Initial energy density of quark-gluon plasma in relativistic heavy-ion collisions

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We estimate the initial rapidity distribution and the initial energy density in the central rapidity region of relativistic heavy-ion collisions by using a multiple-collision model and the nuclear-thickness function of Glauber. The parameter of the rapidity distribution is determined from the experimental multiplicity data of $p\alpha$, dd, $\alpha\alpha$, pA, π^+A, K^+A, Si+Ag, and Ca+C reactions. We find that the initial energy density in the central rapidity region is high. For example, for the head-on collision of ^{238}U on ^{238}U at 30 GeV per nucleon in the center-of-mass system, the maximum energy density is about 10 GeV/fm^3, which may exceed the critical energy density for a phase transition from a confined hadron matter to an unconfined quark-gluon plasma. The initial energy density goes as $A^{1/3}B^{1/3}$ for the collision of two nuclei with mass numbers A and B, and is rather insensitive to impact parameters.

I. INTRODUCTION

Recently, there has been considerable interest in the central rapidity region of highly relativistic heavy-ion collisions.[1–5] Such an interest stems from the possibility of creating hadron matter of high energy density which may exceed the critical energy density for a phase transition between ordinary confined matter and the unconfined quark-gluon plasma.[6] Experimental searches and identification of the quark-gluon plasma may provide new insight into the question of quark confinement. Furthermore, the creation of a domain of high energy density, albeit within a small region of space and time, may allow one to study matter under unusual conditions such as those which exist in the history of the early universe.

High energy density in the central rapidity region is expected for the following reasons. A nucleus-nucleus collision consists of many nucleon-nucleon collisions. For each inelastic nucleon-nucleon collision, particles are produced with a plateau in the central-rapidity region. The width and height increases slowly with energy.[7,8] In a highly relativistic heavy-ion collision, the particles in the central rapidity region are produced outside the colliding nuclei[9] and cannot interact with nucleons in the nuclei. The total number of particles produced after the two nuclei recede from each other is just an additive superposition of those from nucleon-nucleon collisions. Thus, the number of produced particles depends on the frequency of nucleon-nucleon collisions. Because of the leading-particle effect, a nucleon which has suffered a collision will be found as a baryon with comparable momentum and will make further collisions with other nucleons as it transverses the other nucleus. Consequently, in a central collision of two large nuclei (equal nuclei, say), there is a large number of nucleon-nucleon collisions. Furthermore, in the center-of-mass frame, the two colliding nuclei are so much contracted in space due to Lorentz contraction that all the nucleon-nucleon collisions occur at about the same time and the same longitudinal coordinate. With a large number of particles produced within a small space and about the same time, the energy density in the central rapidity region is therefore very high. We shall focus our attention exclusively on the central rapidity region. In this paper, for simplicity of notation, the terms energy density and rapidity distribution will often refer to those in the central rapidity region only.

In the picture developed by Bjorken,[1] the evolution may be divided into two stages. At a time about 1 fm/c after the nuclei interpenetrate each other, there is a domain of high-energy density in the central-rapidity region produced by individual nucleon-nucleon collisions. It has no net baryon number.[10] For ^{238}U + ^{238}U collisions at CERN SPS collider energies (of a few hundred GeV per nucleon), the number of produced particles are numerous enough so that this region rapidly comes into local thermal equilibrium and becomes a quark-gluon plasma. In the second stage, this plasma evolves according to the laws of hydrodynamics, as in the Landau hydrodynamic model.[11] In consequence, the final pion multiplicity should then depend only on the initial entropy density.

The estimate of the initial energy density is quite uncertain. The initial energy density is nonetheless an important physical quantity. It is one of the factors which determine whether the produced matter can undergo phase transition or not. The energy density is inversely proportional to the initial production time t_0, which is taken to be 1 fm/c in Ref. 1. The value of t_0 may be uncertain and is the subject of investigation.[12] We shall examine a different source of uncertainty. In Ref. 1, it was implicitly assumed that the multiplicity of produced particles is directly proportional to the number of wounded nucleons. Multiplicity results were obtained only for equal-mass collisions and could not be compared with the known heavy-ion data for unequal-mass collisions. However, in such a model, the multiplicity predictions for pp and $\alpha\alpha$ differ from the experimental data by a factor of about 8. As the energy density is determined from the multiplicity, the energy density obtained therefrom may be a reasonable approximation for the estimates of the order of magnitude, a better treatment will improve the degree of accuracy. The energy density has also been estimated previously by using the color-neutralization model of Brodsky, Gunion, and Kühn[13]. However, the color-neutralization model gives a central rapidity multiplicity in heavy-ion collisions too low by a factor of 2.[14,4,15] For this reason we wish to obtain a better estimate of the energy density (in the *central rapidity region*). We also wish to examine its spatial dependence, impact-parameter dependence, bombarding-energy dependence, and mass dependence for the collision of equal or unequal nuclei. Many of these dependences have never been investigated before.

As is shown in Ref. 1, the energy density in the central rapidity region at the initial time t_0 is directly proportional to the rapidity density dN^{AB}/dy. It is necessary to evaluate dN^{AB}/dy for the collision of nuclei A and B. As is well known,[16] a simple Glauber multiple-collision model, with additional assumptions concerning the additivity of the rapidity distributions and no secondary collisions of the produced particles, consistently overestimates the total multiplicity and the height of the multiplicity plateau by about 30%. The consistency of this overestimation raises the hope that when one modifies one of the simplifying assumptions, such as the assumption of no energy degradation, the model will give a better description of the dN^{AB}/dy data. Taking into account how a baryon degrades its energy as it travels through a nucleus, we can postulate an incoherent multiple-collision model in which each baryon-baryon collision degrades baryon energies and produces particles in the same way as a baryon-baryon collision in free space.[17,18] We find that this incoherent multiple-collision model with energy-degradation effects can explain the pseudorapidity distribution in nucleon-nucleus collisions.[18] Thus, we understand that the discrepancies between the experimental data and the simple multiple-collision model without the energy-degradation effect arise from the neglect of the energy degradation of the nucleons as they collide. This point is further corroborated from other investigations. The multichain model,[19] which is a multiple-collision model motivated by Reggeon calculus and takes into account the total energy-momentum conservation in a phenomenological way, gives a good description of the rapidity distribution for nucleon-nucleus collisions and for nucleus-nucleus collision as given by the JACEE (Japanese-American cooperative emulsion experiment) data.[14,20] The dual parton model,[21,22] which is another type of multiple-collision model involving quark-partons and effective energy degradation, can also describe the rapidity distributions of the JACEE events.[23] Although the effect of energy degradation needs to be included to describe the experimental data in detail, the basic ingredients of these models are multiple collisions involving the constituents and the additivity of the produced products. It is therefore not surprising that a multiple-collision model including the additivity assumption but without taking into account the effect of energy degradation can still give an approximate description of the multiplicity data consistently within 30%.

