Quarkonium production at $p\bar{p}$ colliders

V. Barger and A. D. Martin*
Physics Department, University of Wisconsin, Madison, Wisconsin 53706
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We evaluate cross sections for $\eta({}^1S_0)$ and $\psi({}^3S_1)$ quarkonium states of $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ from lowest-order $gg\to\eta,\chi_{0,2}({}^3P_{0,2})$ and $gg\to\psi g$ subprocesses, including $\chi_J\to\psi\gamma$ decays. Data on pp production of ψ and Υ are well described. High rates are predicted for η_b and Υ_b production at $p\bar{p}$ colliders. The much smaller $\eta(t\bar{t})$ and $\psi(t\bar{t})$ rates are critically dependent on the singular part of the confining potential.

The importance of hadroproduction searches for bound heavy-flavor states $\mathscr{O}(Q\bar{Q})$ has been dramatically demonstrated in the past by the discoveries of the J/ψ charmonium and the Υ b-quarkonium states in pp collisions. With higher-energy beams now available at $p\bar{p}$ colliders, it is pertinent to investigate the expected production rates for $c\bar{c}$ and $b\bar{b}$ bound states and to estimate whether $t\bar{t}$ bound states are likely to be produced with sufficiently large cross sections to be detected in forthcoming $p\bar{p}$ collider experiments. To address these issues we calculate quarkonium ($\mathscr{O}$) production from leading-order QCD subprocesses ($gg\to\mathscr{O}$ and $gg,g\bar{g}\to\mathscr{O}g$) using $\mathscr{O}$ wave functions at the origin obtained from nonrelativistic potential models. These calculations approximately reproduce the available data on pp, $p\bar{p}\to\psi$, χ_c, and Υ cross sections. Encouraged by this success, we make projections for quarkonium production at $p\bar{p}$ collider energies.

Previous QCD studies of ψ and Υ hadroproduction[1] at low transverse momenta were based on semilocal-duality arguments.[2] The cross section from gg, $q\bar{q}\to Q\bar{Q}$ subprocesses integrated between $2m_Q$ and the $2M(Q\bar{q})$ threshold was attributed to quarkonium production. Allowing for soft-gluon color rearrangements, a fraction of this cross section gives the vector quarkonium state. This calculation successfully described $d\sigma/dy\,(y=0)$-versus-$M/\sqrt{s}$ and $d\sigma/dx_F$-versus-x_F distributions of ψ and Υ hadroproduction data up to $\sqrt{s}=63$ GeV. The fraction of vector states was found to be strongly dependent on the quarkonium mass, so this duality approach does not lead to definite predictions for $\mathscr{O}(t\bar{t})$ production at $p\bar{p}$ collider energies. Also, duality arguments do not specify the relative abundance of the various quarkonium states, such as η_c:ψ and η_b:Υ.

An alternative perturbative QCD approach is to use quarkonia wave functions from a nonrelativistic potential model.[3] The absolute cross sections for individual quarkonium states are then determined. This approach has been used previously[4,5] to explain data on ψ and Υ production at high transverse momenta. For our present application to quarkonium cross sections integrated over p_T we consider the contributions from the lowest-order $\alpha_s{}^2$ or $\alpha_s{}^3$ diagrams shown in Fig. 1. Other contributions of order $\alpha_s{}^3$ with divergent behavior at $p_T=0$ would be included as nonscaling effects in the initial parton distributions according to the factorization theorem, and by a semiempirical multiplicative K factor of order 2 analogous to that needed to account for electroweak $W^\pm$ and Z production rates. In our analysis we neglect the small contribution from $gg\to\chi_1 g$, which may further enhance ψ production via $\chi_1\to\psi\gamma$.

