Second-order corrections to the muon anomalous magnetic moment in alternative electroweak models

S. R. Moore, K. Whisnant, and Bing-Lin Young
Department of Physics and Ames Laboratory, Iowa State University, Ames, Iowa 50011
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We study the second-order weak correction to the muon anomalous magnetic moment in alternative electroweak gauge theories. The gauge-boson contributions are constrained by low-energy weak-interaction phenomenology. We examine the Higgs-boson contributions in three classes of models and find that they are potentially the dominant effect. Contributions of lepton-number-violating Higgs-boson interactions are calculated and included in the analysis. Severe limits on the spontaneous-symmetry-breaking scales and Yukawa couplings are found in some scenarios which have a separate scale for fermion masses, including the standard model with more than one doublet and a left-right model which naturally explains small neutrino masses. Higgs-boson effects may be detectable when order-of-magnitude improvements are made in the measurement of the muon anomalous magnetic moment.

I. INTRODUCTION

The anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$, in addition to being a cornerstone of the experimental verification of quantum electrodynamics, can potentially provide an excellent laboratory for testing electroweak gauge theories. The most recent measurements of a_μ (Ref. 1) are in agreement with the sixth-order quantum-electrodynamics (QED) corrections and hadronic vacuum-polarization effect.[2] Substantial reductions in the present experimental error are currently under consideration[3] which will allow the probing of the eighth-order QED and weak-interaction effects. On the theoretical side, the error in the eighth-order QED contribution to a_μ has recently been reduced significantly; the largest theoretical uncertainty comes from the calculation of the hadronic contribution, and there is hope that it, too, can be reduced. Therefore, the possibility exists that the weak-interaction contribution a_μ^{weak} will be detectable in the near future.

The second-order weak correction has already received much attention in the literature. Weak contributions in the standard electroweak gauge model,[4] including those of the scalar Higgs boson, have been calculated.[5] Some general expressions have been derived for the gauge- and Higgs-boson contributions in an arbitrary gauge model.[6] Limits on physical parameters from a_μ have been found for various nonstandard electroweak models, e.g., the $SU(2)\times U(1)$ model with nonminimal Higgs sector,[7,8] $SU(2)\times U(1)\times G$ natural models,[9] a class of $SU(2)\times U(1)\times U(1)$ models,[10] and certain left-right models.[11] Often, the Higgs-boson contributions to a_μ are either ignored or calculated only in the simplest cases. In this paper we systematically examine the second-order weak correction to a_μ in a wide range of electroweak models. We will place particular emphasis on the Higgs sector and determine general criteria for when the scalar contributions may dominate the weak effect, and thereby be observable when the uncertainties in the measurement and calculation of a_μ are reduced.

Our analysis shows that the gauge-boson contributions to a_μ in any of the popular alternative electroweak gauge models do not vary by more than 50% from the standard-model (SM) value. This is due to constraints on the weak-interaction parameters already imposed by all relevant low-energy data. This variation may be observable if the uncertainty in a_μ can be reduced to the 1 part per million (ppm) level, where the present accuracy is approximately 10 ppm. More interesting, but less certain, are the Higgs-boson contributions to a_μ. They depend crucially on the Higgs-boson Yukawa couplings to the muon and on the Higgs-boson mass. Even in the minimal standard model (MSM) with one Higgs doublet they dominate a_μ^{weak} if the physical-Higgs-boson mass m_H is not much larger than the muon mass, although for $m_H \geq 1$ GeV it is already less than 20% of the gauge boson contribution. Generally, we find that the contributions of the Higgs bosons in various electroweak models fall into one of the following classes:

(1) Fermion mass generation is achieved through large Higgs-field vacuum expectation values (VEV's) and small Yukawa couplings (analogous to the SM). Then the Higgs-boson contribution a_μ^H is comparable to the gauge-boson contributions only when $m_H \lesssim m_\mu$, and current measurements of a_μ provide no useful limit on m_H or the couplings.

(2) Fermion mass generation is achieved through small VEV's and natural-sized (of order the gauge coupling) Yukawa couplings, and there exists a light ($m_H \sim m_\mu$) Higgs boson which contributes to a_μ^H. Then very stringent limits already exist on the VEV and Yukawa coupling.

(3) Same as the case (2), except there is no light Higgs particle with natural-sized Yukawa couplings which contributes to a_μ^H. Then loose limits presently exist on the coupling and mass parameters and a_μ^H is comparable or larger than the gauge boson contribution.

The latter two scenarios offer the intriguing possibility that a_μ^{weak} differs substantially from the SM value, even more than allowed by a nonstandard gauge-boson sector. Thus, a significant improvement in the measurement of a_μ may offer indirect evidence for the existence of Higgs bosons. If we assume that the parameters in the Higgs potential are of the order of gauge coupling and are not arbitrarily tuned, then under certain general conditions there are light Higgs bosons in the model.[12] The light Higgs bosons include neutral scalars; light neutral pseudoscalars can occur in certain models. No existing models contain light charged Higgs bosons, given the above assumptions on the parameters in the Higgs potential. In the case of the SM with one or more light scalar bosons[7] of mass M_H, current limits on a_μ require $hm_\mu/g_L m_H \lesssim 10^{-3}$, where h and g_L are the Yukawa and gauge coupling, respectively. Also, a version of the left-right model in which $m_\nu \propto m_\mu{}^2/m_{W_R}$ (W_R mediates right-handed charge-current interactions) has a light scalar Higgs boson with similar constraints.

Our paper is organized as follows. In Sec. II we discuss the theoretical calculation of second-order weak corrections to a_μ arising from gauge- and Higgs-boson contributions. We pay special attention to the contribution from Higgs bosons which couple to fermions via a $\Delta L=2$ (lepton-number-violating) vertex and verify earlier results. The Feynman rules for the $\Delta L=2$ processes are discussed in the Appendix.

In Sec. III we apply the formulas of the preceding section for the gauge-boson contributions to the general two-Z models and give expressions for a_μ in terms of parameters in the low-energy Hamiltonian. Predictions are obtained for the special cases of left-right and $\mathrm{SU}(2)\times \mathrm{U}(1)\times G$ natural models. The implications of the presence of heavy Majorana neutrinos are discussed.

In Sec. IV we give explicit expressions for the Higgs-boson contributions in a wide class of models. We discuss the importance of the spontaneous-symmetry-breaking scales and Yukawa couplings to fermions. Specific models are classified as outlined above. The phenomenological implications and numerical limits for each model are discussed. Some conclusions are drawn in Sec. V.

II. THEORY

Second-order weak corrections to the muon anomalous magnetic moment are found by calculating the one-loop graphs like those shown in Fig. 1. Contributions from these graphs can be written as

$$\bar{u}(p')e\Gamma_\mu(p',p)u(p) = \bar{u}(p')[e\gamma_\mu F_1(q^2)+ie\sigma_{\mu\nu}q^\nu F_2(q^2)/2m + \gamma_5 \text{ terms}]u(p) \tag{1}$$

in the conventions of Bjorken and Drell.[13] The quantity $F_2(q^2)$ is the magnetic form factor and the muon anomalous magnetic moment is just $F_2(0)$. Since we are dealing with spontaneously broken gauge theories, which are renormalizable, $F_2(q^2)$ will be finite as no Pauli-coupling counterterms are allowed. However, the integrals representing $F_2(q^2)$ are not absolutely convergent; the finite part of the gauge-boson graphs in Figs. 1(a) and 1(b) may be ambiguous, depending on the choice of origin in the loop integral. We use dimensional regularization to define the loop integrals. Moreover, this allows us to work in a particular gauge, such as the unitary gauge, rather than the general R_ξ gauges.[14]

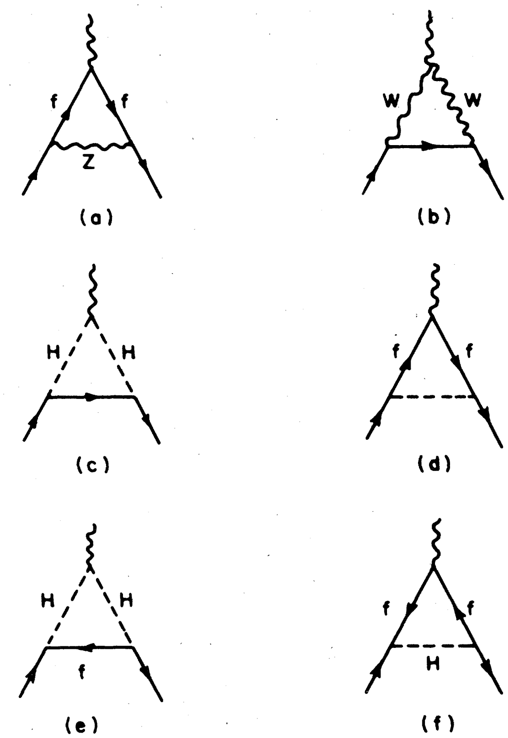

FIG. 1. Diagrams which can contribute to a_μ: (a) Z boson, (b) W boson, (c) lepton-number-conserving (LNC) singly charged Higgs boson, (d) LNC neutral Higgs boson, (e) lepton-number-violating (LNV) singly or doubly charged Higgs boson, and (f) LNV doubly charged Higgs boson.