In view of the uncertainty in the initial production time t_0 and the large number of quantities we wish to examine, a full multiple-collision model with the explicit inclusion of the energy-degradation effects is not warranted at present. We shall adopt the following alternative semiempirical approach to include the effect of energy degradation approximately, in order to have simple analytical results to estimate the various quantities of interest in the vicinity of the *central rapidity region*. Using the multiple-collision model of Glauber[24] and the assumption of additivity but no energy degradation, we obtain analytical functional form for all the quantities in question. To correct for the effect of energy degradation, a single parameter $r_{\rm rms}$ is adjusted to fit the experimental central-rapidity multiplicity data of $p\alpha$, dd, $\alpha\alpha$, Si + Ag, and Ca + C reactions and the total multiplicity data of pA, π^+A, and K^+A reactions. (Bjorken[1] has suggested a similar procedure to determine the length parameter d_0 of his multiplicity formula.) The success of fitting the multiplicity data provides a useful tool to extrapolate to the unknown central rapidity region of heavy-ion collisions.

This paper is organized as follows. We find the energy density by first determining the rapidity distribution in an inelastic nucleus-nucleus collision. In Sec. II, the initial rapidity distribution is given in terms of the nuclear-thickness function. Numerical evaluation of the nuclear-thickness function shows that it can be well approximated by a Gaussian function. We use the Gaussian form of the thickness function and obtain analytic expressions for the initial rapidity distribution. In Sec. III, we determine the parameter of the multiplicity expressions by fitting the experimental multiplicity data. In Sec. IV, we follow the description of Bjorken to obtain the energy density in the central rapidity region from the rapidity distribution. This is first determined as an average over the overlapping area. We then examine the spatial dependence of the energy density in the transverse direction in Sec. V. Section VI concludes the present discussions.

II. INITIAL RAPIDITY DISTRIBUTION IN HEAVY-ION COLLISIONS

In a multiple-collision model, a nucleon in one nucleus makes many elastic collisions with nucleons in the other nucleus, the probability of collision being given by the thickness function and the total nucleon-nucleon inelastic cross section. A nucleon may change its identity during its passage through the other nucleus, but its baryon number remains unchanged. To apply the model to our problem, additional assumptions need to be made. We shall assume that each nucleon-nucleon or baryon-baryon collision degrades the energies and momenta of the colliding baryons and produces particles outside the nucleus[9] in accordance with the experimental nucleon-nucleon data and that there is no secondary collision between the produced particles and the target or projectile nucleons. Thus, the initial rapidity distribution shortly after the two nuclei interpenetrate each other comes from nucleon-nucleon collisions in an additive manner. For a given configuration, the degree of this additivity increases with the number of nucleon-nucleon collisions.

We consider the collision of two nuclei with mass numbers A and B, and focus our attention on inelastic collision events. We shall use the term nucleon-nucleon (or baryon-baryon) collision to refer only to an inelastic nucleon-nucleon (or baryon-baryon) collision *with particle productions,* and shall study first the average number of nucleon-nucleon (or baryon-baryon) collisions when the two nuclei pass through each other. Following Blankenbecler *et al.*,[25] we use the thickness function of Glauber[24] to determine the average number of inelastic baryon-baryon collisions. We introduce a normalized thickness function for nucleus-nucleus collisions $T(\vec{b})$, given by[24]

$$T(\vec{b})=\int \rho_A(\vec{b}_A,z)\rho_B(\vec{b}_B,z_B)t(\vec{b}-\vec{b}_A-\vec{b}_B)\times d\vec{b}_A dz_A d\vec{b}_B dz_B\ , \tag{2.1}$$

where ρ_A (or similarly, ρ_B) is the normalized density distribution for the nucleus A (or B) and $t(\vec{b})$ is the normalized thickness function for nucleon-nucleon collision. The spatial coordinates $\vec{b}_A,z_A$ (or, similarly, $\vec{b}_B,z_B$) are measured with respect to the center of the nucleus A (or B). The functions ρ, T, and t, are normalized as follows:

$$\int \rho_A(\vec{r})d\vec{r}=\int \rho_B(\vec{r})d\vec{r}=\int T(\vec{b})d\vec{b}=\int t(\vec{b})d\vec{b}=1\ . \tag{2.2}$$

In an inelastic collision between nucleus A and nucleus B, the probability for the occurrence of n inelastic nucleon-nucleon collisions at an impact parameter b is given by

$$P(n,\vec{b})=\frac{\begin{bmatrix} AB \\ n \end{bmatrix}[T(b)\sigma_{\rm in}]^n[1-T(b)\sigma_{\rm in}]^{AB-n}}{1-[1-T(b)\sigma_{\rm in}]^{AB}}\ , \tag{2.3}$$

which is normalized according to

$$\sum_{n=1}^{AB} P(n,\vec{b})=1\ .$$

The quantity $\sigma_{\rm in}$ is the total nucleon-nucleon inelastic cross section for particle production. We shall use the notation that quantities such as $\sigma_{\rm in}$, $dN/dy\ldots$ without a superscript refer to those of the nucleon-nucleon system, while the corresponding quantities $\sigma_{\rm in}^{AB}$, $dN^{AB}/dy,\ldots$ with the superscript AB refer to those of the (nucleus A)-(nucleus B) system. Averaging Eq. (2.3) over n, we obtain the average number of inelastic nucleon-nucleon collisions at an impact parameter b as

$$n(b)=\frac{ABT(b)\sigma_{\rm in}}{1-[1-T(b)\sigma_{\rm in}]^{AB}}\ . \tag{2.4}$$

When we further average over the impact parameters with the proper weighting factor of

$$\{1-[1-T(b)\sigma_{\rm in}]^{AB}\}/\sigma_{\rm in}^{AB}\ ,$$

we obtain the mean number of inelastic nucleon-nucleon collisions in a heavy-ion inelastic collision given by

$$\langle n\rangle=\frac{AB\sigma_{\rm in}}{\sigma_{\rm in}^{AB}}\ , \tag{2.5}$$

where $\sigma_{\rm in}^{AB}$ is given by

$$\sigma_{\rm in}^{AB}=\int d\vec{b}\{1-[1-T(b)\sigma_{\rm in}]^{AB}\}\ . \tag{2.6}$$

In the multiple-collision model we have postulated, baryons degrade their energies as they collide and particles are produced in an additive manner depending on the baryon-baryon energy at the time of collision. The rapidity distribution for the collision of nuclei A and B is given by

$$\frac{dN^{AB}}{dy}(b)=\sum_{n=1}^{AB} P(n,b)\sum_{j=1}^{n}\frac{dN}{dy}(\sqrt{s_j})\ , \tag{2.7}$$

where $\sqrt{s_j}$ is the center-of-mass energy of the colliding baryons in the jth baryon-baryon collision. The evaluation of dN^{AB}/dy requires the knowledge of $\sqrt{s_j}$ as a function of j. In the incoherent-multiple-collision model,[17] we know how the energy is degraded as a nucleon passes through a nucleus. Experimental dN/dy data for a nucleon-nucleon collision are also known.[26,27] We can calculate[18] dN^{pA}/dy for the case of nucleon-nucleus collision with Eq. (2.7) which is found to give good description of the experimental data.

