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$\bar{K}N$ inverse scattering problem

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The $\bar{K}N$ inverse scattering problem is solved for a model which assumes that the $\Lambda(1405)$ is an elementary particle rather than a virtual bound state of the $\bar{K}N$ system. This model points to a consistent explanation of the kaonic-hydrogen data and the $\bar{K}N$ scattering results. It is found that the $\bar{K}N$ interaction form factor determined from the real part of the elastic $\bar{K}N$ phase shifts has reasonable behavior and a range that is consistent with that of the $\bar{K}N$ interaction.

I. INTRODUCTION

Recent experiments[1,2] on the x-ray deexcitation spectrum of kaonic hydrogen have focused considerable attention on the low-energy $\bar{K}N$ interaction. The strong interaction is expected to affect the kaonic-hydrogen spectrum; however, only the $1s$ level can be shifted by a detectable amount. The energy shift and width are directly related to the Coulomb-corrected isospin-averaged scattering length of the $\bar{K}N$ system by the equation[3,4]

$$\epsilon+i\frac{\Gamma}{2}=2\alpha^3\mu_r{}^2a_c \ , \qquad (1)$$

where μ_r is the K^-p reduced mass, α is the fine-structure constant, and $\hbar=c=1$. If the Coulomb corrections are neglected, the two measurements of energy shift and width, which do not agree with each other, lead to scattering lengths which are both in sharp disagreement with the value determined from the $\bar{K}N$ scattering data (see Table I).

Although further experimental results are needed to firmly establish the energy shift and width of the $1s$ level of kaonic hydrogen, there have been several theoretical investigations to explain the apparent discrepancy. Deloff and Law[4] argued that Coulomb corrections to the K^-p scattering length could reduce the strong-interaction effect in kaonic hydrogen to bring about agreement. However, we showed that such an anomalously large Coulomb effect, when it is present, affects not only the scattering near zero energy but also at energies substantially different from the threshold energy.[7] Since at medium energy and below threshold isospin is known to be a good quantum number, it is not likely that the anomalous Coulomb effect by itself will explain the discrepancy.

TABLE I. The isospin-averaged scattering length of the $\bar{K}N$ system. The first two entries are obtained from kaonic-hydrogen energy-level shifts, the last two from analysis of $\bar{K}N$ scattering data.

a_s (fm)	Experiment	Reference
$0.10+0i$	Energy shift	1
$0.65+0.68i$	Energy shift	2
$-0.67+0.64i$	$\bar{K}N$ scattering data	5
$-0.73+0.64i$	$\bar{K}N$ scattering data	6

In view of the fact that the $\bar{K}N$ scattering data for $k_{\text{lab}}<100$ MeV/c are not well known,[8] the scattering data and kaonic-hydrogen results are not necessarily contradictory. A strong energy dependence of the scattering amplitude at low energy so that the amplitude at zero energy is very small, or has the opposite sign than that given by $\bar{K}N$ scattering data, could accommodate both the scattering data and the kaonic-hydrogen results. A physical model which yields such behavior of the scattering amplitude[9] is based on the premise that the $\Lambda(1405)$, which lies 27 MeV below the $\bar{K}N$ threshold, is an "elementary" particle rather than a composite bound state. Previous analyses[10,11] favored the composite interpretation; however, according to the quark model of hadrons, the $\Lambda(1405)$ consists of three quarks, and its dynamic origin derives from the interquark forces rather than the $\bar{K}N$ interaction.[12] Hence the $\Lambda(1405)$ is as "elementary" as the nucleon. The recent calculation of Kiang, Kumar, Nogami, and van Dijk[13] leads to the desired strong energy variation in the scattering amplitude while it also predicts the mass of the $\Lambda(1405)$. This work, which incorporates the $\bar{K}N$, $\Lambda(1405)$, and $\pi\Sigma$ channels, predicts the medium- and high-energy scattering data as well as the energy of the $\Lambda(1405)$. Figure 1 shows the $\bar{K}N$ elastic cross section in the isospin $I=0$ state as a function of c.m. energy. At low energy the few data that do exist have large errors.

