New colored particles and the Υ system

G. Eilam
Physics Department, Technion—Israel Institute of Technology, Haifa, Israel

M. Glück
Institut für Physik, Universitat Dortmund, 4600 Dortmund 50, Federal Republic of Germany
(Received 26 September 1983)

We investigate the contributions of new light colored particles to the width and hyperfine splitting of heavy quarkonia, with emphasis on the Υ system. These particles can be unflavored scalars required by models with spontaneous breaking of QCD, light gluinos, or in general any light particles that couple to gluons but not to quarks. In the first case we find for scalars in the $\underline{27}$ representation of $SU(3)_c$, as in the "glow" model of Slansky, Goldman, and Shaw, an effect of 20% on predictions of QCD in the Υ system, while for gluinos the effect is much smaller.

I. INTRODUCTION

The standard model of electroweak and strong interactions has already passed successfully a number of tests, and more will be performed. Nevertheless, it faces quite a few unresolved problems;[1] among them let us mention only two:

(1) It is difficult to incorporate into the standard model the possible existence of free fractionally charged particles, recently reported.[2]

(2) The large mass scale ratio between SU(5) breaking and $SU(2)_L \times U(1)$ breaking is unnatural.

If the existence of fractionally charged particles is confirmed and if it indicates breaking of color symmetry, models describing this phenomenon can be built[3,4] without spoiling the successful predictions of QCD. The scalars breaking the color symmetry appear either as three color triplets[3] (and the conventional triality-charge relation is abandoned) or[4] in a 27-dimensional representation of $SU(3)_c$ (and the above relation is preserved). In the latter model $SU(3)_c$ is broken to $SO(3)_g$, where g stands for "glow," and the lowest-mass observable fractionally charged particles are diquarks, which are produced with a suppressed rate. The scalars in both models are light,[5,6] with masses in the range of tens of MeV or less. They are colored, unflavored, do not couple to quarks, and their coupling to gluons is determined by their color representation.

The second problem was one of the motivations for introducing supersymmetry,[7] in which fermions and bosons are partners in the same representation. Very little is known about supersymmetry breaking, and masses of supersymmetric partners are highly model dependent. Gluinos, the supersymmetric spin-$\frac{1}{2}$ partners of gluons, have in some schemes masses of the order of a few GeV.[7–9] Experimentally,[10] $m_{\tilde{g}} > 3.5$–6 GeV depending on various alternatives. The gluinos are colored, unflavored, do not couple to quarks, and their coupling to gluons is determined by their color representation—which is of course $\underline{8}$.

We picked these two possible expansions of the standard model to indicate that new colored, unflavored particles which couple to gluons only may appear in addition to gluons. If these new particles are light they can, in principle, modify standard QCD predictions, the more so the higher the $SU(3)_c$ representation they span.

In this paper we study the contribution of such particles to the width and hyperfine splitting of heavy quarkonia. The treatment is general, and applies to any light (with masses much smaller than the heavy quark mass), unflavored particles that do not couple to quarks. For scalars required by the glow model[4] we find a 20% effect on standard QCD predictions involving the Υ width and the Υ-η_b splitting; a smaller effect is found when the η_b width is taken instead of the Υ width, which is fortunate since we are still far from measuring the η_b width. For color-triplet scalars proposed in Ref. 3 the effect is smaller; furthermore, this model which has unconfined quarks and gluons may face some problems.[4,11] It is also natural to consider first the most favorable situation of the highest possible contribution, and we therefore limit ourselves to the glow model, while results for the model of Ref. 3 can be trivially obtained from our calculation. The effect of spin-$\frac{1}{2}$ gluinos is about 7% if they are massless, and since we already know that their mass is larger than a few GeV, that number is suppressed even further and there is no chance to observe them through the Υ width and the Υ-η_b splitting. If the gluino mass is in the GeV region, then sensitive experiments of widths and splittings in higher quarkonia systems can reveal its existence; we discuss such systems briefly below.

Our suggestion to hunt for light colored scalars in heavy $Q\bar{Q}$ systems seems more feasible than previous suggestions, but is less practical than other ones in the case of gluinos (except maybe, for $t\bar{t}$ states). It was suggested that light colored scalars form color-singlet bound states.[6] Unfortunately, these new additions to meson spectroscopy resemble glueballs, and estimates of their properties involve large uncertainties inherent in hadronic bound-state problems, especially with low-mass constituents. Other suggestions[5] to search for light colored scalars in high-p_T inclusive and two-jet production suffer from higher-order uncertainties and from our ignorance regarding hadronization. On the other hand, higher-order processes contributing through the modification of the gluon vacuum polarization as discussed here affect significantly the total hadronic widths of heavy $Q\bar{Q}$ states calculated in strong-interaction perturbation theory. Uncertainties such as renormalization-scheme dependence cancel by taking appropriate ratios as presented below, and relativistic corrections are expected to be smaller than the effects of scalars.

The signatures of gluinos should be searched for in observables other than the widths and hyperfine splittings of the Υ system described here, and we refer the reader to Refs. 1 and 7–10 for details about some experimental signatures for gluinos.

