Neutrino oscillations and the atmospheric neutrino fluxes

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We have examined the effects of a possible ν_e-ν_μ oscillation on the atmospheric neutrino fluxes. We have included the influence of the Earth's medium on the oscillations and also considered averaged fluxes in order to account for the inherent uncertainties in (i) the neutrino path lengths and (ii) zenith angles of neutrino arrival. The restrictions provided by our simple criteria for the detectability or for the neglect of oscillation effects in the fluxes are compared with the limits set by a recent reactor experiment. This comparison reveals that the oscillation effects are ignorable in the downward fluxes, but can be sizable in the upward fluxes provided the oscillation length is in the range 600—24 000 km. Incorporating these oscillation effects, we have also obtained lower and upper limits on the ratios of (i) upward and downward fluxes of $(\nu+\bar{\nu})$ of a given flavor, and (ii) fluxes of $(\nu+\bar{\nu})$ of e and μ flavors traveling in the upward direction.

I. INTRODUCTION

In interpreting the observations of deep-mine experiments, it is important to investigate the influence of possible $\nu_e-\nu_\mu$ oscillations[1] on the atmospheric neutrino fluxes.[2,3] Oscillations of the atmospheric neutrinos can lead to unequal fluxes of the upward and downward neutrinos mainly because of the unequal lengths of their flight paths. This up-down asymmetry in the neutrino flux can also be influenced by the medium[4,5] through which the neutrinos have to pass before arriving at the detection level of a deep-mine experiment. An upward-traveling neutrino spends most of its journey going through the terrestrial matter which may affect the oscillations, while the downward-traveling neutrino goes mostly through the Earth's atmosphere, which may be regarded as vacuum. To a good approximation, therefore, we may study the oscillations of the upward neutrinos as occurring only in the Earth's medium while those of the downward neutrinos as only in vacuum. Effects due to geomagnetism and solar modulation can also influence the up-down asymmetry, but these effects are negligible for high-energy neutrinos[6] above a few GeV.

Besides the asymmetries between the fluxes of neutrinos of a given flavor traveling upward and downward, one can also study flavor asymmetries between the fluxes of neutrinos traveling in a given direction as a possible source of information about oscillations. Thus oscillations can make the flux ratios D_μ/U_μ, D_e/U_e, D_μ/D_e, and U_μ/U_e different from their original (production-level) values; here, D_f and U_f, respectively, denote the corresponding fluxes of downward- and upward-traveling neutrinos of flavor f. Experimental measurements of the D's would involve more difficult background problems than those of the U's, and, therefore, information derived from U_μ/U_e would be less ambiguous. We expect the ratio U_μ/U_e to be influenced by the oscillations occurring in matter, D_μ/D_e by oscillations in vacuum, and the remaining two up-down ratios by oscillations taking place in both media.

Our discussion throughout is within the framework of the two-component model incorporating the flavor oscillations $\nu_e \leftrightarrow \nu_\mu$; the various physical effects will be considered as functions of two oscillation parameters: the mixing angle θ_v and the oscillation length

$$l_v = 4\pi E/(\Delta m^2)\ ,\qquad \Delta m^2 = m_1{}^2 - m_2{}^2\ , \tag{1}$$

where $m_{1,2}$ are the masses of the two mass eigenstates, and E is the neutrino energy; the subscript v refers to vacuum as the medium in which oscillations occur. In discussing the flux asymmetries, we have to assume the expected flux ratios in the absence of oscillations as known.

The plan of the paper is as follows. In Sec. II we list the relevant expressions of the oscillation phenomenology including the matter effects for ν and $\bar{\nu}$. Section III is devoted to several interesting questions arranged as subsections: How do we average the probabilities over the neutrino flight paths so as to relate them to the observed neutrino fluxes? What are the criteria for the oscillation effects to be detectable, or be negligible, in a typical deep-mine experiment? At the end of this section we review our considerations in the light of the laboratory experimental limits on oscillations. Section IV discusses the importance of matter corrections to the oscillations in the upward neutrino fluxes. In Sec. V we consider the flux ratios involving D's and U's; due to the reasons of convenience in measurements, we have only considered the D's and U's as referring to the sums of fluxes $(\nu+\bar{\nu})$, going downward and upward. Section VI contains some concluding remarks and a summary.

II. FORMALISM OF THE TWO-COMPONENT MODEL

Let the mass eigenstates $\nu_{1,2}$ mix in vacuum through the angle θ_v to form the flavor eigenstates ν_e and ν_μ as[1]

$$|\nu_e\rangle=|\nu_1\rangle\cos\theta_v+|\nu_2\rangle\sin\theta_v , \quad |\nu_\mu\rangle=-|\nu_1\rangle\sin\theta_v+|\nu_2\rangle\cos\theta_v . \tag{2}$$