In this calculation the scattering amplitude for small positive energies has the opposite sign of that obtained from the analysis of Chao *et al.*[15] and Dalitz *et al.*[6] The sign of the scattering amplitude is in principle determined from the Coulomb-nuclear interference. The experimental situation is not sufficiently clear, however, to unambiguously determine this sign. In previous analysis the sign has not been determined in a model-independent way.[5]

In order to gain further insight into this approach of explaining the kaonic-hydrogen and $\bar{K}N$ scattering data, we consider the inverse $\bar{K}N$ scattering problem, again assuming that an elementary Λ is coupled to the $\bar{K}N$ system. The scattering amplitude is such that its strong variation near threshold is similar to that obtained in Ref. 13 in order to fit $\bar{K}N$ scattering, kaonic-hydrogen, and

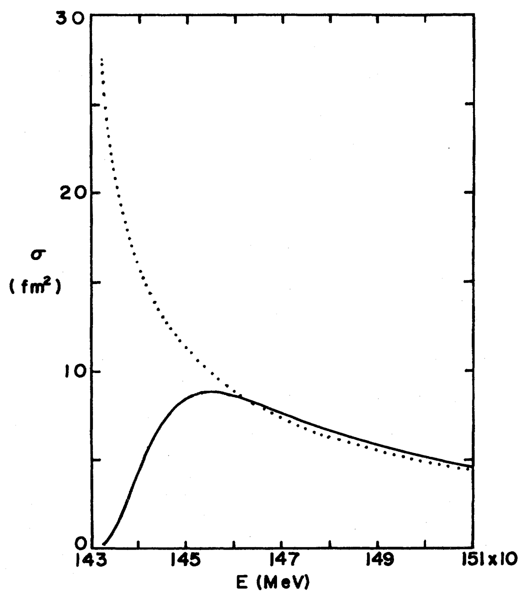

FIG. 1. The $I=0$ $\bar{K}N$ elastic cross section vs c.m. energy. The solid curve represents the results of the model calculation of Kumar (Ref. 14); the dotted curve is the result of the parametrization of Chao *et al.* (Ref. 15).

$\Lambda(1405)$ data. The coupling to the bare Λ [corresponding to the $\Lambda(1405)$] will introduce a Castillejo-Dalitz-Dyson (CDD) zero in the scattering amplitude just below threshold. Since the $\Lambda(1405)$ is an isospin $I=0$ state, most of the variation of the scattering amplitude occurs in this isospin state. The kaonic-hydrogen energy-level shift is proportional to the real part of the K^-p scattering length which is a mixed isospin state.[3] The real part of the $I=1$ scattering length is very small. Since there are no resonances or bound states near threshold in this isospin state, we do not expect unusual behavior in the $I=1$ scattering amplitude. We therefore study only the $I=0$ state. We will neglect the $\pi\Sigma$ channel although the formalism can easily be extended to include it.[13]

Although our long-term interest in this problem derives from gaining an understanding of the experimental results, our purpose at this stage is to obtain a qualitative explanation of the experimental data including the kaonic-hydrogen results. This description can be made more precise in subsequent calculations by the inclusion of additional channels of the K^-p system.

In Sec. II we review the direct $\bar{K}N$ scattering problem for the modified Lee model. The inverse scattering problem for this model is solved in Sec. III. The results of a numerical calculation are given in Sec. IV followed by a summary and conclusions in Sec. V.

II. $\bar{K}N$ SCATTERING

Before outlining the inverse scattering method we review the direct approach and set the notation. We consider the $I=0$ state with the two channels, i.e., $\bar{K}N$ and $\Lambda(1405)$. The Hamiltonian for the system is[9]

$$H=H_0+H_I\ , \tag{2}$$

where

$$H_0=m_N N^\dagger N+m_0\Lambda^\dagger\Lambda+\int d^3k\,\omega_k a_k^\dagger a_k \tag{3}$$

and

$$H_I=g_0\left[\Lambda^\dagger N\int d^3k\,u_k a_k+N^\dagger\Lambda\int d^3k\,u_k a_k^\dagger\right] -GN^\dagger N\int d^3k\,d^3k'u_k u_{k'}a_k^\dagger a_{k'}\ , \tag{4}$$