The contributions to a_μ from graphs 1(a) through 1(d) have already been calculated in Ref. 6, and we have verified the result. For the general gauge-boson interaction given by

$$\mathscr{L}_{\text{int}} = \sum_{f,X} \bar{\mu}(C_V\gamma^\lambda + C_A\gamma^\lambda\gamma^5)fX_\lambda\ , \tag{2}$$

where f sums over all contributing fermions and X over all gauge bosons in the theory, the contribution to a_μ from Fig. 1(a) is

$$a_\mu^Z = \frac{-q_f m_\mu{}^2}{4\pi^2}\int_0^1 dx \left[C_V{}^2 \left\{ x(1-x)\left[x+\frac{2m_f}{m_\mu}-2\right] - \frac{m_\mu{}^2}{2m_Z{}^2}\left[x^3\left[\frac{m_f}{m_\mu}-1\right]^2 + x^2\left[\frac{m_f{}^2}{m_\mu{}^2}-1\right]\left[1-\frac{m_f}{m_\mu}\right]\right]\right\} + C_A{}^2\{m_f \to -m_f\} \right][m_\mu{}^2x^2+m_Z{}^2(1-x)+x(m_f{}^2-m_\mu{}^2)]^{-1} \tag{3}$$

and from Fig. 1(b) is

$$a_\mu^W = \frac{q_W m_\mu^2}{8\pi^2} \int_0^1 dx \left\{ C_V^2 \left[\frac{4m_f}{m_\mu} x^2 - 2x^2(1+x) + \frac{m_\mu^2}{m_W^2} x(1-x) \left[\frac{m_f}{m_\mu} - 1 \right]^2 \left[\frac{m_f}{m_\mu} + x \right] \right] + C_A^2 [m_f \to -m_f] \right\}$$
$$\times [m_\mu^2 x^2 + (m_W^2 - m_\mu^2)x + m_f^2(1-x)]^{-1} . \tag{4}$$

Here the charges q_f of the fermion and q_W of the W are in units of $e>0$, so that $q_\mu = -1$.

For the general lepton-number-conserving (LNC) Higgs-boson interaction

$$\mathscr{L}_{\text{Yukawa}} = \sum_{f,H} \bar{\mu}(C_S + C_P \gamma^5) f H + \text{H.c.} , \tag{5}$$

the contributions to a_μ are

$$a_\mu^H = \frac{-q_H m_\mu^2}{8\pi^2} \int_0^1 dx \frac{C_S^2[x^3 - x^2 + (m_f/m_\mu)(x^2 - x)] + C_P^2[m_f \to -m_f]}{m_\mu^2 x^2 + (m_H^2 - m_\mu^2)x + m_f^2(1-x)} \tag{6}$$

from Fig. 1(c) and

$$a_\mu^H = \frac{-q_f m_\mu^2}{8\pi^2} \int_0^1 dx \frac{C_S^2[x^2 - x^3 + (m_f/m_\mu)x^2] + C_P^2[m_f \to -m_f]}{m_\mu^2 x^2 + (m_f^2 - m_\mu^2)x + m_H^2(1-x)} \tag{7}$$

from Fig. 1(d), where q_H is the charge of the Higgs boson in units of $e>0$ and m_H its mass.

The Higgs boson couplings in Eq. (5) allow only for $\Delta L = 0$ interactions. Many theories also have lepton-number-violating (LNV) $\Delta L = 2$ Higgs-boson Yukawa couplings which take the form

$$\mathscr{L}_{\text{Yukawa}}^{\Delta L=2} = \sum_{f,H} \mu^T C (C_S + C_P \gamma^5) f H + \text{H.c.} , \tag{8}$$

where C is the charge-conjugation matrix $C^T = C^{-1} = C^\dagger = -C$ (Ref. 13). An example of such a theory is the left-right model with a heavy right-handed Majorana neutrino. The interaction given in Eq. (8) leads to the graphs shown in Figs. 1(e) and 1(f). The arrows on the fermion lines indicate the flow of lepton number. Since the Yukawa couplings are not necessarily small, these contributions must also be considered in any theory with $\Delta L = 2$ interactions. We find that the final result is nearly identical to the $\Delta L = 0$ graphs. Equations (6) and (7) hold true for Figs. 1(e) and 1(f), respectively, except that C_S and C_P are replaced, respectively, by $2C_S$ and $2C_P$ when $f = \mu$. The factor 2 arises from the presence of two identical field operators in the individual interaction terms. Once this combinatorial factor is included, the $\Delta L = 0$ expressions may be used for the $\Delta L = 2$ graphs with the help of the charge-conjugation matrix. In the remainder of this section, we give a brief illustration of the statement made above. More details can be found in the Appendix.

Consider the lepton-number-violating Lagrangian

$$\mathscr{L}_{\text{LNV}} = f_1^T C \Gamma f_2 \phi + \bar{f}_2 C \tilde{\Gamma} \bar{f}_1^T \phi^\dagger , \tag{9}$$

where f_1 and f_2 are fermion field operators. The coupling Γ and $\tilde{\Gamma}$ are related by

$$\tilde{\Gamma} = C \bar{\Gamma} C^\dagger, \quad \bar{\Gamma} \equiv \gamma_0 \Gamma^\dagger \gamma_0 . \tag{10}$$

In the present case

$$\Gamma = C_S + C_P \gamma_5 ,$$
$$\tilde{\Gamma} = \bar{\Gamma} = C_S - C_P \gamma_5 . \tag{11}$$

Feynman rules supplementing the usual rules for the lepton-number-conserving interaction can be derived from Eq. (9). However, the presence of the charge-conjugation matrix allows the conversion of the supplemental rules to the usual ones. If we write Eq. (9) in the following form

$$\mathscr{L}_{\text{LNV}} = f_1^T C \Gamma f_2 \phi + \bar{f}_2 C \tilde{\Gamma} C^\dagger C \bar{f}_1^T \phi^\dagger , \tag{12}$$

which can be replaced by the corresponding lepton-number-conserving Lagrangian

$$\mathscr{L}_{\text{LNC}} \equiv \bar{f}_1 \Gamma f_2 \phi + \bar{f}_2 \bar{\Gamma} f_1 \phi^\dagger , \tag{13}$$

in the derivation of the Feynman diagram. One important difference from the lepton-number-conserving interaction exists, however, when $f_1 = f_2$. Then the coupling at the fermion-fermion-scalar boson vertex is 2Γ or $2\bar{\Gamma}$, where the factor 2 comes from the identity of the two fermions involved at the vertex. Details will be given in the Appendix.

III. GAUGE-BOSON CONTRIBUTIONS

The formulas for the gauge-boson contributions to a_μ given in the preceding section may now be applied to practical examples. The neutral-gauge-boson correction a_μ^Z can be found directly from the low-energy neutral-current parameters. The charged-gauge-boson correction a_μ^W can be written in terms of the W boson-masses, the left-right mixing angles, and the muon-neutrino mass. Existing limits on these weak parameters then constrain the allowed values at a_μ^Z and a_μ^W.