The probability for the nondiagonal transition $\nu_e\leftrightarrow\nu_\mu$ in traveling through vacuum is

$$R^{\downarrow}_{e\mu}(x)=\sin^2(2\theta_v)\sin^2\left[\frac{\pi x}{l_v}\right] , \tag{3}$$

where x is the distance traveled, the superscript $\downarrow$ denotes the case of downward travel, and the subscript v refers to vacuum. Similarly, for $\nu_e\to\nu_e$ and $\nu_\mu\to\nu_\mu$,

$$R^{\downarrow}_{ee}(x)=R^{\downarrow}_{\mu\mu}(x)=1-R^{\downarrow}_{e\mu}(x) , \tag{4}$$

where the last equality is just the probability conservation in the two-component model. For antineutrinos, the corresponding probabilities will be denoted by overbars:

$$\bar R^{\downarrow}_{e\mu}(x)=\bar R^{\downarrow}_{\mu e}(x)=R^{\downarrow}_{e\mu}(x) , \quad \bar R^{\downarrow}_{ee}(x)=\bar R^{\downarrow}_{\mu\mu}(x)=R^{\downarrow}_{ee}(x) . \tag{5}$$

Neutrinos and antineutrinos traveling upward through terrestrial matter mix according to the relations[4,5]

$$|\nu_e\rangle=|\nu_{1m}\rangle\cos\theta_m+|\nu_{2m}\rangle\sin\theta_m \quad |\nu_\mu\rangle=-|\nu_{1m}\rangle\sin\theta_m+|\nu_{2m}\rangle\cos\theta_m , \tag{6}$$

$$|\bar\nu_e\rangle=|\bar\nu_{1m}\rangle\cos\bar\theta_m+|\bar\nu_{2m}\rangle\sin\bar\theta_m \quad |\bar\nu_\mu\rangle=-|\bar\nu_{1m}\rangle\sin\bar\theta_m+|\bar\nu_{2m}\rangle\cos\bar\theta_m , \tag{7}$$

where the mixing angles θ_m and $\bar\theta_m$ for ν and $\bar\nu$, respectively, traveling through matter are given by

$$\tan(2\theta_m)=\frac{\tan(2\theta_v)}{1-(l_v/l_W)\sec(2\theta_v)} , \tag{8}$$

$$\tan(2\bar\theta_m)=\frac{\tan(2\theta_v)}{1+(l_v/l_W)\sec(2\theta_v)} , \tag{9}$$

$$l_W\equiv\sqrt{2}\pi/GN_e . \tag{10}$$

Here, the subscript m stands for matter, G is the Fermi constant, and N_e is the mean number of electrons per unit volume in the matter traversed. The Wolfenstein length l_W reflects the effect of matter, due to the coherent forward scattering of ν_e's and $\bar\nu_e$'s by the atomic electrons through the charged weak currents. As the weak neutral currents have been assumed to be ν_e-ν_μ symmetric,[7] they do not influence the value of l_W. For terrestrial matter, $l_W\simeq 8\times10^3$ km, corresponding to the electron density $N_e\simeq 2N_A$, N_A being the Avogadro number.

The nondiagonal probability for upward neutrinos traveling through matter now becomes, for $\nu_e\leftrightarrow\nu_\mu$, in analogy to (3),

$$R^{\uparrow}_{e\mu}(x)=\sin^2(2\theta_m)\sin^2(\pi x/l_m) , \tag{11}$$

where

$$l_m=l_v[1+V^2-2V\cos(2\theta_v)]^{-1/2} , \tag{12}$$

$$V=l_v/l_W\simeq 3.1\times10^{-4}\frac{E\ (\mathrm{GeV})}{\Delta m^2\ (\mathrm{eV}^2)} , \tag{13}$$

$$\sin^2(2\theta_m)=(l_m/l_v)^2\sin^2(2\theta_v) . \tag{14}$$

The corresponding $\bar\nu_e\leftrightarrow\bar\nu_\mu$ probability is given by

$$\bar R^{\uparrow}_{e\mu}(x)=\sin^2(2\bar\theta_m)\sin^2(\pi x/\bar l_m) , \tag{15}$$

with

$$\bar l_m=l_v[1+V^2+2V\cos(2\theta_v)]^{-1/2} , \tag{16}$$

$$\sin^2(2\bar\theta_m)=(\bar l_m/l_v)^2\sin^2(2\theta_v) . \tag{17}$$

The diagonal probabilities are obtained, as in Eqs. (4), by using flux conservation. For neutrinos, for instance,

$$R^{\uparrow}_{\mu\mu}(x)=R^{\uparrow}_{ee}(x)=1-R^{\uparrow}_{e\mu}(x) .$$

This assumes that the absorption of neutrinos in passing through the matter can be ignored, which is trivially valid since the distance traveled $x\simeq10^9$ cm in the Earth's medium is small compared to the absorption mean free path of neutrinos $\lambda\simeq10^{14}$ cm in the few-GeV range.

III. DETECTION OF OSCILLATION EFFECTS

A. Path-length averaging

The probabilities (3), (11), and (15) of the earlier section require a knowledge of the flight path length x, the appropriate oscillation length l, and the mixing angle. The existence of oscillations is not yet experimentally established, and one does not know the mixing angles, nor the oscillation lengths; the latter requires a knowledge of Δm^2 and of E, and E is seldom known precisely for cosmic-ray neutrinos. Even if one knew the oscillation lengths and mixing angles, the path length x is very hard to know because of the uncertainties in the production level and the actual travel direction of the neutrino. One is thus forced to "average" the flux over the path lengths.

