Marciano and Sirlin[7] in the limit $\xi=1$ $(\delta=0)$.

Finally, the contribution of loops of fermions [Fig. 4(g)] of mass $m_f$ and charge $Q_f$ is gauge independent and is given by

$$A(q^2)_f=ig^2\tan\theta_W\frac{q^2}{48\pi^2}(1-4\sin^2\theta_W)$$
$$\times\sum_f Q_f^2\left[\frac{1}{2-n/2}-\ln m_f^2\right]. \tag{9}$$

Therefore $A(q^2)$, the sum of Eqs. (8) and (9), shows a pole at $n=4$ as well as a singularity at $\xi=0$ $(\delta=-\infty)$.

Collecting the results given in Eqs. (3), (4a), (7), (8), and (9), we obtain $F'(0)$ and we can explicitly see that this quantity is infinite and gauge dependent.

## III. DEFINITION OF A CHARACTERISTIC SIZE FOR THE NEUTRINO: THE ELECTROWEAK RADIUS

As was shown in the previous section, the neutrino charge radius is an infinite and gauge-dependent quantity in the linear $R_\xi$ gauge. This is the content of the statement made by Bardeen, Gastmans, and Lautrup[5] when they remarked that the neutrino charge radius is not a static quantity because one cannot measure it with an external electromagnetic field. Therefore in order to look for a neutrino size we have to consider other diagrams contributing to the total amplitude of the physical process $\nu_l l' \to \nu_l l'$.

Marciano and Sirlin,[7] making use of the current-algebra formalism of radiative corrections[11] and working in the context of the standard model, showed that for $q^2 \ll M^2$ and $q \cdot P \ll M^2$ (where $q$ and $P$ are the transferred and initial-charged-lepton momenta, respectively) one can write the total amplitude of the scattering $\nu_l l'$ in terms solely of the currents $l^\gamma$ and $l^Z$ which are defined as

$$l_\gamma^\mu = -\bar{u}_f \gamma^\mu u_i \tag{10}$$

and

$$l_Z^\mu = \bar{u}_f[-\tfrac{1}{4}\gamma^\mu(1-\gamma_5)+\sin^2\theta_W \gamma^\mu]u_i \ , \tag{11}$$

where $u_i$ and $u_f$ are the spinors of the initial and final charged leptons and $\theta_W$ is the Weinberg angle; i.e., it is possible to write the total amplitude of the mentioned process in the following form:

$$M = ie^2 \left[ \frac{F_\gamma(q^2)}{q^2} l_\mu^\gamma + \frac{F_Z(q^2)}{q^2 - M_Z{}^2} l_\mu^Z \right] \bar{\nu}_f \gamma^\mu (1-\gamma_5)\nu_i \ , \tag{12}$$

where $F_\gamma(q^2)$ and $F_Z(q^2)$ are finite and gauge-independent functions separately. $\nu_i$ and $\nu_f$ are the spinors of the initial and final neutrino leptons.

All the diagrams which contribute to the neutrino charge radius (Fig. 5) are proportional to $l_\mu^\gamma$. The diagrams of Fig. 6, the corrections to the charged lepton vertex and the box diagrams, also contribute to $F_\gamma(q^2)$ because these diagrams contain a part proportional to $l_\mu^\gamma$. Furthermore, $F_\gamma(0)=0$ and $F_\gamma(q^2)$ is, as we have already said, a finite and gauge-independent function of $q^2$; thus we opt for defining the electroweak radius of the neutrino as

$$\langle r_\nu{}^2 \rangle_{\rm EW} = 6 \frac{\partial F_\gamma(q^2)}{\partial q^2} \bigg|_{q^2=0} \ . \tag{13}$$

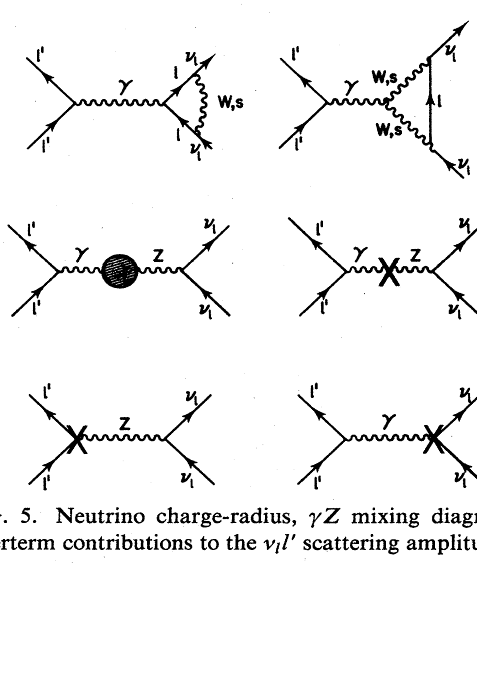


FIG. 5. Neutrino charge-radius, $\gamma Z$ mixing diagrams, and counterterm contributions to the $\nu_l l'$ scattering amplitude.