For our purposes, it is useful to approximate Eq. (2.7) in order to make simple estimates of the rapidity distribution in heavy-ion collisions. We discuss first the crudest approximation of no energy degradation so that the rapidity distributions are the same for each nucleon-nucleon or baryon-baryon collision. Within this approximation, one works in the equal-velocity (EV) frame in which the velocity of the nucleus A is equal and opposite to that of the nucleus B. Then, from Eq. (2.7), the initial rapidity distribution for heavy-ion collision at an impact parameter b is related to the rapidity distribution for nucleon-nucleon collision dN/dy for the same nucleon-nucleon center-of-mass energy by

$$\frac{dN^{AB}}{dy}(b)=\frac{dN}{dy}n(b)\ . \tag{2.8}$$

When we average over the impact parameters, we obtain the average initial rapidity distribution given by

$$\left\langle\frac{dN^{AB}}{dy}\right\rangle=\frac{dN}{dy}\frac{AB\sigma_{\rm in}}{\sigma_{\rm in}^{\rm AB}}\ , \tag{2.9}$$

and the total multiplicity N^{AB} given by

$$N^{AB}=N\frac{AB\sigma_{\rm in}}{\sigma_{\rm in}^{AB}}\ , \tag{2.10}$$

where N is the total multiplicity for the nucleon-nucleon collision.

When Eqs. (2.9) and (2.10) are applied to hadron-nucleus collisions with $\sigma_{\rm in}^{AB}$ taken to be the absorption cross section (which is slightly greater than $\sigma_{\rm in}^{AB}$), they are found to be only approximately correct as they consistently overestimate the multiplicity ratio and the multiplicity plateau by 20—30% (Ref. 16). Furthermore, Eq. (2.9) does not reproduce the asymmetrical experimental rapidity distribution.[16] We can understand this discrepancy and the asymmetry as due to the energy loss of the leading baryon. The fractional loss of energy per nucleon-nucleon collision is not small.[27–29,17] Because the height of the nucleon-nucleon multiplicity plateau decreases logarithmically with the decrease of the colliding energy, the contribution due to each subsequent collision of a nucleon is lower than that due to the first collision. The loss of energy of the colliding baryon also leads to a shift in the rapidity distribution towards the target fragmentation region and hence the asymmetry in the distributions. As we mentioned previously, with the knowledge of how a nucleon loses energy in its passage through the other nucleus,[17] we can investigate the multiplicity data in the framework of Eq. (2.7). We find good agreement with experimental data.[18] However, these treatments are complicated in nature. Furthermore, our interest is in the *multiplicity plateau* and not in the fragmentation region. As far as the *multiplicity plateau* and the total multiplicity are concerned, they are approximately given by Eqs. (2.9) and (2.10) with a systematic underestimation of about 30%. Because of the systematic nature of the underestimation, one can hope to retain the functional forms of these equations and allow the adjustment of a single parameter to represent effects both known and unknown. In such a semiempirical approach, it is important to test the results against known experimental data. If by using ony a single parameter one can succeed in fitting the plateau multiplicity and the total multiplicity data of a large class of relevant reactions, one then has a useful tool to estimate the mass dependence, impact-parameter dependence, spatial dependence, and collision-energy dependence of the initial energy density in the central-rapidity region of heavy-ion collisions.

III. PARAMETRIZATION OF THE RAPIDITY DISTRIBUTION dN^{AB}/dy

For light nuclei, the density distribution can be approximated by a Gaussian distribution. The nucleon-nucleon thickness function $t(b)$ can also be parametrized as a Gaussian function. Its standard deviation β_p can be determined from the nucleon-nucleon elastic cross section by assuming a real profile function. For the region of many tens of GeV in which we are interested, the slope parameter is 12 $(\mathrm{GeV}/c)^{-2}$ (Ref. 30), and we obtain $\beta_p=0.68$ fm. The folding of the density distributions and the nucleon-nucleon thickness function gives a Gaussian thickness function $T(b)$ as follows:

$$T(b)=\exp(-b^2/2\beta^2)/2\pi\beta^2 , \tag{3.1}$$

where

$$\beta^2={\beta_A}^2+{\beta_B}^2+{\beta_p}^2 . \tag{3.2}$$

In terms of the root-mean-squared radius parameter r_{rms} the standard deviation β_A (or, similarly, β_B) is given by

$${\beta_A}^2=(r_{\mathrm{rms}})^2A^{2/3}/3 . \tag{3.3}$$

For a heavy nucleus, the density can be described in the form of a Fermi distribution whose Fourier transform is known analytically.[31] The folding of the density distributions can be carried out by using a fast Fourier transformation. It is found that the thickness function thus obtained can be well approximated by a Gaussian function of the form of Eqs. (3.1)—(3.3). When we use a radius parameter of $r_0=1.2$ fm and a diffusivity $a=0.523$ fm for the density distribution, the parameter r_{rms} has the value of 1.14 fm for ^{40}Ca on ^{40}Ca, 1.08 fm for ^{90}Zr on ^{90}Zr, and 1.04 fm for ^{208}Pb on ^{208}Pb and ^{238}U on ^{238}U. We shall use the Gaussian form of the thickness function for both light and heavy nuclei, but the parameter of the Gaussian function, r_{rms}, will be an effective parameter chosen to fit the experimental multiplicity data.

With a Gaussian thickness function, the total inelastic cross section can be obtained as an analytic function. Using Eq. (2.6), we find

$$\sigma_{\mathrm{in}}^{AB}=2\pi\beta^2\sum_{i=1}^{AB}[1-(1-f)^i]/i , \tag{3.4}$$

where f is a dimensionless quantity given by

$$f=\sigma_{\mathrm{in}}/2\pi\beta^2 . \tag{3.5}$$

The functional form of Eq. (2.8) for the rapidity distribution is then

$$\frac{dN^{AB}}{dy}(b)=\frac{dN}{dy}\frac{ABfe}{[1-(1-fe)^{AB}]} , \tag{3.6}$$

where $e=\exp(-b^2/2\beta^2)$. We average the multiplicity distribution over the impact parameters. The functional form of the ratio of the average rapidity distributions is then

$$R\left[\frac{AB}{pp},y\right]\equiv\frac{\langle dN^{AB}/dy\rangle}{dN/dy} =\frac{ABf}{\sum_{i=1}^{AB}[1-(1-f)^i]/i} . \tag{3.7}$$

The functional form of the ratio of the total multiplicities is also given by the right-hand side of the above equation:

$$R\left[\frac{AB}{pp}\right]\equiv\frac{N^{AB}}{N}=\frac{ABf}{\sum_{i=1}^{AB}[1-(1-f)^i]/i} . \tag{3.8}$$

As we explained before, if we mindlessly apply Eqs. (3.6)—(3.8) by using the root-mean-squared radius parameter as determined by electron scattering, the theoretical results consistently exceed the experimental values. In order to correct for this systematic discrepancy and to have simple and analytical results for use in a variety of situations for the purpose of studying the energy density in the *central rapidity* region in heavy-ion collisions, we shall adopt a semiempirical approach. This consists of assuming the functional forms of Eqs. (3.6)—(3.8) with the only parameter r_{rms} so chosen as to fit the available central-rapidity multiplicity data. If this can be successful, the equations can be applied to study the plateau multiplicity in the central-rapidity region in heavy-ion collisions.