The paper is organized as follows. In Sec. II we present a general scheme-independent formalism for any addition to higher-order corrections. In Sec. III the gluon vacuum-polarization functions for scalars and fermions are presented, and then employed to obtain the contributions of new scalars and fermions to the hyperfine splitting similar to Ref. 12, which gives the standard QCD result. In Sec. IV we apply the methods of Ref. 13 (a calculation in standard QCD) and obtain the next-to-leading-order corrections due to new particles to the decay rate of pseudoscalar quarkonium.[14] The core of our paper is in Sec. V, where we calculate the effect of new scalars and fermions on the hadronic width of the $^3S_1$ state of quarkonium, without using a gluon mass to control infrared divergences, and without the help of involved numerical methods; here we differ from the methods of Ref. 15 (again, a calculation in standard QCD). In Sec. VI we find the contribution of the new particles to the observables formed in Sec. II, and discuss the implications for the Υ and higher $Q\bar{Q}$ systems; a brief discussion of possible uncertainties is also presented. Finally, our results are summarized in Sec. VII.

II. EXPERIMENTAL SIGNATURES OF ADDITIONS TO HIGHER-ORDER CORRECTIONS

A typical result for a decay width or splitting in a heavy quark system has the following form in standard QCD:

$$\Gamma_n = a_n \frac{|\psi(0)|^2}{m^2} \alpha_s{}^n \left[1+\frac{\alpha_s}{\pi}K_n\right], \tag{1}$$

where a_n and K_n are known numerical dimensionless constants, $\psi(0)$ is the wave function at the origin, and m is the heavy quark mass. To provide for an experimental verification of K_n in a renormalization-scheme-independent way, and without reference to a particular potential model one needs three independent observables Γ_n, Γ_p, Γ_q so as to eliminate the two unknowns α_s and $|\psi(0)|^2$. This way, defining

$$r_{npq} \equiv \left[\frac{\Gamma_n/a_n}{\Gamma_q/a_q}\right]^{1/(n-q)} \Big/ \left[\frac{\Gamma_p/a_p}{\Gamma_q/a_q}\right]^{1/(p-q)}, \tag{2}$$

we obtain, within the order considered

$$r_{npq} = 1+\frac{\alpha_s}{\pi}\left[\frac{K_n-K_q}{n-q}-\frac{K_p-K_q}{p-q}\right], \tag{3}$$

where α_s in Eq. 3 is to be understood as a leading-order parameter

$$\alpha_s = \left[\frac{\Gamma_n/a_n}{\Gamma_q/a_q}\right]^{1/(n-q)}. \tag{4}$$

Suppose that new particles contribute an additional term $\tilde{K}_n$, thus modifying the higher-order constant K_n into

$$K_n' = K_n + \tilde{K}_n . \tag{5}$$

Subsequently, the modified observable will be

$$\Gamma_n' = a_n \frac{|\psi(0)|^2}{m^2}\alpha_s{}^n\left[1+\frac{\alpha_s}{\pi}K_n'\right] = \Gamma_n\left[1+\frac{\alpha_s}{\pi}\tilde{K}_n\right], \tag{6}$$

where in the last step we have again neglected terms of order α_s^{n+2}. Defining similarly to Eq. (2),

$$r_{npq}' \equiv \left[\frac{\Gamma_n'/a_n}{\Gamma_q'/a_q}\right]^{1/(n-q)} \Big/ \left[\frac{\Gamma_p'/a_p}{\Gamma_q'/a_q}\right]^{1/(p-q)}, \tag{7}$$

we find

$$r_{npq}' = 1+\frac{\alpha_s}{\pi}\left[\frac{K_n'-K_q'}{n-q}-\frac{K_p'-K_q'}{p-q}\right]. \tag{8}$$

Therefore, for the ratio of r's, the following expression is obtained:

$$\rho_{npq} \equiv \frac{r_{npq}'}{r_{npq}} = 1+\frac{\alpha_s}{\pi}\left[\frac{\tilde{K}_n-\tilde{K}_q}{n-q}-\frac{\tilde{K}_p-\tilde{K}_q}{p-q}\right]. \tag{9}$$

A search for experimental signatures of an addition to higher-order corrections should proceed as follows:

(1) Measure $\Gamma_n, \Gamma_p, \Gamma_q$.

(2) Extract the value of α_s as given in Eq. (4).

(3) Form r_{npq} as defined in Eq. (2).

(4) Compare the measured value of r_{npq} with its expression in standard QCD as in Eq. (3). Any significant discrepancy may indicate a signature for additional contributions to the higher-order terms, in that case r should be given by Eq. (8).

The ratio ρ_{npq} defined in Eq. (9), once calculated within a certain variant of QCD, will enable us to find out whether that variant has a chance of leading to an experimental signature. For example, if one's favorite extension of QCD gives $\rho_{npq}=1.2$, then each one of $\Gamma_n, \Gamma_p, \Gamma_q$ should be measured with an accuracy of the order of 10%, assuming that all the theoretical uncertainties are negligible.

Our aim is to obtain ρ_{npq} in heavy-quark systems, for extensions of QCD which introduce extra light, colored scalars, or fermions that do not couple to quarks, by calculating the modifications they induce in the next-to-leading-order terms. To this end, since these new particles affect widths and hyperfine splittings only through the gluon vacuum-polarization function, we will now compute their contributions to that function.