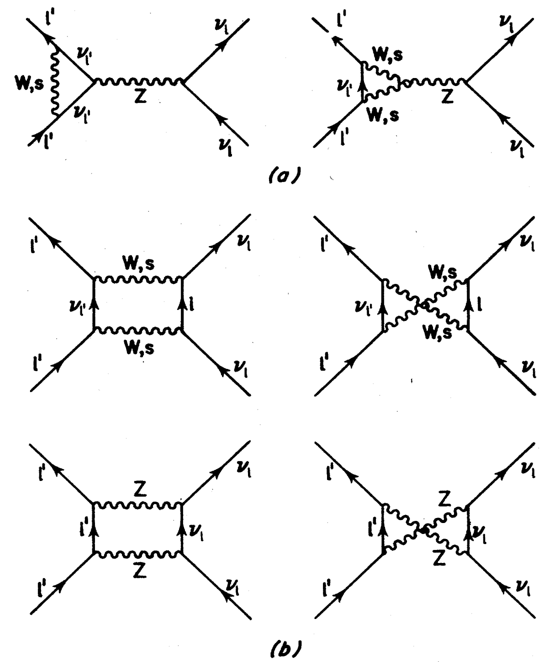


FIG. 6. (a) Lowest-order charged-lepton vertex corrections. (b) Box diagrams.

The electroweak radius is a gauge-independent quantity, therefore we can evaluate it in the 't Hooft—Feynman gauge. Using the results of Ref. 7 (see the Appendix) we obtain

$$\langle r_{\nu_l}{}^2 \rangle_{\rm EW} \approx \frac{g^2}{16\pi^2 M^2} \ln \frac{M^2}{m_l{}^2} \ , \quad l = e, \mu, \tau \ . \tag{14}$$

We can see, from Eq. (14), that the electroweak radius is a finite quantity which does not depend on the properties of the lepton $l'$ used to define it.

Taking $M_W = 83$ GeV, $m_e = 0.5$ MeV, $m_\mu = 105$ MeV, and $m_\tau = 1780$ MeV we get the results

$$\begin{aligned} \langle r_{\nu_e}{}^2 \rangle_{\rm EW} &\approx 3.2\times 10^{-33}\ \text{cm}^2 \ , \\ \langle r_{\nu_\mu}{}^2 \rangle_{\rm EW} &\approx 1.7\times 10^{-33}\ \text{cm}^2 \ , \\ \langle r_{\nu_\tau}{}^2 \rangle_{\rm EW} &\approx 1.0\times 10^{-33}\ \text{cm}^2 \ . \end{aligned} \tag{15}$$

These results agree with the expected values for the neutrino size in composite models.[12]

We should notice the similarity between Eqs. (3) and (14) from which we conclude that the most important contribution to the electroweak radius comes from the diagram depicted in Fig. 3(a).[13]

It is possible to show that the contribution to $F\gamma(q^2)$ which comes from the box diagrams [Fig. 6(b)] is finite but gauge dependent. The contribution to $F\gamma(q^2)$ from the diagrams of Fig. 6(a) contains poles which cancel exactly with the poles of the contribution coming from the diagrams of Fig. 5. S. Y. Lee[6] did not include the box diagrams in his definition of the neutrino charge radius, and therefore he got a finite but gauge-dependent quantity.

## IV. DISCUSSION

We have discussed in detail the neutrino charge radius in the lineal $R_\xi$ gauge in the context of the standard model of the electroweak interactions, and we conclude that it is infinite and also gauge dependent.

The main objective of this paper is to attribute a characteristic size to the neutrino; thus we have defined the neutrino electroweak radius by means of the physical process $\nu_l l' \to \nu_l l'$, in such a way that it has by construction the following properties: (i) it includes the contribution of all the neutrino charge-radius diagrams, and (ii) it is finite and gauge independent. Furthermore, (iii) it does not depend on the properties of the lepton $l'$ used to define it, and (iv) it is of order $10^{-33}$ cm$^2$. Therefore, we believe that it is possible to interpret the neutrino electroweak radius as the neutrino size.

In the case of the neutrino, the electroweak radius is related to the renormalization factor $\kappa(q^2)$ which relates $\sin^2\theta_W(q^2)_{\rm eff}$ and $\sin^2\theta_W$ (see Ref. 7), in the following form:

$$\langle r_{\nu_l}{}^2\rangle_{\rm EW} = \frac{3}{2M^2}[1-\kappa(0)] = \frac{3}{2M^2}\left[1-\frac{\sin^2\theta_W(0)_{\rm eff}}{\sin^2\theta_W}\right] .$$

Thus, taking into account that $M \approx 83$ GeV and that we expect $\langle r_{\nu_l}{}^2\rangle_{\rm EW} \approx 10^{-33}$ cm$^2$, we conclude that if we want to get experimental information about the electroweak radius, one must measure $\kappa(0)$ with a precision of one in a thousand.

