QCD scaling violations for spin-dependent structure functions

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A detailed study of scaling violations for spin-dependent structure functions is worked out by direct resolution of the Altarelli-Parisi equations. Analytic parametrizations of the x and Q^2 dependence of polarized valence-quark, antiquark, and gluon distributions are also given. They are used to make predictions for spin-spin asymmetries in some specific reactions.

I. INTRODUCTION

Scaling violations in unpolarized deep-inelastic scattering have been extensively studied in the framework of perturbative QCD. In order to solve the Altarelli-Parisi equations,[1] two methods are used. The first one involves the moments of the quark and gluon distributions.[2] It is mainly sensitive to the large-x region and therefore not really suitable for sea quarks and gluons which are expected to dominate in the small-x region. The second approach is based on a direct resolution of the Altarelli-Parisi equations.[3] Leaving aside the choice of the resolution method, the large amount of experimental data[4] allows a direct comparison between the different kernels one can use in the evolution equations, i.e., the leading-logarithmic kernel,[1] the second-order one including next-to-leading corrections,[5] and the kernel taking into account soft-gluon emission.[6]

Measurements of polarized electron (muon) scattering off a polarized nucleon target yield information about the polarization inside a proton, but the small number of data points gives only the quark asymmetry A_1 (see below) for $0.1<x<0.64$ and in a limited Q^2 range. The quality of this data from a SLAC-Yale experiment[7] does not allow one to distinguish between the different kernels as in the case of unpolarized structure functions. In order to help the determination of the polarized gluon structure function, spin-spin asymmetries in large-$p_\perp$ production have been considered.[8] The double-spin asymmetry in Drell-Yan processes[9] with polarized protons has also been proposed as a good way to probe the sea-quark spin-dependent distributions. However, these double-spin asymmetries involving both a polarized beam and a polarized target will remain very difficult measurements until the advent of a high-density polarized hydrogen target. Single asymmetries for weak-boson production in collisions between polarized protons and unpolarized antiprotons, which are expected to be large,[10] are perhaps easier to detect and they might give in a not-too-distant future precise information on the distribution of the proton spin among sea and valence quarks.

Our purpose here is to provide a very simple and accurate analytic parametrization of the spin-dependent structure functions to be used in further applications such as those mentioned above. The paper is organized as follows. In Sec. II, we give a brief discussion of partonic spin-dependent structure functions and the set we will use as input. In Sec. III we will describe the relevant formalism to solve the Altarelli-Parisi equations governed by the leading-logarithmic kernel. An analytic parametrization valid for all x and for $5<Q^2<5000$ GeV^2 is presented in Sec. IV, where we also stress different ways to detect these scaling violations.

II. PARTONIC SPIN-DEPENDENT STRUCTURE FUNCTIONS

The first model which was used to describe spin-dependent structure functions is the so-called conservative SU(6) model,[11] based on the three-quarks SU(6) wave function. Assuming all the spin of the proton is carried by its valence quarks, one has

$$\Delta u_v(x)=\tfrac{2}{3}u_v(x)\ , \qquad \Delta d_v(x)=-\tfrac{1}{3}d_v(x) \tag{1}$$

with the definition $\Delta q(x)=q_+(x)-q_-(x)$, where + (−) denotes the quark helicity parallel (antiparallel) to the parent proton helicity. Clearly this simple choice does not satisfy the Bjorken sum rule[12]

$$\int_0^1 dx[\Delta u(x)+\Delta\bar{u}(x)-\Delta d(x)-\Delta\bar{d}(x)]=\frac{G_A}{G_V}\ , \tag{2}$$

where G_A/G_V is the ratio of the axial vector to the vector coupling in neutron β decay, whose experimental value is 1.254 ± 0.006 and not $\frac{5}{3}$, as predicted by SU(6). After the modification

$$\Delta u_v(x)=0.44u_v(x)\ , \qquad \Delta d_v(x)=-0.35d_v(x)\ , \tag{3}$$

Eq. (2) is truly satisfied, provided one still assumes no polarization for the sea quarks (i.e., $\Delta\bar{u}=\Delta\bar{d}=0$) or alternatively, if the sea quarks are polarized, one must assume isospin invariance of the sea.