where m_N, m_0 are the nucleon and bare Λ mass [corresponding to the $\Lambda(1405)$], and $\omega_k=(\mu^2+k^2)^{1/2}$ with μ the kaon mass. The noninteracting part of the Hamiltonian is in the static approximation for the nucleon and Λ. Recoil can be taken into account in a straightforward manner, i.e., by replacing ω_p by $\omega_p+p^2/2m_N$ in the energy denominators. In this model the Schrödinger equation leads to a simple solution because the form factor u_k is the same in both terms of the interaction Hamiltonian. The interaction involves two processes, the nucleon kaon transforming into Λ or vice versa and a direct nucleon-kaon interaction of the separable kind. The form factor u_k is related to the more convenient function v_k by the equation

$$v_k=(2\pi)^{3/2}\sqrt{2\omega_k}\,u_k$$

and is normalized so that $v_0=1$. u_k is a function of k; hence only S-wave scattering occurs.

The on-shell scattering amplitude for elastic $\bar{K}N$ scattering is[9]

$$f(k)=\frac{1}{k}e^{i\delta}\sin\delta \tag{5}$$

$$=\frac{\lambda_k v_k{}^2}{1-\lambda_k I_k}\ , \tag{6}$$

where

$$\lambda_k=\frac{1}{4\pi}\left[\mathcal{G}+\frac{g_0{}^2}{\Delta-\omega_k}\right]$$

and

$$I_k=\frac{1}{\pi}\int_0^\infty\frac{dk'k'^2v_{k'}{}^2}{\omega_{k'}(\omega_{k'}-\omega_k-i\epsilon)} \tag{7}$$

and

$$\Delta=m_0-m_N\ . \tag{8}$$

We use units so that $\mu=1$, where μ is the kaon mass.

Since we are investigating the low-energy behavior of the scattering amplitude, we employ nonrelativistic kinematics for the nucleon. Furthermore we assume no absorption occurs. In the real $\bar{K}N$ system there is absorption since $\bar{K}N$ can transform into $\pi\Sigma$. However, in the isospin $I=0$ state, no inelastic channels other than the $\Lambda(1405)$ open in the energy region of interest; the threshold energy of the $\pi\Sigma$ channel is around 1330 MeV. When the inelasticity is a smoothly varying function of energy,

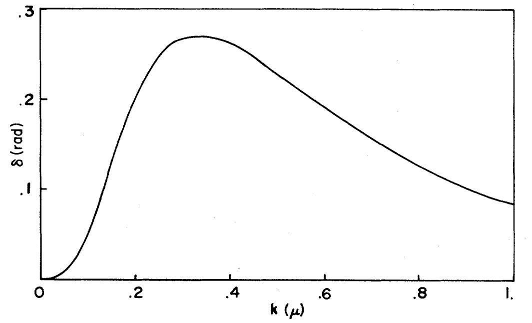

FIG. 2. Phase shifts calculated using a modified Lee model with Yamaguchi form factors when parameters were chosen (in units of μ); $\gamma=0.56$, $\tilde{\Delta}'=-0.011$, $\mathcal{G}=20$, and $g_0=0.469$.

one expects no rapid variation of the elastic scattering amplitude on account of inelastic processes[16] and consequently the neglect of the inelastic channel will not qualitatively affect the conclusions.

If the parameters of the interaction and v_k (or u_k) are specified, the phase shift can be determined,

$$k\cot\delta(k)=\frac{\lambda_k^{-1}-J_k}{v_k^2}\ , \tag{9}$$

where

$$J_k=\frac{2}{\pi}\mathrm{P}\int_0^\infty \frac{dk'k'^2v_{k'}^2}{k'^2-k^2}\ .$$

The function λ_k has the form

$$\lambda_k=\frac{1}{4\pi}\left[\mathcal{G}+\frac{g_0^2}{\tilde{\Delta}'-\dfrac{k^2}{2\mu}}\right]\ , \tag{10}$$

where $\tilde{\Delta}'=\Delta-\mu$. Choosing parameters $\tilde{\Delta}', \mathcal{G}, g_0^2$ appropriate for the $\bar{K}N$ system and a Yamaguchi form factor $v_k=\gamma^2/(k^2+\gamma^2)$, we obtain phase shifts as a function of momentum (see Fig. 2). This graph illustrates the general features of the behavior of the phase shift as a function of energy.

