Fits to the hadron masses in the chiral bag model

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The influence of the center-of-mass correction and quark-pion interactions on the hadron mass spectrum is investigated. Various possibilities of treating gluonic and pionic corrections in the pressure-balance equation are discussed. With the correction for the finite size of the pion, excellent agreement with experimental data is obtained.

I. INTRODUCTION

Since 1974, when the MIT bag model[1,2] was formulated, several improvements have been proposed.[3,4,5] In the original formulation of the model, the axial-vector current is not conserved on the bag surface even for massless quarks and the chiral symmetry is explicitly broken. On the other hand, one expects from QCD spontaneously broken chiral symmetry and the presence of Goldstone bosons. To restore chiral symmetry in the bag model it is possible to introduce an elementary pion field coupled to the quarks.[3,6] The resulting energy of the pion cloud has now to be added to the bag-model Hamiltonian. In this way, treating the pion approximately as an elementary field, modification of the properties of "heavy" qqq and $\bar{q}q$ states due to the coupling to Goldstone modes can be calculated.

Another correction that one has to take into account is the center-of-mass (c.m.) motion correction. It has been realized[4,5] that quarks in the bag moving independently are not in an eigenstate of the total momentum. A part of the bag energy is spurious and comes from the motion of the center of mass. To get the mass of the physical particle it is necessary to subtract, in some way, the spurious c.m. motion.

In this paper we want to investigate the influence on the mass spectrum of these two important corrections, namely, the pionic correction coming from the quark-pion interaction and the c.m.-motion correction. There have been many papers on this subject.[4,7–11] We differ from most of them in the way of treating the pionic correction. Our calculation of pionic correction is based on the model formulated in Ref. 12. Contrary to the so-called cloudy bag model,[7,13] we consider the Lagrangian for the quark-pion interactions as a phenomenological Lagrangian and use it exactly in the same way as phenomenological Lagrangians were used in the past to describe pion-nucleon interactions. That means that we will consider only tree diagrams and neglect all pion quark loops. One should not forget that phenomenological Lagrangians for πN interactions are only a convenient way of summarizing results of current algebra for low-energy many-pion emission and not a basic theory. It is difficult to imagine that quark-pion (elementary—composite-object) coupling with the same pion nonlinearity as for the πN system could be fundamental. Taking tree diagrams and no loops means that only exchange of pions between *different* quarks is considered.

Following the idea of Ref. 14, in addition to the standard approach to gluonic correction, we have also made fits treating interaction with gluons perturbatively (not including gluon energy in the equation $dE/dR=0$). The results in both cases are very similar.

It is known that the pionic pressure is too large[7–9,11,15] (leads to destabilization of hadrons). On the other hand, we believe that the multigluonic interactions that are responsible for quark confinement are described in the bag model by the constant B and pions should not influence the structure of the bag very strongly.[8] So we treat pionic corrections in two ways: perturbatively and, trying to take into account the finite pion radius $[(\langle r^2\rangle_{\rm EM}^{\pi})^{1/2}=0.56$ fm$]$ modifying the interactions of quarks with pions in a rather *ad hoc* way proposed by De Tar.[7] In the first case, reasonable results are obtained, comparable to those already obtained by other authors, and in the second one, the fit to the hadron masses is surprisingly good.

It is also shown that center-of-mass corrections improve fits slightly when only gluonic corrections are taken into account and are very important numerically when pionic corrections are considered.

Our results are compared with other fits to hadron masses given in the literature.

