Q^2-dependent parametrizations of pion parton distribution functions

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Pion distribution functions have been obtained by performing fits to data on J/ψ and dimuon production. Simple Q^2-dependent parametrizations of the results are presented. Some of the implications for high-p_T phenomenology are discussed.

I. INTRODUCTION

Our knowledge of the nature of the parton distributions in nucleon targets comes from a variety of sources, including deep-inelastic lepton-nucleon scattering, high-mass dilepton production, and the production of massive particles such as the J/ψ and Υ. For pions, however, the available information is somewhat limited and our knowledge of the parton distributions is derived solely from the last two sources. Several experimental groups have published pion quark distributions determined using their own dimuon data.[1-3] Also, several determinations[4-6] of the shape of the gluon distribution function have been made using data on J/ψ production. However, in no case has a determination of the distributions been made that takes into account the relevant scaling violations. The purpose of this paper is to report the results of an analysis in which the pion distribution functions were determined by simultaneously fitting both dimuon and J/ψ data. Scaling violations, calculated to leading-logarithm accuracy, have been included. Simple parametrizations of the results are also presented.

In a previous paper,[7] two sets of Q^2-dependent parametrizations for nucleon distribution functions were presented. The two sets differed in the choice of the gluon distribution and the resulting fitted value of the QCD scale parameter Λ. In fitting the pion distribution functions the results of the preceding analysis were used. Two analogous sets of pion distributions are presented here. This analysis is, therefore, complementary to the first.

In Sec. II the details of the data set used, the fitting procedure, and the resulting parametrizations are given. Some of the implications for the phenomenology of high-p_T pion-induced reactions are discussed in Sec. III.

II. DATA SET AND PARAMETRIZATIONS

In this analysis dimuon data for $d\sigma/dx_F dM^2$ and $M^3 d\sigma/dM$ (Refs. 8 and 9) as well as J/ψ data for $d\sigma/dx_F$ (Refs. 4 and 10) have been used. The relevant theoretical expressions are well known and can be found, for example, in Refs. 11 and 12, respectively. In some instances, especially for the lower-energy data sets, it was found that it makes a difference as to which definition of x_F is used. Throughout this paper the definition, in the overall center of momentum frame, $x_F = p_{||}/p_{||\max}$ has been used. This introduces some extra factors of $1-\tau$ where $\tau = M^2/s$ as is discussed in Ref. 6.

It is by now well known that the leading-logarithm predictions for dimuon production lie about a factor of 2 below the data.[13] Accordingly, an arbitrary normalization (or K factor) was allowed in order not to overly bias the shape of the fitted distribution functions. The "semilocal-duality" formalism used in fitting the J/ψ data also has an overall normalization factor associated with it. In addition, it was found necessary in several instances to allow for relative normalization shifts between data sets.

The distributions used here have been evolved from $Q_0{}^2 = 4$ (GeV/c)^2 assuming four quark flavors. At the input value of Q^2 the charm-quark distribution was assumed to be zero and an SU(3)-symmetric sea was used. The valence quark distribution v_π was constrained to satisfy the sum rule

$$\int_0^1 v_\pi dx = 1 .$$

The data used in the fits are sensitive to the pion distributions down to values of the momentum fraction x near 0.2. Therefore, there is relatively little constraint on the sea terms. The results presented here were obtained by normalizing the momentum fraction of the sea to 15% and assuming a form at $Q_0{}^2$ proportional to $(1-x)^5$. The valence quark distribution was assumed to be proportional to $x^a(1-x)^b$ at $Q_0{}^2$. Again, the lack of low-x data precluded a precise determination of the power a. It was fixed at the value 0.4 in both of the fits reported here. Values between 0.3 and 0.5 yielded fits of approximately the same quality, although a strong correlation was noted between the value of the power a and the fitted K factor. The input gluon distribution was assumed to be proportional to $(1+\gamma_1 x)(1-x)^\beta$. As in the nucleon case,[6] a significant improvement in the fits was noted for nonzero values of γ_1. However, the minimum in χ^2 was extremely broad so that a precise value could not be obtained. Both fits presented here used $\gamma_1 = 6$.