The polarization of the sea quarks can be obtained from a model of gluon bremsstrahlung and quark-pair creation.[13] If one takes for the unpolarized sea quarks

$$\bar{u}(x)=\bar{d}(x)=\frac{0.4}{x}[1+(1-x)^2](1-x)^{10}\ , \tag{4}$$

one predicts

$$\Delta\bar{u}(x)=\Delta\bar{d}(x)=\frac{0.4}{3}(2-x)(1-x)^{10}\,. \tag{5}$$

Similarly it is possible to generate a polarization for gluons radiated by a polarized valence quark, and from the gluon distribution

$$G(x)=\frac{2}{x}(1-x)^6[1+(1-x)^2]\,, \tag{6}$$

one obtains

$$\Delta G(x)=\tfrac{2}{3}(2-x)(1-x)^6\,. \tag{7}$$

The distributions Eqs. (5) and (7) are properly normalized and they fulfill the important constraint

$$\langle J_z\rangle=\tfrac{1}{2}=\tfrac{1}{2}\sum_i\int_0^1 dx[\Delta q_i(x)+\Delta\bar{q}_i(x)]+\int_0^1 dx\,\Delta G(x)\,, \tag{8}$$

namely, the projection of the third component of the total angular momentum summed over all the constituents must be equal to $\frac{1}{2}$, if one neglects the orbital angular momentum. In this model, the valence quarks carry 53% of the proton spin, the sea quarks 13%, and the gluons 34% of it. Unfortunately, the SLAC-Yale data on polarized electroproduction show that the quark asymmetry defined as

$$A_1(x)=\frac{\sum_i e_i{}^2\Delta q_i(x)}{\sum_i e_i{}^2 q_i(x)} \tag{9}$$

is larger than predicted by the conservative SU(6) model for $x>0.4$. This means that in a realistic model, the valence quarks must carry a larger fraction of the proton spin.

Consequently, it seems preferable to use the model of Carlitz and Kaur[14] based on the idea that valence quarks carry most of the proton helicity *only* at large x values. Therefore they introduce a spin-dilution factor

$$\cos2\theta=\left[1+H_0\frac{(1-x)^2}{\sqrt{x}}\right]^{-1} \tag{10}$$

and the quark spin distributions are taken to be

$$\begin{aligned}\Delta u(x)&=\cos2\theta[u_v(x)-\tfrac{2}{3}d_v(x)]\,,\\ \Delta d(x)&=\cos2\theta[-\tfrac{1}{3}d_v(x)]\,.\end{aligned} \tag{11}$$

The spin-dilution factor becomes important for small x and the free parameter H_0 was fixed by using the Bjorken sum rule [Eq. (2)]. The value $H_0=0.052$ is obtained with the Field-Feynman valence distributions.[15]

Let us now describe our model, which is inspired by the Carlitz-Kaur model, but with specific differences. Clearly Ref. 15 is not the best choice for studying scaling violations and we will use instead the parametrization of Gluck, Hoffman, and Reya.[16] We will also take into account the fact that there is a QCD correction to the Bjorken sum rule,[17] which now reads

$$\int_0^1 dx[\Delta u(x)+\Delta\bar{u}(x)-\Delta d(x)-\Delta\bar{d}(x)]=\frac{G_A}{G_V}\left[1-\frac{\alpha_s}{\pi}\right]. \tag{12}$$

In order to satisfy this new sum rule for $Q^2=5$ GeV^2, we must have $H_0=0.114$, which is very different from the value used in Ref. 14. Assuming Eq. (11) for the valence-quark spin distributions, we find that they carry 73% of the proton spin and the remaining 27% must be shared between sea quarks and gluons. Following the bremsstrahlung model,[13] we construct the sea-quark spin distributions from the unpolarized sea. The existing deep-inelastic data at $Q^2=4$ GeV^2 favors a distribution like $(1-x)^7$ carrying 14% of the nucleon momentum.[16] For the total sea contribution

$$S(x)=2\bar{u}(x)+2\bar{d}(x)+s(x)+\bar{s}(x)\,,$$

we will use

$$xS(x)=0.588[1+(1-x)^2](1-x)^{6.5}\,, \tag{13}$$

which leads to the corresponding spin-dependent sea component

$$x\,\Delta S(x)=\frac{0.588}{3}x(2-x)(1-x)^{6.5}\,, \tag{14}$$

which is found to carry 5% of the proton spin. Similarly, by choosing for the unpolarized gluon distribution