The leading-order subprocess of Fig. 1(a) for η, χ_0, or χ_2 production has the cross section (see, for example, Ref. 6)

$$\hat{\sigma}(gg\to\mathscr{O})=\frac{(2J+1)\pi^2}{8M^3}\Gamma(\mathscr{O}\to gg)\delta\left[\frac{\hat{s}}{M^2}-1\right], \qquad (1)$$

where M is the quarkonium mass, J is its spin, and the widths in lowest order[7] are given in terms of wave functions evaluated at the origin by

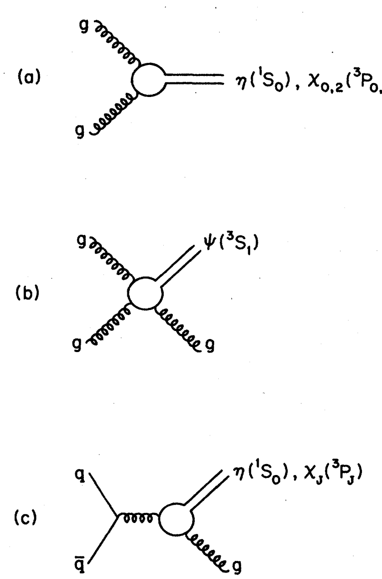

FIG. 1. Lowest-order QCD diagrams for quarkonium hadroproduction.

TABLE I. Partial widths and radiative branching fractions for quarkonium decays calculated using the Wisconsin (Cornell) quarkonium potential. The $c\bar{c}$ $\chi_J \to \psi\gamma$ branching fractions are taken from the Particle Data Group tables [Rev. Mod. Phys. **56**, S1 (1984)]. To calculate $B(\chi_J \to {}^3S_1\psi)$ for t-quarkonium we have taken the partial width for χ single-quark decay to be 47 keV (Ref. 19).

	$\Gamma(\eta\to gg)$ (MeV)	$\Gamma(\chi_2\to gg)$ (MeV)	$B(\chi_2\to {}^3S_1\gamma)$ (%)	$B(\chi_0\to {}^3S_1\gamma)$ (%)
$c\bar{c}$	18 (18)	1.2 (1.2)	15.5	4.3
$b\bar{b}$	6 (12)	0.14 (0.14)	20 (20)	5 (5)
$t\bar{t}$	2 (26)	0.004 (0.05)	33 (64)	29 (42)

$$\Gamma(\eta\to 2g)=\tfrac{8}{3}\alpha_s^2\,|R_S(0)|^2/M^2\,,$$
$$\Gamma(\chi_2\to 2g)=\tfrac{4}{15}\Gamma(\chi_0\to 2g)=\tfrac{128}{5}\alpha_s^2\,|R_P'(0)|^2/M^4\,, \tag{2}$$

where

$$\alpha_s=12\pi/25\ln(M^2/\Lambda^2)\;;$$

we choose $\Lambda=0.2$ GeV. Table I lists widths obtained from the wave functions of two representative potentials that describe ψ and Υ mass spectra. The more singular r^{-1} Cornell potential[8] gives larger widths at high M than the $(r\ln r)^{-1}$ singular behavior of the Wisconsin potential[9] that takes into account the Q^2 dependence of α_s. For the Cornell potential, $V_c(r)=-\kappa/r+ar$, we use the parameter choice[10] $\kappa=0.494$ and $a=0.173$ GeV^2.

The hadroproduction cross sections are obtained by folding Eq. (1) with the QCD-evolved gluon distributions evaluated at $Q^2=M^2$. Figure 2 shows the results obtained for the universal cross-section-to-width ratio