II. BAG ENERGY WITH VARIOUS CORRECTIONS

In the limit of static, spherical cavity[2] the energy of the bag state of a radius R is given by

$$E=E_K+E_0+E_V+E_g+E_\pi\ , \tag{1}$$

where E_K is the quark kinetic energy

$$E_K=\sum_i [x_i^2+(m_iR)^2]^{1/2}/R \ , \quad (2)$$

$$\tan x_i = x_i/\{1-m_iR-[x_i^2+(m_iR)^2]^{1/2}\} \ , \quad (3)$$

E_0 is a phenomenological term originally attributed to zero-point energy and recently associated with center-of-mass[16] and color-electric contributions[17]

$$E_0=-Z_0/R \ , \quad (4)$$

E_V is a volume energy

$$E_V=\tfrac{4}{3}\pi R^3 B \ , \quad (5)$$

and E_g is the color-magnetic interaction associated with the exchange of a single gluon between two quarks in the bag,

$$E_g=\tfrac{8}{3}\sum_{i<j}\alpha_c \frac{M(m_iR,m_jR)}{R}(\lambda^a\vec{\sigma})^i\cdot(\lambda^a\vec{\sigma})^j \ . \quad (6)$$

The function M was calculated in Ref. 2 and for $mR \le 1.5$ can be approximated[9] as

$$M(0,mR)=0.175-0.025mR \ ,$$
$$M(mR,mR)=0.175-0.043mR \ , \quad (7)$$

where λ^a and $\vec{\sigma}$ are the color and spin matrices. We neglect the electrostatic part of the one-gluon-exchange energy.

The last term is the pionic interaction associated with the exchange of a single pion between two quarks in the bag

$$E_\pi=-\frac{1}{f_\pi^2R^3}f\sum_{i<j}(\vec{\sigma}\underline{\tau})^i\cdot(\vec{\sigma}\underline{\tau})^j \ , \quad (8)$$

where i,j denote nonstrange quarks. Here $\vec{\sigma}$ and $\underline{\tau}$ are the quark spin and isospin matrices, m_π is the pion mass, f_π equals the pion-to-vacuum transition amplitude induced by the axial-vector current, and

$$f=\left[\frac{x_0}{2(x_0-1)}\right]^2\frac{1}{24\pi}\frac{\exp(-m_\pi R)}{m_\pi R}\times\left[1+\frac{1}{m_\pi R}\right]\left[\cosh m_\pi R-\frac{\sinh m_\pi R}{m_\pi R}\right] . \quad (9)$$

As was already mentioned in the Introduction, we do not include any self-energy diagrams for quarks. The contribution with $i=j$ is absent. The values of

$$\hat{\Omega}=\sum_{i<j}(\vec{\sigma}\underline{\tau})^i\cdot(\vec{\sigma}\underline{\tau})^j$$

for different particles are given in Table I.

The bag radius can be eliminated from the above equations by demanding that

$$\frac{dE}{dR}=0 \ . \quad (10)$$

We will follow the method from the original MIT-group paper.[2] We set the masses of the nonstrange quarks equal to zero and from the masses of $N(938)$, $\Delta(1236)$, $\Omega^-(1672)$, and $\omega(783)$ calculate the parameters of the model: B, Z_0, α_c, and m_s.

The mass of the physical hadron is found by correcting for the c.m. motion:

$$M_{\rm bag}{}^2=E^2-\sum_i x_i^2/R^2 \ . \quad (11)$$

However, one should not forget that there exist different methods of correcting for the center-of-mass motion. One can minimize the energy of the bag,[7,11] mass squared,[5,10] or correct only quark energy for the c.m. motion.[4] Using the equation $dE/dR=0$ the spurious pressure of the center-of-mass motion is taken into account in the nonlinear boundary condition (the other methods are probably more physical). That means that the radius R is connected with the extension in space of the single-particle wave function in the independent-particle model rather than with the actual radius of the physical hadron. On the other hand, this method has some advantages. It seems that working with the independent-particle model, it is relatively easy to correct electroweak parameters for the c.m. motion.[5,7]

$$\chi^2=\sum_{\rm hadrons}^{N}(M_{\rm expt}-M_{\rm bag})^2 \ . \quad (12)$$

The sum is extended over the octet and decuplet of baryons and the octet of vector mesons. In the chiral bag, pions are treated as elementary Goldstone-boson fields. If one takes into account strange particles it would be natural to extend SU(2)×SU(2) chiral symmetry to SU(3)×SU(3).[18] That would mean that the whole pseudoscalar octet should be treated as an octet of elementary Goldstone bosons. So we will not include these particles in the sum in formula (12). Another quantity we use to compare between different fits to hadron masses is an average mass deviation per particle[11]

$$\delta M=(\chi^2/N)^{1/2} \ , \quad (13)$$

where N is the number of considered hadrons.