$$xG(x)=2.82(1-x)^5\,, \tag{15}$$

one obtains

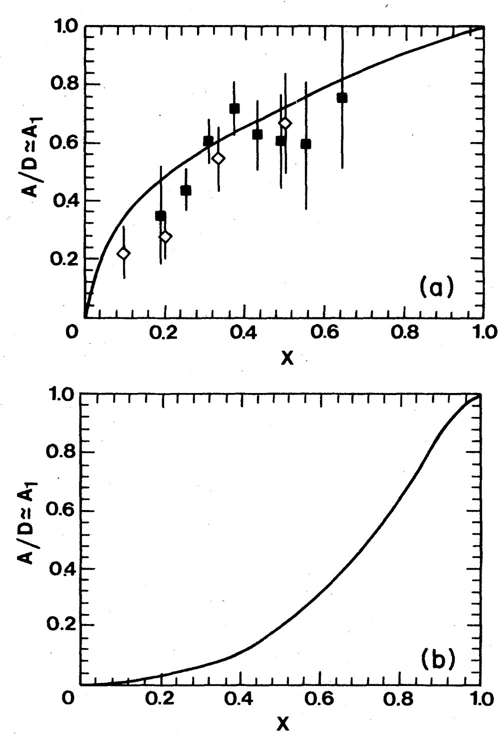

FIG. 1. (a) The quark asymmetry A_1 on a proton target as a function of x. The data are extracted from Ref. 7. (b) Predictions for the quark asymmetry A_1 on a neutron target.

$$x\,\Delta G(x)=\frac{0.15}{3}x(5-2x)xG(x)\ , \tag{16}$$

where the normalization is fixed to satisfy the proton-spin sum rule [see Eq. (3)]. This completes the description of our input parametrization and we are now ready to study the evolution of the nonsinglet, singlet, and glue distribution functions.

Before doing so, we ought to make a simple comparison of our starting point with the quark asymmetry A_1 determined by the SLAC-Yale experiment.[7] As shown in Fig. 1(a), the agreement is good although it is clear that more accurate data is required, mainly in the low-x region which is crucial for the precise determination of the sea-quark polarization. This question will be answered soon by the European Muon Collaboration in the NA2 experiment at CERN which will investigate accurately the kinematic region $0.04<x<0.2$ for Q^2 up to 20 GeV.^{2} We also give in Fig. 1(b) our prediction for A_1 on a neutron target that one merely obtains by exchanging u and d. Of course, a measurement of this quantity is extremely important.[18]

III. EVOLUTION OF THE DISTRIBUTION FUNCTIONS

We have to solve the following set of integrodifferential equations:

$$\begin{aligned}
\frac{d\,\Delta Q_v(x,t)}{dt}&=\int_x^1 dz\,\Delta P_{qq}(z)\Delta Q_v\left[\frac{x}{z},t\right]\ ,\\
\frac{d\,\Delta Q(x,t)}{dt}&=\int_x^1 dz\left[\Delta P_{qq}(z)\Delta Q\left[\frac{x}{z},t\right]+\Delta P_{qG}(z)2N_f\Delta\widetilde{G}\left[\frac{x}{z},t\right]\right]\\
\frac{d\,\Delta\widetilde{G}(x,t)}{dt}&=\int_x^1 dz\left[\Delta P_{Gq}(z)\Delta Q\left[\frac{x}{z},t\right]+\Delta P_{GG}(z)\Delta\widetilde{G}\left[\frac{x}{z},t\right]\right]\ ,
\end{aligned} \tag{17}$$

where we have defined

$$\Delta Q_v(x,t)=x\Delta q_v(x,t)\ ,$$

$$\Delta Q(x,t)=x[\,\Delta u_v(x,t)+\Delta d_v(x,t)+\Delta S(x,t)]\ ,$$

$$\Delta\widetilde{G}(x,t)=x\Delta G(x,t)\ ,$$

and

$$t=\frac{2}{(11-\frac{2}{3}N_f)}\ln[\ln(Q^2/\Lambda^2)/\ln(Q_0{}^2/\Lambda^2)]\ .$$