Let us consider this averaging first for the downward neutrinos. We shall define the uncertainty η_v in the flight path such that the maximum, the minimum, and the average values of the path length are, respectively, $x_0+\eta_v$, $x_0-\eta_v$, and x_0. The nondiagonal oscillation probability relevant for observations should then be an average of $R^{\downarrow}_{e\mu}(x)$ over the *downward* path length:

$$P_d=\frac{1}{2\eta_v}\int_{x_0-\eta_v}^{x_0+\eta_v}dx\,R^{\downarrow}_{e\mu}(x)=P_{0v}\left[1-\frac{\sin\psi_v}{\psi_v}\cos\phi_v\right] , \tag{18}$$

where

$$P_{0v}=\tfrac{1}{2}\sin^2(2\theta_v),\ \psi_v=\frac{2\pi\eta_v}{l_v},\ \ \phi_v=\frac{2\pi x_0}{l_v} . \tag{19}$$

For small enough uncertainties in the path lengths, $\eta_v\sim0$ and P_d reduces to $R^{\downarrow}_{e\mu}(x)$, as expected. However when

$\psi_v \to \infty$, i.e., when the path-length uncertainty is very much larger than the oscillation length l_v, P_d reduces to the oft-quoted[2,3] "average value" P_{0v}, which is obtained by replacing $\sin^2(\pi x/l_v)$ by $\frac{1}{2}$ in Eq. (3).

The corresponding "smeared-out" probabilities for the *upward* travel, for neutrinos and antineutrinos, respectively, can be written down analogously

$$P_u = P_{0m}\left[1 - \frac{\sin\psi_m}{\psi_m}\cos\phi_m\right], \quad (20)$$

$$P_{0m} = \tfrac{1}{2}\sin^2(2\theta_m), \quad \psi_m = \frac{2\pi\eta_m}{l_m}, \quad \phi_m = \frac{2\pi x_0}{l_m}, \quad (21)$$

$$\bar{P}_u = \bar{P}_{0m}\left[1 - \frac{\sin\bar{\psi}_m}{\bar{\psi}_m}\cos\bar{\phi}_m\right], \quad (22)$$

$$\bar{P}_{0m} = \tfrac{1}{2}\sin^2(2\bar{\theta}_m), \quad \bar{\psi}_m = \frac{2\pi\eta_m}{\bar{l}_m}, \quad \bar{\phi}_m = \frac{2\pi x_0}{\bar{l}_m}, \quad (23)$$

where η_m is the uncertainty in the path length of the upward travel.

In our discussions to follow, whenever we wish to avoid any dependence on a specific value of x_0, we shall utilize the following simple bounds satisfied by the averaged probabilities:

$$P_{0v}\left[1 - \frac{|\sin\psi_v|}{\psi_v}\right] \le P_d \le P_{0v}\left[1 + \frac{|\sin\psi_v|}{\psi_v}\right], \quad (24)$$

$$P_{0m}\left[1 - \frac{|\sin\psi_m|}{\psi_m}\right] \le P_u \le P_{0m}\left[1 + \frac{|\sin\psi_m|}{\psi_m}\right], \quad (25)$$

$$\bar{P}_{0m}\left[1 - \frac{|\sin\bar{\psi}_m|}{\bar{\psi}_m}\right] \le \bar{P}_u \le \bar{P}_{0m}\left[1 + \frac{|\sin\bar{\psi}_m|}{\bar{\psi}_m}\right]. \quad (26)$$

What are the reasonable values to be assumed for η_v and η_m? Taking the total extent of the atmosphere to be about 50 km, the intrinsic uncertainty η_0 in the production levels is at most 25 km. For a neutrino traveling vertically downward, we may take $\eta_v = \eta_0$; then x_0 and η_v can be at most 25 km each. In practice, the value of η_0 for a neutrino of a few GeV is probably only a few km: first, the atmospheric neutrinos cannot originate too close to the Earth's surface as most of the parent hadrons which would have decayed into neutrinos in the requisite energy range would have been already absorbed; nor can they originate too far above the Earth's surface because of the low density of matter in the high atmospheric layers. Strictly speaking, we should write

$$\eta_v = [\eta_0{}^2 + (\eta_\theta^\downarrow)^2]^{1/2}, \quad (27)$$

to account for the extra uncertainty due to the neutrino arrival in nonvertical directions. Here $\eta_\theta^\downarrow$ is the uncertainty originating from the finite size of the angular cone around the vertical. For a cone of half-angle θ,

$2\eta_\theta^\downarrow =$ difference between the maximum and minimum path lengths arising from the angular spread (28)

$= x_0(\sec\theta - 1)$, (28a)

where x_0 is the average height of the production layer. Since x_0 and η_0 are of the same order, and both are ill determined, we take $\eta_v \simeq \eta_0$ and conservatively assume

$$\eta_v = 10 \text{ km} \quad (29)$$

as an illustration for our numerical work.

On the other hand, for the neutrinos traveling upward in matter,

$$\eta_m = [\eta_0{}^2 + (\eta_\theta^\uparrow)^2]^{1/2}, \quad (30)$$

where $\eta_\theta^\uparrow$ is the uncertainty due to the angular spread, defined as in Eq. (28). One gets

$$\eta_\theta^\uparrow = (D_E/2)(1 - \cos\theta), \quad (31)$$

where $D_E \simeq 1.3 \times 10^4$ km is the Earth's diameter. Since the values of θ cannot be too small in practice, the $\eta_\theta^\uparrow$ term in Eq. (30) generally dominates, and hence we take

$$\eta_m \simeq D_E \sin^2\frac{\theta}{2}. \quad (32)$$

We shall take $\theta = 20°$ and use $\eta_m = 400$ km in our numerical work.[8]

B. Criterion for detecting flux oscillations

To detect the variations in the atmospheric neutrino fluxes due to oscillations, the variations should have to be larger than the uncertainty involved in a typical cosmic-ray experiment.