The scattering amplitude has a CDD zero[17] at negative energy arising from the bare Λ below threshold. It gives rise to the strong variation of the phase shift with energy around threshold. Levinson's theorem for a scattering amplitude with a CDD pole of a system with a bound state,[18] i.e., $\delta(0)-\delta(\infty)=0$, is satisfied. Although one might consider this type of strong variation of the scattering amplitude over a small energy range to be unphysical,[6] the position of the "bare" Λ just below the threshold gives rise to such a scattering amplitude.

III. INVERSE $\bar{K}N$ SCATTERING

Within the context of this model, we investigate the inverse scattering problem, i.e., given the phase shifts and the mass of the bare Λ and the physical $\Lambda(1405)$ mass, we determine the form factor v_k (or u_k). The scattering amplitude, Eq. (6), has a similar form as that obtained from a nonrelativistic scattering problem with a separable potential. The only difference is that the potential strength is a specified function of energy arising from the presence of the bare Λ. The usual method of the inverse scattering problem will apply here if we take special care with the zero and pole of the denominator of the scattering amplitude.[16,19]

The scattering amplitude can be written in the form

$$f(k)=\frac{\lambda_k v_k^2}{D(k+i\epsilon)}\ , \tag{11}$$

where

$$D(z)=1-\frac{2\lambda_z}{\pi}\int_0^\infty \frac{dk\,k^2v_k^2}{k^2-z^2}\ . \tag{12}$$

Consider D as a function of the complex variable $s=z^2$. Then

$$D(s)=1-\frac{2}{\pi}\lambda(s)\int_0^\infty \frac{dk\,k^2v_k^2}{k^2-s} \tag{13}$$

with

$$\lambda(s)=\frac{1}{4\pi}\left[\mathcal{G}+\frac{2\mu g_0^2}{\tilde{\Delta}-s}\right] \tag{14}$$

and $\tilde{\Delta}=2\mu(\Delta-\mu)$. The analytic properties of $D(s)$ on the first, or physical, sheet of the s plane are such that the inversion procedure follows in a straightforward manner.

$D(s)$ has a branch cut along the positive real s axis, with a discontinuity across the cut so that

$$D(k^2+i\epsilon)-D(k^2-i\epsilon)=-2i\lambda(k^2)kv_k^2\ . \tag{15}$$

Comparing Eqs. (5) and (11), we obtain the phase of $D(k^2+i\epsilon)$, i.e.,

$$D(k^2+i\epsilon)=|D(k^2+i\epsilon)|e^{-i\delta}\ . \tag{16}$$

$D(s)$ has zeros at negative real values of s corresponding to bound states. We let one such zero occur at $s=-k_b^2$; then the condition for bound states is the existence of positive real values of k_b^2 that satisfy the equation

$$\frac{1}{4\pi}\left[\mathcal{G}+\frac{2\mu g_0^2}{\tilde{\Delta}+k_b^2}\right]=\frac{1}{\dfrac{2}{\pi}\displaystyle\int_0^\infty dk\,k^2v_k^2/(k^2+k_b^2)}\ . \tag{17}$$

There can be no, one, or two bound states depending on the values of $\mathcal{G}$ and $\lambda(0)$. The energy-dependent coupling constant gives rise to a pole at $s=\tilde{\Delta}$ of $D(s)$. We will assume that there is one bound-state zero as well as the simple pole on the first Riemann sheet of the cut s plane. Since the $\Lambda(1405)$ occur below the $\bar{K}N$ elastic threshold, we take $\tilde{\Delta}$ to be real and negative.