III. CONTRIBUTIONS OF SCALARS AND FERMIONS TO THE GLUON VACUUM-POLARIZATION FUNCTION AND TO HYPERFINE SPLITTING

In this section we will present the contributions of scalars and fermions to the gluon vacuum-polarization function $\Pi(k^2)$, using the dimensional-regularization scheme[16] in D dimensions with

$$D=4-\epsilon\ . \quad (10)$$

By using Feynman rules for the coupling of scalars [Fig. 1(a)] and fermions [Fig. 1(b)] to gluons

$$\text{scalar-scalar-gluon:}\quad -igT^a_{ij}(p+p')_\mu\ ,$$
$$\text{scalar-scalar-gluon-gluon:}\quad ig^2\{T^a,T^b\}_{ij}g_{\mu\nu}\ , \quad (11)$$

$$\text{fermion-fermion-gluon:}\quad -igT^a_{ij}\gamma_\mu\ , \quad (12)$$

where g is related to α_s through $\alpha_s=g^2/4\pi$ and T is a matrix representation of $SU(3)_c$, we compute their contribution to $\Pi(k^2)$ defined by

$$i\Pi^{ab}_{\mu\nu}=i\delta_{ab}(-k_\mu k_\nu+k^2g_{\mu\nu})\Pi(k^2)\ . \quad (13)$$

With the help of standard techniques of dimensional regularization[16] one obtains for the contribution of massless scalars and fermions shown in Fig. 2 (mass singularities are absent, of course, from all observables) the following expressions:

$$\Pi^{\text{scalars}}(k^2)=-\frac{\alpha_s}{4\pi}T(R_s)f\left[\frac{\epsilon}{2}\right]\Gamma\left[\frac{\epsilon}{2}\right]\left[\frac{-k^2}{4m^2}\right]^{-\epsilon/2}\int_0^1 d\alpha(1-2\alpha)^2[\alpha(1-\alpha)]^{-\epsilon/2}\ , \quad (14)$$

$$\Pi^{\text{fermions}}(k^2)=-\frac{2\alpha_s}{\pi}T(R_f)f\left[\frac{\epsilon}{2}\right]\Gamma\left[\frac{\epsilon}{2}\right]\left[\frac{-k^2}{4m^2}\right]^{-\epsilon/2}\int_0^1 d\alpha[\alpha(1-\alpha)]^{1-\epsilon/2}\ , \quad (15)$$

where

$$f\left[\frac{\epsilon}{2}\right]\equiv\left[\frac{4\pi\mu^2}{4m^2}\right]^{\epsilon/2}\Gamma\left[1+\frac{\epsilon}{2}\right]\ , \quad (16)$$

with μ a unit of mass, absent from physical observables. The group-theoretical factors[17] $T(R)$ refer to scalars and fermions in the $SU(3)_c$ representations R_s and R_f, respectively, where it is understood that for a real representation one should replace $T(R)$ by $T(R)/2$; therefore for scalars in the $\underline{27}$ representation $T(R_s)/2=\frac{27}{2}$, while for gluinos $T(R_f)/2=\frac{3}{2}$. Performing the integration in Eqs. (14) and (15), we find

$$\Pi^{\text{scalars}}(k^2)=-\frac{\alpha_s}{4\pi}T(R_s)f\left[\frac{\epsilon}{2}\right]\Gamma\left[\frac{\epsilon}{2}\right]\left[\frac{-k^2}{4m^2}\right]^{-\epsilon/2}\times\frac{[\Gamma(1-\epsilon/2)]^2}{\Gamma(2-\epsilon)}\frac{1}{3-\epsilon}\ , \quad (17)$$

$$\Pi^{\text{fermions}}(k^2)=-\frac{2\alpha_s}{\pi}T(R_f)f\left[\frac{\epsilon}{2}\right]\Gamma\left[\frac{\epsilon}{2}\right]\left[\frac{-k^2}{4m^2}\right]^{-\epsilon/2}\times\frac{[\Gamma(2-\epsilon/2)]^2}{\Gamma(4-\epsilon)}\ . \quad (18)$$

We let $\epsilon\rightarrow0$, and write in a unified notation

$$\Pi(k^2)=C(-k^2)^{-\epsilon/2} \quad (19)$$

with

$$C=-\frac{\alpha_s}{\pi}f\left[\frac{\epsilon}{2}\right](4m^2)^{\epsilon/2}\left[\frac{a}{\epsilon}+b\right]\ , \quad (20)$$

where

$$a^{\text{scalars}}=T(R_s)\tfrac{1}{6}\ ,$$
$$b^{\text{scalars}}=T(R_s)[\tfrac{1}{12}\Gamma'(1)+\tfrac{2}{9}]\ , \quad (21)$$

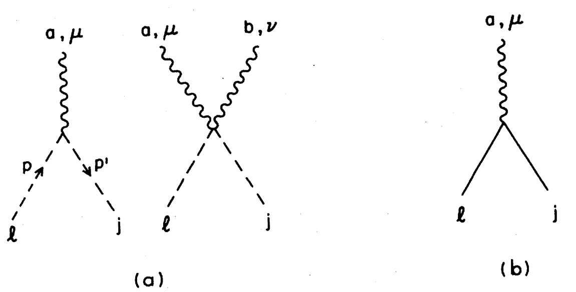

FIG. 1. The coupling of colored scalars (a) and fermions (b) to gluons. Wavy, dashed, and solid lines denote gluons, scalars, and fermions, respectively.