1. Detectability in downward fluxes

The stipulation that the averaged off-diagonal probability (18) should be "detectably large" leads us naturally to a condition of the type

$$P_d > \epsilon, \quad (33)$$

here ϵ is a small dimensionless parameter denoting the tolerance limit (optimistically of the order of 10% but lower than 50%). To avoid any reference to the average path length x_0, we shall use the inequality (24) and postulate the detectability criterion to be

$$\tfrac{1}{2}\sin^2(2\theta_v)\left[1 - \frac{1}{\psi_v}|\sin\psi_v|\right] > \epsilon. \quad (34)$$

This can be recast in the convenient form

$$\frac{1}{\psi_v}|\sin\psi_v| < \left[1 - \frac{2\epsilon}{\sin^2(2\theta_v)}\right], \quad (35)$$

where

$$\psi_v = \frac{2\pi\eta_v}{V l_W}. \quad (36)$$

Taking $\eta_v = 10$ km and $\epsilon = 0.1$ the condition (35) yields the region of allowed values of V and $\sin^2(2\theta_v)$ enclosed by the curve labeled (a) in Fig. 1.[9] This region corresponds to the detectability of the oscillations in downward fluxes of ν as well as $\bar{\nu}$. The maximum allowed value of $V = 0.007$ (or $l_v = 56$ km) occurs for maximal mixing, $\sin^2(2\theta_v) = 1$. The curve (a) is not extended into the region

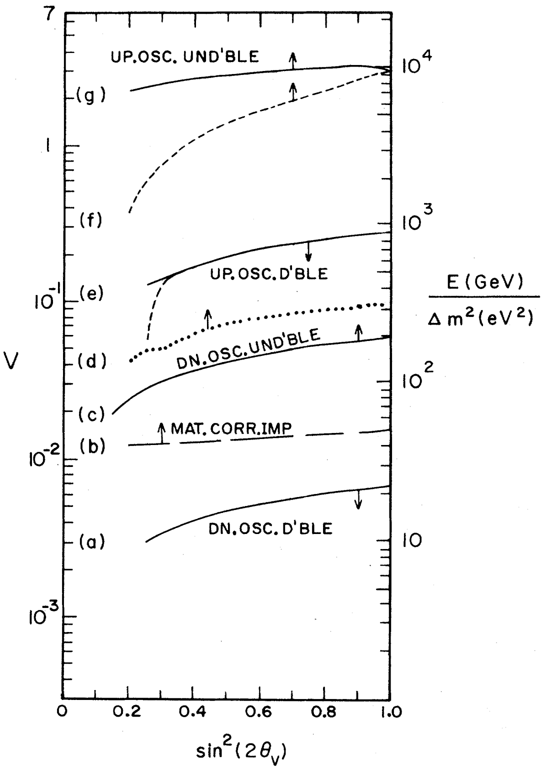

FIG. 1. Boundaries of the allowed regions (indicated by arrows) in the V-$\sin^2(2\theta_v)$ plot. The numerical values used are $\eta_v=10$ km, $\eta_m=400$ km, $\epsilon=0.1$, and $\mu=0.1$. The curve labeled (a) denotes the boundary of V values below which the oscillations in the downward fluxes are "detectable." Curve (e) is its analog for the upward neutrino flux (dashed curve is for $\bar{\nu}$). For the values of V above the curve (c) oscillations in the downward fluxes are "undetectable"; curve (g) is its analog for the upward fluxes for ν, and dashed curve (f) is for $\bar{\nu}$. Values of V above the curve labeled (b) lead to matter corrections which are important according to the 10% criterion. Dotted curve (d) is the 90%-C.L. lower limit on V for a 5-GeV neutrino to exhibit oscillations, as implied by the results of the Gösgen reactor experiment. The vertical axis on the right is marked in units of [E (in GeV)/Δm^2 (in eV^2)].

$\sin^2(2\theta_v)<2.55\epsilon\simeq0.25$ because in this region the curve breaks into several disjoint pieces due to the oscillatory nature of the function ($|\sin\psi|/\psi$). It does not appear worthwhile displaying these details as this region corresponds to tiny mixing angles ($\theta_v<15°$). In any case it should be noted that one could also start with a simpler criterion[10] which avoids these pathological features, by replacing $|\sin\psi|$ in (34) by unity and requiring

$$\frac{1}{\psi_v}<\left[1-\frac{2\epsilon}{\sin^2(2\theta_v)}\right] . \tag{37}$$

Although this condition is stronger than the condition (35) which we had used, it yields a curve which is roughly the same as the curve (a) in Fig. 1: for $\sin^2(2\theta_v)\gtrsim0.35$ the two curves are hardly distinguishable; at smaller θ_v the curve from (37) lies slightly lower than the curve (a); e.g., at $\sin^2(2\theta_v)=0.3$, the criterion (37) gives $V<2.6\times10^{-3}$ compared to the better limit $V<3.4\times10^{-3}$ from the criterion (35).

