Calculation of exclusive decay modes of the tau

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We update the calculation of the decays of the τ lepton into various modes using recent data on electron-positron annihilation into hadrons plus bounds that follow from isotopic-spin conservation. Comparison is made with exclusive branching-ratio data and inclusive charged-prong-distribution measurements in τ decays, and the difficulty in accounting for all the one-charged-prong decays as a sum of exclusive decay modes is discussed.

I. INTRODUCTION

The observed properties of the τ lepton are consistent with it being a sequential lepton,[1] a heavier version of the electron or muon, with its own neutrino partner ν_τ. Particularly with the coming into operation of the DESY PETRA/SLAC PEP generation of electron-positron colliding-beam machines, the separation of $\tau^-\tau^+$ pair production from production of hadrons has become very clean, and more accurate measurements of some properties of the τ have become possible. In particular, measurements of the τ lifetime[2] show that the strength of the charged-weak-current coupling between τ and ν_τ is consistent with being of universal strength, further supporting the standard assignment of the τ (and ν_τ).

The clean separation of τ events has allowed the accurate measurement of the distribution of charged-prong multiplicity in its decay.[3] In this regard it is interesting to reexamine the branching ratios for τ decay into each of its exclusive decay modes, both to check that the individual modes occur at the predicted rate and to see that the exclusive modes which yield one charged prong, three charged prongs, etc., sum up to give the measured inclusive charged-prong multiplicities. In this way we can perform a further check to see if everything is attributable to the decays expected in the standard model, or if there is some small percentage of τ decays which is "unexplained."

In the next section we go through τ decays mode by mode and establish their branching ratio relative to that for $\tau\to\nu_\tau e\bar{\nu}_e$ by using experimental data (from outside of τ decay) wherever possible:[4,5] $\pi^-\to\mu^-\bar{\nu}_\mu$ to determine $\tau^-\to\nu_\tau\pi^-$, $e^+e^-\to 4\pi$ to determine $\tau\to\nu_\tau 4\pi$, etc. In a number of cases we work out bounds that follow from conservation of isotopic spin which allow us to put limits on as-yet-unmeasured branching ratios, e.g., we show that the rate for $\tau^-\to\nu_\tau\pi^-4\pi^0$ can be bounded in terms of the rate for $\tau^-\to\nu_\tau 3\pi^-2\pi^+$ (for which an excellent experimental upper bound exists).

In Sec. III we compare the sum of the exclusive modes with the inclusive charged-prong multiplicity measurements. We find that it is difficult to account for all the one-charged-prong decays and still be consistent with the number of decays going into three charged prongs. We then examine various "cures" for this problem ranging from statistical fluctuations in some measurements to new physics and indicate how various possibilities may be eliminated.

II. CALCULATION OF τ DECAY MODES

We calculate τ decay modes assuming the standard model with a $V-A$ interaction of universal strength between the τ and ν_τ, whose masses we take to be[6] 1784 MeV and zero, respectively. With an eye to the next section, where we compare the sum of exclusive modes with inclusive charged-prong branching ratios, in a number of cases the breakdown of a given mode into charged-prong multiplicities will be examined in some detail.

A. $\tau^-\to\nu_\tau e^-\bar{\nu}_e$

Neglecting the mass of the electron, the width for this decay is

$$\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)=\frac{G_F{}^2 m_\tau{}^5}{192\pi^3}=\left[\frac{1}{1.595\times 10^{-12}\ \text{sec}}\right]. \tag{1}$$

The lifetime of the τ is then

$$\tau_\tau=(1.595\times 10^{-12}\ \text{sec})B(\tau\to\nu_\tau e\bar{\nu}_e)\ . \tag{2}$$

The present most accurate measurement[2] of the lifetime, $(2.86\pm0.16\pm0.25)\times10^{-13}$ sec, implies $B(\tau\to\nu_\tau e\bar{\nu}_e)=(17.9\pm1.0\pm1.6)\%$, in agreement with the direct measurements[6] of this branching ratio. Conversely, to within the errors of the branching-ratio measurements, the predicted lifetime in Eq. (2) agrees with the measured one, providing support for the standard-model assumptions that lead to Eqs. (1) and (2). We shall return in the next section to the question of how much $B(\tau\to\nu_\tau e\bar{\nu}_e)$ can be "stretched" within the experimental errors. We will normalize all other calculations of decay widths to the theoretical value for $\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)$ in Eq. (1).

B. $\tau^-\to\nu_\tau\mu^-\bar{\nu}_\mu$

Taking account of the mass of the muon, we have

$$\Gamma(\tau\to\nu_\tau\mu^-\bar{\nu}_\mu)=\frac{G_F{}^2 m_\tau{}^5}{192\pi^3}F(m_\mu/m_\tau)\ , \tag{3}$$

where

$$F(\Delta)=1-8\Delta^2+8\Delta^6-\Delta^8-12\Delta^4\ln\Delta^2 .$$

Thus

$$\frac{\Gamma(\tau\to\nu_\tau\mu^-\bar{\nu}_\mu)}{\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)}=F(m_\mu/m_\tau)=0.97 .$$

C. $\tau^-\to\nu_\tau\pi^-$

The strength of the pion's coupling to the axial-vector current is directly determined in the well-measured pion decay, $\pi^-\to\mu^-\bar{\nu}_\mu$. Exactly the same quantity, $f_\pi\cos\theta_C$, is relevant in $\tau^-\to\nu_\tau\pi^-$:[4,5]

$$\frac{\Gamma(\tau^-\to\nu_\tau\pi^-)}{\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)}=\frac{(f_\pi\cos\theta_C)^2}{m_\tau^2}12\pi^2\left[1-\frac{m_\pi^2}{m_\tau^2}\right]^2$$

$$=0.607 . \quad (4)$$

D. $\tau^-\to\nu_\tau K^-$

Just as for $\tau^-\to\nu_\tau\pi^-$, the quantity of relevance, $f_K\sin\theta_C$, is directly measured elsewhere and very accurately so, in the dominant decay of the charged kaon, $K^-\to\mu^-\bar{\nu}_\mu$. Inserting this information[6] gives