We use the formalism of the general two-Z model described in detail elsewhere.[15] The effective low-energy neutral-current (NC) interaction is given by

$$\mathscr{L}_{\rm NC}^{\rm eff}=\frac{4G_F}{\sqrt{2}}[\rho_1{}^2(J^{\mu}_{Z_L})^2+(\rho_2 J^{\mu}_{Z_L}+\eta J^{\mu}_{Z_R})^2]\ , \tag{14}$$

where $J^{\mu}_{L,R}=J^{\mu}_{3_{L,R}}-x_{L,R}J^{\mu}_{Q}$ and $J^{\mu}_{3_{L,R}}$ are the usual $V\mp A$ neutral currents, J^{μ}_{Q} is the electromagnetic current, $x_L=e^2/g_L{}^2$, and g_L is the $SU(2)_L$ gauge coupling. In a left-right model $x_R=e^2/g_R{}^2$ and is constrained to lie between 0 and $1-x_L$. The neutral current then contributes to a_μ via Fig. 1(a). We ignore any flavor-changing neutral currents since they are not present in the above interaction. For $m_\mu{}^2 \ll m_Z{}^2$ we may apply Eq. (3) with $m_f=m_\mu$ and find

$$a_\mu^Z=\frac{m_\mu{}^2 g_Z{}^2}{12\pi^2}\sum_{i=1}^{2}\frac{(g_V^i)^2-5(g_A^i)^2}{m_{Z_i}{}^2}\ , \tag{15}$$

where

$$g_Z=ex_L{}^{-1/2}(1-x_L)^{-1/2}$$

and $g^i_{V,A}$ are the actual vector and axial-vector couplings of the muon to Z_i, with g_V^1 and g_A^1 normalized to $-\frac{1}{4}+x_L$ and $\frac{1}{4}$, respectively, in the limit of the SM. The vector bosons Z_1 and Z_2 are the mass eigenstates, and are equal to Z_L and Z_R, respectively, in the absence of mixing. Thus, a_μ^Z is proportional to a sum of effective vector and axial-vector couplings. The values of the effective couplings may be deduced from Eq. (14):

$$g_Z{}^2\sum_{i=1}^{2}\frac{(g_V^i)^2}{m_{Z_i}{}^2}=\frac{8G_F}{\sqrt{2}}[(\rho_1{}^2+\rho_2{}^2)(-\tfrac{1}{4}+x_L)^2+\eta^2(-\tfrac{1}{4}+x_R)^2+2\rho_2\eta(-\tfrac{1}{4}+x_L)(-\tfrac{1}{4}+x_R)] \tag{16}$$

and

$$g_Z{}^2\sum_{i=1}^{2}\frac{(g_A^i)^2}{m_{Z_i}{}^2}=\frac{8G_F}{\sqrt{2}}[\rho_1{}^2+(\rho_2-\eta)^2](\tfrac{1}{4})^2\ , \tag{17}$$

so that

$$a_\mu^Z=\frac{m_\mu{}^2 G_F}{6\sqrt{2}\pi^2}[(\rho_1{}^2+\rho_2{}^2)(-1-2x_L+4x_L{}^2)+\eta^2(-1-2x_R+4x_R)^2+\rho_2\eta(3-2x_L-2x_R+8x_Lx_R)]\ . \tag{18}$$

Equation (18) reduces to the SM result in the limit $\rho_1=1$, $\eta=\rho_2=0$. In left-right models, limits on the low-energy parameters are known, and hence a prediction for a_μ^Z can be made. Using the allowed region in parameter space from Ref. 15, we find

$$-2.35\times10^{-9}\le(a_\mu^Z)_{LR}\le-1.65\times10^{-9} \tag{19}$$

at the 1σ level, where the SM value is $(a_\mu^Z)_{\rm SM}=-1.92\times10^{-9}$ for $x_L=0.233$ and $M_Z=88.7$ GeV is the unrenormalized Z mass. Adding in the constraints on the W_1 and Z_1 mass from the CERN $\bar{p}p$ experiments[16] does not significantly affect the range in Eq. (19).

In models with no nonstandard fermion currents, i.e., only J_Q, J_{3L}, and $J_Y=2(J_Q-J_{3L})$, Eq. (14) reduces to[17]

$$\mathscr{L}_{\rm NC}^{\rm eff}=\frac{4G_F}{\sqrt{2}}\rho^2[(J^{\mu}_{Z_L})^2+C(J^{\mu}_{Q})^2] \tag{20}$$

with $\rho=1$ for the case of the $SU(2)\times U(1)\times G$ natural models.[9] Then it is easy to show that

$$\left[\frac{a_\mu^Z-(a_\mu^Z)_{\rm SM}}{(a_\mu^Z)_{\rm SM}}\right]=\frac{4C}{-1-2x_L+4x_L{}^2}\ . \tag{21}$$

The quantity C is constrained by $e^+e^-\to\mu^+\mu^-$ measurements of DESY PETRA[18] and SLAC PEP[19] to $C\le0.010$, so the change from the SM value is only a few percent for models with $\mathscr{L}_{\rm NC}^{\rm eff}$ given by Eq. (20).

The W boson contributions arise from diagrams like Fig. 1(b). The virtual fermion may be a (light) left-handed neutrino (denoted ν) or a (possibly heavy) right-handed neutrino (N), and the two cases must be treated separately. Given the general interaction of Eq. (2), the W contribution to a_μ may be expressed as

$$a_\mu^W=\frac{m_\mu{}^2 G_F}{8\sqrt{2}\pi^2}\sum_{i,f}\left[\frac{m_W}{m_{W_i}}\right]^2\left[(C^i_{V_f})^2 I_1\left[\frac{m_f}{m_{W_i}},\frac{m_\mu}{m_{W_i}}\right]+(C^i_{A_f})^2 I_1\left[-\frac{m_f}{m_{W_i}},\frac{m_\mu}{m_{W_i}}\right]\right]\ , \tag{22}$$

where m_f is the mass of the intermediate state ν or N, m_W is the W mass in the SM determined from x_L,

$$m_W{}^2=(\sqrt{2}e^2)/(8G_Fx_L)\ ,$$

C_{V_f} and C_{A_f} are normalized to unity in the SM, and

$$I_1(z,\epsilon)=\frac{1}{\epsilon}\int_0^1 dx\frac{[-4zx^2+2x^2(1+x)\epsilon+x(x-1)(z+\epsilon x)(z-\epsilon)^2]}{[\epsilon^2x(x-1)+x+z^2(1-x)]}\ . \tag{23}$$

If $m_f \gg m_\mu$ to give $z \gtrsim 1$ this integral is potentially large. However, the interaction of N is right-handed so that $(C_V^i)^2 = (C_A^i)^2$ and terms odd in m_N/m_{W_i} will cancel. The surviving terms in Eq. (23) are at most of order one; the distinct handedness of the N limits its contribution to a_μ^W regardless of the value of m_N. The only deviation from this result occurs when mixing effects are included. If we define a W_L-W_R mixing angle ζ and a ν-N mixing angle θ, deviations from the exact $C_V = C_A$ result are of order

$$\sin\zeta \sin\theta (m_N/m_\mu) ,$$

so that the existence of non-negligible mixings could, in principle, lead to substantial modifications to the zero-mixing result.

To estimate the size of this effect, we consider the left-right model with Higgs multiplets that transform as (1,0,2), (0,1,2), and $(\frac{1}{2},\frac{1}{2},0)$ under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Two other Higgs multiplets, a $(\frac{1}{2},0,1)$ and a $(0,\frac{1}{2},1)$, have been considered in the literature[20] in various combinations with the above three multiplets, but cannot couple directly to fermions. Therefore, we do not consider them here, although, in principle, they could affect the Higgs-boson self-couplings. There are two popular scenarios[21] and in each case the contribution due to the mixing terms is small:

(i) There is only one of each type of multiplet, and the mixing angles ζ and θ are of order m_{W_L}/m_{W_R} or smaller. We also have $m_\nu/m_N \sim (m_{W_L}/m_{W_R})^2$ in this case so that

$$\sin\zeta \sin\theta (m_N/m_\mu) \sim m_\nu/m_\mu \ll 1$$

and the mixing effect can be ignored.

(ii) There are two $(\frac{1}{2},\frac{1}{2},0)$ multiplets with large and small VEV's, respectively, and only the latter couples to fermions. There is one each of the other multiplets. Then $\theta \sim m_\mu/m_N$ and the mixing effect is of order ζ, which is known phenomenologically to be small.[22]

As we are interested only in the leading effects in a_μ^{weak}, we can safely ignore neutrino mixing and W mixing and consider only terms even in (m_f/m_{W_i}) in Eqs. (22) and (23).