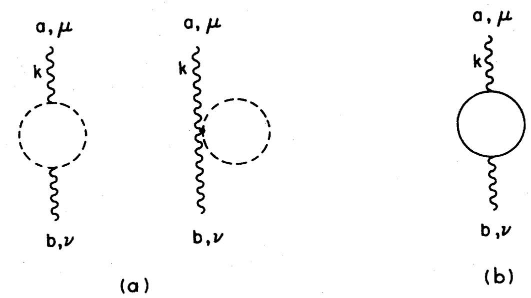

FIG. 2. The contribution of colored scalars (a) and fermions (b) to the gluon vacuum-polarization function.

$$a^{\text{fermions}}=T(R_f)\tfrac{2}{3}\ ,$$
$$b^{\text{fermions}}=T(R_f)[\tfrac{1}{3}\Gamma'(1)+\tfrac{5}{9}]\ . \tag{22}$$

In the following, $\Gamma'(1)$ is irrelevant since it is absorbed in the renormalized coupling constant.

The hyperfine splitting

$$\Gamma_1\equiv\Delta E(^3S_1-{}^1S_0)\ , \tag{23}$$

is affected by extra scalars and fermions. This effect is simply obtained from Eqs. (21) and (22), similarly to the hyperfine splitting calculated in Ref. 12 for standard QCD; in fact, the calculation here is even simpler, since only the vacuum-polarization graphs contribute. Writing the modified splitting as in Eq. (6), we have

$$\widetilde{K}_1^{\text{scalars}}=-\tfrac{2}{9}T(R_s)\ , \tag{24}$$

$$\widetilde{K}_1^{\text{fermions}}=-\tfrac{5}{9}T(R_f)\ . \tag{25}$$

The sign is of course as for quarks, and opposite to the sign coming from gluons.

The latter form of $\Pi(k^2)$ will also enable us to calculate the contributions of colored scalars and fermions to the hadronic decay widths of 0^{-+} and 1^{--} heavy-quark states. That calculation will be more involved than for hyperfine splitting, especially for 1^{--} states, since phase-space integrations should be performed.

IV. EFFECT ON THE HADRONIC WIDTH OF 0^{-+} STATES

Radiative corrections to the hadronic width of ground-state heavy paraquarkonium, which in lowest order decays via two gluons,[18] were calculated in standard perturbative QCD.[19,13] Although for the charmonium system the perturbation expansion is unreliable, the next-to-leading order correction becomes small enough for a $b\bar{b}\ 0^{-+}$ state.

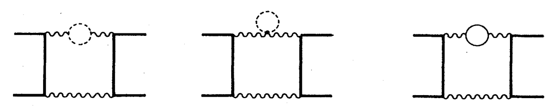

FIG. 3. The contribution of colored scalars (a) and fermions (b) to imaginary parts relevant for the first-order radiative corrections to 0^{-+} decay. Heavy lines denote heavy quarks. For 1^{--} decay there is an extra gluon connecting the heavy quark lines.

In Ref. 19 a gluon mass is introduced to control infrared divergences. This procedure complicates the calculation, and it was subsequently observed[13] that a much simpler and more elegant calculational procedure results from using massless kinematics with dimensional regularization. Needless to say, the results agree with each other, and the agreement takes place for the contribution from each infrared and mass singularity-free set of diagrams. In this section we employ the methods of Ref. 13 to calculate the modifications to the standard radiative-correction result, from colored, unflavored scalars and fermions.

Scalars and fermions discussed in this paper will contribute to the next-to-leading order of the imaginary parts of forward elastic $Q\bar{Q}$ scattering only through the diagrams in Fig. 3. These diagrams have both two- and three-particle cuts. The width of an S-wave quarkonium is

$$\Gamma(Q\bar{Q}\to X)=v\sigma(Q\bar{Q}\to X)\,|\,\psi(0)\,|^2\ . \tag{26}$$

Denoting by P, k, and k_1 the momenta of the $Q\bar{Q}$ state, and of gluon lines with and without corrections to the vacuum-polarization function, respectively, and denoting by $\Phi(2)$ the two-body massless phase space, the contributions to two- and three-body final states are

$$v\sigma^{(1)}(2\text{ body})=v\sigma^{(0)}\frac{1}{\Phi(2)(2\pi)^{D-2}}\int d^Dk\,d^Dk_1\delta(k^2)\theta(k_0)\delta(k_1{}^2)\theta(k_{10})\delta^D(P-k_1-k)[\Pi(k^2+i0)+\Pi(k^2-i0)]\ , \tag{27}$$

$$v\sigma^{(1)}(3\text{ body})=v\sigma^{(0)}\frac{1}{\Phi(2)(2\pi)^{D-2}}\int d^Dk\,d^Dk_1\theta(k_0)\delta(k_1{}^2)\theta(k_{10})\delta^D(P-k_1-k)\frac{1}{k^2}[-\epsilon\Pi(-k^2)]\ . \tag{28}$$

The superscripts (1) and (0) on σ denote next-to-leading and leading orders, respectively.