## ACKNOWLEDGMENTS

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## APPENDIX

In this appendix we will give the explicit evaluation of the electroweak radius, in the 't Hooft—Feynman gauge.

From the results of Ref. 7 we get the part of the total amplitude of the process $\nu_l l' \to \nu_l l'$ proportional to $l_\gamma^\mu$:

$$\delta M_{l_\gamma} = \frac{ie^2}{4\cos^2\theta_W(q^2-M_Z{}^2)}\langle u_f \mid l_\gamma^\mu \mid u_i\rangle \bar{\nu}(p_2)\gamma_\mu(1-\gamma_5)\nu(p_1)[\delta_\theta^{(b)}(q^2)+\delta_\theta^{(l)}(q^2)] , \quad \text{(A1)}$$

where (for $\sin^2\theta_W = 0.23$)

$$\delta_\theta^{(b)}(q^2) = \frac{\alpha}{2\pi\sin^2\theta_W}\left\{1.28+2\int_0^1 dx\, x(1-x)\ln\frac{M^2}{m_l{}^2-q^2x(1-x)}+\frac{1}{2\sin^2\theta_W}\left[H(\rho)-\cos^2\theta_W H\left[\frac{\rho}{\cos^2\theta_W}\right]\right]\right\} \quad \text{(A2)}$$

with

$$H(\rho) = \int_0^1 dx\left[1-\frac{x^2}{2}-\frac{\rho}{2}(1-x)\right]\ln[x^2+\rho(1-x)]+\frac{\rho}{4}(\ln\rho-\tfrac{1}{2})$$

and $\rho = M_\phi{}^2/M_Z$ ($M_\phi$ is the mass of the physical Higgs scalar), and

$$\delta_\theta^{(l)}(q^2) = \frac{\alpha}{2\pi\sin^2\theta_W}\left[0.87+(1-4\sin^2\theta_W)\sum_{j=e,\mu,\tau}\int_0^1 dx\, x(1-x)\ln\frac{m_j{}^2-q^2x(1-x)}{M_Z{}^2x(1-x)}\right] . \quad \text{(A3)}$$

From Eqs. (13) and (A1) we obtain

$$F_\gamma(q^2) = \frac{-q^2}{4\cos^2\theta_W(q^2-M_Z{}^2)}[\delta_\theta^{(b)}(q^2)+\delta_\theta^{(l)}(q^2)] . \quad \text{(A4)}$$

Now using our definition of the electroweak radius, Eqs. (13) and (A4), we get

$$\langle r_{\nu_l}{}^2\rangle_{\rm EW} = \frac{3}{2M^2}[\delta_\theta^{(b)}(0)+\delta_\theta^{(l)}(0)]$$
$$= \frac{3\alpha}{4\pi M^2\sin^2\theta_W}\left[\tfrac{1}{3}\ln\frac{M^2}{m_l{}^2}+1.15+\frac{1}{2\sin^2\theta_W}[H(\rho)-\cos^2\theta_W H(\rho/\cos^2\theta_W)]\right.$$
$$\left. +\frac{1-4\sin^2\theta_W}{6}\left[\ln\frac{m_e{}^2}{M_Z{}^2}+\ln\frac{m_\mu{}^2}{M_Z{}^2}+\ln\frac{m^2}{M_Z{}^2}+\frac{5}{18}\right]\right] , \quad \text{(A5)}$$

where $l = e, \mu, \tau$. It is possible to show that

$$| [H(\rho) - \cos^2\theta_W H(\rho/\cos^2\theta_W)]/(2\sin^2\theta_W) |$$

is a quantity smaller than 1.2 for values of $\rho$ between 0 and 100. Therefore, we can finally write

$$\langle r_{\nu_l}{}^2 \rangle_{\rm EW} \approx \frac{g^2}{16\pi^2 M^2} \ln \frac{M^2}{m_l{}^2}, \quad l = e, \mu, \tau \, .$$

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[1]If we consider an electrically charged particle of mass $m$, the classical size is defined (Ref. 2) as the radius of the charge distribution such that its electrostatic energy equals $m$. In the case of neutral particles, such as the neutron and the neutrino, we can apply the same definition changing the charge by a dipolar magnetic moment. Obviously this definition makes sense only if the particles considered are massive. The structure size has been discussed recently in the literature (Ref. 3) and we do not have anything to add.

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[13]M. A. B. Bég, W. J. Marciano, and M. Ruderman [Phys. Rev. D **17**, 1395 (1978)] showed that the leading contribution to the neutrino charge radius in the 't Hooft—Feynman gauge is gauge independent and comes from this diagram.