We define the function

$$A(s)=\frac{s-k_b^2}{s+k_b^2}\,\frac{s-\tilde{\Delta}}{s+\tilde{\Delta}}D(s)\ . \tag{18}$$

$A(s)$ is an analytic function in the cut s plane without any zero. Since $|A(s)|\to 1$ as $|s|\to\infty$, we can apply

Cauchy's theorem to the function $\ln A(s)$ when the contour is an infinite circle with a detour about the branch cut as shown in Fig. 3,

$$\ln A(s)=\frac{1}{2\pi i}\oint_c \frac{\ln A(s')ds'}{s'-s}$$
$$=\frac{1}{2\pi i}\int_0^\infty \frac{\ln A(k'^2+i\epsilon)-\ln A(k'^2-i\epsilon)}{k'^2-s}dk'^2 . \quad (19)$$

Since

$$\ln A(k'^2+i\epsilon)-\ln A(k'^2-i\epsilon) =2i[\pi\theta(k_b^2-k'^2)-\pi\theta(-\tilde{\Delta}-k'^2) +\mathrm{Im}\ln D(k'^2+i\epsilon)] , \quad (20)$$

we evaluate $A(k^2+i\epsilon)$ in Eq. (19) to obtain

$$\ln A(k^2+i\epsilon) =\ln\left[\frac{k_b^2-k^2}{\tilde{\Delta}+k^2}\right]-\frac{\mathrm{P}}{\pi}\int_0^\infty \frac{\delta(k')dk'^2}{k'^2-k^2}-i\delta(k) , \quad (21)$$

where we have used the result that

$$\mathrm{Im}\ln D(k^2+i\epsilon)=-\delta(k) . \quad (22)$$

From Eq. (15) we get

$$\mathrm{Im}D(k^2+i\epsilon)=-\lambda(k^2)kv_k^2 . \quad (23)$$

Using this result when we have substituted the expression Eq. (18) for $A(k^2+i\epsilon)$ in Eq. (20), we obtain the final result,

$$\lambda(k^2)v_k^2=\frac{1}{k}\frac{k^2+k_b^2}{k^2-\tilde{\Delta}}\sin\delta(k) \times\exp\left[-\frac{\mathrm{P}}{\pi}\int_0^\infty \frac{\delta(k')dk'^2}{k'^2-k^2}\right] . \quad (24)$$

The equation is valid also when $\tilde{\Delta}>0$. This can be shown by defining

$$A(s)=\frac{s-k_b^2}{s+k_b^2}D(s)=\frac{s-k_b^2}{s+k_b^2}\frac{d(s)}{s-\tilde{\Delta}} \quad (25)$$

and using Cauchy's theorem for the function $\ln A(s)$ as before. In the case that there is no bound state and/or no pole in $\lambda(k^2)$ we merely set $k_b^2=0$ and/or $\tilde{\Delta}=0$ in Eq. (24) and we have the correct equation. Thus given the phase shifts at all positive energies, the position of the bound state, and the pole in $\lambda(k^2)$, we are able to determine the form factor v_k. The form factor in coordinate space is

$$v(r)=\frac{1}{2\pi^2 r}\int_0^\infty \sin(kr)v_k k\, dk . \quad (26)$$

By considering $D(s)$ as a function in the cut complex s plane with n_b zeros and n_p simple poles inside the contour of Fig. 3, we obtain

$$\frac{1}{2\pi i}\oint_c \frac{D'(s)}{D(s)}ds=n_b-n_p , \quad (27)$$

which leads to Levinson's theorem, viz.,

$$\delta(0)-\delta(\infty)=(n_b-n_p)\pi . \quad (28)$$

IV. RESULTS

We calculate the form factor $v(r)$ using as input the real part of the elastic phase shifts in the $I=0$ isospin state, obtained from the K-matrix parametrization of Dalitz *et al.*[6] These authors use an effective-range expansion form for the K matrix and have made a careful search of the parameters so that the K matrix fits the known experimental data with the exception of the energy shift of kaonic hydrogen. We choose the effective-range expansion of Dalitz *et al.*[6] rather than the zero-range expansion of Chao *et al.*[15] since the phase shifts corresponding to the latter's parametrization do not satisfy Levinson's theorem because a zero-range interaction is assumed. We take the interaction to be of finite range. Since the K matrix includes the $\pi\Sigma$ channel in the $I=0$ state, we determine the elastic $\bar{K}N$ amplitude and corresponding complex phase shift from the K matrix.[20] The real part of the phase shifts are shown in Fig. 4, curve I. They display