$$\frac{\Gamma(\tau^-\to\nu_\tau K^-)}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}=\frac{(f_K\sin\theta_C)^2}{m_\tau^2}12\pi^2\left[1-\frac{m_K^2}{m_\tau^2}\right]^2$$

$$=0.0395 . \quad (5)$$

E. $\tau^-\to\nu_\tau\pi^-\pi^0$

Here the strength of the charged-vector-current coupling to $\pi\pi$ can be related, via the conserved-vector-current hypothesis, to that of the electromagnetic (neutral vector) current to $\pi\pi$. The latter is measured by $\sigma(e^+e^-\to\gamma\to\pi^+\pi^-)$. The precise relation between the two processes[4,5] is

$$\frac{\Gamma(\tau^-\to\nu_\tau\pi^-\pi^0)}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}=\frac{3}{2\pi\alpha^2m_\tau^8}\int^{m_\tau^2}dQ^2Q^2(m_\tau^2-Q^2)^2(m_\tau^2+2Q^2)\times\sigma_{e^+e^-\to\pi^+\pi^-}(Q^2) , \quad (6)$$

where the integration variable Q is the center-of-mass energy of the e^+e^- (=the invariant mass of the $\pi\pi$ pair).

Of course the $\pi\pi$ system is dominated by the ρ resonance and an approximate result for $\Gamma(\tau^-\to\nu_\tau\pi^-\pi^0)$ can be obtained[4] from computing $\tau^-\to\nu_\tau\rho^-$ in the narrow-resonance approximation with the coupling to the vector current extracted from $e^+e^-\to\rho^0$ experiments. A more accurate result is obtained by integrating directly over the $e^+e^-\to\pi\pi$ cross section (or actually a fit to it) using Eq. (6). With the present τ mass, one finds[7]

$$\frac{\Gamma(\tau^-\to\nu_\tau\pi^-\pi^0)}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}=1.23 , \quad (7)$$

with an error which is due principally to the possible overall normalization error in measurement of the $e^+e^-\to\pi^+\pi^-$ cross sections.

F. $\tau^-\to\nu_\tau(K\pi)^-$

Just as the $\pi\pi$ system is dominated by the ρ, we expect this Cabibbo-suppressed decay to be dominated by the $K^*(890)$, as is indeed observed experimentally.[6] The rate can be obtained from that for $\tau^-\to\nu_\tau\rho^-$ by multiplying by $\tan^2\theta_C$ due to the strangeness-changing current and

$$m_{K^*}^{-2}(1-m_{K^*}^2/m_\tau^2)^2(1+2m_{K^*}^2/m_\tau^2)\times[m_\rho^{-2}(1-m_\rho^2/m_\tau^2)^2(1+2m_\rho^2/m_\tau^2)]^{-1}$$

from phase space, aside from SU(3) breaking in the coupling strengths to the respective vector currents.[4] This yields the prediction

$$\frac{\Gamma(\tau^-\to\nu_\tau K^{*-})}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}=0.047 . \quad (8)$$

If we incorporate SU(3) breaking by setting[8] $g_{K^*}^2/m_{K^*}^2=g_\rho^2/m_\rho^2$, then the prediction becomes

$$\frac{\Gamma(\tau^-\to\nu_\tau K^{*-})}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}=0.064 . \quad (9)$$

Because of the decay $K_S^0\to\pi^+\pi^-$, two ninths of the decays $\tau^-\to\nu_\tau(K\pi)^-$ will appear as decays with three charged prongs.

G. $\tau^-\to\nu_\tau(4\pi)^-$

Inasmuch as this decay proceeds through the vector current, we can again directly relate the decay rate to an integral over e^+e^- cross sections, as in Eq. (6). There are two possible final states in e^+e^-, $2\pi^-2\pi^+$, and $\pi^-\pi^+2\pi^0$, and two in τ decay as well, $\nu_\tau 2\pi^-\pi^+\pi^0$ and $\nu_\tau\pi^-3\pi^0$. The constraint of being produced by different I_3 components of the same $I=1$ weak current forces one linear relation between the rates for producing the respective charge states at each 4π invariant mass. As a result, not only can we write the $\tau^-\to\nu_\tau(4\pi)^-$ total decay rate as an integral over $\sigma_{e^+e^-\to4\pi}$, but more specifically:[7]

$$\frac{\Gamma(\tau^-\to\nu_\tau\pi^-3\pi^0)}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}=\frac{3}{2\pi\alpha^2m_\tau^8}\int_0^{m_\tau^2}dQ^2Q^2(m_\tau^2-Q^2)^2(m_\tau^2+2Q^2)\times[\tfrac{1}{2}\sigma_{e^+e^-\to2\pi^-2\pi^+}(Q^2)] \quad (10a)$$

and

$$\frac{\Gamma(\tau^- \to \nu_\tau 2\pi^- \pi^+ \pi^0)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{3}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \times [\tfrac{1}{2}\sigma_{e^+e^- \to 2\pi^- 2\pi^+}(Q^2) + \sigma_{e^+e^- \to \pi^+\pi^- 2\pi^0}(Q^2)] \; . \quad (10b)$$

The $e^+e^- \to 4\pi$ cross sections are dominated by the ρ' resonance and rough results may be obtained by approximating the integrand using a single narrow resonance. This is somewhat dangerous in that the mass ($\sim$1550 MeV) and width ($\sim$300 MeV) of the ρ' make the factor $(m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2)$ vary strongly over the resonance, considerably distorting its shape in τ decay.[9]

A more accurate result for $\Gamma(\tau \to \nu_\tau 4\pi)$ can be obtained by integrating directly over the e^+e^- cross sections. Recent data[10–12] for $e^+e^- \to 2\pi^+ 2\pi^-$ are shown in Fig. 1, and they represent a considerable improvement, both with respect to statistics and systematics, over the data that were available for a previous calculation of $\Gamma(\tau \to \nu_\tau 4\pi)$.[7] Carrying out the integration using the curve drawn through the data in Fig. 1 gives

$$\frac{3}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \times \sigma_{e^+e^- \to 2\pi^- 2\pi^+}(Q^2) = 0.11 \; . \quad (11)$$