In the leading-logarithmic approximation, and assuming three flavors, the kernels read

$$\begin{aligned}
\Delta P_{qq}&=\frac{4}{3}\left[\frac{1+z^2}{1-z}\right]_+\ ,\\
\Delta P_{qG}&=\tfrac{1}{2}(2z-1)\ ,\\
\Delta P_{Gq}&=\tfrac{4}{3}(2-z)\ ,\\
\Delta P_{GG}&=3\left[\left[\frac{1+z^4}{1-z}\right]_+ +(3-3z+z^2+z^3)-\tfrac{7}{12}\delta(1-z)\right]\ .
\end{aligned} \tag{18}$$

The analysis of scaling violations has been done by means of moment inversion,[19] linear approximation[20] (only valid for very small t), or expansion in Chebyshev polynomials.[11] To solve these equations we will follow the method of Ref. 21 based on a direct-resolution procedure, which is much simpler than the other methods. Let us consider first the nonsinglet case. According to the definition of the + distribution we are able to write

$$\int_x^1 dz\,\Delta P_{qq}(z)\Delta Q_v\left[\frac{x}{z},t\right]=\int_x^1 z\,\Delta P_{qq}(z)\left[\Delta Q_v\left[\frac{x}{z},t\right]-\Delta Q_v(x,t)\right]-\Delta Q_v(x,t)\int_0^x dz\,\Delta P_{qq}(z)\ . \tag{19}$$

Therefore one has to solve the first equation of Eqs. (17), that is,

$$\frac{d\,\Delta Q_v(x,t)}{dt}=\tfrac{4}{3}\left\{\int_x^1 dz\left[\frac{1+z^2}{1-z}\right]\left[\frac{\Delta Q_v(x/z,t)}{\Delta Q_v(x,t)}-1\right]+\left[2\ln(1-x)+x+\frac{x^2}{2}\right]\right\}\Delta Q_v(x,t)\ . \tag{20}$$

Expanding the expression into curly brackets as a Taylor series and keeping only the first terms, one gets

$$\Delta Q_v(x,t)=\Delta Q_v(x,0)\exp\tfrac{4}{3}t\left\{\int_x^1 dz\left[\frac{1+z^2}{1-z}\right]\left[\frac{\Delta Q_v(x/z,0)}{\Delta Q_v(x,0)}-1\right]+\left[2\ln(1-x)+x+\frac{x^2}{2}\right]\right\}\ . \tag{21}$$

This approximation is justified because, as in Ref. 21, it can be shown that the second-order term in the Taylor expansion gives a negligible contribution.

Let us now turn to the singlet and glue distribution functions. We have to solve the last two coupled equations of (17) in the form

$$\begin{bmatrix} \dfrac{d\,\Delta Q(x,t)}{dt} \\ \dfrac{d\,\Delta\widetilde{G}(x,t)}{dt} \end{bmatrix} = \begin{bmatrix} R_{11}+S_{11} & R_{12} \\ R_{21} & R_{22}+S_{22} \end{bmatrix} \begin{bmatrix} \Delta Q(x,t) \\ \Delta\widetilde{G}(x,t) \end{bmatrix}, \tag{22}$$

where

$$R_{11}=\tfrac{4}{3}\int_x^1 dz \left[\frac{1+z^2}{1-z}\right]\left[\frac{\Delta Q\left[\frac{x}{z},t\right]}{\Delta Q(x,t)}-1\right],$$

$$S_{11}=\tfrac{4}{3}\left[2\ln(1-x)+x+\frac{x^2}{2}\right],$$

$$R_{12}=3\int_x^1 dz(2z-1)\frac{\Delta\widetilde{G}\left[\frac{x}{z},t\right]}{\Delta\widetilde{G}(x,t)},$$

$$R_{21}=\tfrac{4}{3}\int_x^1 dz(2-z)\frac{\Delta Q\left[\frac{x}{z},t\right]}{\Delta Q(x,t)}, \tag{23}$$

$$R_{22}=3\int_x^1 dz\left[(3-3z+z^2+z^3)\frac{\Delta\widetilde{G}\left[\frac{x}{z},t\right]}{\Delta\widetilde{G}(x,t)} + \left[\frac{1+z^4}{1-z}\right]\left[\frac{\Delta G\left[\frac{x}{z},t\right]}{\Delta G(x,t)}-1\right]\right],$$

$$S_{22}=3\left[2\ln(1-x)+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{7}{12}\right].$$

Analogously to the nonsinglet case, one makes a Taylor expansion and, keeping the first term only, one gets the solutions of Eqs. (22) in an exponential form which is very easy to handle.