Two sets of distributions have been determined, corresponding to the two sets of nucleon distributions presented in Ref. 7. The first, referred to hereafter as set 1, has $\Lambda = 200$ MeV/c, while the second, set 2, has $\Lambda = 400$

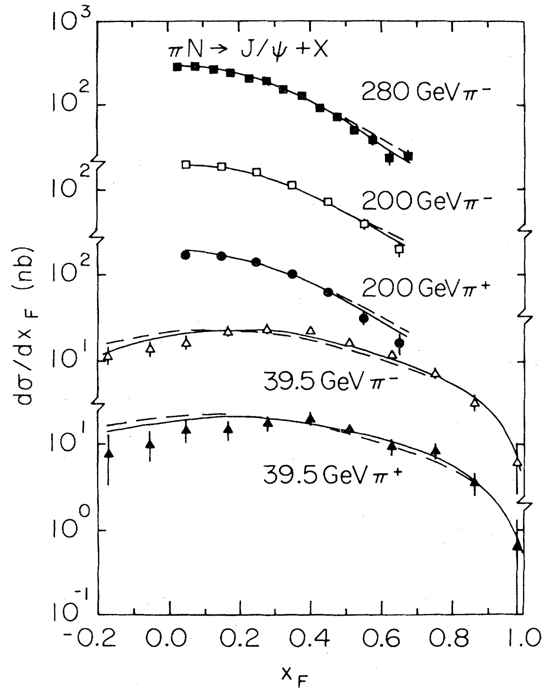

FIG. 1. Fits to a representative sample of the data for x_F distributions in J/ψ production. The solid and dashed curves are for set 1 and set 2, respectively. The data are from Ref. 4 (squares and circles) and Ref. 10 (triangles).

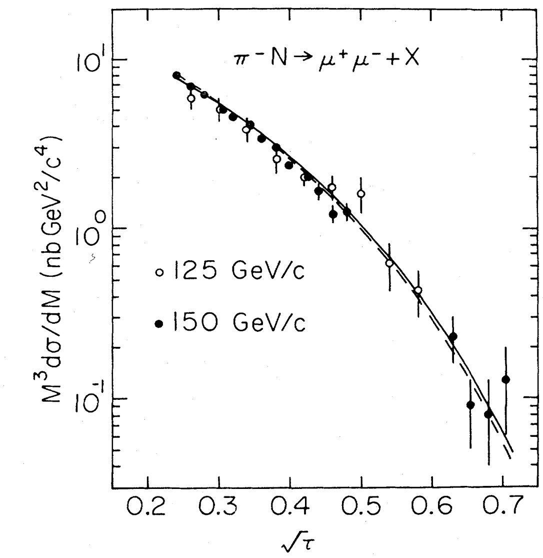

FIG. 2. Fits to a representative sample of the data for $M^3 d\sigma/dM$ ($x_F > 0$). The solid and dashed curves are for set 1 and set 2, respectively. The data are from Ref. 8 (open circles) and Ref. 9 (solid circles).

MeV/c. In Fig. 1 the results of the fits to the J/ψ data are shown. The solid and dashed curves correspond to set 1 and set 2, respectively. There is clearly very little difference between the two fits. In Fig. 2 the results of the fits to the dimuon data for $M^3 d\sigma/dM$ are shown. Again, the two fits are virtually indistinguishable. Figure 3 shows the results of the fits to the dimuon data for $d\sigma/dM\,dx_F$. Only the results from the set-1 fit are shown as the two fits are essentially identical.

Parametrizations for the valence quark distribution v_π, the gluon distribution G, the charm-quark distribution c, and the sea-quark distribution $S=2(\bar{u}+\bar{d}+\bar{s})$ have been determined by fitting the evolved distributions for values of Q^2 between 4 and approximately 2000 (GeV/c)^2. This range should be sufficient for experiments to be performed in the foreseeable future. As in the previous analysis,[7] the results are parametrized in terms of polynomials which depend on

$$s=\ln[\ln(Q^2/\Lambda^2)/\ln(Q_0{}^2/\Lambda^2)] .$$

The valence term is parametrized as

$$xv_\pi = x^a(1-x)^b/B(a,b+1) ,$$

where the Euler Beta function $B(a,b+1)$ ensures the proper normalization as mentioned above. The gluon, sea, and charm distributions all have the basic form

$$xG,xS,xc = Ax^\alpha(1-x)^\beta(1+\gamma_1 x+\gamma_2 x^2) .$$

The various Q^2-dependent polynomials for the two sets are given below. For set 1 ($\Lambda=200$ MeV/c), we have