2. *Detectability in upward fluxes*

Considering first the case of neutrinos, we write, in analogy to the condition (34) of the vacuum case, the detectability criterion for travel in matter as

$$\tfrac{1}{2}\sin^2(2\theta_m)\left[1-\frac{1}{\psi_m}|\sin\psi_m|\right]>\epsilon . \tag{38}$$

This also may be rewritten in the form

$$\frac{1}{\psi_m}|\sin\psi_m|<\left\{1-\frac{2\epsilon}{\sin^2(2\theta_v)}[1+V^2-2V\cos(2\theta_v)]\right\} , \tag{39}$$

where

$$\psi_m=\frac{2\pi\eta_m}{Vl_W}[1+V^2-2V\cos(2\theta_v)]^{1/2} . \tag{40}$$

For the flight-path uncertainty of the upward neutrinos we shall take a typical value of 400 km corresponding to neutrino directions lying within 20° to the vertical [see Eq. (32)]. The allowed region implied by the condition (39) for detectability of oscillations in the upward neutrino fluxes has the boundary shown by the curve (e) in Fig. 1. This region includes the region allowed by the relation (35) for the downward fluxes. For maximal mixing, we see that $V<0.28$; this upper limit on V is, however, sensitive to the assumed η_m; for $\eta_m=1200$ km it turns out that $V<0.82$. Just as the curve (a) for vacuum travel moves up in proportion to η_v,[9] the boundary (e) also moves up with η_m, but only in rough proportionality.

For antineutrinos traveling upward, the detectability criterion would be

$$\tfrac{1}{2}\sin^2(2\bar{\theta}_m)\left[1-\frac{1}{\bar{\psi}_m}|\sin\bar{\psi}_m|\right]>\epsilon , \tag{41}$$

which can also be rewritten in the form of (39) simply by reversing the signs of the $\cos(2\theta_v)$ terms. In fact, as this is the only change in going from the ν to $\bar{\nu}$ case the boundaries of the allowed regions coincide for small values of $\cos(2\theta_v)$; this feature is evident in the curve (e) of Fig. 1 for $\sin^2(2\theta_v)\gtrsim0.4$, where the full curve is for ν and dashed curve for $\bar{\nu}$.[11]

It should be emphasized that the difference between the boundaries (a) and (e) of the detectability regions for the downward and upward cases in Fig. 1 arises mainly from the difference in the values of the respective η's and not so much from the difference in the media. This is easily seen by verifying that we obtain nearly the same allowed region from the criterion (35) (meant for the detectability of oscillations in vacuum) provided we appropriately take the path length uncertainty η_v to be 400 km. The reason for this concidence is that the two criteria (34) and (38), or (41), differ only by the absence or the presence of a factor which is close to unity,

$$1+V^2\pm2V\cos(2\theta_v)\simeq1 ,$$

in the relevant region of V values. Nevertheless it should be noted that it does not imply that the matter corrections are negligible in this region, for it is only the *regions* of detectability which are nearly the same. This point will be elaborated in Sec. IV.

Once again at small enough θ_v, instead of studying the complexities of the criterion (39), we may undertake to simply replace the factor $|\sin\psi_m|$ on the left side by unity and evaluate a slightly poorer bound. The ensuing discussion parallels the one concerning the earlier relation (37), and the modifications in the curve (e) are not large.[10]

C. Criterion for neglecting flux oscillations

In contrast to the spirit of Sec. III B, we shall now discuss the conditions under which oscillation effects can be ignored. Once again the discussion breaks up naturally into two cases.

1. Neglect of oscillation effects in downward fluxes

For this we shall stipulate that the probability for $\nu_e \leftrightarrow \nu_\mu$ transition in traversing a path length x in vacuum is below the level of detectability characterized by a parameter ϵ

$$\sin^2(2\theta_v)\sin^2(\pi x/l_v) < \epsilon \ . \tag{42}$$

Since x cannot in any case exceed the height of the first collision of the cosmic primary (which is $\lesssim 50$ km), we can view the above criterion as providing a lower limit on the value of l_v, above which oscillations do not have noticeable influence on the fluxes. In terms of the variable V therefore, we have

$$V > \frac{\pi x}{l_W} \frac{1}{\sin^{-1}\{[\epsilon/\sin^2(2\theta_v)]^{1/2}\}} \ . \tag{43}$$

The resulting boundary of V values above which downward flux oscillations become undetectable is shown as curve (c) in Fig. 1, for $\epsilon=0.1$ and $x=50$ km.