Let us now describe our numerical results with $\Lambda=0.2$ GeV. We show in Fig. 2 the antiquark spin distribution $\Delta\bar{Q}=x\,\Delta S/6$ for four scales $Q^2=5$, 50, 500, and 5000 GeV^2. Its behavior is very similar to that of the unpolarized distributions, exhibiting an increase with Q^2 in the low-x region which is compensated by lower values in the large-x region. As we can see from Fig. 3, we get similar conclusions for $\Delta\widetilde{G}$, but with a more pronounced increase at very small x.

To make more practical a quantitative analysis of some hadron asymmetries, we will give now an analytic parametrization of the x and Q^2 dependence of the various spin-dependent distribution functions.

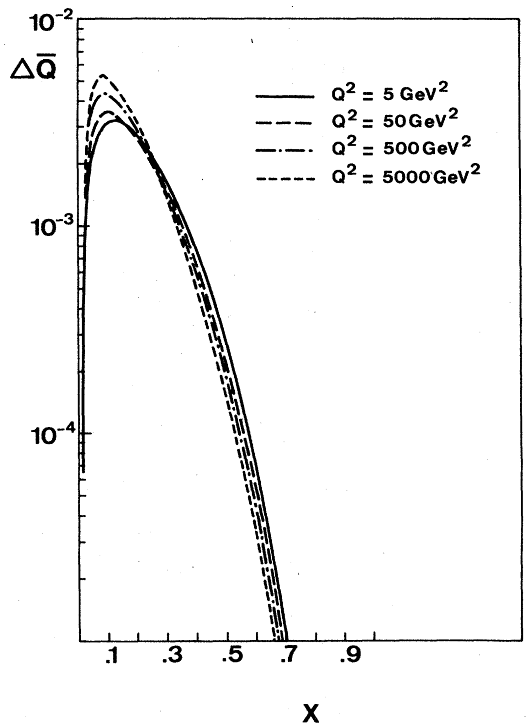

FIG. 2. The antiquark spin distribution $\Delta\bar{Q}$ as a function of x for four scales $Q^2=5$, 50, 500, and 5000 GeV^2.

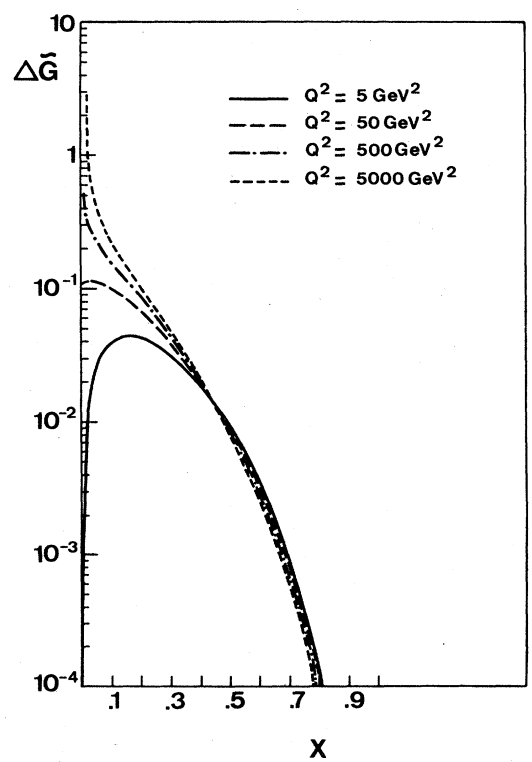

FIG. 3. The same as Fig. 2 for the gluon spin distribution $\Delta\widetilde{G}$.

IV. PARAMETRIZATION OF THE SPIN-DISTRIBUTION FUNCTIONS AND APPLICATIONS

The polarized valence-quark structure functions being expressed in terms of unpolarized nonsinglet distribution functions [see Eq. (11)], they can be parametrized according to an updated analysis,[16] provided one allows a Q^2 dependence for H_0 as we will see below.