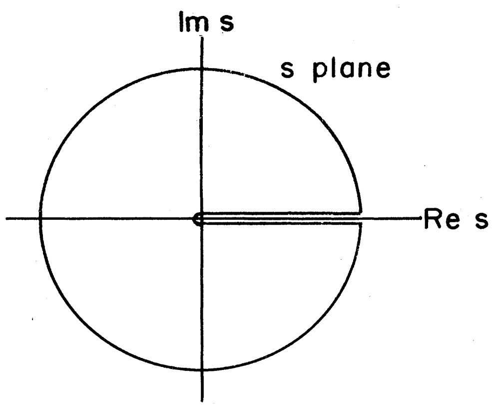

FIG. 3. The cut s plane and contour used to apply Cauchy's theorem to the function $\ln A(s)$.

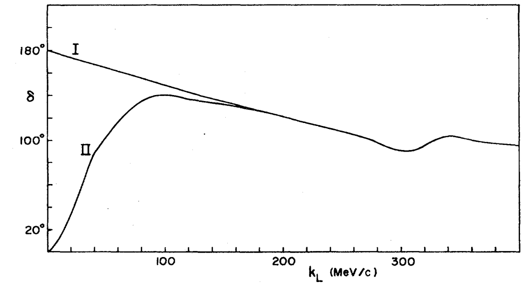

FIG. 4. Real part of phase shift as a function of laboratory momentum. Curve I is obtained from the K-matrix parametrization of Dalitz *et al.* (see Table 1, sixth column, of Ref. 6). Curve II is curve I modified by factor $(1+e^{-\alpha k}-2e^{-\sigma\alpha k})$, where $\alpha=50.00$ and $\sigma=25.06$ (in units of μ).

typical behavior of phase shifts due to an interaction supporting one bound state, i.e., $\delta_R(0)-\delta_R(\infty)=\pi$.

In our calculation the CDD zero modifies Levinson's theorem so that with one bound state, $\delta(0)-\delta(\infty)=0$. Furthermore, in order to fit the kaonic-hydrogen energy shift, the scattering amplitude at zero energy should be small or of opposite sign than at laboratory momentum of 100 MeV/c or higher. In order to fit both the higher-energy data and the kaonic-hydrogen data we define the phase shifts to be used in the inverse scattering calculation as

$$\delta(k)=(1+e^{-\alpha k}-2e^{-\sigma\alpha k})\delta_R(k)\ , \qquad (29)$$

where $\delta_R(k)$ are real phase shifts obtained from the K matrix of Dalitz *et al.*[6] The factor $(1+e^{-\alpha k}-2e^{-\sigma\alpha k})$ is introduced so that, by an appropriate choice of parameters α and σ, the phase shifts $\delta(k)$ are zero at zero momentum and at momenta greater than 100 MeV/c are approximately equal to the phase shifts obtained from experimental data. In the momentum region from 0 to 100 MeV/c not much experimental data is available, and the data that do exist have large errors associated with them. Thus the phase shifts $\delta(k)$ simulate experimental results including the scattering and the kaonic-hydrogen data. The graph of $\delta(k)$ for a typical set of α and σ is curve II in Fig. 4. The parameters α and σ are adjusted to yield the appropriate scattering length as well as the rise of the phase shifts from zero to low momenta.

The phase shifts $\delta(k)$ approach zero slowly as k goes to infinity. In order to enhance computational efficiency the phase shifts are adjusted so that they approach zero more rapidly for $k>10$ by means of a diffuse cutoff. The phase shifts in the energy region in which experimental data are available are not affected by the high-energy cutoff. In summary, the phase shifts $\delta(k)$ used in the inverse scattering problem are obtained by modifying the real part of elastic $\bar{K}N$ phase shifts at low and at very high energy without affecting them in the energy region for which they are experimentally determined.