We are interested in the total width, which is, of course, finite. Since the contributions of the diagrams in Fig. 3 to two-particle cuts [Eq. (27)] are all divergent, it is sufficient to consider only three-particle cuts as given by Eq. (28). As a check of the calculation, one can make sure that for two-particle cuts infinities cancel as required. Furthermore, we can easily verify the results of Ref. 13 in standard QCD. Inserting in Eqs. (28) the universal form of Π as given in Eq. (19), we get, after performing the integration over the D-dimensional δ function in the rest frame of the decaying particle,

$$\begin{aligned}v\sigma^{(1)}(3\text{ body})&=-v\sigma^{(0)}\frac{C\epsilon}{\Phi(2)(2\pi)^{D-2}}\int d^Dk_1\theta(2m-k_{10})\theta(k_{10})\delta(k_1{}^2)[(P-k_1)^2]^{-\epsilon/2-1}\\&=-v\sigma^{(0)}\frac{C\epsilon}{\Phi(2)(2\pi)^{D-2}}\frac{1}{2}\int d\Omega(D)(4m)^{-\epsilon/2-1}\int_0^m dk_{10}k_{10}{}^{1-\epsilon}(m-k_{10})^{-\epsilon/2-1}\ .\end{aligned} \tag{29}$$

The limits $0\le k_{10}\le m$ in the last step follow from the θ functions and from $k_1{}^2=0$, $k^2>0$. The two-body massless phase space in the rest frame of the decaying particle can be expressed in terms of the volume element in D dimensions as follows:

$$\Phi(2)=\frac{m^{-\epsilon}}{8(2\pi)^{D-2}}\int d\Omega(D)\ . \tag{30}$$

Therefore,

$$v\sigma^{(1)}(3\text{ body})=-v\sigma^{(0)}(4m^2)^{-\epsilon/2}C\epsilon\frac{\Gamma(2-\epsilon)\Gamma(-\epsilon/2)}{\Gamma(2-3\epsilon/2)}\ . \tag{31}$$

Letting $\epsilon\to 0$, we find with the help of Eq. (20) the final result for this section, where only the finite part will be relevant for future applications:

$$v\sigma^{(1)}(3\text{ body})=-v\sigma^{(0)}\frac{\alpha_s}{\pi}f\left[\frac{\epsilon}{2}\right]\left[\frac{2a}{\epsilon}+a+2b\right]\ . \tag{32}$$

This result is applicable to any modification of a 0^{-+} total width resulting from additional contributions to the vacuum-polarization function of gluons.

Before inserting numbers into the last equation, we turn now to a similar calculation for the total width of a 1^{--} state.

V. EFFECT ON THE HADRONIC WIDTH OF 1^{--} STATES

In lowest-order perturbative QCD, an orthoquarkonium state decays hadronically by the emission of three gluons.[18] The next-to-leading order in standard QCD was calculated[15] and found to be manageable for the $b\bar{b}$ and higher systems. That calculation involved multidimensional numerical integrations, and is rather lengthy. As we will show, at least the part of the radiative corrections coming from gluon vacuum-polarization graphs is relatively simple and involves only a single two-dimensional numerical integral; it is only slightly more complicated than the calculation for a 0^{-+} state in the previous section.

Again, new unflavored, colored scalars and fermions will modify the total 1^{--} hadronic width only through the graphs in Fig. 3, but now with an extra gluon connecting the two heavy-quark parts. Since we are interested in the finite part of the calculation, only four-body final states count; it is the presence of these final states that complicates the calculation as compared to the previous one. Experimentally, however, it will be long before widths of 0^{-+} states of heavy quarks will be known at the required accuracy, thus making 1^{--} states much more practical as probes for new physics.

Denoting by P the momentum of the $Q\bar{Q}$ state, by k the momentum of the gluon line with the modified vacuum-polarization function, and by k_1, k_2 the momentum of the remaining two gluon lines, we obtain—with $\Phi(3)$ standing for the three-body massless phase space—the expression

$$v\sigma^{(1)}(4\text{ body})=v\sigma^{(0)}\frac{1}{\Phi(3)}\int dP_{\mathrm{LI}}(P=k+k_1+k_2)\frac{1}{\delta(k^2)}\frac{1}{k^2}[-\epsilon\Pi(-k^2)]\ , \tag{33}$$

where dP_{LI} is the Lorentz-invariant phase-space volume element. The division by $\delta(k^2)$ indicates the absence of the $k^2=0$ condition from the Lorentz-invariant phase-space integration. By using Eq. (19) and decomposing the three-body phase space into a pair of two-body phase spaces, we find

$$v\sigma^{(1)}(4\text{ body})=-v\sigma^{(0)}\frac{C\epsilon}{\Phi(3)}\int dP_{\mathrm{LI}}(P=k+K)\frac{1}{\delta(k^2)\delta(K^2)}dP_{\mathrm{LI}}(K=k_1+k_2)(k^2)^{-\epsilon/2-1}\ . \tag{34}$$