Given this simplification, we can assume left-handed ν-W_L and right-handed N-W_R interactions. Using $m_\nu \ll m_\mu \ll m_{W_i}$ and the observed fact that m_{W_1} is approximately the SM value,[16] we have

$$a_\mu^W \approx \frac{5 m_\mu{}^2 G_F}{12\sqrt{2}\pi^2} \left[1 + \frac{3}{5} \frac{x_L}{x_R} \left[\frac{m_{W_L}}{m_{W_R}} \right]^2 I_2 \left[\frac{m_N}{m_{W_R}} \right] \right] , \quad (24)$$

where

$$I_2(z) = \int_0^1 dx \frac{[2x^2(1+x) + z^2 x(x-1)(x-2)]}{x + z^2(1-x)} . \quad (25)$$

The integral I_2 varies from $I_2(0) = \frac{5}{3}$ to $I_2(\infty) = \frac{2}{3}$. Thus, to leading order, the W contributions to a_μ in a left-right model are the SM result plus a term suppressed by two powers of (m_{W_L}/m_{W_R}). Note that the W contribution is always positive. If the left-right model is embedded in an SO(10) grand-unified model, the fact that $SU(2)_R$ is broken before $SU(2)_L$ implies $g_R < g_L$ or $x_R > x_L$. If in addition we have $m_{W_R} \geq 2 m_{W_L}$, which is derived from the low-energy constraints on the left-right models,[22] a_μ^W is at most 25% bigger than the SM value. The combined gauge-boson contribution in a left-right model then lies in the range

$$15 \times 10^{-10} \leq a_\mu^Z + a_\mu^W \leq 32 \times 10^{-10} . \quad (26)$$

If the right-handed neutrino mass is less than about 100 MeV, stricter limits can be placed on m_{W_R}. The recent muon-decay experiment at TRIUMF[23] in this case gives the limit $m_{W_R} > 5 m_{W_L}$, which reduces the upper limit of Eq. (26) to 24×10^{-10}. However, the most thoroughly investigated models[21] favor a very heavy right-handed neutrino, and thereby circumvent this stricter limit.

In the natural models with the gauge group $SU(2) \times U(1) \times G$, the low-energy charge-current interaction is identical to that of the SM and there are no right-handed charge currents. It is easy to show that a_μ^W is then the same as in the standard model, so the total gauge boson contribution $a_\mu^Z + a_\mu^W$ varies only slightly from the standard result in the natural models. Also, models which differ from the minimal SM only in the Higgs sector and have no heavy neutrino, such as the Gelmini-Roncadelli model[8] or the SM with more than one doublet,[7] will agree with the SM in their predictions for $a_\mu^Z + a_\mu^W$.

IV. HIGGS-BOSON CONTRIBUTIONS

While the gauge-boson contributions to a_μ are constrained by low-energy weak-interaction data, very little is known about the Higgs sector. Higgs-boson masses in gauge theories are generally independent parameters, depending upon the form of the Higgs potential and the value of the coupling parameters. As is evident from Eqs. (6) and (7), contributions to a_μ from Higgs bosons are critically dependent on the size of the Yukawa coupling and Higgs-boson mass. Typically, Yukawa couplings are assumed to be of order m_f/m_W (when fermion and gauge boson masses are derived from the same VEV) or of order the gauge coupling (when different mass scales are explained by the existence of different symmetry breaking scales). The Higgs-boson mass will be of the order of a symmetry-breaking scale if the parameters in the Higgs potential are not too large or too small (i.e., of the order of the gauge coupling) and in the absence of fortuitous cancellations. If the fermion masses arise from a separate VEV, called the fermion mass scale, a light Higgs boson $(m_H \sim m_f)$ may result and a_μ^H will be enhanced. Models which have large Yukawa couplings and small Higgs-boson masses tend to increase a_μ^H and are therefore severely restricted by current a_μ measurements; models in which one of these situations occurs will be tested by future measurements.[3] It is therefore important to know which situation exists in any given model.

Physical Higgs-boson masses may all be heavy even if there is a small symmetry-breaking scale. The scale of the Higgs-boson masses depends on the gauge group in the model and any additional symmetries of the Higgs potential. For example, consider an $SU(2)\times U(1)$ model which has two Higgs doublets Φ_1 and Φ_2, with VEV's $\langle\phi_2^0\rangle=v_2 \ll \langle\phi_1^0\rangle=v_1$. The most general $SU(2)\times U(1)$-invariant Higgs potential which conserves CP is

$$V=\mu_1\Phi_1^\dagger\Phi_1+\mu_2\Phi_2^\dagger\Phi_2+\mu_3(\Phi_2^\dagger\Phi_1+\Phi_2^\dagger\Phi_2)+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2$$
$$+[\lambda_4\Phi_1^\dagger\Phi_1+\lambda_5\Phi_2^\dagger\Phi_2+\lambda_6(\Phi_2^\dagger\Phi_1+\Phi_1^\dagger\Phi_2)](\Phi_2^\dagger\Phi_1+\Phi_1^\dagger\Phi_2)+\lambda_7(\Phi_2^\dagger\Phi_1-\Phi_1^\dagger\Phi_2)^2\ . \tag{27}$$

When the minimum of the Higgs potential is found and the physical Higgs-boson masses solved for, one discovers that, barring fortuitous cancellations, the masses are all of order v_1. If we want v_2 to be the scale for fermion masses, and assume that the Yukawa couplings are about the same size as the gauge couplings, we must prevent the fermions from coupling to Φ_1. The discrete symmetry

$$\Phi_1\to-\Phi_1 \tag{28}$$

will accomplish this. The μ_3, λ_4, and λ_5 terms in the Higgs potential must vanish under this symmetry, and one neutral scalar boson with a mass proportional to v_2 appears. If the coupling parameters in the Higgs potential are natural sized, i.e., comparable to the gauge coupling, then this Higgs boson will be light, of the order of a fermion mass.[24] Roughly speaking, as long as the Higgs potential was complex enough, all physical Higgs particles received a mass at the larger symmetry-breaking scale; when an additional symmetry is introduced which eliminates certain terms in the potential, one Higgs boson will be shielded from the heavy mass scale and falls to the lower scale.[12] This shielding occurs only for neutral Higgs fields.

We now calculate the contributions of the Higgs bosons to a_μ in the SM with two Higgs doublets. In the limit $v_2 \ll v_1$, the physical Higgs particles are $\phi_2^\pm$, $\mathrm{Im}(\phi_2^0)$, $\mathrm{Re}(\phi_2^0)$, and $\mathrm{Re}(\phi_1^0)$; the $\phi_1^\pm$ and $\mathrm{Im}(\phi_1^0)$ become the longitudinal parts of $W^\pm$ and Z^0, respectively. With the discrete symmetry in Eq. (28) the scalar boson $\mathrm{Re}(\phi_2^0)$ has a mass of order v_2, while the others are all at the scale v_1, i.e., of order m_W. We define

$$m_H=rg_Lv_2 \tag{29}$$

for the mass of $\mathrm{Re}(\phi_2^0)$, with r expected to be of order one. With the discrete symmetry of Eq. (28) imposed, the allowed Yukawa interaction for the muon is

$$\mathscr{L}_{\mathrm{Yukawa}}=h[\bar\mu\mu\,\mathrm{Re}(\phi_2^0)+i\bar\mu\gamma_5\mu\,\mathrm{Im}(\phi_2^0)+\bar\mu L\nu_\mu\phi_2^- +\bar\nu_\mu R\mu\phi_2^+]\ , \tag{30}$$

where $L,R=\frac{1}{2}(1\mp\gamma_5)$ and h is arbitrary but in the present scenario is presumed to be of order g_L, the gauge coupling. We therefore define

$$\beta=h/g_L \tag{31}$$

and expect $\beta\sim1$ in this scenario. The parameters r and β measure how far the Higgs-boson mass and muon Yukawa coupling deviate from their "natural" values. The muon mass is given by hv_2. We can now use the interaction of Eq. (30) to find a_μ^H from Eqs. (6) and (7). In the limit $m_\mu \ll m_W$

$$a_\mu^H=\frac{{g_L}^2}{8\pi^2}\beta^2\left[2I_3\left[\frac{r^2}{\beta^2}\right]-I_4\left[\frac{r^2}{\beta^2}\right]+I_4\left[\frac{{m_W}^2}{{m_\mu}^2}\right]-\frac{1}{12}\frac{{m_\mu}^2}{{m_W}^2}\right]\ , \tag{32}$$

where

$$I_3(z)\equiv\int_0^1 dx\frac{x^2}{x^2+z(1-x)}\simeq\frac{1}{z}(\ln z-\tfrac{3}{2}) \tag{33}$$

and

$$I_4(z)\equiv\int_0^1 dx\frac{x^3}{x^2+z(1-x)}\simeq\frac{1}{z}(\ln z-\tfrac{11}{6})\ . \tag{34}$$

The approximation is valid only for $z\gg1$. For simplicity we put the masses of the charged- and neutral-pseudoscalar-Higgs-boson masses to be m_W. Our result is independent of this precise identification of the heavy-Higgs-boson masses. The first two terms in Eq. (32) are from the $\mathrm{Re}(\phi_2^0)$ contribution and dominate for $r\sim1$. However, the coefficient in front of the bracket is the size of a typical weak contribution so that existing data will put strong limits on β and r. Either β is small, which means that v_2 is larger than the fermion mass scale, or $r\gg1$, i.e., there are large parameters such as λ_1 in the Higgs potential of Eq. (27) that make m_H much larger than the scale of v_2.