To search for r_{rms}, we calculate the multiplicity ratios for many reactions with Eqs. (3.6)—(3.8) and found $r_{\mathrm{rms}}=1.15$ fm to give good fits to many pieces of experimental data (Table I) which we shall discuss below. The nucleon-nucleon inelastic cross section is $\sigma_{\rm in}=30$ mb (Ref. 32); we get from Eq. (3.7)

$$R\left[\frac{\alpha p}{pp},y\right]=1.19\ , \tag{3.9}$$

which should be compared with the experimental value[33] of 1.18 ± 0.07 over a large range of y in the central rapidity region at a nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}}=44$ GeV. We get from Eq. (3.7)

$$R\left[\frac{\alpha\alpha}{dd},y\right]=1.42\ , \tag{3.10}$$

which should be compared with the experimental value[34] of about 1.4 in the central rapidity region at $\sqrt{s_{NN}}=31$ GeV. We also get from Eq. (3.7)

$$R\left[\frac{\alpha\alpha}{pp},y\right]=1.71\ , \tag{3.11}$$

which should be compared with the experimental value[33] of 1.74 ± 0.09 in the central rapidity region at $\sqrt{s_{NN}}=31$ GeV.

We can use the present model to estimate the rapidity distribution for the highly central collision events of Si on Ag and Ca on C observed in cosmic-ray experiments.[14] The impact parameter b in these events has been estimated[35] to be about 1 fm. For the collision of Si on Ag at this impact parameter, Eq. (3.6) gives

$$\frac{dN^{AB}}{dy}(b=1\text{ fm})=96.0\frac{dN}{dy}\ . \tag{3.12}$$

We can represent dN/dy for nucleon-nucleon collision by a Fermi-type distribution:[7]

$$\frac{dN}{dy}=C\left[1+\exp\left[\frac{y-y_0}{\Delta}\right]\right]^{-1}\ . \tag{3.13}$$

The constants C and y_0 can be parametrized as a function of the nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}}$. From the data of Refs. 7 and 8 and taking into account the small difference between the rapidity variable and the pseudorapidity variable, we have

$$\Delta=0.55\ , \tag{3.14}$$

$$C(\sqrt{s_{NN}})=0.48\ln\sqrt{s_{NN}}+0.038 \tag{3.15}$$

$$=0.48\ln(\sqrt{s_{NN}}/2)+0.37\ , \tag{3.16}$$

and

$$y_0(\sqrt{s_{NN}})=0.45\ln\sqrt{s_{NN}}+1.40 \tag{3.17}$$

$$=0.45\ln(\sqrt{s_{NN}}/2)+1.71\ . \tag{3.18}$$

In the above equations, the quantity $\sqrt{s_{NN}}$ is in units of GeV, and we recognize the quantity $(\sqrt{s_{NN}}/2)$ as the center-of-mass energy per nucleon in the collision of equal nuclei and as half the nucleon-nucleon center-of-mass energy in the collision of unequal nuclei. At the reported incident energy of 5 TeV per Si nucleon in the laboratory frame, the nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}}$ is 96.8 GeV. As extrapolated from Eqs. (3.15)—(3.18), the parameters in Eq. (3.13) are $C=2.23$ and $y_0=3.46$. The pseudorapidity distribution for this event, given by Eq. (3.12), is shown in Fig. 1. There is an overall approximate agreement between the theoretical and experimental multiplicity near the central rapidity region which is the main concern of the present work. The agreement near the fragmentation regions is not as good because of the effects of energy degradation.

For the collision of Ca on C at $b=1$ fm, Eq. (3.6) gives

$$\frac{dN^{AB}}{dy}=27.1\frac{dN}{dy}\ . \tag{3.19}$$

At the reported incident energy of 200 TeV/nucleon, the nucleon-nucleon center-of-mass energy is 613 GeV and the extrapolated parameters of Eq. (3.13) are $C=3.12$ and $y_0=4.29$. The pseudorapidity distribution for this event

TABLE I. Comparison of experimental and theoretical ratios of dN^{AB}/dy and dN/dy in the central rapidity region for the collision of nuclei A and B. The experimental data of $p\alpha$ (Ref. 33), dd (Refs. 34 and 33), and $\alpha\alpha$ (Ref. 33) are averaged over the impact parameters, and the theoretical formula (3.7) is used. The experimental data of Si + Ag and Ca + C (Ref. 14) are central collisions with impact parameter of 1 fm and the theoretical formula (3.6) is used. The theoretical results are obtained with the value of the parameter $r_{\rm rms}=1.15$ fm.

Nuclei		$(dN^{AB}/dy)/(dN/dy)$	
A	B	exp	Theory
p	p	1	1
p	α	1.18 ± 0.07	1.19
d	d	1.24 ± 0.10	1.21
α	α	1.24 ± 0.09	1.71
Si	Ag	$\sim$90	96.0
Ca	C	$\sim$25	27.1

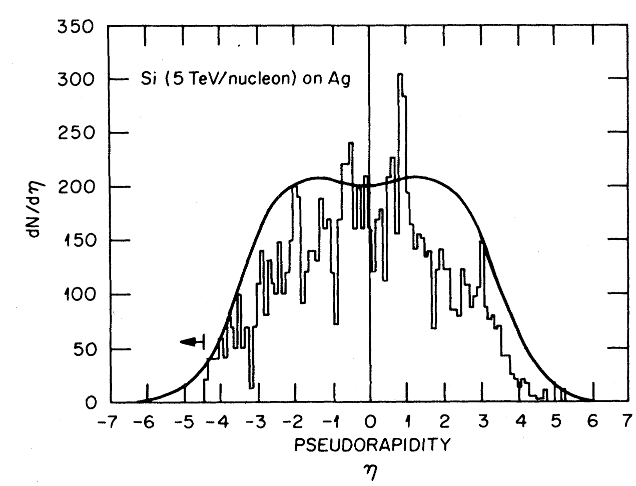

FIG. 1. The pseudorapidity distribution for the cosmic-ray event of Si (5 TeV/nucleon) on Ag. The histogram is the experimental data (Ref. 14) and the solid smooth curve is the theoretical result for $b=1$ fm from Eq. (3.6).

is shown in Fig. 2. There is a good agreement of the theoretical and the experimental distributions in the central rapidity region.