Unfortunately, the situation with respect to the data from $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ is not as good. The data[10,12,13] from recent experiments are shown in Fig. 2 along with a dashed curve which passes through the data below $Q = 1.4$ GeV and is scaled up from the curve for $e^+e^- \to 2\pi^- 2\pi^+$ in Fig. 1. It lies above much of the available data for $Q > 1.4$ GeV. If the dashed curve represented the data,

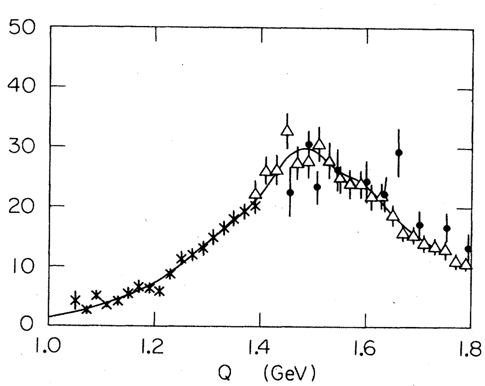

FIG. 1. Data for $\sigma(e^+e^- \to 2\pi^+ 2\pi^-)$ from Novosibirsk (Ref. 10) (✱), Orsay (Ref. 11) (△), and Frascati (Ref. 12) (●) as a function of center-of-mass energy Q.

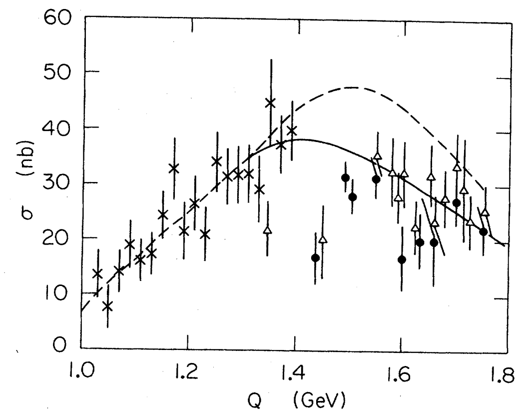

FIG. 2. Data for $\sigma(e^+e^- \to \pi^+\pi^-\pi^0\pi^0)$ from Novosibirsk (Ref. 10) (✱), Frascati (Ref. 12) (●), and Orsay (Ref. 13) (△) as a function of center-of-mass energy Q.

$$\frac{3}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \times \sigma_{e^+e^- \to \pi^+\pi^- 2\pi^0}(Q^2) = 0.25 \; . \quad (12a)$$

The solid curve, which is a much better representation of the data, gives

$$\frac{3}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \times \sigma_{e^+e^- \to \pi^+\pi^- 2\pi^0}(Q^2) = 0.22 \; , \quad (12b)$$

while taking the integration only up to $Q = 1.4$ GeV (certainly a minimum value for the integral) yields

$$\frac{3}{2\pi\alpha^2 m_\tau^8} \int_0^{(1.4\ \mathrm{GeV})^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \times \sigma_{e^+e^- \to \pi^+\pi^- 2\pi^0}(Q^2) = 0.13 \; . \quad (12c)$$

We will use (12a) and (12c) as bracketing the actual value of the integral, whose value we take as that given in (12b).

If we now insert the numerical-integration results in Eqs. (11) and (12) back into Eq. (10) we find

$$\frac{\Gamma(\tau^- \to \nu_\tau \pi^- 3\pi^0)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.055 \; , \quad (13a)$$

$$\frac{\Gamma(\tau^- \to \nu_\tau 2\pi^- \pi^+ \pi^0)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.275 \; , \quad (13b)$$

and the sum,

$$\frac{\Gamma[\tau^- \to \nu_\tau (4\pi)^-]}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.33 \; . \quad (13c)$$

The result for the one-charged-prong decay $\tau^- \to \nu_\tau \pi^- 3\pi^0$ in Eq. (13a) is rather certain as it depends only on the integral over the well-measured cross section for $e^+e^- \to 2\pi^+ 2\pi^-$. The numerical result in Eq. (13b) is more uncertain, but only varies from 0.185 to 0.305 if we use the extreme values for the cross section for $e^+e^-\rightarrow\pi^+\pi^-2\pi^0$ discussed above to bracket the data in Fig. 2.

These results, which correspond to $a\sim 6\%$ branching ratio for $\tau^-\rightarrow\nu_\tau(4\pi)^-$ are 30% to 50% smaller than values[7] reported a number of years ago. This is almost entirely due to the change in experimental data for $e^+e^-\rightarrow 4\pi$.

H. $\tau^-\rightarrow\nu_\tau(\pi\pi\pi)^-$

The 3π system is generated through the axial-vector current and may have $J^P=0^-$ or 1^+. There is no other directly measured quantity which can be used to predict the branching ratio for this mode, which is presumably dominated by the A_1 and possible π' resonances.[14] Older calculations[15] led to branching ratios of order 10% for $\tau^-\rightarrow\nu_\tau A_1^-$, in very rough accord with the data.[6,14] Inasmuch as these predictions should only be trusted at the factor of two level, they are now superseded for our purposes by the rather accurate measurements, particularly of $\tau^-\rightarrow\nu_\tau\pi^-\pi^-\pi^+$, which now exist.[14]

Independent of dynamics, isotopic spin forces an important constraint on the different 3π charge states. Since the total isospin is one, it follows that in the decay $\tau^-\rightarrow\nu_\tau(3\pi)^-$,

$$\tfrac{1}{5}\le f_1=\frac{\pi^-\pi^0\pi^0}{\text{all }(3\pi)^-}\le\tfrac{1}{2}\,, \tag{14a}$$

and

$$\tfrac{1}{2}\le f_3=\frac{2\pi^-\pi^+}{\text{all }(3\pi)^-}\le\tfrac{4}{5}\,. \tag{14b}$$

Thus the number of three-charged-prong τ decays must be greater than that of one-charged-prong decays when $\tau^-\rightarrow\nu_\tau(3\pi)^-$, something which will have a role to play in the next section.