K is of course the total momentum of the two gluon lines without the Π. The second two-body phase space can be easily performed with the help of [compare with Eq. (30)]

$$dP_{\mathrm{LI}}(K=k_1+k_2)=\frac{(K^2)^{-\epsilon/2}}{8(2\pi)^{D-2}}\frac{1}{2^{D-4}}\int d\Omega(D)\ , \tag{35}$$

and for the first one we use

$$dP_{\mathrm{LI}}(P=k+K)=(2\pi)^{2-D}\int d^Dk\,d^DK\,\delta^D(P-k-K)\theta(k_0)\theta(K_0)\delta(k^2)\delta(K^2)\ , \tag{36}$$

to obtain, after performing the integration over K with the help of the D-dimensional δ function

$$v\sigma^{(1)}(4\text{ body})=-v\sigma^{(0)}\frac{C\epsilon}{\Phi(3)(2\pi)^{2D-3}}\frac{1}{2^{D-4}8}\int d\Omega(D)\int d^Dk\frac{1}{(k^2-2P\cdot k+P^2)^{-\epsilon/2}}\frac{1}{(k^2)^{\epsilon/2+1}}\ , \tag{37}$$

where the θ functions were dropped for brevity, but the conditions they set will be fulfilled. Specifying now to the rest system of the decaying state and employing Feynman parametrization, we have

$$v\sigma^{(1)}(4\text{ body})=-v\sigma^{(0)}\frac{C\epsilon}{\Phi(3)(2\pi)^{2D-3}}\frac{1}{2^{D-4}8}\int d\Omega(D)\frac{\Gamma(1+\epsilon)}{\Gamma(\epsilon/2)\Gamma(1+\epsilon/2)}\int d^Dk\,dz\frac{z^{\epsilon/2-1}(1-z)^{\epsilon/2}}{(k^2-4zmk_0+4zm^2)^{1+\epsilon}}\ . \tag{38}$$

We now integrate over the square of the length of the three-vector from 0 to $k_0{}^2-4zmk_0+4zm^2$, and then make a change of variable from k_0 to $x=k_0/2m$ to obtain

$$v\sigma^{(1)}(4\text{ body})=-v\sigma^{(0)}\frac{C\epsilon}{\Phi(3)(2\pi)^{2D-3}}\frac{(2m)^{2-3\epsilon}}{2^{D-4}16}\left[\int d\Omega(D)\right]^2\frac{\Gamma(1+\epsilon)B(\frac{3}{2}-\epsilon/2,-\epsilon)}{\Gamma(\epsilon/2)\Gamma(1+\epsilon/2)}I\ , \tag{39}$$

where

$$I=\int_0^1 dz\, z^{\epsilon/2-1}(1-z)^{\epsilon/2}\int_0^1 dx(x^2-2zx+z)^{1/2-3\epsilon/2}\ . \qquad (40)$$

The three-body massless phase space in the rest frame of the decaying particle is

$$\Phi(3)=\frac{\Gamma(1-\epsilon/2)\Gamma(2-\epsilon)m^{2-2\epsilon}(2\pi)^{2\epsilon-5}}{16\Gamma(3-3\epsilon/2)}\left[\int d\Omega(D)\right]^2 . \qquad (41)$$

Therefore,

$$v\sigma^{(1)}(4\text{ body})=-v\sigma^{(0)}(4m^2)^{-\epsilon/2}C\epsilon\frac{4^{1-\epsilon/2}\Gamma(3-3\epsilon/2)\Gamma(1+\epsilon)B(\frac{3}{2}-\epsilon/2,-\epsilon)}{\Gamma(1-\epsilon/2)\Gamma(2-\epsilon)\Gamma(\epsilon/2)\Gamma(1+\epsilon/2)}I\ . \qquad (42)$$

The only numerical integration left is the double integral I in Eq. (40). It is most simply evaluated by introducing a parameter $y>2$ and then changing variables from z to $t^{y/\epsilon}$. Therefore,

$$I=\frac{y}{\epsilon}\int_0^1 dt\, t^{y/2-1}(1-t^{y/\epsilon})^{\epsilon/2}\times\int_0^1 dx(x^2-2t^{y/\epsilon}x+t^{y/\epsilon})^{1/2-3\epsilon/2}\ , \qquad (43)$$

where I is independent of y. We find

$$I=\frac{1}{\epsilon}+1.69\pm0.01\ . \qquad (44)$$

Finally, letting $\epsilon\to0$ in Eq. (42), we obtain with the help of Eq. (20) that the contribution of new scalars and fermions to the total hadronic width of a $^3S_1$ state is

$$v\sigma^{(1)}(4\text{ body})=-v\sigma^{(0)}\frac{\alpha_s}{\pi}f\left[\frac{\epsilon}{2}\right]\left[\frac{4a}{\epsilon}+9.76a-12a\ln2+4b\right]. \qquad (45)$$

Since a and b are known for any given modification to the gluon vacuum-polarization function, one can use Eq. (45) to get the corresponding change in the width of 1^{--} states.

In the next section we will discuss the implications of Eqs. (32) and (45) for heavy $Q\bar{Q}$ systems.

VI. NUMERICAL RESULTS AND DISCUSSION

In Sec. II we formed observables which are subtraction scheme independent. Quantities of this type are suitable for an unambiguous test of the higher-order terms calculated in the previous three sections. In order to find out the size of the effects expected in extensions of standard QCD requiring new scalars or fermions that modify widths through the gluon vacuum-polarization function, one should calculate the ratio ρ defined in Eq. (9). First, we present in Table I the values of $\tilde{K}_n$ [defined in Eq. (5)] for four observables, both for new scalars and for new fermions. The observables are as follows.

(a) The hyperfine splitting $\Delta E(^3S_1-{}^1S_0)$ discussed in Sec. III [see Eqs. (24) and (25)]. For this process $n=1$, as defined in Eq. (1).

(b) The total hadronic width of $^1S_0$ states. The value for $\tilde{K}$ is obtained using Eqs. (21), (22), and (32). Here $n=2$.

(c) The total hadronic width of $^3S_1$. Its $\tilde{K}$ value is inferred from Eqs. (21), (22), and (45). In this case $n=3$.