To find quantitative limits on r and β in this scenario, we must first subtract the nonweak theoretical values for a_μ from the experimental number. The current experimental limits are[1]

$$a_{\mu^-}^{\mathrm{expt}}=(11\,659\,370\pm120)\times10^{-10}\ ,$$
$$a_{\mu^+}^{\mathrm{expt}}=(11\,659\,110\pm110)\times10^{-10}\ , \tag{35}$$

where the μ^- and μ^+ result should be identical if CPT is conserved. The most recent theoretical calculations of the QED (up to eighth order) and hadronic and τ vacuum polarization contributions to a_μ are[3]

$$a_\mu^{\mathrm{QED}}=(11\,658\,476\pm3)\times10^{-10}\ ,$$
$$a_\mu^{\mathrm{had}}=(702\pm19)\times10^{-10}\ , \tag{36}$$
$$a_\mu^{\tau}=4.2\times10^{-10}\ .$$

If we combine the results of Eq. (35), and subtract the theoretical contributions of Eq. (36), we deduce the weak contribution

$$a_\mu^{\rm weak}=(47\pm69)\times10^{-10}\ . \tag{37}$$

In the SM with two Higgs multiplets considered here the gauge-boson sector is identical to the minimal SM, so we can subtract the standard weak contribution[5] $a_\mu^{\rm weak}({\rm SM})=20\times10^{-10}$ to get the limit

$$a_\mu^H=(27\pm69)\times10^{-10}\ . \tag{38}$$

Applying Eq. (38) to the calculation of Eq. (34), the allowed values of $m_H/m_\mu=r/\beta$ and $h/g_L=\beta$ can be found. The 2σ limits are shown in Fig. 2.

In the left-right model with a separate symmetry-breaking mass scale for fermions,[21] light neutral Higgs bosons can appear,[12] and so we expect that similar limits on the Higgs-boson mass and coupling will be obtained. Because the Higgs-boson sector is much more complicated, we examine it in detail. The theory has four Higgs multiplets $\Phi_w(\frac{1}{2},\frac{1}{2},0)$, $\Phi_f(\frac{1}{2},\frac{1}{2},0)$, $\Delta_L(1,0,2)$, and $\Delta_R(0,1,2)$, where the three numbers in parentheses denote, respectively the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ quantum numbers. These Higgs multiplets are given explicitly below:

$$\Phi_w=\begin{bmatrix}\phi^0_{w_1} & \phi^+_{w_2}\\ \phi^-_{w_1} & \phi^0_{w_2}\end{bmatrix},\quad \Phi_f=\begin{bmatrix}\phi^0_{f_1} & \phi^+_{f_2}\\ \phi^-_{f_1} & \phi^0_{f_2}\end{bmatrix}$$
$$\Delta_L=\begin{bmatrix}\delta_L^+/\sqrt{2} & \delta_L^{++}\\ \delta_L^0 & -\delta_L^+/\sqrt{2}\end{bmatrix}, \tag{39}$$
$$\Delta_R=\begin{bmatrix}\delta_R^+/\sqrt{2} & \delta_R^{++}\\ \delta_R^0 & -\delta_R^+/\sqrt{2}\end{bmatrix}.$$

The nonzero VEV's will be denoted as $\langle\delta_R^0\rangle=v_R$, $\langle\delta_L^0\rangle=v_L$, $\langle\phi^0_{w1}\rangle=\kappa_w$, and $\langle\phi^0_{f1}\rangle=\kappa_f$. For simplicity of computation we set $\langle\phi^0_{f2}\rangle=0$ and $\langle\phi^0_{w2}\rangle=0$. Our results are independent of $\langle\phi^0_{f2}\rangle$ and $\langle\phi^0_{w2}\rangle$ in so far as they are of the same order of κ_f and κ_w, respectively. The leptonic Yukawa potential is

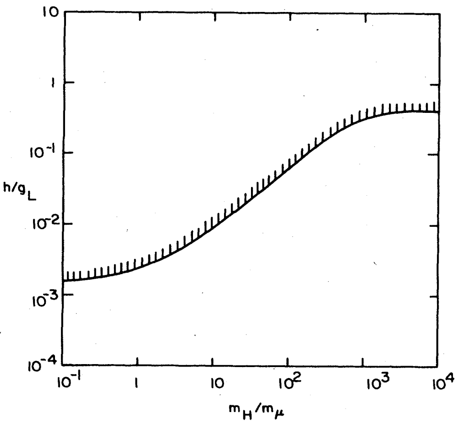

FIG. 2. Upper limits, derived from a_μ, on the light-Higgs-boson coupling to muons versus Higgs-boson mass in the standard model with two doublets.

$$\mathcal{L}_{\rm Yukawa}=h_1\bar\psi_L\Phi_f\psi_R+h_2\bar\psi_L\tilde\Phi_f\psi_R +ih_5[\psi_L^TC\tau_2\Delta_L\psi_L+(L\leftrightarrow R)]+{\rm H.c.}\ , \tag{40}$$

where $\psi_L=(\nu_L,e_L)$, $\psi_R=(N^c,e_R)$, N^c is the charge conjugate to $N=C(\bar\nu_R)^T$, and $\tilde\Phi_f\equiv\tau_2\Phi_f^*\tau_2$. In this scenario, the VEV's obey the relation $v_L\ll\kappa_f\ll\kappa_w$, v_R. The discrete symmetry $\Phi_w\to i\Phi_w$ has been imposed to give the desired VEV hierarchy and to forbid couplings of Φ_w to fermions. Thus fermion Dirac masses arise from κ_f, which is small. Majorana masses to left- and right-handed neutrinos come from v_L and v_R, respectively. In the limit $v_L\ll\kappa_f\ll\kappa_w$, v_R the physical Higgs bosons which can contribute to a_μ^H are listed in Table I. The δ_R^+ contribution is just the part of the physical Higgs boson

$$H^+=(\kappa_w\delta_R^++\sqrt{2}v_R\phi^+_{f2})/(\kappa_w{}^2+2v_R{}^2)^{1/2}$$

TABLE I. Higgs-boson Yukawa couplings which contribute to a_μ in the left-right model with a separate fermion mass scale.

Higgs particle	Diagram	Coupling
δ_L^{++}	(e),(f)	$-\frac{h_5}{2}\mu^TC(1-\gamma_5)\mu$
δ_L^+	(e)	$-\frac{h_5}{\sqrt{2}}\mu^TC(1-\gamma_5)\nu$
δ_R^{++}	(e),(f)	$-\frac{h_5}{\sqrt{2}}\mu^TC(1+\gamma_5)\mu$
$\frac{\langle\phi^0_{w1}\rangle}{(\langle\phi^0_{w1}\rangle^2+\langle\delta_R^0\rangle^2)^{1/2}}\delta_R^+$	(e)	$-\frac{h_5}{\sqrt{2}}\mu^TC(1+\gamma_5)N^c$
${\rm Re}(\phi^0_{f1})$	(d)	$h_1\bar\mu\mu$
${\rm Im}(\phi^0_{f1})$	(d)	$h_1\bar\mu i\gamma_5\mu$
${\rm Re}(\phi^0_{f2})$	(d)	$h_2\bar\mu\mu$
${\rm Im}(\phi^0_{f2})$	(d)	$-h_2\bar\mu i\gamma_5\mu$
ϕ^+_{f1}	(c)	$\frac{h_1}{2}\bar\nu(1+\gamma_5)\mu-\frac{h_2}{2}\bar N^c(1-\gamma_5)\mu$
ϕ^+_{f2}	(c)	$\frac{h_1}{2}\bar N^c(1-\gamma_5)\mu-\frac{h_2}{2}\bar\nu(1+\gamma_5)\mu$

which couples to fermions. The couplings given in Table I can be applied to the formulas derived in Sec. II. Of course, the Higgs-boson masses are not known, but if we assume that all parameters are natural-sized, i.e., of the order of g_L, then the symmetry-breaking scales will roughly determine the masses. For simplicity, all Higgs-boson masses at the weak boson scale are set at m_{W_L}. An analysis of the Higgs potential shows that there are two light Higgs bosons, $\mathrm{Re}(\phi_{f_1})$ and $\mathrm{Im}(\phi_{f_1})$, with masses of order κ_f, which will be taken to be equal in the following discussion. For this model we redefine β and r as

$$\begin{aligned} m_H &= r g_L \kappa_f \ , \\ h_1 &= h_2 = \beta g_L \ , \end{aligned} \tag{41}$$