The above comparisons have been made for the rapidity distribution in the central-rapidity region in collisions where the projectile and target masses are not disparagingly different. Two specific cases considered have been heavy-ion collisions with mass numbers much greater than unity. We have examined cases where there is no impact-parameter selection and also cases where there is an impact-parameter selection. There is a good agreement between the experimental and theoretical results. This good agreement lends support to the use of the present theoretical results for the multiplicity in the central rapidity region of heavy-ion collisions.

The value of the phenomenological parameter $r_{\rm rms}=1.15$ fm is larger than what is obtained by folding the nuclear density where $r_{\rm rms}$ is 1 fm for light nuclei[36] and ranges from 1.04 to 1.14 fm for heavy nuclei. As its number of collisions increases, the energy of a nucleon decreases; the height of the multiplicity plateau in baryon-baryon collision also decreases with the energy of the colliding baryons. This slightly larger value of $r_{\rm rms}$ is needed to take into account effectively the decrease of the height of the multiplicity plateau for the collision of those nucleons which are degraded in energy due to prior collisions.

We may well ask whether the semiempirical results are consistent with other multiplicity data. A comparison of the results of Eq. (3.8) with the experimental pA data[16] for the ratio of the total multiplicity at 100 GeV/c is shown in Fig. 3. As one observes, the agreement between theory and experiment is good. We can further compare other hadron-nucleus experimental total-multiplicity data. In this case, $B=1$ and β_B and β_p take on the proper values appropriate for the hadron in question. The quantity $\sigma_{\rm in}$ becomes the hadron-nucleus inelastic cross section. We apply Eq. (3.8) to π^+A and K^+A reactions at 100 GeV/c and compare with the experimental data of Ref. 16. As $A \gg 1$, we can neglect minor differences in the values of β_B (and t_p) for protons and for π^+ and K^+. We use $\sigma_{\rm in}=20$ mb for π^+p inelastic collisions and $\sigma_{\rm in}=17$ mb for K^+p inelastic collisions.[32] The results from Eq. (3.8) are shown in Fig. 3 and compared with the experimental data. The theoretical results can be represented approximately by

$$R\left[\frac{pA}{pp}\right] \sim 0.314A^{1/3}+0.686\ , \tag{3.20}$$

$$R\left[\frac{\pi^+A}{\pi^+p}\right] \sim 0.212A^{1/3}+0.788\ , \tag{3.21}$$

and

$$R\left[\frac{K^+A}{K^+p}\right] \sim 0.181A^{1/3}+0.819\ . \tag{3.22}$$

There is a good agreement between the theoretical and the experimental results.

Strictly speaking, while a Gaussian thickness function is a good representation for the collision of two nuclei whose masses are not disparagingly different from each other, the thickness function for hadron-nucleus collision is better represented by a function of the form $(R^2-b^2)^{1/2}\theta(R-b)$. For completeness, we give results for this type of thickness function in the Appendix. The total multiplicities for this type of thickness function differ from those of Gaussian thickness function only by at most 10%.

With our interest focused exclusively in the central-rapidity region in heavy-ion reactions, we seek simple, analytical results which can be applied to a large class of problems concerning the initial energy density in the central rapidity region: its spatial dependence, its impact-parameter dependence, its bombarding-energy dependence, its target-mass dependence, and its projectile-mass dependence. From the comparison with experimental data given above, we see that the analytical results of Eqs.

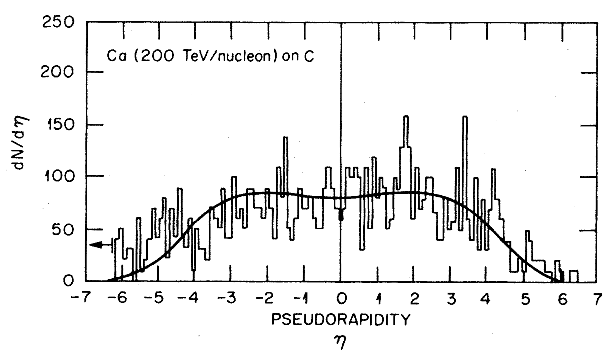

FIG. 2. The pseudorapidity distribution for the cosmic-ray event of Ca (200 TeV/nucleon) on C. The histogram is the experimental data (Ref. 14) and the solid smooth curve is the theoretical result for $b=1$ fm from Eq. (3.6).

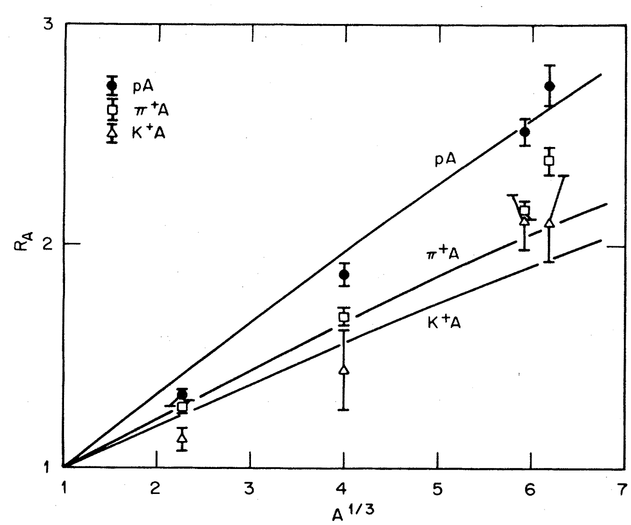

FIG. 3. Comparison of the ratios of the total multiplicities. The data points are the experimental data of R_A for hadron-nucleus reactions at 100 GeV/c (Ref. 16). The solid-circle points are for pA reactions, open squares for π^+A reactions, and the triangle points for K^+A reactions. The curves are theoretical results obtained from Eq. (3.8) with $r_{\rm rms}=1.15$ fm.

(3.6)—(3.8), with an effective parameter of $r_{\rm rms}=1.15$ fm, give an adequate description of the plateau multiplicity data for a large number of reactions; they are therefore useful tools for our purposes.

We show in Fig. 4 the ratio

$$R(AA/pp,y)=R(AA/pp)$$

calculated with Eq. (3.7) for the case of equal-nuclei collisions. We find that for $A>25$

$$R\left[\frac{AA}{pp}\right]\sim 0.238A^{1.12}\ , \tag{3.23}$$

which goes approximately as $A^{4/3}$.

It is easy to obtain the general mass dependence of R for the collision of a nucleus A with $A\gg 1$ and a nucleus B (or a hadron with $B=1$). The denominator in the right-hand side of Eqs. (3.7) and (3.8) is a function that is logarithmic in f and AB and has therefore only weak mass dependence. The ratio of the rapidity distributions is approximately

$$R\left[\frac{AB}{pp}\right]\propto\frac{AB}{\beta^2}\propto\frac{AB}{A^{2/3}+B^{2/3}}\ , \tag{3.24}$$

which is approximately consistent with Eqs. (3.20)—(3.23).