(d) Furthermore, we will also use $\tilde{K}=0$ (and of course $n=0$) for the electromagnetic process $^3S_1\to e^+e^-$.

From Table I and Eq. (9), we find for scalars

$$\rho_{021}^{\text{scalars}}=1+\frac{\alpha_s}{\pi}T(R_s)\tfrac{1}{6}\ , \qquad (46)$$

$$\rho_{031}^{\text{scalars}}=1+\frac{\alpha_s}{\pi}T(R_s)0.23 \qquad (47)$$

and for fermions

$$\rho_{021}^{\text{fermions}}=1+\frac{\alpha_s}{\pi}T(R_f)\tfrac{2}{3}\ , \qquad (48)$$

$$\rho_{031}^{\text{fermions}}=1+\frac{\alpha_s}{\pi}T(R_f)0.76\ . \qquad (49)$$

Certainly the values for ρ_{031} are more interesting from the experimental point of view than for ρ_{021}, since pseudosca-

TABLE I. Modifications to physical quantities in heavy-quark systems from new scalars and fermions. $\tilde{K}_n$ and n are defined in Eqs. (5) and (1), respectively.

Physical quantity	$\tilde{K}_n$	n
$\Gamma(^3S_1\to e^+e^-)$	0	0
$\Delta E(^3S_1-{}^1S_0)$	$-\frac{2}{9}T(R_s)-\frac{5}{9}T(R_f)$	1
$\Gamma(^1S_0\to\text{hadrons})$	$-\frac{11}{18}T(R_s)-\frac{16}{9}T(R_f)$	2
$\Gamma(^3S_1\to\text{hadrons})$	$-1.13T(R_s)-3.18T(R_f)$	3

TABLE II. The ratio ρ_{npq} built from three physical quantities outside and in standard QCD [Eqs. (9)] estimated for $b\bar{b}$ and $t\bar{t}$ systems with $m_t=30$ GeV and $\alpha_s=0.2$ for $b\bar{b}$. The subscripts of ρ can be deduced from Table I.

Observable	$b\bar{b}$	$t\bar{t}$
$\rho_{021}^{\text{scalars}}$	1.14	1.10
$\rho_{031}^{\text{scalars}}$	1.20	1.14
$\rho_{021}^{\text{fermions}}$	1.06	1.05
$\rho_{031}^{\text{fermions}}$	1.07	1.05

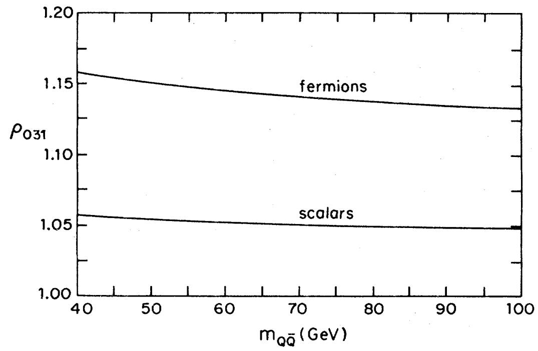

FIG. 4. The ratio ρ_{031} built from three physical quantities outside and in standard QCD [Eq. (9)] estimated for heavy $Q\bar{Q}$ systems, as a function of $m_{Q\bar{Q}}$.

lar quarkonia are more elusive than vector states.

Specifying to new scalars and fermions in the $\underline{27}$ and $\underline{8}$ representations of color, we should insert in Eqs. (46)–(49) $\frac{27}{2}$ and $\frac{3}{2}$ for $T(R_s)$ and $T(R_f)$, respectively, since the representations are real. The value of α_s should be, of course, taken from experiment as argued in Sec. III, using Eq. (4). Nevertheless, to get a feeling for the experimental accuracies needed we use the "educated guess" $\alpha_s=0.20$ for the Υ system, and present in Table II the corresponding values for ρ. We extrapolate to higher $Q\bar{Q}$ systems by using the q^2 dependence of α_s, and as an example present in Table II ρ values for $m_t=30$ GeV where t is a heavy quark, yet to be observed.

In Fig. 4 we present ρ_{031} for gluinos and scalars as a function of $m_{Q\bar{Q}}$ for heavy-quark systems, where we use α_s as in standard QCD. Of course the recommended procedure is as outlined in Sec. II. If one is more ambitious and considers $\Gamma(Q\bar{Q}\to g\tilde{g}\tilde{g}g)/\Gamma(Q\bar{Q}\to ggg)$ for $m_{\tilde{g}}/m_Q \to 0$ with α_s from QCD, then our results extrapolate nicely to the results of Ref. 9.

We see that the largest ρ value involves the Υ hadronic width and scalars in the $\underline{27}$ representation of $SU(3)_c$, i.e., $\rho_{031}=1.20$ which means a 20% correction to standard QCD results. This is rather fortunate since an order of 10% accuracy can be expected for each one of the physical quantities in ρ_{031}, i.e, the widths for $\Upsilon\to e^+e^-$ and $\Upsilon\to$hadrons, and the hyperfine splitting $m(\Upsilon)-m(\eta_b)$. This may not be the case for ρ_{021} which involves the η_b width. Furthermore, the scalars that are supposed to break QCD spontaneously are probably very light if they exist[5,6], and thus our estimates of ρ^{scalars} are reliable, while gluinos are much more massive,[10] which turns ρ^{fermions} in Table II into upper limits.