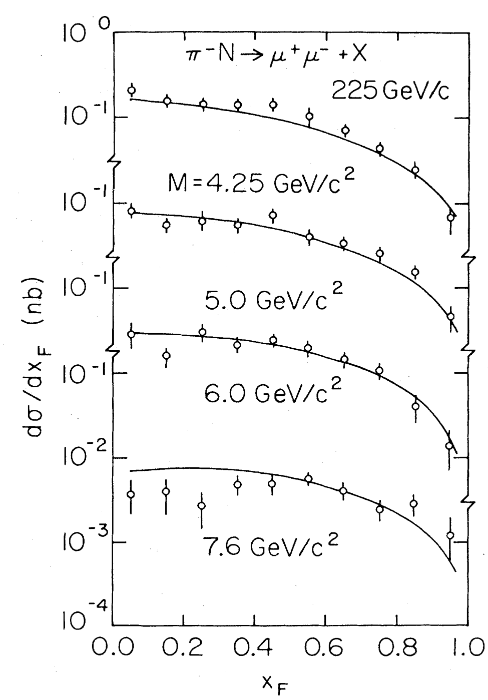

FIG. 3. Fit to the x_F distribution for dimuon production. The results for set 1 are shown while those for set 2 are virtually identical. The data are from Ref. 2.