It is useful to note that the quantity R is an average over the impact parameters. An individual event with an impact parameter b will have a multiplicity ratio given by $n(b)$ as in Eqs. (2.8) and (3.6). We should note that upon introducing an effective parameter $r_{\rm rms}$, the quantity $n(b)$ is no longer the actual number of nucleon-nucleon collisions, but becomes an effective average number of nucleon-nucleon collisions. It is the multiplicative factor which, when multiplied by dN/dy, gives the rapidity distribution for nucleus-nucleus collisions. To illustrate the variation of this multiplicative factor, we show in Fig. 5(a) the quantity $n(b)$ for the collision of ^{238}U on ^{238}U. It is approximately a Gaussian function. Its peak value is many times greater than its average value of R.

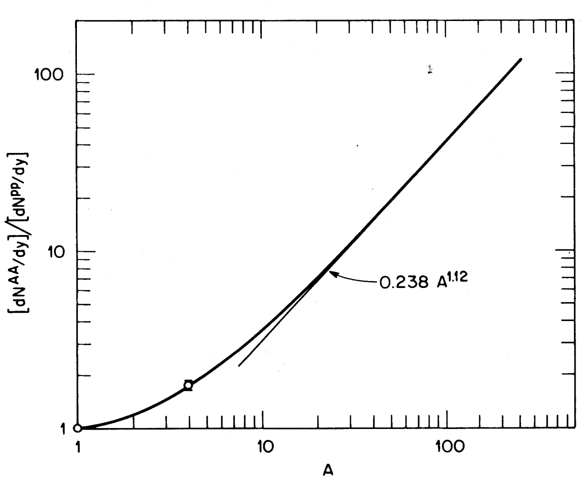

FIG. 4. The ratio of multiplicity $R(AA/pp)$ for the collision of two equal nuclei. The data point on the $\alpha\alpha$ reaction is from Ref. 33. The theoretical curve is obtained from Eq. (3.7) with $r_{\rm rms}=1.15$ fm.

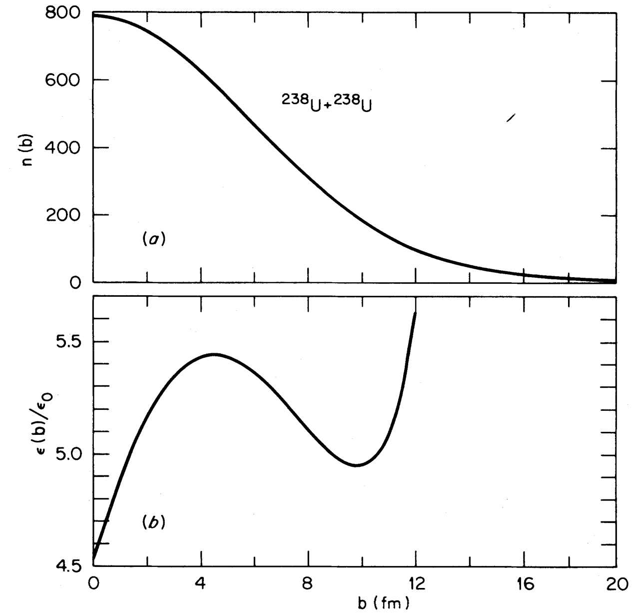

FIG. 5. (a) The quantity $n(b)$ as a function of b in the collision of ^{238}U on ^{238}U. It is the multiplicative factor which, when multiplied by dN^{pp}/dy gives the rapidity distribution for nucleus-nucleus collisions. (b) The energy density $\epsilon(b)$ as a function of impact parameters in the collision of ^{238}U on ^{238}U in units of ϵ_0.

IV. INITIAL ENERGY DENSITY IN HIGHLY RELATIVISTIC HEAVY-ION COLLISIONS

From the rapidity distribution, we can obtain the energy density in heavy-ion collisions. We consider a nucleus-nucleus collision at an impact parameter of b in the equal-velocity frame. We shall follow the general description of Bjorken and assume that the evolution may be divided into two stages. The initial stage commences when the two pancake nuclei interpenetrate each other and ends at a time $t_0=1$ fm/c thereafter. At the end of this initial stage, there is a domain of high energy density in the central rapidity region as a result of nucleon-nucleon collisions. As in Ref. 1, we shall neglect the thickness of the nuclei in this frame. Then at the time $t_0=1$ fm/c, the longitudinal coordinate of a produced particle is related to its rapidity by

$$z=t_0\tanh y\ . \tag{4.1}$$

The energy produced in the region of thickness Δz between the two pancakes is[37]

$$\Delta E=\frac{dN^{AB}}{dy}\frac{\Delta z}{t_0}m_\perp[1-(z/t_0)^2]^{-3/2}\ , \tag{4.2}$$

where $m_\perp$ is the transverse mass of the produced particle. From the transverse momentum distribution,[38] the quantity $m_\perp$ can be estimated to be $\sim$400 MeV. The (*proper*) energy density of the quark-gluon plasma is defined in the frame in which the fluid element is at rest. The energy density is the region $y=0$ (i.e., $z=0$) can be examined in the center-of-mass system, while the energy density in the other regions can be obtained by going to other uniformly moving frames of reference. The plateau structure of the rapidity distribution [Eq. (3.13)] indicates that for a range of $-y_0 < y < y_0$, the energy density is the same for these different regions, the only difference being the velocity of the fluid element. It suffices to study the energy density at $y=0$ in the equal-velocity frame as a representative point for $-y_0 < y < y_0$. For a given time t_0 in the equal-velocity frame, there is a longitudinal region $-z_0 < z < z_0$ related to this range of y by Eq. (4.1). Within this region of z, the energy density in the Lagrangian sense is the same. The value of y_0 for a collision energy of many tens of GeV per nucleon in the equal-velocity frame is approximately 3.

The energy density is not uniform in the transverse direction. We shall discuss the nonuniformity in the next section. Here, we shall examine the energy density $\epsilon(b)$ at $z=0$, averaged over the overlapping area $\mathscr{A}(b)$, as a function of the impact parameter b. We have

$$\frac{\epsilon(b)}{\epsilon_0}=\frac{n(b)}{\mathscr{A}(b)}\ \mathrm{fm}^2\ , \tag{4.3}$$

where ϵ_0 is the energy density in a nucleon-nucleon collision averaged over a unit area of 1 fm^2,

$$\epsilon_0=\frac{dN}{dy}\frac{m_\perp}{t_0}\ \mathrm{fm}^{-2}\ . \tag{4.4}$$

As the central-rapidity distribution increases with energy, ϵ_0 also increases with energy. Assuming that the number of neutral particles is half as much as the charged particles and taking t_0 to be 1 fm/c,[1] we get from the experimental values[7,8] of dN/dy, as parametrized in Eqs. (3.13)–(3.18), the following parametrization[39] of ϵ_0:

$$\epsilon_0=0.6(0.48\ \ln E^*+0.37)\ \mathrm{GeV/fm}^3\ , \tag{4.5}$$

where E^* is the center-of-mass energy per nucleon in GeV in the collision of equal nuclei and is half of the nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}}/2$ in the collision of unequal nuclei. We can list below the values of ϵ_0 for a few bombarding energies of interest:

$$\epsilon_0=\begin{cases} 0.89\ \mathrm{GeV/fm}^3 & \dfrac{\sqrt{s}}{A}=10\ \mathrm{GeV\ per\ nucleon}\ ,\\ 1.20\ \mathrm{GeV/fm}^3\ \ \mathrm{for} & \dfrac{\sqrt{s}}{A}=30\ \mathrm{GeV\ per\ nucleon}\ ,\\ 1.83\ \mathrm{GeV/fm}^3\ \ \mathrm{for} & \dfrac{\sqrt{s}}{A}=270\ \mathrm{GeV\ per\ nucleon}\ . \end{cases} \tag{4.6}$$