III. NUMERICAL RESULTS

Since 1975, when the first fit to hadron masses based on the original MIT bag model was performed,[2] giving, by the way, good agreement with experimental values, many other fits, enriched with pionic or center-of-mass correction, have been made, and published in the literature.[4,8–11] Unfortunately, to our knowledge, except for

TABLE I. The values of $\hat{\Omega}$ for the lightest hadron states.

Particle	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω^-	ρ	K^*	ω	ϕ	K	π
$\hat{\Omega}$	30	18	2	0	6	2	0	0	2	0	−6	0	0	−6

TABLE II. The values of δM (MeV) obtained in earlier fits.

Reference	2	4	10	8	9	11 (a)	11 (b)	11 (c)
δM	24	25	27	38	25	17	20	16

Ref. 11 there exists no publication with a complete fit with both of these important corrections applied together. The Nordita paper,[9] to be sure, takes both of them into account, but unfortunately limits its presentation to baryons only. The fit by Liu and Wong[4] and that by Carlson, Hansson, and Peterson[10] include the c.m.-motion contribution, but the authors do not discuss the Goldstone pions' influence on the bag mass. And, conversely, Mulders and Thomas in their fit[8] consider the pion energy but do not correct the masses with respect to the c.m. motion. And finally, Ref. 11 gives the fits with both center-of-mass and chiral pion corrections, but stands out with an unconventional method of fitting. Of course, the above works differ from the present paper not only in the indicated points, but there are many other essential distinctions. For example, all those that include chiral symmetry[8,9,11] calculate off-diagonal and diagonal terms of $\hat{\Omega}$ as well, allowing in such a way, besides tree diagrams, also quark self-interaction processes. The forms of c.m. correction are also different in various fits. The results mentioned above are all quoted in Table II in terms of δM (and not of χ^2 because the numbers of the particles in consideration are not the same in different fits). In the values of δM there is no contribution from pseudoscalar mesons, as argued in the previous section, even in the case where their masses were present in the tables of original papers.

The fits of Ref. 11 are distinguished from earlier ones with better agreement with experiment thanks to the method of fitting: nonstrange quarks are admitted to have a nonzero mass, and five parameters m_0, m_s, α_c, Z_0, and B are established by the requirement that χ^2 be minimal. [In this paper there were shown three fits with (a) H_{MIT}, (b) H_{MIT} and center-of-mass corrections (CMC), (c) $H_{\text{MIT}}+E_\pi$ and CMC, the coupling constant $1/f_\pi$ being lowered to $0.56(1/f_\pi)$ in the last case.]

We would like to begin the presentation of our calculations by recalling the nonlinear boundary condition which, in the case of the nonchiral MIT bag, takes the form

$$B=\tfrac{1}{2}n^\mu\partial_\mu(\bar{q}q)-\tfrac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}\ . \tag{14}$$

We expect this to be equivalent to the equation $dE/dR=0$ and, actually, it is exactly true when quarks are in the lowest cavity mode, and when we omit their interactions. But, unfortunately, it is not so sure if we consider the full equation (14), since color-SU(3) interactions contribute in a nonspherical way. In this case we do not know, in fact, what the nonlinear boundary condition means and what equation we should use to find the bag radius. For this and also since we do not expect one-

TABLE III. The fits with MIT bag Hamiltonian. (a) From Ref. 2, (b) gluons treated perturbatively.