2. Neglect of oscillation effects in upward fluxes

For large enough values of V ($\gg 1$) matter can however damp out oscillations altogether[4,5] because the mixing angles θ_m, $\bar{\theta}_m$ become negligibly small irrespective of θ_v; see Eqs. (8) and (9). This means the probability of oscillation into the "other" type of neutrino becomes negligible for large V

$$P_u < \epsilon \ . \tag{44}$$

We write this, using the bound (25), as

$$\tfrac{1}{2}\sin^2(2\theta_m)\left[1+\frac{1}{\psi_m}|\sin\psi_m|\right] < \epsilon \ , \tag{45}$$

which may be recast in the style of (39) for doing numerical work. For the previously chosen values of the parameters ($\epsilon=0.1$, $\eta_m=400$ km), the region of the V-$\sin^2(2\theta_v)$ plane characterizing the near absence of the oscillation effects in the upward neutrino fluxes is shown by curve (g) of Fig. 1. The case for the antineutrinos is discussed similarly by using the constraint

$$\tfrac{1}{2}\sin^2(2\bar{\theta}_m)\left[1+\frac{1}{\bar{\psi}_m}|\sin\bar{\psi}_m|\right] < \epsilon \ , \tag{46}$$

and the results are shown by the dashed curve (f). A comparison of the two curves (f) and (g) shows that among the upward fluxes the $\bar{\nu}$ oscillations are undetectable at relatively smaller values of V (or at smaller energies) than those for the ν oscillations.

It may be noted that the boundaries (f) and (g) for "matter damping" in Fig. 1 are not sensitive to the assumed values of η_m; the quantities in parentheses in the conditions (45) and (46) (which contain the entire η_m dependence) can vary at most by a factor 2. In fact, the criterion (44), or its stronger version (45), is always valid provided

$$\sin^2(2\theta_m) < \epsilon \ , \tag{47}$$

the implication of such a naive restriction as (47) turns out to be nearly the same as the one shown in Fig. 1 for ν; these statements are also valid for the case of $\bar{\nu}$. Note that the curve (g) for the ν case has its maximum at $\sin^2(2\theta_v) \simeq 1-\epsilon$ rather than at 1.

D. Use of experimental limits: An illustration

It is perhaps instructive to view the considerations of Secs. III B and III C in the light of the recent results of the Gösgen reactor experiment of Vuilleumier *et al.*[12] This experiment finds no evidence for $\bar{\nu}_e$ oscillations by the "depletion" method, and thus provides useful restrictions on θ_v and Δm^2. We shall assume that these restrictions are relevant for the ν_e-ν_μ oscillation and convert them as limits on $\sin^2(2\theta_v)$ and V. For illustration, we have displayed in Fig. 1 the 90% C.L. limits on θ_v and Δm^2 in the range $\sin^2(2\theta_v)=0.2-1.0$ as the lower limits on V [dotted curve (d)]. For converting the quoted experimental limits on Δm^2 into limits on the variable V, we have used $E=5$ GeV as a typical energy for the neutrinos of interest in the present context.[13]

Obviously, a comparison of the curves (c) and (d) in Fig. 1 shows that the downward neutrino fluxes around 5 GeV and higher are unlikely to be influenced by oscillations.

For the detection of the oscillation effects in the upward fluxes the Gösgen limits favor values of V in the small region between the curves (d) and (g). For a typical value of $\sin^2(2\theta_v)=0.5$ this range is $V=0.075-3$ (or $l_v=600-24\,000$ km), which corresponds, for $E=5$ GeV, to the range $\Delta m^2=5\times10^{-4}-0.02$ (eV).^{2} Consequently matter corrections, if any, would be important only in such restricted ranges.

It should however be emphasized that the boundaries given in Fig. 1 provide only necessary constraints as they have been obtained by using the *limits* (24)–(26) rather than the probabilities P_d, P_u, and $\bar{P}_u$ themselves. Thus, for instance, the oscillation effects may be extinguished by matter at some values of V situated even below the curve (f) or (g).

IV. IMPORTANCE OF MATTER CORRECTIONS

Matter corrections arise if the ν_e-ν_μ oscillations occurring in traversing a piece of matter are different from the corresponding oscillations in vacuum, keeping the path lengths the same in both the media. We regard matter corrections to be significant if the ratio ρ of the averaged nondiagonal probabilities in vacuum and in matter, differs appreciably from unity; that means ρ differs from 1 by more than a quantity μ (of the order of 0.1). Using therefore the same value of the mean path length x_0 in the two cases, and also its uncertainty

$$\eta_m = \eta_v \ , \tag{48}$$

we define the ratio ρ

$$\rho \equiv \frac{P_d}{P_u} = [1+V^2-2V\cos(2\theta_v)] \times \frac{1-(1/\psi_v)\sin\psi_v \cos\phi_v}{1-(1/\psi_m)\sin\psi_m \cos\phi_m} \ , \tag{49}$$

and require the condition

$$|\rho - 1| > \mu \tag{50}$$

for matter corrections to be important. In obtaining the form (49) which refers to neutrinos, we have used Eqs. (12)—(14) and (18)—(21). Noting that the maximum and minimum values of ($\sin\psi \cos\phi$) are ± 1 we can express the criterion (50) as two conditions:

$$1-\mu > \rho \geq [1+V^2-2V\cos(2\theta_v)] \frac{1-(1/\psi_v)}{1+(1/\psi_m)} \ , \tag{51}$$

$$[1+V^2-2V\cos(2\theta_v)] \frac{1+(1/\psi_v)}{1-(1/\psi_m)} \geq \rho > 1+\mu \ . \tag{52}$$

The value of V above which matter corrections are important for a given θ_v, can be determined from the constraints (51) and (52). The latter represent the condition (50) only if ψ_v and ψ_m exceed unity. We have taken $\mu = 0.1$ and $\eta = 400$ km and obtained the dashed curve (b) in Fig. 1; sizable matter corrections are expected above this curve (note that the V values of this curve correspond to $\psi_v \simeq \psi_m \simeq 20$).