From the previous results on the number of collisions at an impact parameter b, we have

$$\frac{\epsilon(b)}{\epsilon_0}=\frac{AB\sigma_{\mathrm{in}}T(b)}{\mathscr{A}(b)\{1-[1-T(b)\sigma_{\mathrm{in}}]^{AB}\}}\ \mathrm{fm}^2\ . \tag{4.7}$$

Using a Gaussian form of $T(b)$ and an effective rms radius parameter of $r_{\mathrm{rms}}=1.15$ fm, we calculate the initial energy density for head-on collisions of two equal nuclei at $t_0=1$ fm/c and $z=0$ fm. In Fig. 6, we show this quantity as a function of A. The numerical result can be parametrized as

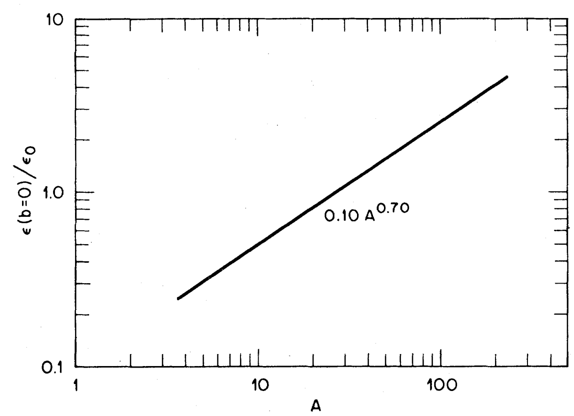

FIG. 6. The energy density $\epsilon(b=0)$ reached in the head-on collision of two equal nuclei and averaged over the overlapping area as a function of the mass number A. The energy unit ϵ_0 is given by Eqs. (4.4) and (4.5).

$$\epsilon(b=0)=0.06A^{0.70}(0.48\ln E^*+0.37)\ \mathrm{GeV/fm}^3)\ , \tag{4.8}$$

which goes approximately as $A^{2/3}$, as expected from Eq. (4.7).

With the same Gaussian form of $T(b)$, we calculate the number of collisions and the initial energy density in units of ϵ_0 as a function of the impact parameter, for the collision of ^{238}U on ^{238}U. The results are shown in Fig. 5(b). The energy density is about 5 units of ϵ_0. It oscillates as a function of the impact parameter with an amplitude of about $0.5\epsilon_0$. For peripheral collisions with large impact parameters ≥ 12 fm, the overlapping area loses its meaning; so does the energy density. We do not need to consider them here. For collisions with impact parameter less than 12 fm, the energy density is insensitive to the change of impact parameters. We can understand this in the following way. The overlapping area $\mathscr{A}(b)$ is an integral of two overlapping step functions. When modified to allow for a diffused density, it has the same form as the thickness function $T(b)$. The two functions are therefore approximately related by a normalization constant as follows:

$$\mathscr{A}(b)\sim\pi R_A{}^2\pi R_B{}^2T(b)\ . \tag{4.9}$$

As the radius parameter $r_{\mathrm{rms}}=1.15$ fm for $T(b)$ is slightly larger than what one obtains with $r_0=1.2$ fm, the corresponding radius parameter r_0 for R_A and R_B of Eq. (4.9) should also be slightly larger. We use $r_0=1.3$ fm to evaluate the radii R_A and R_B. Within this approximation, we obtain from Eqs. (4.7) and (4.9)

$$\epsilon(b)\sim 0.064A^{1/3}B^{1/3}(0.48\ln E^*+0.37)\ \mathrm{GeV/fm}^3\ , \tag{4.10}$$

which is independent of the impact parameter. Its special case for $b=0$ also agrees with Eq. (4.8).

V. ENERGY-DENSITY DISTRIBUTION IN THE TRANSVERSE DIRECTION

The energy distribution in the transverse direction is not uniform. We denote by $\epsilon(\vec{b}_A,b)$ the energy density at the point $\vec{b}_A$ measured from the center of nucleus A for a collision with impact parameter b. It will be evaluated at $z=0$ in the equal-velocity frame. This energy density is the same as the energy density at other points of z in the Lagrangian sense, within the range of $-z_0<z<z_0$. From the definition of $T(b)$ and Eq. (2.4), it is easy to see that over an area element of $\Delta\vec{b}_A$, the (effective) number of inelastic nucleon-nucleon collisions is

$$\Delta n=\frac{AB\sigma_{\rm in}D_A\Delta\vec{b}_A\int D_B(\vec{b}_B)t(\vec{b}-\vec{b}_A-\vec{b}_B)d\vec{b}_B}{1-[1-T(b)\sigma_{\rm in}]^{AB}}\ , \tag{5.1}$$

where the function $D_A(\vec{b}_A)$ [or, similarly, $D_B(\vec{b}_B)$] is the density function integrated over the longitudinal coordinate:

$$D_A(\vec{b}_A)=\int\rho_A(\vec{b}_A,z_A)dz_A\ . \tag{5.2}$$

In consequence, the local energy density at $\vec{b}_A$ is

$$\frac{\epsilon(\vec{b}_A,b)}{\epsilon_0}=\frac{AB\sigma_{\rm in}D_A(\vec{b}_A)\int D_B(\vec{b}_B)t(\vec{b}-\vec{b}_A-\vec{b}_B)d\vec{b}_B}{1-[1-T(b)\sigma_{\rm in}]^{AB}}\ . \tag{5.3}$$

For large nuclei, the nucleon-nucleon thickness function can be approximated by a delta function and the nuclear density function by a step function θ. We then obtain

$$\frac{\epsilon(\vec{b}_A,b)}{\epsilon_0}=\frac{AB\sigma_{\rm in}D_A(\vec{b}_A)D_B(\vec{b}-\vec{b}_B)}{1-[1-T(b)\sigma_{\rm in}]^{AB}}\ , \tag{5.4}$$

where the function $D_A(\vec{b}_A)$ [or, similarly, $D_B(\vec{b}_B)$] is given by

$$D_A(\vec{b}_A)=3(1-b_A{}^2/R_A{}^2)^{1/2}\theta(R_A-b_A)/2\pi R_A{}^2\ . \tag{5.5}$$

As we explained before, the effective radius parameter $r_{\rm rms}=1.15$ fm for $T(b)$ is slightly larger than what one obtains with $r_0=1.2$ fm, so the corresponding effective radius parameter r_0 for R_A and R_B in Eq. (5.5) should also be slightly larger. We use $r_0=1.3$ fm to evaluate the radii R_A and R_B. The local energy density in units of ϵ_0 at $\vec{b}_A$ and $z=0$ for $\vec{b}_A$ parallel to $\vec{b}$ in the collision of ^{238}U on ^{238}U is shown in Fig. 7. It peaks in the region where the projected number of nucleons is the largest, as it should. The maximum energy density is high. It has the value of about 9 units of ϵ_0 for head-on collisions. As the impact parameter increases, the maximum energy density decreases but the energy density becomes a steeper function of b_A. It centers at $b_A=b/2$ and extends over a region of R_A+R_B-b.