I. $\tau^-\rightarrow\nu_\tau(5\pi)^-$

Although theoretical estimates[16] of the rate for $\tau^-\rightarrow\nu_\tau(5\pi)^-$ exist, this decay has yet to be observed and there is no independent way to experimentally determine the strength of the axial-vector current to five-pion transition which is involved here. Standard methods[17] do allow one to find the constraints due to isotopic-spin conservation:

$$0\le f_1=\frac{\pi^-4\pi^0}{\text{all }(5\pi)^-}\le\tfrac{3}{10}\,, \tag{15a}$$

$$\tfrac{8}{35}\le f_3=\frac{2\pi^-\pi^+2\pi^0}{\text{all }(5\pi)^-}\le 1\,, \tag{15b}$$

$$0\le f_5=\frac{3\pi^-2\pi^+}{\text{all }(5\pi)^-}\le\tfrac{24}{35}\,. \tag{15c}$$

All of this decay mode could go into three charged prongs (e.g., $\tau^-\rightarrow\nu_\tau\omega^0\rho^-$), as implied in (15b), and hence the very good experimental limit[18] on five-charged-prong decays of the τ does not necessarily imply the overall branching ratio for $\tau^-\rightarrow\nu_\tau(5\pi)^-$ is very small. However, for our purposes later we will want to have a bound on the one-charged-prong mode $\tau^-\rightarrow\nu_\tau\pi^-4\pi^0$, and this is obtainable from an upper limit on $\tau^-\rightarrow\nu_\tau 3\pi^-2\pi^+$.

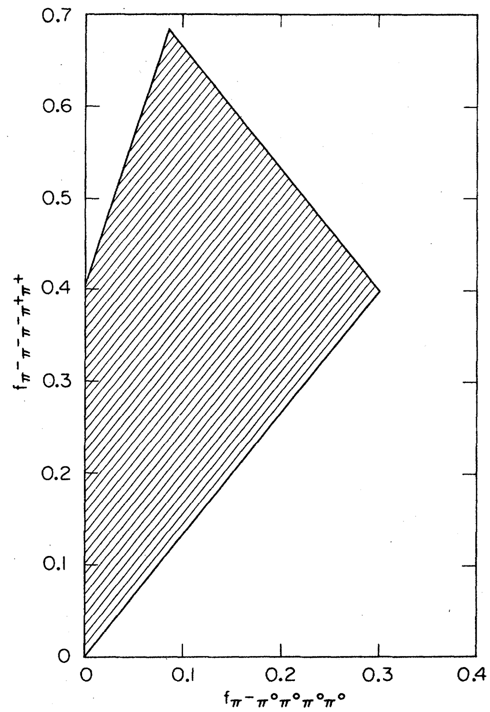

FIG. 3. Region (shaded) allowed by the constraint of isospin conservation in $\tau^-\rightarrow\nu_\tau(5\pi)^-$ for the fraction $f_{\pi^-\pi^-\pi^-\pi^+\pi^+}$ versus the fraction $f_{\pi^-\pi^0\pi^0\pi^0\pi^0}$.

For this purpose we need not the bound in Eq. (15), but the joint distribution on the fraction of $\pi^-4\pi^0$ versus the fraction of $3\pi^-2\pi^+$. The region allowed by isotopic-spin conservation for these two fractions is shown in Fig. 3, and from this we see that

$$\frac{f_{3\pi^-2\pi^+}}{f_{\pi^-4\pi^0}}\ge\tfrac{4}{3}\,.$$

Consequently we have the bound

$$B(\tau^-\rightarrow\nu_\tau\pi^-4\pi^0)\le\tfrac{3}{4}B(\tau^-\rightarrow\nu_\tau 3\pi^-2\pi^+)$$
$$\le\tfrac{3}{4}B(\tau^-\rightarrow 5\text{ charged prongs})\,. \tag{16}$$

J. $\tau^-\rightarrow\nu_\tau(6\pi)^-$

In this case we can use either isotopic-spin conservation plus the experimental bound[18] on $\tau\rightarrow 5$ charge prongs to bound either the decay rate for the complete mode $\tau^-\rightarrow\nu_\tau(6\pi)^-$ or the particular decay $\tau^-\rightarrow\nu_\tau\pi^-5\pi^0$, for the constraints from isotopic-spin force[17]

$$0\le f_1=\frac{\pi^-5\pi^0}{\text{all }(6\pi)^-}\le\tfrac{9}{35}\,, \tag{17a}$$

$$\tfrac{1}{5}\le f_3=\frac{2\pi^-\pi^+3\pi^0}{\text{all }(6\pi)^-}\le\tfrac{4}{5}\,, \tag{17b}$$

$$\tfrac{1}{5}\le f_5=\frac{3\pi^-2\pi^+\pi^0}{\text{all }(6\pi)^-}\le\tfrac{4}{5}\,. \tag{17c}$$

From (17c)

$$B[\tau^- \to \nu_\tau(6\pi)^-] \le 5B(\tau^- \to \nu_\tau 3\pi^- 2\pi^+ \pi^0)$$
$$\le 5B(\tau^- \to 5 \text{ charged prongs}) , \quad (18)$$

and from (17a) and (17c),

$$B(\tau \to \nu_\tau \pi^- 5\pi^0) \le (\tfrac{9}{7}) B(\tau^- \to \nu_\tau 3\pi^- 2\pi^+ \pi^0)$$
$$\le (\tfrac{9}{7}) B(\tau^- \to 5 \text{ charged prongs}) . \quad (19)$$

Alternately, since six pions are produced through the hadronic vector current we can use $e^+e^- \to 6\pi$ data to directly calculate $\tau^- \to \nu_\tau(6\pi)^-$. The e^+e^- data[19] are not very accurate, but they indicate a cross section for 6π production of a few nanobarns for center-of-mass energies below m_τ. If we take 10 nb as a reasonable upper limit for $\sigma(e^+e^- \to 6\pi)$ from 1.4 GeV to m_τ, then $\Gamma[\tau^- \to \nu_\tau(6\pi)^-]/\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e) < 0.024$. This is negligible from the point of view of having an impact on the considerations in the next section.