The main uncertainty in our results comes from neglect of relativistic corrections. Although we have not used any specific potential model, our results certainly depend upon the validity of the nonrelativistic treatment common to all applications of perturbative QCD to heavy-quark systems. As pointed out in Ref. 20, the task of evaluating relativistic corrections is hopeless at present as long as a much deeper understanding of bound states in QCD is missing. Nevertheless, it is estimated[20] that these corrections will be less than 10% for the Υ system (see also Ref. 21).

VII. SUMMARY

We have calculated the contributions of new light colored particles—scalars and fermions—to observables formed from widths and hyperfine splittings in heavy-quark systems. Our results are summarized in Table II, with ρ defined in Eq. (9). The most significant result is $\rho_{031}^{\text{scalars}}=1.20$ for the Υ system where the subscripts 0, 3, 1 indicate that the scheme-independent ρ contains the following physical quantities: $\Gamma(^3S_1\to e^+e^-)$, $\Gamma(^3S_1\to$hadrons), and $\Delta E(^3S_1-{}^1S_0)$. To observe the influence of scalars required in the glow model of Slansky, Goldman, and Shaw,[4] a measurement of each of the above quantities on a level of 10% accuracy should enable us, following the method described in Sec. II, to see the influence of such scalars. It is interesting that QCD breaking can manifest itself significantly not only through the appearance of free fractional charges. If the speculative suggestion of QCD breaking will turn into reality, then more accurate experiments such as angular distributions in $1^{--}\to 4$ jets will be necessary to confirm the properties of the conjectured scalars.

For gluinos we cannot offer bright prospects in total widths of heavy-quark systems, as evident from ρ^{fermions} in Table II. Even for the unrealistic case of massless gluinos which we consider here, uncertainties from relativistic corrections will not allow a meaningful test of supersymmetry in the Υ system. There is a slight chance that for higher $Q\bar{Q}$ states if $m_Q \gg m_{\tilde{g}}$ and if a few percent experimental accuracies are possible, the effect of gluinos will be visible; uncertainties from relativistic corrections are expected to be negligible by then.

Finally, let us point out that our methods could be applied to any extension of QCD. We have demonstrated that massless kinematics[13] simplifies significantly width calculations for heavy-quark systems.

ACKNOWLEDGMENTS

We would like to thank E. Reya for helpful discussions. The research of G. E. was supported in part by the Fund for Basic Research administered by the Israel Academy of Sciences and Humanities.

[1]G. L. Kane and M. L. Perl, SLAC report, 1982 (unpublished), and references therein.

[2]G. S. LaRue, W. M. Fairbank, and A. F. Hebard, Phys. Rev. Lett. 38, 1011 (1977); G. S. LaRue, J. D. Phillips, and W. M. Fairbank, *ibid.* 42, 142 (1979); 42, 1019E (1979); 46, 967 (1981).

[3]A. De Rújula, R. C. Giles, and R. L. Jaffe, Phys. Rev. D 17, 285 (1978); *ibid.* 22, 227 (1980).

[4]R. Slansky, T. Goldman, and G. Shaw, Phys. Rev. Lett. 47, 887 (1981).

[5]M. Glück and E. Reya, Phys. Rev. Lett. 48, 662 (1982).

[6]S. Dawson and C. T. Hill, Ann. Phys. (N.Y.) 148, 282 (1983).

[7]P. Fayet, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982,* edited by P. Petiau and M.

Porneuf [J. Phys. (Paris) Colloq. 43 (1982)], and references therein.
[8]P. Fayet, Phys. Lett. 78B, 417 (1978); G. Farrar and P. Fayet, *ibid.* 69B, 442 (1978); 76B, 575 (1978).
[9]B. A. Campbell, J. Ellis, and S. Rudaz, Nucl. Phys. B198, 1 (1982).
[10]G. L. Kane and J. P. Leveille, Phys. Lett. 112B, 227 (1982).
[11]H. Georgi, Phys. Rev. D 22, 228 (1980).
[12]R. Barbieri, R. Gatto, and E. Remiddi, Phys. Lett. 106B, 497 (1981); W. Buchmüller, Y. J. Ng, and S.-H. H. Tye, Phys. Rev. D 24, 3003 (1981).
[13]K. Hagiwara, C. B. Kim, and T. Yoshino, Nucl. Phys. B177, 46 (1981).
[14]G. Eilam and M. Glück, Phys. Lett. 120B, 423 (1983).
[15]P. B. Mackenzie and G. P. Lepage, Phys. Rev. Lett. 47, 1244 (1981).
[16]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).
[17]D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).
[18]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); Phys. Rev. D 12, 1404 (1975).
[19]R. Barbieri, G. Curci, E. d'Emilio, and E. Remiddi, Nucl. Phys. B154, 535 (1979); E. Remiddi, *Theory of Fundamental Interactions,* proceedings of the International School of Physics, "Enrico Fermi," Course LXXXI, Varenna, Italy, 1980, edited by G. Costa and R. R. Gatto (North-Holland, Amsterdam, 1982), p. 18.
[20]W. Buchmüller and S.-H. H. Tye. Phys. Rev. D 24, 132 (1981).
[21]W.-Y. Keung and I. J. Muzinich, Phys. Rev. D 27, 1518 (1983).