The corresponding antineutrino region is obtained by replacing $\cos(2\theta_v)$ by $-\cos(2\theta_v)$ and ψ_m by $\bar{\psi}_m$ in the above relations; the region derived turns out to be almost indistinguishable from the region bounded by the curve (b).

In summary, matter corrections are expected to be important at the level of 10% in the entire region bounded by the curves (b) and (e) in Fig. 1. Again for a 5-GeV neutrino, for $\sin^2(2\theta_v) = 0.5$ this corresponds to the range $\Delta m^2 = 0.008$–0.12 (eV)^2.

To get an idea of the sensitivity of this region to values of μ and η_m, we observe that, to a good approximation, we can drop the $O(V^2)$ terms in handling the conditions (51) and (52); thus we obtain the approximate lower limit on V,

$$V > \frac{\mu}{2\left[l_W/2\pi\eta_m \pm \cos(2\theta_v) \right]} \ , \tag{53}$$

which increases with μ and also with η_m. The presence of a lower limit on V for the matter corrections to become important is understandable; for $V \ll 1$ one has $(l_m, \bar{l}_m) \simeq l_v$, implying no essential difference between oscillations in matter and in vacuum.

V. RATIOS OF FLUXES

From the point of view of the cosmic-ray experiments deep underground, it would be useful to consider path-averaged neutrino fluxes which refer to a particular flavor. Let $f_e, f_\mu, \bar{f}_e, \bar{f}_\mu$ denote, respectively, the relative fractions of $\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu$ fluxes at a given energy in the absence of oscillations. Due to oscillations, the path-averaged fluxes of the upgoing and downgoing neutrinos at the detector will then be modified as

$$\begin{aligned} d_e &= f_e(1-P_d) + f_\mu P_d \ , \\ d_\mu &= f_\mu(1-P_d) + f_e P_d \ , \\ u_e &= f_e(1-P_u) + f_\mu P_u \ , \\ u_\mu &= f_\mu(1-P_u) + f_e P_u \ , \end{aligned} \tag{54}$$

where u and d denote the relative upward and downward fluxes of neutrinos, and the subscripts μ and e stand for the neutrino flavor. For antineutrinos, similar expressions hold, using bars over the various quantities:

$$\bar{u}_e = \bar{f}_e(1-\bar{P}_u) + \bar{f}_\mu \bar{P}_u, \text{ etc.} \tag{55}$$

One can easily write down expressions for various experimental observables, such as $(u_e - d_e), (\bar{u}_e - \bar{u}_\mu), \ldots,$ in terms of θ_v, l_v, and other parameters. General relations characteristic of the two-component model, such as

$$u_e - d_e = d_\mu - u_\mu \tag{56}$$

will follow simply because neutrino absorption in the Earth's medium is negligible. We shall, however, concentrate not on $u_e, \bar{d}_\mu, \ldots,$ but on sums of the neutrino and antineutrino fluxes of a given flavor, going either upward or going downward; these will be denoted by capital letters U and D: $D_e = d_e + \bar{d}_e$, etc.

It is of interest to consider the case when the neutrino energies are above a few GeV so that the geomagnetic effects are not important. For neutrinos of these energies, in view of the present experimental limit [given by curve (d) of Fig. 1 for 5-GeV neutrinos], we expect the downward flux sums D_e and D_μ to be unaffected by oscillations, but the flux sums U_e and U_μ to get modified by them; see curves (c) and (g) of Fig. 1. Therefore in what follows we shall examine the interesting situation in which D_e and D_μ do not vary:

$$D_e = d_e + \bar{d}_e = (f_\mu + \bar{f}_\mu)(1-\alpha-\bar{\alpha}) \ , \tag{57}$$

$$D_\mu = d_\mu + \bar{d}_\mu = (f_\mu + \bar{f}_\mu) \ , \tag{58}$$

$$U_e = u_e + \bar{u}_e = (f_\mu + \bar{f}_\mu)[1-\alpha(1-P_u) - \bar{\alpha}(1-\bar{P}_u)] \ , \tag{59}$$

$$U_\mu = u_\mu + \bar{u}_\mu = (f_\mu + \bar{f}_\mu)[1-\alpha P_u - \bar{\alpha}\bar{P}_u] \ , \tag{60}$$

where

$$\alpha = \frac{f_\mu - f_e}{f_\mu + \bar{f}_\mu}$$

and

$$\bar{\alpha}=\frac{\bar{f}_\mu-\bar{f}_e}{f_\mu+\bar{f}_\mu}\ . \tag{61}$$

Instead of displaying the rapid fluctuations of the fluxes U_e and U_μ (arising from the factor $\cos\phi$ in P_u and $\bar{P}_u$), it would be more advantageous to look at the upper and lower limits on them by using the bounds (25) and (26). These limits on the ratios of flux sums, which are of direct experimental interest, are

$$\frac{D_e}{D_\mu}=\beta\ , \tag{62}$$

$$\frac{\beta+Q_-}{1-Q_-}\le\frac{U_e}{U_\mu}\le\frac{\beta+Q_+}{1-Q_+}\ , \tag{63}$$

$$1+\frac{1}{\beta}Q_-\le\frac{U_e}{D_e}\le 1+\frac{1}{\beta}Q_+\ , \tag{64}$$

$$1-Q_+\le\frac{U_\mu}{D_\mu}\le 1-Q_-\ , \tag{65}$$

where we used the abbreviations

$$\beta\equiv 1-\alpha-\bar{\alpha}\ , \tag{66}$$

$$Q_\pm\equiv\alpha P_{0m}\left[1\pm\frac{|\sin\psi_m|}{\psi_m}\right]+\bar{\alpha}\bar{P}_{0m}\left[1\pm\frac{|\sin\bar{\psi}_m|}{\bar{\psi}_m}\right]\ . \tag{67}$$