K. $\tau \to \nu_\tau(K\bar{K})^-$

The decay $\tau^- \to \nu_\tau K^0 \bar{K}^-$ occurs through the weak vector current and hence the rate for this process can be related to the cross section for $e^+e^- \to K\bar{K}$ occurring through the action of the isovector part of the electromagnetic current. Unfortunately the process $e^+e^- \to K\bar{K}$ also occurs through the action of the isoscalar part of the electromagnetic current: in fact for $Q \approx 1$ GeV it is dominated by the presence of the ϕ. Sorting out the two contributions requires at least data on both $e^+e^- \to K^+K^-$ and $e^+e^- \to K_S^0 K_L^0$, but the data on the latter process are sparse. If we nevertheless blindly proceed and assume that above 1.2 GeV the isoscalar and isovector contributions are equal (adding in $e^+e^- \to K^+K^-$ and canceling in $e^+e^- \to K_S^0 K_L^0$), then integration over the measured $\sigma(e^+e^- \to K^+K^-)$ as in Eq. (6) yields a branching ratio for $\tau^- \to \nu_\tau K^0 K^-$ of $\sim 0.5\%$.

We note that due to the decay $K_S^0 \to \pi^+\pi^-$, one third of these involve three charged prongs. The upper limit[20] of 0.6% on $B(\tau^- \to \nu_\tau K^- \pi^+ \pi^-)$ then puts an experimental limit of 1.8% on $B(\tau^- \to \nu_\tau K^0 K^-)$.

L. $\tau \to \nu_\tau(K\bar{K}\pi)^-$

This mode must be present inasmuch as the ρ', which is an important part of the four-pion state in $\tau^- \to \nu_\tau(4\pi)^-$ also decays into $K^*\bar{K}$ and $\bar{K}^*K$ and thus feeds $\tau^- \to \nu_\tau(K^*\bar{K})^-$ and $\tau^- \to \nu_\tau(\bar{K}^*K)^-$ at an expected branching ratio of $\lesssim 1\%$. Indeed the observation[21–23] of $\tau^- \to \nu_\tau K^+ K^- \pi^-$ is consistent with being of this origin with a branching ratio $\sim 0.2\%$.

The constraints due to isotopic-spin conservation (which hold whatever is the dynamics) limit the $K\bar{K}\pi$ charge states to obey[17]

$$0 \le f_{K^0 K^- \pi^0} = \frac{K^0 K^- \pi^0}{\text{all } (K\bar{K}\pi)} \le \tfrac{1}{2} , \quad (20a)$$

$$0 \le f_{K^0 \bar{K}^0 \pi^-} = \frac{K^0 \bar{K}^0 \pi^-}{\text{all } (K\bar{K}\pi)^-} \le \tfrac{3}{4} , \quad (20b)$$

$$0 \le f_{K^+ K^- \pi^-} = \frac{K^+ K^- \pi^-}{\text{all } (K\bar{K}\pi)^-} \le \tfrac{3}{4} . \quad (20c)$$

However, one-charged-prong decays arise both from $K^0 K^- \pi^0$ (two-thirds of the time) and from $K^0 \bar{K}^0 \pi^-$ (four-ninths of the time), and a calculation of the maximum (or minimum) fraction of $\tau \to K\bar{K}\pi$ decays which result in one charged prong demands that we look at their joint distribution.

This is shown in Fig. 4, where the shaded interior of the ellipse is the allowed region. The fraction of one-charged-prong decays is given by

$$f_1 = \tfrac{4}{9} f_{K^0 \bar{K}^0 \pi^-} + \tfrac{2}{3} f_{K^0 K^- \pi^0} ,$$

and would appear as a diagonal line with slope $-\frac{3}{2}$ in Fig. 4. The quantity f_1 is maximal when this line is just tangent to the ellipse, which occurs when

$$f_1 = (1+\sqrt{3})/(3\sqrt{3}) = 0.526 ,$$
$$f_3 = (5-\sqrt{3})/8 ,$$
and
$$f_5 = (1+\sqrt{3})/(24\sqrt{3}) . \quad (21)$$

M. $\tau^- \to \nu_\tau(K\pi\pi)^-$

This is the Cabibbo-suppressed analog of $\tau^- \to \nu_\tau(3\pi)^-$. The constraints[17] due to isospin conservation are

$$\tfrac{1}{3} \le f_{K^-\pi^+\pi^-} = \frac{K^-\pi^+\pi^-}{\text{all } (K\pi\pi)^-} \le \tfrac{2}{3} , \quad (22a)$$

$$0 \le f_{K^-\pi^0\pi^0} = \frac{K^-\pi^0\pi^0}{\text{all } (K\pi\pi)^-} \le \tfrac{1}{3} , \quad (22b)$$

$$0 \le f_{\bar{K}^0\pi^0\pi^-} = \frac{\bar{K}^0\pi^0\pi^-}{\text{all } (K\pi\pi)^-} \le \tfrac{2}{3} . \quad (22c)$$

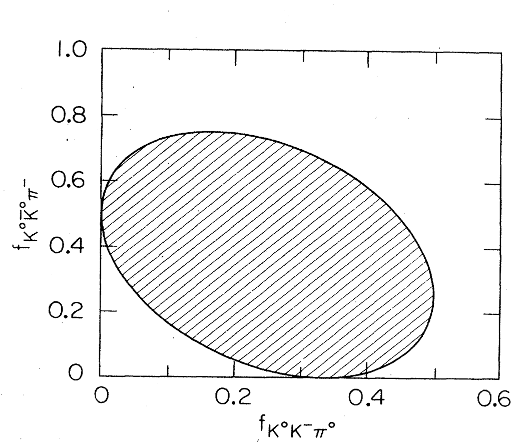

FIG. 4. Region (shaded) allowed by the constraints of isospin conservation in $\tau^- \to \nu_\tau(K\bar{K}\pi)$ for the fraction $f_{K^0\bar{K}^0\pi^-}$ versus the fraction $f_{K^0K^-\pi^0}$.