where for simplicity, we have set $h_1 = h_2$. We also have the relation

$$m_\mu = \beta g_L \kappa_f \ . \tag{42a}$$

Furthermore, in the left-right model the neutrino masses are approximately

$$\begin{aligned} m_N &\approx h_5 v_R \ , \\ m_\nu &\sim \beta g_L \kappa_f{}^2 / v_R \ , \end{aligned} \tag{42b}$$

where we expect $v_R \approx m_{W_R}/g_R$. If $m_N \sim m_{W_R}$, then $h_5 \sim g_R$ and these Yukawa couplings are also natural-sized. The total Higgs-boson contribution to a_μ can then be written

$$a_\mu^H = \frac{m_\mu{}^2 g_L{}^2}{96\pi^2 m_{W_L}{}^2} \left\{ -34\frac{h_5{}^2}{g_L{}^2} + 24\beta^2 \frac{m_{W_L}{}^2}{m_\mu{}^2} \left[I_3 \left[\frac{r^2}{\beta^2} \right] + I_3 \left[\frac{m_{W_L}{}^2}{m_\mu{}^2} \right] \right] \right.$$
$$\left. - 4\beta^2 \left[1 + I_5 \left[\frac{m_N{}^2}{m_{W_L}{}^2} \right] \right] - 8 \frac{\kappa_w{}^2}{\kappa_w{}^2 + 2v_R{}^2} \frac{h_5{}^2}{g_L{}^2} I_5 \left[\frac{m_N{}^2}{m_{W_L}{}^2} \right] \right\} , \tag{43}$$

where $I_3(z)$ is defined in Eq. (33) and

$$I_5(z) = \int_0^1 \frac{6x^2(1-x)}{x + z(1-x)} \ . \tag{44}$$

Since $m_{W_L} \gg m_\mu$, the second term in the bracket dominates. To find the numerical limits we must first take a_μ^{weak} from Eq. (37) and allow for the variation in gauge boson contributions given in Eq. (26) for left-right models. This yields the allowed range

$$-123 \times 10^{-10} < a_\mu^H < 170 \times 10^{-10} \tag{45}$$

at the 2σ level. The corresponding limits on $m_H/m_\mu = r/\beta$ and $h/g_L = \beta$ are very similar to those shown in Fig. 2 for the SM with two Higgs doublets since each is dominated by a term proportional to

$$(\beta^4/r^2)(m_W{}^2/m_\mu{}^2)\ln(r^2/\beta^2) \ .$$

Another possible scenario in left-right models is to omit the Φ_f multiplet. Then fermion masses are the result of small couplings to Φ_w; the Yukawa potential is the same as Eq. (40) except Φ_f is replaced by Φ_w. No discrete symmetries are necessary so that all Higgs-boson masses are expected to be of order m_{W_L} (Ref. 25) for natural-sized parameters. In the limit $v_L \ll \kappa_w \ll v_R$, the physical Higgs bosons which can contribute to a_μ^H are listed in Table II. We assume for simplicity that the Higgs-boson masses are approximately the same and introduce the parametrizations for this scenario

$$\begin{aligned} m_H &= r m_{W_L} \ , \\ h_1 &= h_2 = \beta g_2 \frac{m_\mu}{m_{W_L}} \ , \end{aligned} \tag{46}$$

where again we expect $r \sim \beta \sim 1$ for natural-sized couplings. The muon mass and neutrino masses are given by Eq. (42), respectively, with κ_f replaced by κ_w. Then using the formulas of Sec. II we find

$$a_\mu^H = \frac{m_\mu{}^2 g_L{}^2}{96\pi^2 m_{W_L}{}^2} \frac{1}{r^2} \left\{ -34\frac{h_5{}^2}{g_L{}^2} + 12\beta^2 r^2 \frac{m_{W_L}{}^2}{m_\mu{}^2} \left[4I_3 \left[r^2 \frac{m_{W_L}{}^2}{m_\mu{}^2} \right] - I_4 \left[r^2 \frac{m_{W_L}{}^2}{m_\mu{}^2} \right] \right] - \beta^2 \frac{4v_R{}^2}{\kappa_w{}^2 + 2v_R{}^2} \left[\frac{m_\mu{}^2}{m_{W_L}} \right] \right.$$
$$\left. - 8 \frac{v_R{}^2 \beta^2 (m_\mu{}^2/m_{W_L}{}^2) + \kappa_w{}^2 (h_5{}^2/g_L{}^2)}{\kappa_w{}^2 + 2v_R{}^2} I_5 \left[\frac{m_N{}^2}{m_H{}^2} \right] - 48 \frac{m_N}{m_{W_L}} \beta^2 \frac{h_5}{g_L} \frac{\kappa_w v_R}{\kappa_w{}^2 + 2v_R{}^2} I_6 \left[\frac{m_N{}^2}{m_H{}^2} \right] \right\} , \tag{47}$$

TABLE II. Higgs-boson Yukawa couplings which contribute to a_μ in the left-right model without a separate fermion mass scale.

Higgs particle	Diagram	Coupling
δ_L^{++}	(e),(f)	$-\frac{h_5}{2}\mu^T C(1-\gamma_5)\mu$
δ_L^{+}	(e)	$-\frac{h_5}{\sqrt{2}}\mu^T C(1-\gamma_5)\nu$
δ_R^{++}	(e),(f)	$-\frac{h_5}{2}\mu^T C(1+\gamma_5)\mu$
$\frac{\langle\phi_{w1}^0\rangle\delta_R^+ + \sqrt{2}\langle\delta_R^0\rangle\phi_{w1}^+}{(\langle\phi_{w1}^0\rangle^2+2\langle\delta_R^0\rangle^2)^{1/2}}$	(c),(e)	$\frac{1}{\sqrt{2}}(\langle\phi_{w1}^0\rangle^2+2\langle\delta_R^0\rangle^2)^{1/2}[h_1\bar{\nu}(1+\gamma_5)\mu\langle\delta_R^0\rangle - h_2 N^T(1-\gamma_5)\mu\langle\delta_R^0\rangle - h_5\bar{N}(1+\gamma_5)\mu\langle\phi_{w1}^0\rangle]$
$\mathrm{Re}(\phi_{w1}^0)$	(d)	$h_1\bar{\mu}\mu$
$\mathrm{Im}(\phi_{w1}^0)$	(d)	$h_1\bar{\mu}i\gamma_5\mu$
$\mathrm{Re}(\phi_{w2}^0)$	(d)	$h_2\bar{\mu}\mu$

where $I_5(z)$ is given in Eq. (44) and

$$I_6(z)\equiv\int_0^1\frac{2x(1-x)dx}{x+z(1-x)}\ . \tag{48}$$

In order to have a light ν, we require $\kappa_w \ll v_R$, which implies $m_{W_L}\ll m_{W_R}$, i.e., parity is broken at a very high energy. Because the muon mass is achieved with very small couplings of order m_μ/m_W, the terms in a_μ^H involving these couplings contribute very little. If we assume $g_L\kappa_w/g_R v_R \approx m_{W_L}/m_{W_R}$,

$$a_\mu^H \approx \frac{m_\mu^2 g_L^2}{96\pi^2 m_{W_L}^2}\frac{1}{r^2}\frac{h_5^2}{g_L^2}\times\left[-34-24\frac{m_N}{m_{W_R}}\frac{\beta}{h_5}g_R I_6\left[\frac{m_N^2}{m_H^2}\right]\right]. \tag{49}$$

For $\beta\sim r\sim 1$, only weak limits are placed on the parameters from the current experimental measurements.