$$a=0.4-0.062\,12s-0.007\,109s^2\,,$$

$$b=0.7+0.6478s+0.013\,35s^2\,;$$

for the gluon distribution,

$$A=0.888-1.802s+1.812s^2\,,$$

$$\alpha=0.-1.576s+1.20s^2\,,$$

$$\beta=3.11-0.1317s+0.5068s^2\,,$$

$$\gamma_1=6.0+2.801s-12.16s^2\,,$$

$$\gamma_2=0.-17.28s+20.49s^2\,;$$

for the sea distribution,

$$A=0.9-0.2428s+0.1386s^2\,,$$

$$\alpha=0.-0.2120s+0.003\,671s^2\,,$$

$$\beta=5.0+0.8673s+0.047\,47s^2\,,$$

$$\gamma_1=0.+1.266s-2.215s^2\,,$$

$$\gamma_2=0.+2.382s+0.3482s^2\,;$$

for the charm distribution,

$$A=0.+0.079\,28s-0.061\,34s^2\,,$$

$$\alpha=-0.022\,12-0.3785s-0.1088s^2\,,$$

$$\beta=2.894+9.433s-10.852s^2\,,$$

$$\gamma_1=0.+5.248s-7.187s^2\,,$$

$$\gamma_2=0.+8.388s-11.61s^2\,.$$

For set 2 ($\Lambda=400$ MeV/c), we have

$$a=0.40-0.059\,09s-0.006\,524s^2\,,$$

$$b=0.628+0.6436s+0.014\,51s^2\,;$$

for the gluon distribution,

$$A=0.794-0.9144s+0.5966s^2\,,$$

$$\alpha=0.-1.237s+0.6582s^2\,,$$

$$\beta=2.89+0.5966s-0.2550s^2\,,$$

$$\gamma_1=6.0-3.671s-2.304s^2\,,$$

$$\gamma_2=0.-8.191s+7.758s^2\,;$$

for the sea distribution,

$$A=0.90-0.1417s-0.1740s^2\,,$$

$$\alpha=0.-0.1697s-0.096\,23s^2\,,$$

$$\beta=5.0-2.474s+1.575s^2\,,$$

$$\gamma_1=0.-2.534s+1.378s^2\,,$$

$$\gamma_2=0.+0.5621s-0.2701s^2\,;$$

for the charm distribution,

$$A=0.+0.062\,29s-0.040\,99s^2\,,$$

$$\alpha=-0.0882-0.2892s-0.1082s^2\,,$$

$$\beta=1.924+0.2424s+2.036s^2\,,$$

$$\gamma_1=0.-4.463s+5.209s^2\,,$$

$$\gamma_2=0.-0.8367s-0.048\,40s^2\,.$$

III. DISCUSSION

Existing Q^2-dependent pion distributions[14] have been characterized by having x dependences as given by the counting rules[15] at the input value of Q^2. This corresponds to $b=1$ for the valence term and $\beta=3$ for the gluon. Recent analyses by various experimental groups[1–3] have shown that for the valence term $b\approx 1$ for $Q^2\approx 25$ (GeV/c)^2 and for the gluon[4,5] $\beta\approx 2$ at $Q^2\approx 10$ (GeV/c)^2. The values for b given in the preceding section are consistent with these results. It is somewhat harder to compare the gluon exponents since the experimental analyses used simpler parametrizations, i.e., $\gamma_1=\gamma_2=0$. However, the gluon distribution parametrizations presented here, when evaluated at $Q^2=10$ (GeV/c)^2, are very close to that obtained in Ref. 4 where an effective power of $(1-x)$ of $2.38\pm0.06\pm0.10$ was found at $Q^2=10$ (GeV/c)^2. A slightly lower exponent, 1.9 ± 0.3, was quoted in Ref. 5. This difference is due, in part, to the fact that in Ref. 4 a small diffractive contribution to J/ψ production at large x_F was removed, thereby giving a slightly steeper x_F distribution. These differences are, however, rather small.

Estimates of the pion gluon distribution, obtained using J/ψ data, have been presented in Ref. 6. The parametrizations presented here agree well in shape with the results shown as data points in Fig. 4 of Ref. 6. In particular, the low-x region is described better by the form used here than by a single power of $(1-x)$.

The use of harder parton distributions has several interesting effects for predictions of high-p_T pion-induced reactions. First, there will be an increase in the predicted rates due, mostly, to the increase in the gluon distribution at modest x values. The increased role of the gluon-initiated subprocesses will also have a noticeable effect on the predictions for particle ratios. An example of this is the angular dependence of the π^-/π^+ ratio in π^--

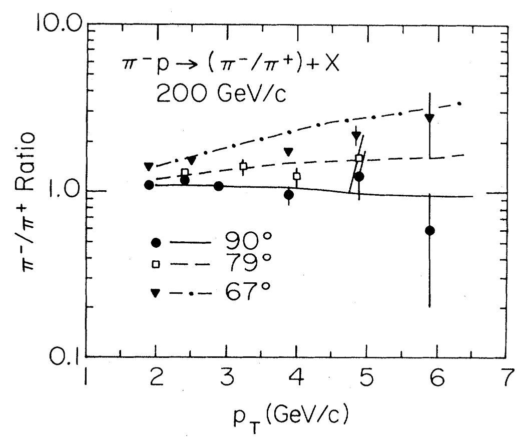

FIG. 4. Predictions for the π^-/π^+ ratio in π^--induced reactions. The data are from Ref. 17.

induced reactions. The original predictions[16] for this ratio, as quoted in Ref. 17, rose much too rapidly as the scattering angle was decreased. New predictions for this ratio are compared with data[17] in Fig. 4. The curve for 67° has been lowered by about 40% and is in better agreement with the trend of the data.

Several experiments are now or will soon be taking data for the production of high-p_T photons produced by pion beams.[19] As a result of the harder gluon distributions reported here, one can anticipate important contributions from both gluon- and quark-initiated subprocesses. A large $gq \rightarrow \gamma q$ contribution will result in the ratio of π^+- and π^--induced cross sections being nearer to one than would be the case if the $q\bar{q} \rightarrow \gamma g$ subprocess were dominant.[20] Data for this beam ratio should be particularly interesting.

In summary, the results presented here are representative of our current knowledge concerning pion distributions. They will provide convenient benchmarks against which new experimental determinations can be tested. The parametrizations are presented in a form similar to that used previously for the nucleon distributions and it should, therefore, be an easy task to incorporate them into existing programs.

ACKNOWLEDGMENTS

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