TABLE I. τ decay branching ratios assuming $B(\tau\to\nu_\tau e\bar{\nu}_e)=17.9\%$.

Decay mode	Branching ratio (%) 1 charged prong	3 charged prongs	Source
$\tau^-\to\nu_\tau e^-\bar{\nu}_e$	17.9		Input
$\tau^-\to\nu_\tau \mu^-\bar{\nu}_\mu$	17.4		Eq. (3)
$\tau^-\to\nu_\tau\pi^-$	10.9		Eq. (4)
$\tau^-\to\nu_\tau K^-$	0.7		Eq. (5)
$\tau^-\to\nu_\tau(\pi\pi)^-$	22.0		Eq. (7)
$\tau^-\to\nu_\tau(K\pi)^-$	0.9	0.3	Eq. (9)
$\tau^-\to\nu_\tau(4\pi)^-$	1.0	4.9	Eqs. (13)
$\tau^-\to\nu_\tau(3\pi)^-$	$\leq x$	x	Eqs. (14)
$\tau^-\to\nu_\tau(5\pi)^-$	<0.12	y	Eq. (16) and Ref. 18
$\tau^-\to\nu_\tau(6\pi)^-$	<0.21	<0.64	Eqs. (17) and (19) and Ref. 18
$\tau^-\to\nu_\tau(K\bar{K})^-$	<1.2	<0.6	Ref. 20
$\tau^-\to\nu_\tau(K\bar{K}\pi)^-$	$<1.29z$	z	Eq. (21)
$\tau^-\to\nu_\tau(K\pi\pi)^-$	$<0.8w$	w	Eq. (25)

The upper bound on $\tau\to\nu_\tau K^-\pi^+\pi^-$ noted above[20,23] and Eq. (22a) imply $B(\tau\to\nu_\tau K\pi\pi)<1.8\%$.

Both the channels $K^-\pi^0\pi^0$ (always) and $\bar{K}^0\pi^0\pi^-$ (two-thirds of the time) generate one-charged-prong $\tau^-\to\nu_\tau(K\pi\pi)^-$ decays. Their joint distribution is very simply the linear relation

$$3f_{K^-\pi^0\pi^0}+\tfrac{3}{2}f_{\bar{K}^0\pi^0\pi^-}=1\ , \tag{23}$$

while the fraction of one-charged-prong decays is

$$f_1=f_{K^-\pi^0\pi^0}+\tfrac{2}{3}f_{\bar{K}^0\pi^0\pi^-}\ . \tag{24}$$

The maximum of f_1 occurs when $f_{\bar{K}^0\pi^0\pi^-}$ is a maximum $(=\frac{2}{3})$, at which point

$$f_1=\tfrac{4}{9}\ \text{and}\ f_3=\tfrac{5}{9}\ . \tag{25}$$

III. SUMMARY AND COMPARISON WITH EXPERIMENT

The various τ decay modes we have considered are summarized in Table I. For the seven modes in the upper part of the table we have calculated the branching ratios in terms of the branching ratio for $\tau\to\nu_\tau e\bar{\nu}_e$, which we have assumed to be 17.9%. In contrast, for the six modes listed in the lower part of the table we have chosen either to give upper limits on their branching ratios or to list the three-charged-prong branching ratio as an unknown quantity (x,y,z, or w) and to insert the corresponding upper bound that ensues for the one-charged-prong branching ratio.

The reason for normalizing the rates in the upper part to that for $\tau\to\nu_\tau e\bar{\nu}_e$ is that they are all calculable from data obtained *outside* of τ decay. These data are mostly *more accurately known* experimentally than are the corresponding τ branching ratios. In particular $\tau\to\nu_\tau\mu\bar{\nu}_\mu$, $\tau\to\nu_\tau\pi$, and $\pi\to\nu_\tau K$ are calculable in terms of $\tau\to\nu_\tau e\bar{\nu}_e$ to very high accuracy and these four modes together account for almost half of all τ decays. For one-charged-prong decays only $\tau\to\nu_\tau\pi\pi$, $\tau\to\nu_\tau K\pi$, and $\tau\to\nu_\tau 4\pi$ have predicted branching ratios in terms of $\tau\to\nu_\tau e\nu_\tau$ for which the input data do not have tiny errors. In the case of $\tau\to\nu_\tau K\pi$ and $\tau\to\nu_\tau 4\pi$ the one-charged-prong portions of their branching ratios are of order 1%, and even a $\pm 20\%$ error in the prediction has little effect on the overall one-charged-prong branching fraction of the τ. Only for $\tau\to\nu_\tau\pi\pi$ is the branching ratio big and the error on the input $\sigma(e^+e^-\to\pi\pi)$ not tiny. Here a $\pm 10\%$ error on the input cross section would mean a $\pm 2\%$ error in the prediction for $B(\tau\to\nu_\tau\pi\pi)$ and thus in the one-charged-prong branching fraction. Furthermore, while the decay rates themselves depend strongly on m_τ, their ratios to $\Gamma(\tau\to\nu_\tau e\bar{\nu}_e)$ depend very weakly on m_τ.

It should be immediately pointed out, moreover, that within errors all the predicted branching ratios in the upper part of Table I are in agreement with experiment.[6] For example, $B(\tau\to\nu_\tau\pi)=10.9\%$ in Table I is to be compared to $(10.3\pm1.2)\%$ of the Particle Data Group[6] and $B(\tau\to\nu_\tau\pi\pi)=22.0\%$ is to be compared to[6] $(22.1\pm2.4)\%$ or to the new Mark II number reported in Ref. 3 of $(22.0\pm0.8\pm1.9)\%$.