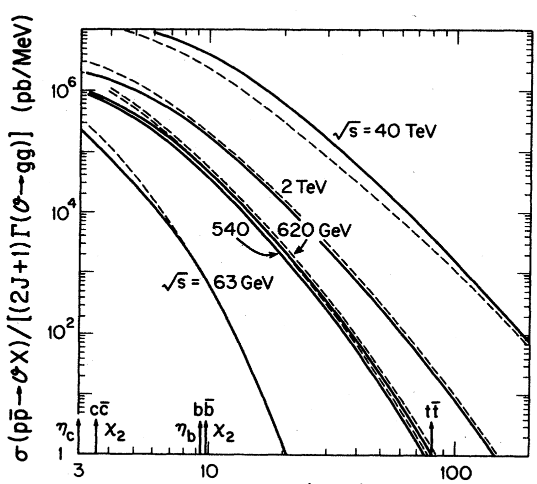

FIG. 2. Universal cross section to width ratio $\sigma(p\bar{p}\to gg\to\mathcal{O})/[(2J+1)\Gamma(\mathcal{O}\to gg)]$ for gluon-gluon-fusion production (with $K=1$) of heavy-quarkonia states $\mathcal{O}(Q\bar{Q})$ versus the quarkonium mass for several c.m. energies $\sqrt{s}$. Solid (dashed) curves are obtained with Duke-Owens (Eichten *et al.*) structure functions.

$$\frac{\sigma(p\bar{p}\to gg\to\mathcal{O})}{(2J+1)\Gamma(\mathcal{O}\to gg)K}=\frac{\pi^2}{8M^3}\tau\int_\tau^1\frac{dx}{x}D_g(x,Q^2)D_g\left[\frac{\tau}{x},Q^2\right]\,, \tag{3}$$

where $\tau=M^2/s$ and D_g is the gluon distribution in a proton and K is a QCD-motivated enhancement factor. For our numerical evaluations we used two recent sets of structure functions, namely, the parametrizations of Duke and Owens[11] and Eichten, Hinchliffe, Lane, and Quigg[12] with $\Lambda=0.2$ GeV. These structure functions give significantly differing results only at low $M/\sqrt{s}$ as shown by the two sets of curves in Fig. 2. For quarkonium hadronic widths that are measured the pp or $p\bar{p}$ production cross sections can be directly read off from Fig. 2.

One source of the $^3S_1$ states $\psi(c\bar{c})$, $\Upsilon(b\bar{b})$, $\psi(t\bar{t})$, collectively denoted by $\psi(Q\bar{Q})$, is $\chi_J(Q\bar{Q})\to\psi(Q\bar{Q})\gamma$ decays. The $\chi_{0,2}(c\bar{c})\to\psi\gamma$ and $\chi_2(b\bar{b})\to\Upsilon\gamma$ branching fractions have been measured. For the other transitions we use potential-model calculations of the partial width as expressed by

$$\Gamma(\chi_J\to\psi\gamma)=(\tfrac{4}{9})\alpha e_Q^2k_\gamma^3\,|\langle R_S\,|\,r\,|\,R_P\rangle|^2\,. \tag{4}$$

For $t\bar{t}$ the photon momentum k_γ is calculated from the χ_J and ψ_t masses obtained from the potential models with $m_t=40$ GeV as input.[13] The branching fractions are summarized in Table I.

The lowest-order direct production of $\psi(Q\bar{Q})$ states occurs via the so-called bleaching-gluon α_s^3 subprocess[3] of Fig. 1(b), whose cross section is given in terms of the $\psi\to 3g$ width

$$\Gamma(\psi\to 3g)=\frac{40(\pi^2-9)\alpha_s^3(M)\,|R_S(0)|^2}{81\pi M^2} \tag{5}$$

by

$$\hat{\sigma}(gg\to\psi g)=\frac{9\pi^2}{8M^3(\pi^2-9)}\Gamma(\psi\to 3g)I(\hat{s}/M^2)\,, \tag{6}$$

where

$$I(\gamma)=\frac{2}{\gamma^2}\left[\frac{\gamma+1}{\gamma-1}-\frac{2\gamma\ln\gamma}{(\gamma-1)^2}\right]+\frac{2(\gamma-1)}{\gamma(\gamma+1)^2}+\frac{4\ln\gamma}{(\gamma+1)^3}\,. \tag{7}$$

The calculated $p\bar{p}\to\psi X$ cross section due to the bleaching-gluon mechanism is comparable in size to that of ψ production via $\chi\to\psi\gamma$ decay. The values for $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ $\psi(^3S_1)$ production are given in Table II for two

TABLE II. The cross sections (in nb) for $\psi(^3S_1)$ production via $gg\to\psi g$ and $\chi_J\to\psi\gamma$, respectively at $\sqrt{s}=63$ GeV (620 GeV). The calculations are based on the Wisconsin quarkonium potential. For the Cornell potential, the $t\bar{t}$ cross sections would be an order of magnitude larger.