Particle	M_{expt} (MeV)	(a) R (GeV^{-1})	(a) M_{bag} (MeV)	(a) ΔM (MeV)	(b) R (GeV^{-1})	(b) M_{bag} (MeV)	(b) ΔM (MeV)
N	938	5.00	938	0	5.22	938	0
Λ	1116	4.95	1105	11	5.18	1099	17
Σ	1189	4.95	1144	45	5.18	1137	52
Ξ	1321	4.91	1289	32	5.14	1280	41
Δ	1236	5.48	1233	3	5.22	1236	0
Σ^*	1385	5.43	1382	3	5.18	1380	5
Ξ^*	1533	5.39	1529	4	5.14	1526	7
Ω^-	1672	5.35	1672	0	5.10	1672	0
ρ	770	4.71	783	-13	4.44	783	-13
K^*	892	4.65	928	-36	4.39	923	-31
ω	783	4.71	783	0	4.44	783	0
ϕ	1019	4.61	1068	-49	4.34	1066	-47
$B^{1/4}$ (MeV)			145			146	
α_c			0.55			0.556	
Z_0			1.84			1.87	
m_s (MeV)			279			278	
m_0 (MeV)			0			0	
χ^2 (MeV^2)			0.71×10^4			0.81×10^4	
δM (MeV)			24			26	

TABLE IV. The fits with CMC. Gluons treated (a) nonperturbatively and (b) perturbatively.

Particle	M_{expt} (MeV)	(a) R (GeV^{-1})	(a) M_{bag} (MeV)	(a) ΔM (MeV)	(b) R (GeV^{-1})	(b) M_{bag} (MeV)	(b) ΔM (MeV)
N	938	5.42	938	0	5.67	938	0
Λ	1116	5.37	1100	16	5.62	1104	12
Σ	1189	5.37	1139	50	5.62	1147	42
Ξ	1321	5.33	1283	38	5.58	1293	28
Δ	1236	5.77	1236	0	5.67	1236	0
Σ^*	1385	5.73	1378	7	5.62	1380	5
Ξ^*	1533	5.69	1523	10	5.58	1526	7
Ω^-	1672	5.65	1670	2	5.53	1675	−3
ρ	770	5.12	783	−13	5.02	783	−13
K^*	892	5.07	915	−23	4.97	916	−24
ω	783	5.12	783	0	5.02	783	0
ϕ	1019	5.03	1054	−35	4.92	1056	−37
$B^{1/4}$ (MeV)			144			142	
α_c			0.474			0.522	
Z_0			0.822			0.792	
m_s (MeV)			284			290	
m_0 (MeV)			0			0	
χ^2 (MeV^2)			0.63×10^4			0.49×10^4	
δM (MeV)			23			20	

gluon-exchange processes to influence strongly the structure of the bag, we have decided to check up on the role of gluons in determining the bag size by making a fit in which E_g is not present in equation $dE/dR=0$, and to compare it with the results of De Grand, Jaffe, Johnson, and Kiskis.[2] Both fits are shown in Table III. The masses and parameters are quoted in MeV, radii in GeV^{-1}.

A glance at Table III allows us to draw the conclusion that gluons are not of great importance in establishing the bag radius and we need not bother about the exact form of the nonlinear boundary condition: the differentiating of E_g is almost as good as nondifferentiating. This welcome conclusion is, as we shall see, true also in all further cases.

While doing this fit, as well as the others, we have been investigating also the behavior of pseudoscalar mesons

TABLE V. The fit with E_π but without CMC.