The energy density has been given in terms of ϵ_0 which depend on the energy of the colliding nuclei [Eqs. (4.4) and (4.5)]. In the collision of ^{238}U with ^{238}U at an energy of $\sqrt{s}/A=30$ GeV, the maximum energy density achieved for a head-on collision is about 10 GeV/fm^3 along the axis of collision, with an average of about 5 GeV/fm^3 when averaged over the overlapping area. This average energy density is insensitive to the impact parameter and has a value of about 5 to 6 GeV/fm^3 over a large range of impact parameters [Fig. 5(b)]. This value of energy density is substantially higher than the critical energy density of 2 GeV/fm^3 estimated for the onset of phase transition.[4] Thus, a relativistic heavy-ion collision may bring about the interesting phenomenon of phase transition from the confined hadron matter to the unconfined quark-gluon matter.

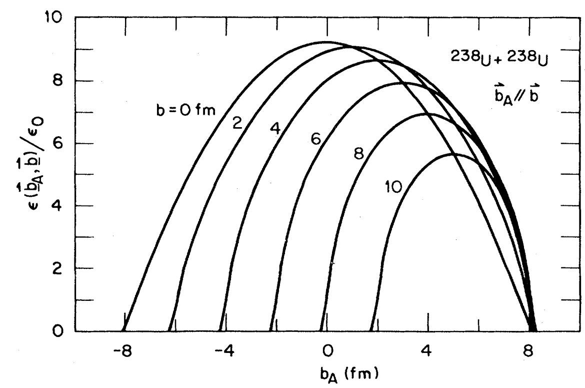

FIG. 7. The spatial dependence of the energy density $\epsilon(\vec{b}_A,\vec{b})$ in the transverse direction at $\vec{b}_A$ with $\vec{b}_A$ parallel to $\vec{b}$ for various impact parameters b. The energy unit ϵ_0 is given by Eqs. (4.4) and (4.5).

The energy density decreases by about 26% when the colliding energy decreases from $\sqrt{s}/A=30$ GeV per nucleon to 10 GeV per nucleon, and increases by a factor of 1.5 when the colliding energy increases from $\sqrt{s}/A=30$ GeV per nucleon to 270 GeV per nucleon.

VI. DISCUSSIONS AND CONCLUSIONS

In a highly relativistic heavy-ion collision, reaction products can come from the fragmentation region or the central rapidity region. We focus our attention on the central rapidity region and divide the dynamics into the production stage and the evolution stage. We estimate the rapidity distribution and energy density at the end point of the production stage, which can be used to provide the initial conditions for the evolution stage.

Our study indicates that the energy density is not spatially uniform. In the transverse direction, the energy density depends on the thickness of the colliding nuclear matter. The thicker the colliding nuclear matter, the greater is the energy density. In a head-on collision, the energy density decreases with transverse distance away from the center. For the head-on collision of ^{238}U on ^{238}U at an energy of 30 GeV per nucleon, the energy density is ~ 5 GeV/fm^3 when one averages over the overlapping area but is as high as 10 GeV/fm^3 in the center of the overlapping region. These estimates are greater than those of previous estimates[1,4] as the detail treatments are different. In the longitudinal direction, the energy density increases as a function of a $|z|$ as the region of large $|z|$ contains particles of higher longitudinal momenta. However, if one follows these particles by making a boost of the coordinate system (within a range specified by y_0), the energy density is the same.

The study of the evolution stage using the initial energy density we have obtained will allow the exploration into the unknown region of quark-gluon matter. In the picture proposed by Bjorken, the system rapidly comes into local thermal equilibrium and the plasma is likely to be in the deconfined quark-gluon phase. During the subsequent expansion, the energy density drops but the entropy per unit of rapidity is conserved and thus the particle production per unit of rapidity does not depend on the detail of the hydrodynamical evolution but on the initial energy deposition. This conclusion, however, will be modified if thermal equilibrium is not reached or if the effect of viscosity is not negligible. Detail hydrodynamical studies of the evolution of the quark-gluon plasma have already been carried out.[40,41] The use of different types of particles[1,42,43] as a probe of the history of the plasma during its expansion may allow one to explore some properties of this unusual quark-gluon matter.

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APPENDIX: MULTIPLICITY FOR THE CASE WITH A SHARP-CUTOFF DENSITY

For hadron-nucleus collisions with a target mass $A \gg 1$ or for nucleus-nucleus collisions with $A \gg B$ and $B \sim 1$, the thickness function obtained by folding the densities of the projectile and the target has the form of $(R^2-b^2)^{1/2}$ joining on to an exponential tail near $b \sim R$. It is useful to study the case where we approximate the thickness function by assuming a sharp-cutoff density and obtain

$$T(b)=(3/2\pi R^3)(R^2-b^2)^{1/2}\theta(R-b)\ , \tag{A1}$$

where R is the sum of the radii of the target nucleus A and the projectile nucleus B:

$$R=r_0(A^{1/3}+B^{1/3})\ . \tag{A2}$$

Then, from Eq. (2.6), the quantity $\sigma_{\rm in}^{AB}$ is

$$\sigma_{\rm in}^{AB}=\pi R^2\left\{1+\frac{2}{F^2}\left[\frac{1-(1-F)^{A+2}}{A+2}-\frac{1-(1-F)^{A+1}}{A+1}\right]\right\}, \tag{A3}$$

where F is a dimensionless ratio

$$F=3\sigma_{\rm in}/2\pi R^2\ . \tag{A4}$$

The functional form for the ratio of the total multiplicity and the multiplicity plateau is

$$\begin{aligned}\frac{N^{hA}}{N}&=\frac{dN^{hA}/dy(\text{plateau})}{dN/dy(\text{plateau})}\\&=(2AF/3)\left\{1+\frac{2}{F^2}\left[\frac{1-(1-F)^{A+2}}{A+2}-\frac{1-(1-F)^{A+1}}{A+1}\right]\right\}.\end{aligned} \tag{A5}$$

A phenomenological parameter of $r_0=1.35$ fm in the above formula (A5) gives a good agreement of the ratios of total multiplicities in pA, π^+A, and K^+A reactions that is slightly better than the agreement obtained with the Gaussian thickness function. The difference is, however, at most 10%.

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