Now comes the problem. Accepting the input hypotheses of Table I, the sum of the one-charged-prong branching ratios for the decay modes above the dashed line is 70.8%, while the corresponding three-charged-prong number is 5.2%. On the other hand, the world-average value[3] of the total three-charged-prong branching fraction of the τ is $(13.7\pm0.5)\%$, and correspondingly[18] for the one charged prongs it is $(86.3\pm0.5)\%$. Therefore 15.5% of the one-charged-prong decays and 8.5% of the three-charged-prong decays must come from modes in the lower part of Table I (or other modes yet). However, of the modes in the lower part of the table, only $\tau\to\nu_\tau 3\pi$ is sizeable and its contribution to three charged prongs (denoted by x in Table I) is always larger (by isospin) than its contribution to one charged prong.

Thus if $\tau\to\nu_\tau 3\pi$ accounted for the remaining 8.5% of three-charged-prong decays, one would still have at least $(15.5-8.5)\%=7.0\%$ of τ decays which go to one charged prong to account for. The remaining τ decay modes below $\tau \to \nu_\tau 3\pi$ in Table I all are small, have small contributions to the one-charged-prong branching fractions, and even if they were not small, all have at most comparable contributions to one-charged-prong and three-charged-prong decays.

In fact, the average of recent measurements[3] of $B(\tau^- \to \nu_\tau \pi^- \pi^- \pi^+)$ from DELCO and MAC is $(6.4 \pm 0.7)\%$, so that the three-charged-prong decays of the τ are almost accounted for within errors.[24] As this decay is known to be overwhelmingly $\tau \to \nu_\tau \pi \rho$,

$$B(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0) = B(\tau^- \to \nu_\tau \pi^- \pi^- \pi^+) \; .$$

Even allowing the sum of the modes listed below $\tau \to \nu_\tau 3\pi$ to contribute a total of 2% to one-charged-prong τ decays, we have $\sim 6\%$ of τ decays going into one charged prong for which we do not account.

This situation has been examined previously,[25–27] with some indication of a problem.[27] Previous summations[25,26] of exclusive τ modes purely from experimental data for each mode involve a considerably larger statistical error since they generally do not take into account the correlation in branching ratios imposed by data from outside τ decay, as well as some individual modes involving multineutrals have large errors. The discrepancy in the one-charged-prong decays is then much less statistically significant when things are done in this way. Also, without the bounds on $\tau \to \nu_\tau 5\pi$ and $\tau \to \nu_\tau 6\pi$ we derived, these modes could have made up any discrepancy. In addition, the error bars on the measurement of the inclusive one- and three-charged-prong branching fractions have recently shrunk considerably,[3] making the problem more acute. A possible discrepancy of $\sim 10\%$ in one-charged-prong decays was pointed out in Ref. 27. However, the recent data do not indicate (see Table I) a significant difference between the contributions from the vector current ($\sim 28\%$ known) and the axial-vector current ($\sim 24\%$ known), which was used in Ref. 27 to indicate that the effect originated in the axial-vector current.

What are the possible explanations? First, the branching ratio for $\tau \to \nu_\tau e \bar{\nu}_e$ could be larger than the 17.9% used in Table I. For example, Table II shows what happens when $B(\tau \to \nu_\tau e \bar{\nu}_e) = 19.0\%$. The sum of one-charged-prong branching fractions in the upper part of the table goes up to 75.1%. Including $B(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0)$ at the 6.4% level and taking 2% for the sum of the one-charged-prong contributions of the modes listed below $\tau \to \nu_\tau 3\pi$ almost removes any statistically significant discrepancy. It should be noted, however, that many of the branching ratios in the upper part of Table II are straining against the upper limits of the experimental error bars.

Second, note in particular the large mode $\tau \to \nu_\tau \pi\pi$, for which the prediction relative to $\tau \to \nu_\tau e \bar{\nu}_e$ has the possibility of error due to errors in $\sigma(e^+e^- \to \pi\pi)$. However, if in Table I the discrepancy is to be "solved" by increasing this mode alone [raising $B(\tau \to \nu_\tau \pi\pi)$ by $\sim 6\%$], then the input $\sigma(e^+e^- \to \pi^+\pi^-)$ must have been too low by $\sim 30\%$ and the "true" $B(\tau \to \nu_\tau \pi\pi)$ would be three or more standard deviations above the present measurements.[3]

Third, the τ could have conventional decay modes which we have not considered so far, e.g., $\tau^- \to \nu_\tau \eta \pi^- \pi^0$, or the mode $\tau \to \nu_\tau \eta\eta\pi$ considered in Ref. 27. Such decays would mostly appear as one-charged-prong decays and although the former process is related in strength to $e^+e^- \to \eta\pi^+\pi^-$, it seems this might have been missed. There is furthermore no reason to assume that such τ decays would have comparatively large branching fractions, aside from fixing up the discrepancy in the one-charged-prong branching fraction.

Fourth, the τ could have decays which are unconventional. Decays such as $\tau^- \to \nu_\tau S^-$, where S^- is "elementary" (i.e., pointlike) and either stable or unstable, are ruled out by the lack of evidence for $e^+e^- \to S^+S^-$ and an increase in R above that which is expected from the known quarks at high energies. If the S^- were virtual however, and coupled mostly to particles which manifest themselves as one charged prong at low masses, it might provide an explanation.

The experimental path to settle the question of a possi-

TABLE II. τ decay branching ratios assuming $B(\tau \to \nu_\tau e \bar{\nu}_e) = 19.0\%$.