From the preceding examples, we see that *CP*-conserving models which have a separate symmetry-breaking scale for fermion masses often will have a light Higgs boson if the same type of Higgs multiplet is necessary for both fermion and gauge-boson masses. If a discrete symmetry is imposed to shield the fermion masses from the large VEV, the Higgs potential is simplified and a light Higgs boson will be the result. The measurement of a_μ then puts severe limits on the Higgs-boson couplings and masses. One might speculate whether a model can be constructed which avoids this problem. In fact, one already exists: the SU(2)×U(1)×SU(2)′ natural model[9] where the fermions transform in the standard way under SU(2)×U(1) and are neutral under SU(2)′. The Higgs multiplets are $\phi(\frac{1}{2},1,0)$, $\psi(0,1,\frac{1}{2})$, and $\eta(\frac{1}{2},0,\frac{1}{2})$, and the latter two are sufficient for giving large gauge-boson masses. The fermions couple exclusively to the $\phi(\frac{1}{2},1,0)$, so there can be a separate fermion mass scale without imposing any discrete symmetries. Therefore no Higgs bosons are shielded from the large mass scale and all Higgs-boson masses are likely to be of order m_W. The fermion—Higgs-boson Yukawa potential relevant to a_μ is

$$\mathcal{L}_{\mathrm{Yukawa}}=h[\bar{\nu}(1+\gamma_5)\mu\phi^+ + \bar{\mu}\mu\,\mathrm{Re}(\phi^0)+\bar{\mu}i\gamma_5\mu\,\mathrm{Im}(\phi^0) + \bar{\mu}(1-\gamma_5)\nu\bar{\phi}]\ , \tag{50}$$

where $m_\mu = h\langle\phi^0\rangle$. In this model the three Higgs bosons that contribute to a_μ can be shown to have the same mass,[9] which we denote as m_H. For this model, we parametrize the mass and coupling unknowns as

$$m_H = r m_W\ ,\qquad h=\beta g_L\ . \tag{51}$$

The contributions to a_μ come from diagrams 1c and 1d and are easily calculated to be

$$a_\mu^H=\frac{m_\mu^2 g_L^2}{96\pi^2 m_W^2}\frac{\beta^2}{r^2}\left[-1+24r^2\frac{m_W^2}{m_\mu^2}I_3\left[r^2\frac{m_W^2}{m_\mu^2}\right]\right]. \tag{52}$$

This gives only loose constraints on r and β. Thus a model which has a separate symmetry-breaking scale for fermions need not have a light Higgs boson, and therefore will not be severely restricted by current data. However, the Higgs-boson contributions are potentially as large or larger than the gauge-boson contributions and might be detectable when the limits on a_μ are tightened.

As a final example, we consider a model first proposed by Gelmini and Roncadelli,[8] an $\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$ model with standard Higgs doublet and an additional lepton-number-violating Higgs triplet. The VEV of the triplet, v_T, is constrained by the known equality of charged and neutral-current strengths to be much smaller than the doublet VEV v_D. The gauge-boson mass scale is determined by v_D. Although the triplet cannot give the usual Dirac masses to the fermions, it can give a light Majorana mass to the left-handed neutrino, of order v_T, which can be made quite small ($\sim$1 eV) if desired. Lepton-number conservation is imposed on the unbroken Lagrangian and is violated only by the spontaneous symmetry breaking of the triplet. This additional condition eliminates trilinear terms in the Higgs potential and leads to a light neutral scalar, with a mass of order v_T. The breakdown of global

$U(1)_{\rm lepton}$ symmetry also leads to a massless Goldstone boson, the majoron; the other physical Higgs bosons have mass of order m_W. Details of the Higgs sector are given in Ref. 8. Because the muon mass does not come from v_T, the light Higgs boson does not contribute to a_μ^H with any appreciable strength (there is a small coupling of order $m_\nu m_\mu / m_W{}^2$ due to mixing). The muon coupling of the doublet Higgs boson is standard and gives also a very small contribution; the effect of the heavy Higgs boson from the triplet has been calculated and shown to be small.[26] Therefore, we see that a_μ^H will be detectable only if the muon mass comes from a small VEV.

In the models we have considered the Higgs Yukawa couplings to fermions are directly proportional to the fermion mass, so that processes involving heavy fermions might put additional restrictions on the Higgs couplings and masses. The most stringent limits would come from charged Higgs-boson contributions to the K_L-K_S mass difference[27] and radiatively induced flavor-changing neutral-Higgs-boson interactions.[28] Both of these processes can put a limit on the Higgs-boson coupling but not on the light-neutral-Higgs-boson mass. These processes also involve virtual t quarks in a one-loop diagram so that the limit they provide is subject to uncertainties. Although greatly dependent on six-quark mixing angles, a K_L-K_S mass-difference calculation[27] implies that

$$\frac{h}{g_L} \leq 10 \frac{m_\mu}{m_W} \tag{53}$$

when the charged-Higgs-boson mass is near its natural scale of m_W. Saturation of the above limit plus the a_μ constraint of Eq. (38) then implies $m_H \geq 10 m_\mu$. If the neutral Higgs boson is light enough, it can be produced in the flavor-changing decay $b \rightarrow sH^0$. The H^0 can then decay to $\mu^+\mu^-$ or perhaps $K\bar{K}$ if its mass is above the appropriate threshold. Limits on such decays, in principle, will restrict h/g_L,[28] although cancellations can occur in the induced coupling for certain values of charged-Higgs-boson and t-quark masses.[29]

It has been suggested that the state $\xi(2.2)$ observed[30] in the decay $\psi \rightarrow \gamma \xi$, $\xi \rightarrow K^+K^-, K_sK_s$ is a Higgs boson.[29] The branching ratio for this decay would imply $h/g_L \approx 4 m_\mu / m_W$. With this coupling and a mass of 2.2 GeV, if the ξ is in fact a neutral Higgs boson, its contribution to a_μ could be as much as $+7 \times 10^{-10}$ in the scenario of the Higgs potentials considered in Sec. IV. This could be detectable once the improvements on the measurement and calculation of a_μ are implemented.

More complicated scenarios are also possible. For instance, there could be separate Higgs multiplets for generating lepton and quark masses. Then the Higgs couplings which contribute to a_μ are not constrained by processes involving quarks. Thus a signal for Higgs bosons would not necessarily be present in both the quark and lepton sectors. Another scenario is to have the weak-isospin-up members of the fermions to couple to one Higgs multiplet and the isospin-down members to another. Two types of models can be constructed in this scenario: (a) There is a third multiplet which is responsible for the gauge-boson masses and the first two multiplets are used to give masses to fermions. Then there are Higgs bosons with masses proportional to the light mass scales. Such models have been used in theories of spontaneous CP violation,[31] although Higgs-boson masses and spontaneous-symmetry-breaking scales are not discussed in this context. (b) There are only two multiplets; then no naturally light Higgs bosons will result from spontaneous symmetry breaking.[32] A scenario which proliferates with Higgs fields is to have a separate Higgs multiplet (and hence a separate mass scale) for each generation. However, there would be strict limits on Yukawa couplings with first-generation fermions, which were previously suppressed. Also, tree-level flavor-changing neutral currents,[33] due to quark mixing, would put severe limits on the couplings.

V. CONCLUSIONS

We have examined in detail the weak contribution to the anomalous magnetic moment of the muon in a variety of electroweak models, including the effects of Higgs scalars. We find that gauge-boson contributions to a_μ may vary up to 50% from the standard result in left-right models, while models which do not have any additional fermion currents, such as the $SU(2)\times U(1)\times G$ natural models, give virtually the same result as the standard model for the gauge boson graphs. As for the Higgs sector, models in which light-fermion masses are a consequence of small Yukawa couplings (as in the standard model) and not small vacuum expectation values will not have any significant contribution to $a_\mu^{\rm weak}$ unless the Higgs-boson mass is of order the muon mass, a situation which requires small parameters or fortuitous cancellation among the coupling parameters in the Higgs potential. On the other hand, in models where fermion masses are generated through natural-sized couplings (of order the gauge coupling) and small vacuum expectation values, i.e., a separate symmetry-breaking scale, the contribution to a_μ from Higgs particles can be just as large or larger than that from gauge bosons. If there are one or more light Higgs bosons with masses of order the muon mass the corrections to a_μ are potentially quite large. Examples of this situation are the standard model with an additional doublet that undergoes spontaneous symmetry breaking at a fermion mass scale and the left-right-model scenario which explains light left-handed Majorana neutrinos and Dirac fermions as resulting from separate spontaneous-symmetry-breaking scales. Current limits on a_μ put severe limits on the Higgs-boson masses and coupling in these models, so that the original motivation of them is to some extent lost. Models which have a separate fermion mass scale but do not have a light Higgs boson, such as one scenario in the $SU(2)\times U(1)\times G$ natural models, are modestly restricted by current data, but could have significant deviations from the standard value for $a_\mu^{\rm weak}$, almost all of which come from the Higgs sector.

The implications of the above analysis are intriguing. Although the effects of the Higgs boson in the standard model on the muon anomalous magnetic moment are probably negligible, we find that in many alternative models they can be a significant, if not dominant, fraction of the total weak contribution. Therefore, despite the many uncertainties about the exact structure of the Higgs sector, any significant deviation of $a_\mu^{\rm weak}$ from the minimal-standard-model results could be a signal for some kind of richer Higgs structure, and may provide a guide as to where to look for them. In light of this, improvements to the experimental measurements and theoretical calculations of a_μ by an order of magnitude could lead to a much greater understanding of the electroweak interactions.