In order to parametrize the sea and glue polarized structure functions, the following scheme was adopted. We first evaluate the distribution functions for a set of points in the x range (0,1) at three evolution scales Q^2 equal to 50, 500, and 5000 GeV^2. We parametrize the antiquarks and the glue according to

$$\Delta\bar{Q}(x,t)=Ee^{-Fx}(x-0.07)+0.03266(1-SN)\times(2-x)x^{1+\alpha}(1-x)^{6.5+\beta} \qquad (24)$$

and

$$\Delta\widetilde{G}(x,t)=\widetilde{E}e^{-\widetilde{F}x}+0.425(\tfrac{5}{3}-\tfrac{2}{3}x)x^{1+\widetilde{\alpha}}\times(1-\widetilde{SN})(1-x)^{5+\widetilde{\beta}}\ .$$

We then make a fit of E ($\widetilde{E}$), F ($\widetilde{F}$), SN ($\widetilde{SN}$), α ($\widetilde{\alpha}$), and β ($\widetilde{\beta}$) using the minimization procedure MINUIT and parametrize the fitted coefficients in terms of

$$\bar{S}=\ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]\ .$$

We find

$$\begin{aligned}
E&=0.15197\bar{S}-1.11515\bar{S}^{1.8}+1.16576\bar{S}^2\ ,\\
\widetilde{E}&=1.34848\bar{S}-12.96224\bar{S}^{1.8}+12.44553\bar{S}^2\ ,\\
F&=143.117\bar{S}-587.045\bar{S}^{1.8}+465.662\bar{S}^2\ ,\\
\widetilde{F}&=236.201\bar{S}-1170.258\bar{S}^{1.8}+974.181\bar{S}^2\ ,\\
SN&=0.5948\bar{S}+0.66941\bar{S}^{1.8}-0.66793\bar{S}^2\ ,\\
\widetilde{SN}&=2.83253\bar{S}-9.46122\bar{S}^{1.8}+7.33766\bar{S}^2\ ,\\
\alpha&=-0.36407\bar{S}-0.253278\bar{S}^2\ ,\\
\widetilde{\alpha}&=-3.01178\bar{S}+7.90763\bar{S}^{1.8}-6.20342\bar{S}^2\ ,\\
\beta&=0.52554\bar{S}-0.48334\bar{S}^{1.8}+0.14164\bar{S}^2\ ,\\
\widetilde{\beta}&=-2.59368\bar{S}+9.80538\bar{S}^{1.8}-7.44397\bar{S}^2\ .
\end{aligned} \qquad (25)$$

A similar method was used to determine the valence-quark spin-dependent structure functions and we found the following expression for H_0:

$$H_0=0.114-0.10581\bar{S}+0.48144\bar{S}^{1.8}-0.46111\bar{S}^2\ . \qquad (26)$$

Note that when Q^2 increases H_0 decreases and approaches the value of Ref. 14.

Finally, we have checked that this parametrization is valid within a few per cent in the range $0.03<x<0.9$ and $5<Q^2<5000$ GeV^2.

In order to test our evaluation of the sea polarization we will now give some predictions for two hadron asym-

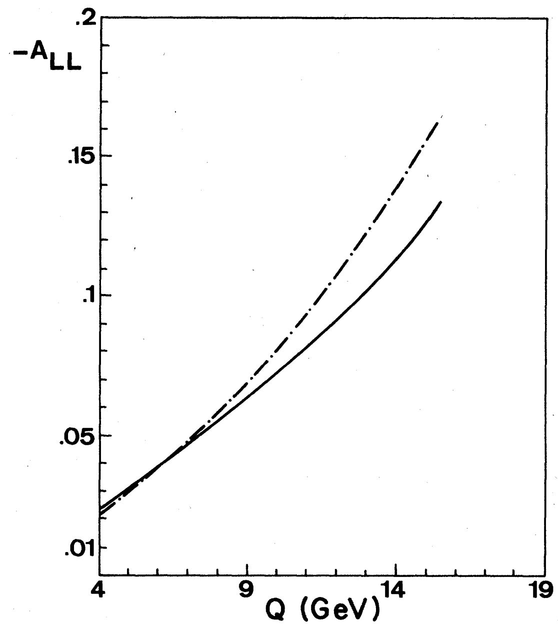

FIG. 4. Predictions for the double helicity asymmetry A_{LL} in $\vec{p}\vec{p}\rightarrow\mu^+\mu^-X$ as a function of the lepton-pair mass Q at $\sqrt{s}=27$ GeV. Dotted-dashed curve: scaling prediction. Full curve: leading prediction.