The phase shifts $\delta(k)$ defined by Eq. (29) and the energy of the bound state $k_b^2/2\mu=27$ MeV are used as input in the inversion problem. In order that the energy dependence of the phase shift near zero energy is the result of energy variation of $\lambda(k^2)$ rather than variation of v_k, we define $\mathscr{G}$ using Eq. (24) for $k=0$. Thus k_b^2, $\tilde{\Delta}$, $\delta(k)$, g_0^2, and a, the scattering length, are given, and the form factors v_k and $v(r)$ are determined.

The resulting form factor in coordinate space is shown in Fig. 5. It consists of a short-range part, with a range less than 0.2 fm, and a weaker portion with a range of approximately 0.4 fm. This is consistent with an understanding of the $\bar{K}N$ interaction in terms of two-pion exchange and exchange of heavier particles.

The small wiggles in the form factor for $r>0.5$ are due to the high-energy cutoff of the phase shift. If a sharp cutoff is used, the form factor will have zeros and have the shape of a diffraction pattern. For a more diffuse cutoff, e.g., we divide $\delta(k)$ by $1+\exp(-0.5k+5)$, the dips are filled in and the bumps are smoothed out. The overall decay of the form factor for increasing r is not qualitatively altered by changing the diffuseness of the cutoff. In any case, regardless of the cutoff the "experimentally" determined phase shifts are not affected.

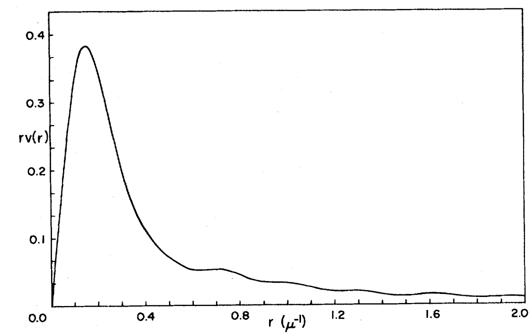

FIG. 5. Form factor as a function of r with $\mathscr{G}=19.14$, $g^2=0.2126$, $\tilde{\Delta}'=-0.0115$, $a_s=1.626$, $\alpha=50.00$, $\sigma=25.26$ (in units such that $\mu=1$).

The form factors for a set of phase shifts which lead to a scattering length of 0.10 fm, instead of 0.65 fm, is identical, to two significant figures, to the one shown in Fig. 5. However, the value of $\mathscr{G}$ is now $\mathscr{G}=18.6$. This is consistent with the constraint that the threshold behavior of the phase shift is largely determined by the energy variation of $\lambda(k^2)$.

There are still two degrees of freedom in the model; $\tilde{\Delta}$ and g_0^2 are not known *a priori*. However, in order that the bound state be an elementary particle, the coupling constant g_0 is small, and consequently $|\tilde{\Delta}|$ needs to be small so that the scattering amplitude rises rapidly above threshold. In calculating the form factor it is important to choose the parameters of λ_k so that the small value of the scattering length is due to a small value of λ_0. Otherwise the inverse scattering method is still valid, but one obtains form factors in k space which are unity when $k=0$ but become very large at values of k for which the scattering amplitude is not small.

V. CONCLUSION

Employing phase shifts derived from the $I=0$ K-matrix parametrization of the $\bar{K}N$ scattering process, we are able to determine the source function by the inverse scattering method. We find that although there is a strong variation in the scattering amplitude as a function of energy, the source function has no unphysical properties. It is a smoothly varying function that has a sufficiently short range to make it consistent with the strong $\bar{K}N$ interaction. The interpretation of the $\Lambda(1405)$ as an elementary particle does not lead to undesirable properties, and yet it allows one to fit simultaneously the kaonic-hydrogen and scattering data.

Since the coupled-channel inverse scattering problem has also been solved,[19] this approach can be generalized to include the $\pi\Sigma$ channel. Alternatively one can introduce an inelasticity parameter in the elastic channel to account for absorption by other channels and determine the interaction form factor using the inverse scattering technique.[16] In either case the interaction is obtained directly from the scattering data.

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