Decay mode	Branching ratio (%) 1 charged prong	3 charged prongs	Source
$\tau^- \to \nu_\tau e^- \bar{\nu}_e$	19.0		Input
$\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$	18.4		Eq. (3)
$\tau^- \to \nu_\tau \pi^-$	11.6		Eq. (4)
$\tau^- \to \nu_\tau K^-$	0.8		Eq. (5)
$\tau^- \to \nu_\tau (\pi\pi)^-$	23.4		Eq. (7)
$\tau^- \to \nu_\tau (K\pi)^-$	0.9	0.3	Eq. (9)
$\tau^- \to \nu_\tau (4\pi)^-$	1.0	5.2	Eqs. (13)
$\tau^- \to \nu_\tau (3\pi)^-$	$\leq x$	x	Eqs. (14)
$\tau^- \to \nu_\tau (5\pi)^-$	<0.12	y	Eq. (16) and Ref. 18
$\tau^- \to \nu_\tau (6\pi)^-$	<0.21	<0.64	Eqs. (17) and (19) and Ref. 18
$\tau^- \to \nu_\tau (K\bar{K})^-$	<1.2	<0.6	Ref. 20
$\tau^- \to \nu_\tau (K\bar{K}\pi)^-$	$<1.29z$	z	Eq. (21)
$\tau^- \to \nu_\tau (K\pi\pi)^-$	$<0.8w$	w	Eq. (25)

ble discrepancy is fairly clear. We need a better determination of $B(\tau \to \nu_\tau e \bar{\nu}_e)$ and/or $B(\tau \to \nu_\tau \mu \bar{\nu}_\mu)$ in the clean PETRA/PEP environment. If the discrepancy persists, one should then check whether the "extra" decays are in $\tau \to$one charged prong with photons or without photons, with one π^0 or with more than one π^0, etc. With thousands of clear $e^+e^- \to \tau^+\tau^-$ events produced, a decay mode with a branching ratio of order 5% should be fairly easily detected. Experiments in the relatively near future should tell us whether the discrepancy in one-charged-prong decays of the τ is a statistical accident or will lead us to interesting new decays of the τ.

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[1]M. L. Perl, Annu. Rev. Nucl. Part. Sci. **30**, 299 (1980).

[2]J. Jaros, talk at the Topical Conference of the 1984 SLAC Summer Institute on Particle Physics, 1984 (unpublished) reviews the measurements of the τ lifetime.

[3]W. Ruckstuhl, talk at the Topical Conference of the 1984 SLAC Summer Institute on Particle Physics, 1984 (unpublished) reviews the data on τ decays.

[4]Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971); H. B. Thacker and J. J. Sakurai, Phys. Lett. **36B**, 103 (1971).

[5]K. Gaemers and R. Raitio, Phys. Rev. D **14**, 1262 (1976); T. Hagiwara *et al.*, Ann. Phys. (N.Y.) **106**, 134 (1977); F. J. Gilman and D. H. Miller, Phys. Rev. D **17**, 1846 (1978); T. N. Pham *et al.*, Phys. Lett. **78B**, 623 (1978); N. Kawamoto and A. Sanda, *ibid.* **76B**, 446 (1978).

[6]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[7]Gilman and Miller (Ref. 5), and unpublished calculations with the present value of m_τ.

[8]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. **18**, 761 (1967).

[9]The numerical integration results in Eqs. (12a) and (12c) show that over half the contribution comes from 4π masses below 1.4 GeV. We thank Jim Smith for discussions on this point.

[10]V. Sidorov, in *Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, Fermilab,* edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1980), p. 490.

[11]J. E. Augustin *et al.*, Orsay Report No. LAL/83-21, 1983 (unpublished).

[12]C. Bacci *et al.*, Nucl. Phys. **B184**, 31 (1981).

[13]G. Cosme *et al.*, Nucl. Phys. **B152**, 215 (1979).

[14]Recent measurements from the Mark II and DELCO collaborations show the 3π mass distribution to be dominated by a broad $A_1(J^P=1^+)$ peak (with mass $\sim$1100 MeV). See Ref. 3 and the previous measurements in Ref. 6.

[15]Y. S. Tsai, Ref. 4; T. N. Pham *et al.*, Ref. 5; N. Kawamoto and A. Sanda, Ref. 5.

[16]T. N. Pham *et al.*, Ref. 5.

[17]M. Peshkin and J. L. Rosner, Nucl. Phys. **B122**, 144 (1977). We thank M. Peshkin and J. L. Rosner for the use of their computer program to find isospin restrictions on charge distributions and several discussions.

[18]The MAC collaboration reports a 95%-C.L. limit: $B(\tau \to 5$ charged prongs$) < 0.17\%$. See Ref. 3 and E. Fernandez *et al.*, Report No. SLAC-PUB-3524, 1984 (unpublished).

[19]B. Delcourt *et al.*, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn, 1981), p. 205; Bacci *et al.*, Ref. 12.

[20]H. Aihara *et al.*, Phys. Rev. D **30**, 2436 (1984). We thank R. Madaras for a discussion of the applicability of the Time Projection Chamber result to $\tau^- \to \nu_\tau K^- K^0_S$.

[21]DELCO collaboration, see Ref. 3, and also the event reported in Ref. 20.

[22]Both the vector and axial-vector currents can contribute to this mode.

[23]DELCO collaboration reports $B(\tau^- \to \nu_\tau K^+ K^- \pi) = (0.22^{+0.17}_{-0.11})\%$ and $B(\tau^- \to \nu_\tau K^- \pi^+ \pi^-) = (0.22^{+0.16}_{-0.13})\%$. See Ref. 3 and G. B. Mills *et al.*, Caltech Report No. CALT-68-1196, 1984 (unpublished).

[24]Combining the possible error in the calculation of $B(\tau^- \to \nu_\tau \pi^- \pi^- \pi^+ \pi^0)$ with the measurement error in $B(\tau^- \to \nu_\tau \pi^- \pi^- \pi^+)$ already allows consistency within errors with the branching fraction for three-charged prongs, together with small contributions from $\tau \to \nu_\tau 5\pi$, $\tau \to \nu_\tau K\pi\pi$, etc.

[25]C. A. Blocker *et al.*, Phys. Rev. Lett. **49**, 1369 (1982).

[26]H. J. Behrend *et al.*, Z. Phys. C **23**, 103 (1984).

[27]T. N. Truong, Phys. Rev. D **30**, 1509 (1984).