Particle	M_{expt} (MeV)	R (GeV^{-1})	M_{bag} (MeV)	ΔM (MeV)
N	938	3.91	938	0
Λ	1116	3.89	1163	−47
Σ	1189	3.89	1297	−108
Ξ	1321	3.87	1444	−123
Δ	1236	4.03	1236	0
Σ^*	1385	4.01	1395	−10
Ξ^*	1533	3.99	1540	−7
Ω^-	1672	3.97	1672	0
ρ	770	3.30	678	92
K^*	892	3.28	832	60
ω	783	3.30	782	1
ϕ	1019	3.26	961	58
$B^{1/4}$ (MeV)			185	
α_c			0.155	
Z_0			2.49	
m_s (MeV)			246	
m_0 (MeV)			0	
χ^2 (MeV^2)			0.45×10^5	
δM (MeV)			61	

and we have found them to be very sensitive to the small changes of parameters. This is the consequence of the fact that for π and K large contributions give in the sum small masses. As we do not know, in fact, the exact values of parameters (we may take other particles than N, Δ, Ω^-, ω, which are chosen for the reason of ease of fitting, obtaining in general as good a fit as before, but with quite different π and K masses), the model gives us no prediction of pseudoscalar-octet masses. The very good result for the K mass in the MIT-group fit[2] seems therefore to be rather accidental.

The next calculations we have performed are quite similar to those shown in Table III, but enriched with center-of-mass corrections in the form defined in the previous section. They are given in Table IV.

Our conclusion that gluons have only weak influence on the solutions of the pressure-balance equations, drawn on the basis of Table III, is confirmed by the comparison of both fits of Table IV. Another nice observation is some small improvement of the results after having corrected the masses for the center-of-mass motion. The sensitivity of the pseudoscalar-meson masses, about which we have already told, can be seen here as well. For example, the value of ΔM for kaons leaps from 184 to 19 MeV while going from the left column to the right one, which one can compare with the numbers obtained for other particles.

The other important effect, the influence of which on the hadron mass spectrum we want to investigate, is the chiral pions interaction. In this case, however, some serious problems arise. While fitting with the full pion pressure, as given by dE_π/dR, we cannot obtain the correct ω-Δ mass difference with a stable nucleon ($\Omega=30$) of proper mass; hence there are no numerical solutions. The pion pressure is therefore too big causing the instability of the model for small R and possible collapse of the bag. This is in contradiction with theoretical expectations,[8] because the essence of the bag is quarks and external pressure and not pions, and consequently they should not play the major role in the model. Besides, it is not obvious that the pion contribution to the nonlinear boundary condition gives $dE/dR=0$. All that suggests that we should try to apply to pions the same scheme as we have already done for gluons—not to differentiate E_π. It does not seem to be less reasonable than nonperturbative treating. However, if we add pion energy to the Hamiltonian, but forget about c.m.-motion correction we obtain a discouraging fit as shown in Table V.

This, together with Table VI, where CMC is already included, gives the confirmation of the necessity of this correction, which, by the way, has not aroused any doubt from the theoretical point of view. (The same can also be seen in Table II.) What one can and should discuss is its particular form.

Table VI gives us, after all, quite satisfactory fits, although we observe some small deterioration of the results in relation to those of the prechiral bag. However we do not limit ourselves to perturbative treating of pions because the bag model describes quark-pion interactions very coarsely and it is likely that not only is our formula for pionic pressure exerted on the bag surface wrong, but also even that for the pionic energy is wrong. One could then modify the expressions taking into account the fact that pions are not structureless, but, on the contrary, they

TABLE VI. The fits with E_π and CMC. Gluons treated (a) nonperturbatively and (b) perturbatively. In both cases E_π is not differentiated.