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
$\sigma(p\bar{p}\to gg\to\psi g)$	280 (3000)	0.13 (19)	(3×10^{-5})
$\sigma(p\bar{p}\to gg\to\chi_J\to\psi\gamma)$	300 (1800)	0.24 (15)	(2.5×10^{-5})

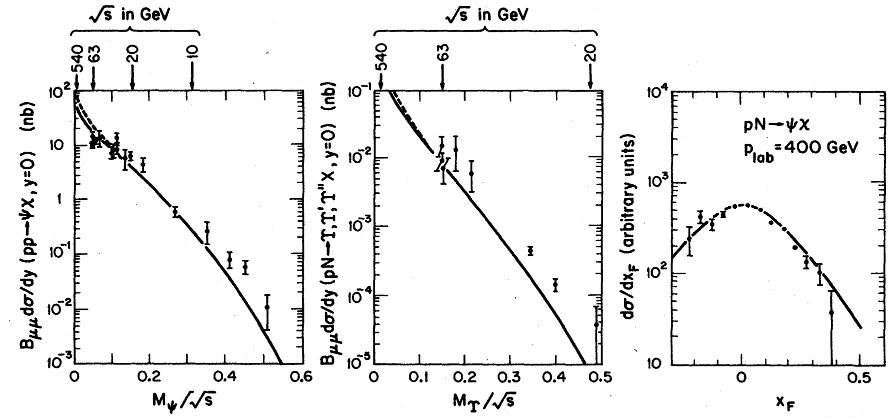

FIG. 3. Comparison of $gg \to \chi \to \psi(Q\bar{Q})\gamma$ and $gg \to \psi(Q\bar{Q})g$ model predictions with data on $\psi(3.1)$ and $\Upsilon(9.46)$ production (a), (b) $d\sigma/dy$ at $y=0$ versus $\sqrt{s}$, (c) $d\sigma/dx_F$ versus x_F. Leptonic branching fractions $B(\psi \to \mu\bar{\mu})=0.074$ and $B(\Upsilon \to \mu\bar{\mu})=0.03$ are used. Data are from Ref. 14.

typical energies, $\sqrt{s}=63$ and 620 GeV. For $c\bar{c}$ at $\sqrt{s}=63$ GeV the bleaching-gluon and χ-decay contributions are about equal in accord with experiment.[14]

An additional bleaching-gluon contribution originates from $q\bar{q}$ fusion [Fig. 1(c)]. The threshold divergence in $q\bar{q} \to \chi_J g$ at $s=M^2$ (equivalent to the singularity in $\chi_J \to q\bar{q}g$ decays) is due to the breakdown of the nonrelativistic model for quarkonium states and can be regularized by requiring $(\hat{s})^{1/2} > M+\Delta$ with $\Delta \sim 0.3$ GeV. However, the η and ψ (from $\chi \to \psi\gamma$) cross sections obtained from $q\bar{q} \to \mathscr{O}g$ were found to be insignificant in comparison with the gg initiated subprocesses. As a consequence the quarkonium production rates in pp and $p\bar{p}$ collisions should be equal, which is compatible with the available $\psi(3.1)$ production data[15] at accelerator energies.

This model for vector-quarkonium production is compared with data on $\psi(3.1)$ and $\Upsilon(9.8)$ production[14–16] in Fig. 3, taking $K=2$. The normalization of the cross section, the $M/\sqrt{s}$ dependence of $d\sigma/dy$ at $y=0$, and the shape of the x_F distribution are reasonably well reproduced (Υ', Υ'' contributions are not included in the calculation). The y and x_F variables are defined as in Ref. 1.