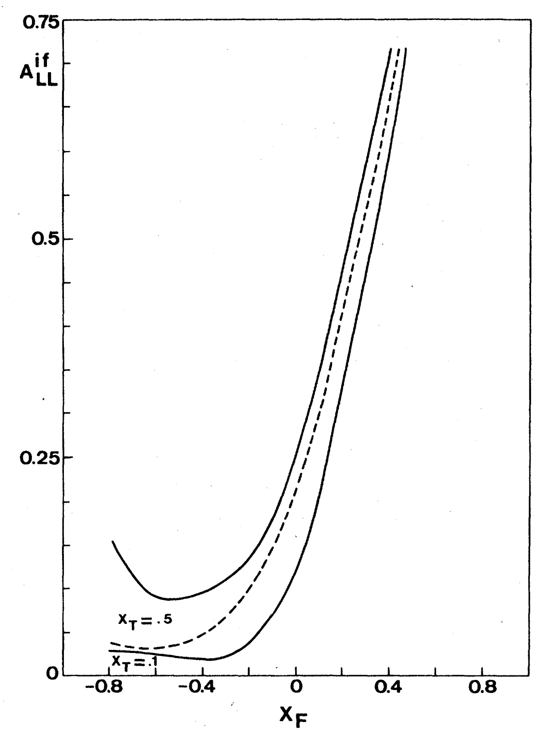

FIG. 5. Prediction for the transmitted asymmetry A_{LL}^{if} in $\vec{p}p\rightarrow\vec{\gamma}X$ as a function of x_F and x_T at $\sqrt{s}=27.4$ GeV. Solid curves: first QCD order with evolved structure functions. Dashed curve: first QCD order with scaling structure functions at $Q_0^2=5$ GeV^2. For $x_T=0.1$ dashed and solid curves are indistinguishable.

metries. Let us first study the double-helicity asymmetry for lepton-pair production from polarized-proton—polarized-proton collisions. In the leading approximation we have

$$A_{LL}=-\frac{\sum_{q=u,d,s} e_q{}^2[\Delta q(x_1,Q^2)\Delta\bar{q}(x_2,Q^2)+(q\leftrightarrow\bar{q})]}{\sum_{q=u,d,s} e_q{}^2[q(x_1,Q^2)\bar{q}(x_2,Q^2)+(q\leftrightarrow\bar{q})]}\,, \tag{27}$$

where Q is the lepton pair mass and $x_1=x_2=Q/\sqrt{s}$, when one calculates the cross section $d\sigma/dQ^2$. If the sea is unpolarized, A_{LL} is zero and consequently the detection of a nonzero asymmetry is a very clear indication for sea polarization. The large subleading corrections which are present in the Drell-Yan process[22] happen to factorize and therefore do not affect this leading approximation. As exhibited in Fig. 4, scaling violations are important and increase with the lepton pair mass, giving a correction of about 30% for $Q=14$ GeV. The observation of this effect will be tough, but it is certainly worth trying in the future appropriate experimental programs both at Fermilab and at CERN.[23]

Next let us consider the transmitted asymmetry in prompt-photon production at large $p_\perp$ in pp collisions with one initial proton polarized. This reaction has been advocated as a good test for spin-dependent structure functions.[24] We show in Fig. 5 that the negative X_F region at large $p_\perp$ $(\sim\sqrt{s}/4)$, which is dominated by the gluon structure function, is very sensitive to the scaling violations of $\Delta\widetilde{G}$ leading to deviations of a factor 5 at $X_F=-0.8$ and a factor 2 at $X_F=-0.4$.

Clearly there are other examples where one can explore the expected effects of these scaling violations described by our analytic parametrization. In particular, the asymmetries in heavy-boson production, which will be studied in a forthcoming paper.

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