Particle	$M_{\rm expt}$ (MeV)	(a) R (GeV^{-1})	(a) $M_{\rm bag}$ (MeV)	(a) ΔM (MeV)	(b) R (GeV^{-1})	(b) $M_{\rm bag}$ (MeV)	(b) ΔM (MeV)
N	938	4.85	938	0	4.98	938	0
Λ	1116	4.82	1142	−26	4.95	1137	−21
Σ	1189	4.82	1238	−49	4.95	1229	−40
Ξ	1321	4.79	1386	−65	4.92	1374	−53
Δ	1236	5.01	1236	0	4.98	1236	0
Σ^*	1385	4.99	1388	−3	4.95	1387	−2
Ξ^*	1533	4.96	1535	−2	4.92	1532	1
Ω^-	1672	4.93	1675	−3	4.89	1670	2
ρ	770	4.42	727	43	4.38	726	44
K^*	892	4.39	869	23	4.34	867	25
ω	783	4.42	782	1	4.38	783	0
ϕ	1019	4.35	1003	16	4.31	998	21
$B^{1/4}$ (MeV)			162			160	
α_c			0.246			0.291	
Z_0			1.04			1.03	
m_s (MeV)			269			268	
m_0 (MeV)			0			0	
χ^2 (MeV^2)			0.10×10^5			0.79×10^4	
δM (MeV)			29			26	

TABLE VII. Fit with E_π and CMC. R replaced with $(R^2+R_\pi^2)^{1/2}$ in the formula for E_π. Both E_g and E_π differentiated.

Particle	M_{expt} (MeV)	R (GeV^{-1})	M_{bag} (MeV)	ΔM (MeV)
N	938	5.05	938	0
Λ	1116	5.06	1119	−3
Σ	1189	5.13	1181	8
Ξ	1321	5.10	1327	−6
Δ	1236	5.41	1236	0
Σ^*	1385	5.39	1383	2
Ξ^*	1533	5.36	1528	5
Ω^-	1672	5.33	1672	0
ρ	770	4.80	763	7
K^*	892	4.77	898	−6
ω	783	4.83	782	1
ϕ	1019	4.73	1034	−15
$B^{1/4}$ (MeV)			151	
α_c			0.367	
Z_0			0.914	
m_s (MeV)			278	
m_0 (MeV)			0	
χ^2 (MeV^2)			0.45×10^3	
δM (MeV)			6	

are of the finite sizes and have their $\bar{q}q$ contents.[7,15] As pointed out in the Introduction, we use the De Tar modification,[7] which consists in replacing R with $(R^2+R_\pi^2)^{1/2}$ in the expression for E_π. In the opposition to earlier fits presented in this paper, we now do use E_π, rectified in the above way, in the equation $dE/dR=0$ since the pion pressure is weaker and there is no more instability. In this case there exists a numerical solution (for reasonable values of R_π). We have looked at the fits obtained for a few values of R_π and here we present the best one, corresponding to $R_\pi=4.25\ \text{GeV}^{-1}$ (Table VII).

The agreement with experimental masses after this modification is excellent, and in spite of its prescription formulated in an *ad hoc* manner, the "goodness" of the fit is very attractive. In any case, the pion-finite-size effects are worth being investigated in detail.

IV. SUMMARY

We have studied the influence of center-of-mass-motion corrections and pion-quark interactions on fits to the masses of octet and decuplet of baryons and octet of vector mesons. In the pion energy only exchange of pions between different quarks was included.

Taking into account the center-of-mass correction improves fits only slightly: δM changes from 24 MeV for the original MIT fit[2] to 23 MeV or from 26 to 20 MeV if the gluons are treated perturbatively. In the presence of pionic corrections if one does not take into account c.m. corrections the fit is very bad: $\delta M=61$ MeV. With both center-of-mass and pionic corrections (pions treated perturbatively) the obtained value of $\delta M=29$ MeV (26 MeV) is not very different from those obtained by other authors.[4,8–11]

There is no great difference whether gluonic contributions are treated perturbatively or are taken into account in a pressure-balance equation. In general, comparing our fits with those obtained by other authors, it seems that the results for δM do not depend strongly on the details of treating various corrections and values of δM obtained in different fits are between 20 and 30 MeV.

If one corrects, the pionic contribution for the finite pion radius with the center-of-mass corrections included, an exceptionally good fit to hadron masses is obtained with the value of $\delta M=6$ MeV. The way of taking into account of finite pion radius is not very well justified theoretically.

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