The predicted energy dependence of the η and ψ quarkonium cross sections are given in Fig. 4. The following statements can be made about the results.

(i) For any hidden flavor, η production is larger than ψ by about an order of magnitude (see also Ref. 4).

(ii) With the present CERN $p\bar{p}$ collider integrated luminosity of approximately 100 nb^{-1} at $\sqrt{s}=540$ GeV about 100 000 η_b would have been produced. The rare decay mode $\eta_b \to \gamma\gamma$, with branching fraction of order 0.1% or less, or the expected $\eta_b \to \Lambda\bar{\Lambda}$ mode may provide a possible means to search for the η_b in $p\bar{p}$ data.

(iii) An Υ cross section of about 30 nb is predicted at $\sqrt{s}=540$ GeV and so the existing data sample should contain about 100 $\Upsilon \to \mu^+\mu^-$ events. Detection of these requires muon identification down to transverse momentum of $p_T \simeq 5$ GeV.

(iv) Enormous numbers of η_c and $\psi(c\bar{c})$ events are produced at $p\bar{p}$ collider energies but only those at high p_T are likely to be detected.[4,5]

(v) The predictions for $\mathscr{O}(t\bar{t})$ production depend very sensitively on the singular part of the potential, with larger cross sections expected for a $1/r$ behavior at small r. t-quarkonium detection will be difficult at collider energies up to $\sqrt{s}=2$ TeV. The higher-cross-sections predictions in Fig. 4 at 2 TeV are $\sigma(\eta_t) \sim 1$ nb and $\sigma(\psi_t) \sim 20$ pb. The expected branching fractions[4,17] $B(\eta_t \to \gamma\gamma) \sim 0.2\%$ and $B(\psi_t \to \mu^+\mu^-) \sim 3\%$ suppress these detectable modes.

(vi) If a light Higgs boson exists, the radiative decay[18] $\psi_t \to H\gamma$ provides a good t-quarkonium signature of a high-p_T photon. In the standard model $B(\psi_t(80) \to H\gamma) \lesssim 1\%$.

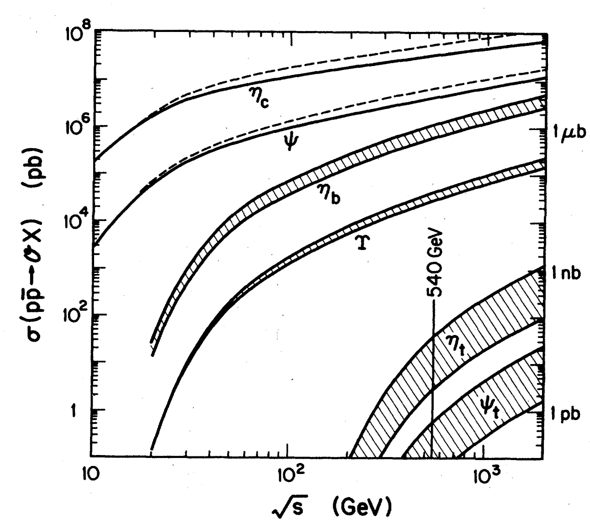

FIG. 4. Predicted cross sections with $K=2$ for heavy-quark bound states, resulting from $gg \to \eta, \chi_{0,2}$ and $gg \to \psi g$ production with $\chi_{0,2} \to \psi\gamma$ decays, versus the c.m. energy $\sqrt{s}$. The shaded regions for production of $b\bar{b}$ and $t\bar{t}$ (80 GeV) states are representative of the uncertainty in potential model calculations; the lower (higher) curves correspond to the Wisconsin (Cornell) potential (Refs. 8–10). Solid (dashed) curves correspond to use of Duke-Owens (Eichten *et al.*) structure functions.

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*Permanent address: University of Durham, Physics Department, Durham, England.

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