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APPENDIX

For lepton-number-violating (LNV) interactions like Eq. (9), the graphic rules for the external lines can be read off directly from the expression of the Lagrangian, but a slight complication arises for fermion propagators. The necessary rules can be derived using the standard Wick expansion and no new elements are present in the derivation. However, in view of the fact that the LNV-diagram rules are less well known, some details will be given in this appendix. It will be shown also, by examples, that these LNV-diagram rules can be reduced to corresponding conventional rules for lepton-number-conserving (LNC) interactions.

Recall that in writing down the amplitude of a LNC process from its Feynman diagram, two directions can be defined for each segment of a fermion line. The first is the direction of the fermion number flow which is in the same direction of the momentum for leptons, but in the opposite direction of the momentum for antileptons. We note that a fermion number can still be defined for a fermion line even though the fermion number is not conserved at the vertex. The second direction is a direction along which a fermion line is read as in the amplitude; this direction is fixed for a given fermion line. For LNC interactions, the direction of the fermion number flow is always opposite to the direction of reading the line segment. In the case of LNV interactions the direction of the fermion number flow is reversed when a LNV vertex is passed. Therefore the line-reading direction and the fermion-number-flow direction for some of the line segments will be parallel. These are the parts of a diagram which require new rules. These new rules are stated in the following.

(a) For an external fermion line, the transpose of a suitable spinor wave function is associated: $u^T(p)$, $\bar{u}^T(p)$, $v^T(p)$, or $\bar{v}^T(p)$.

(b) For an internal line segment the transpose of the negative of the corresponding fermion propagator is associated. Recall that for an internal fermion line segment, its momentum is assigned in the same direction of the fermion number flow.

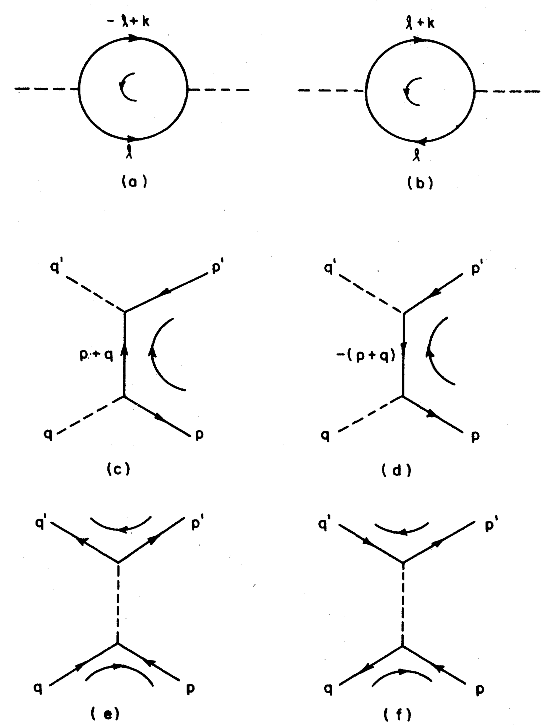

FIG. 3. Diagrams of three LNV processes and their corresponding equivalent LNC processes.

(c) At a LNV vertex, fermion numbers of the associated lines either flow into or flow out of the vertex. When lepton numbers flow into [out] a vertex, the factor $i(n!)C\Gamma$ [$i(n!)C\tilde{\Gamma}$] is associated, where n is the number of identical particles occurring at the vertex, where C is the charge conjugation matrix, and Γ and $\tilde{\Gamma}$ are given in Eqs. (10) and (11). In Eq. (9), $n=1$ if $f_1 \neq f_2$ and $n=2$ if $f_1=f_2$.

In the following we give three examples to show that the above rules can be reduced to the conventional ones. We also show at the end of the appendix how the fermion statistics is satisfied.

A. Self-energy of a doubly charged boson

The Feynman graph, drawn according to the LNV interaction, is given in Fig. 3(a). The arrow on a fermion line indicates the direction of the fermion number flow. The direction in the loop indicates the line-reading direction. According to the Feynman rules stated above the amplitude is

$$-\tfrac{1}{2}\int \frac{d^n l}{(2\pi)^n}\mathrm{Tr}\left[2iC\Gamma\frac{i}{-\not{l}+\not{k}-m}2iC\tilde{\Gamma}\left[\frac{-i}{\not{l}-m}\right]^T\right]^T = -\tfrac{1}{2}\int \frac{d^n l}{(2\pi)^n}\mathrm{Tr}\left[2i\Gamma\frac{i}{-\not{l}-\not{k}-m}2i(C\tilde{\Gamma}C^{-1})C\left[\frac{-i}{\not{l}-m}\right]^T C\right], \qquad \text{(A1)}$$

where the factor $-\frac{1}{2}$ is a conventional factor associated with the fermion-loop contribution to a boson self-energy diagram. The momentum integrals are defined in n dimensions so that the integral is regularized. From Eq. (10) and the properties of the charge conjugation matrix[13]

$$C^{-1}=C^{\dagger}=C^{T}=-C$$

and

$$C\gamma_{\mu}C^{-1}=-\gamma_{\mu}^{T}\ .$$

We can rewrite Eq. (A1) as

$$-\tfrac{1}{2}\int \frac{d^{n}l}{(2\pi)^{n}}\mathrm{Tr}\left[2i\Gamma\frac{i}{-\not{l}+\not{k}-m}2i\overline{\Gamma}\frac{i}{-\not{l}-m}\right]$$

$$=-\tfrac{1}{2}\int \frac{d^{n}l}{(2\pi)^{n}}\mathrm{Tr}\left[2i\Gamma\frac{i}{\not{l}+\not{k}-m}2i\overline{\Gamma}\frac{i}{\not{l}-m}\right]. \tag{A2}$$

This final expression corresponds to the diagram in Fig. 3(b), which is the LNC equivalent graph, except for the factors of 2 multiplying the couplings due to identity of the two fermion lines.

B. $e^{-}+e^{-}\to e^{-}+e^{-}$

The LNV interaction contributes to the diagram in Fig. 3(c). The amplitude for this diagram is

$$\overline{u}(p')2iC\widetilde{\Gamma}\overline{u}^{\,T}(q')\frac{i}{(p+q)^{2}-m_{\delta}{}^{2}}u^{T}(q)2iC\Gamma u(p)$$

$$=\overline{u}(p')2i\overline{\Gamma}v(q')\frac{i}{(p+q)^{2}-m_{\delta}{}^{2}}\overline{v}(q)2i\Gamma u(p)\ , \tag{A3}$$

where the following identities are used:

$$C\overline{u}^{\,T}(p)=v(p),\quad C\overline{v}^{\,T}(p)=u(p)\ ,$$

$$u^{T}(p)C=\overline{v}(p),\quad v^{T}(p)C=\overline{u}(p)\ .$$

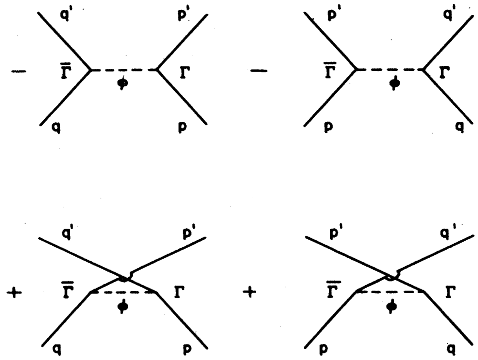

FIG. 4. Relative sign of the LNC process in relation to the LNV one [see (c)] for the reaction $e^{-}e^{-}\to e^{-}e^{-}$.

This final expression corresponds to the diagram in Fig. 3(d), obtained from the corresponding LNC interaction.

Figure 3(c) alone already satisfies Fermi statistics. This can be demonstrated as follows. Consider the upper vertex of Fig. 3(c) and interchange p' and q':

$$\overline{u}(q')2iC\widetilde{\Gamma}\overline{u}^{\,T}(p')=\overline{u}(p')2iC(C^{\dagger}\widetilde{\Gamma}^{\,T}C^{T})\overline{u}^{\,T}(q')$$

$$=-\overline{u}(p')2iC\widetilde{\Gamma}\overline{u}^{\,T}(q')\ ,$$

where Eq. (12) is used. Similarly, it can be shown that the lower vertex also satisfies Fermi statistics.

Note that the overall sign of the expression given in Eq. (A3) is arbitrary. However, the relative sign between Eq. (A3) and the amplitude of the corresponding LNC contributions is not arbitrary. Take as an example the reaction $e^{-}e^{-}\to e^{-}e^{-}$. The phase relationships between Fig. 3(c) as given by Eq. (A3) and the diagrams resulting from the LNC interaction Lagrangian given in Eq. (12) are shown in Fig. 4.

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