In addition to the parameters which appear in the quantities P's, values of the parameters α and $\bar{\alpha}$ (which depend on the ratios of fluxes at the production level) are also required for evaluating the bounds (63)–(65). One can take them from the calculations of atmospheric neutrino fluxes at sea level.[14] We take them from the calculations of Cowsik *et al.*,[14]

$$\alpha=0.473,\quad \bar{\alpha}=0.269 \tag{68}$$

corresponding to a neutrino energy $E=5$ GeV. This immediately implies that the ratio

$$\frac{D_e}{D_\mu}=0.258 \tag{69}$$

for the situation being considered wherein there are no oscillation effects for the downward fluxes.

It should be noted that when the ratios (U_e/D_e) and (U_μ/D_μ) are known from Eqs. (57)–(61) there is no new information obtainable on (U_e/U_μ), since (D_e/D_μ) has been assumed to be a constant. However this is not true of the bounds (63)–(65), and therefore we shall consider the upper and lower limits on the three ratios (U_e/D_e), (U_μ/D_μ), and (U_e/U_μ) separately.

The bounds (64) and (65) on the "up-down" ratios of fluxes are shown in Fig. 2. The bounds (63) on the ratio of neutrino fluxes in the upward direction are shown in Fig. 3. For convenience the x axis on the top is marked off in units of $E/\Delta m^2$ when E is in GeV and Δm^2 is in $(\text{eV})^2$, using Eq. (13). In order to show the variation of the

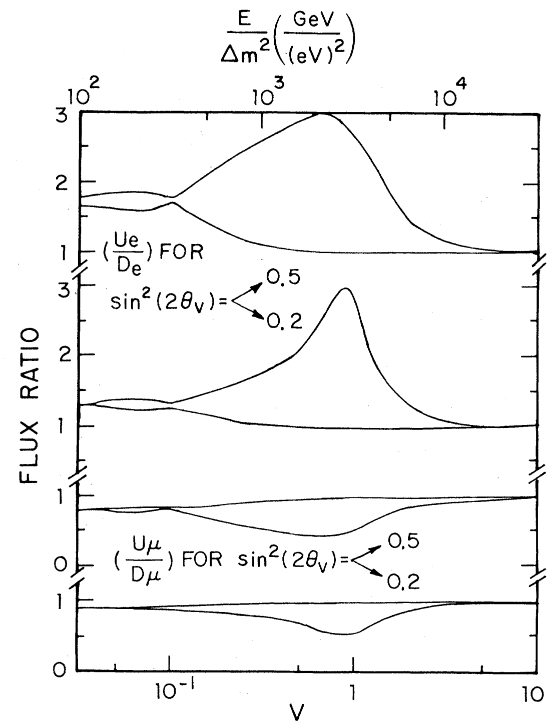

FIG. 2. Upper and lower bounds on the ratios of $(\nu+\bar{\nu})$ fluxes in upward and downward directions as functions of V. Fluxes in the downward direction are regarded as constants. These bounds are shown for two values $\sin^2(2\theta_v)=0.2$ and 0.5. The upper abscissa scale gives $[E\text{ (in GeV)}/\Delta m^2\text{ (in eV}^2)]$.

bounds as a function of θ_v, they have been evaluated at two values: $\sin^2(2\theta_v)=0.2$ and 0.5. The effects of oscillations are smaller for the smaller mixing angle. It should be remarked that although a change in the $(\nu_e+\bar{\nu}_e)$ flux due to oscillations is compensated by an opposite change in the $(\nu_\mu+\bar{\nu}_\mu)$ flux, the ratio (U_e/D_e) according to Fig. 2

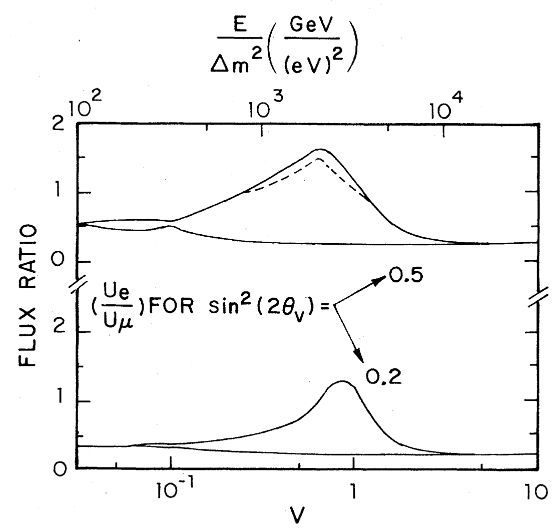

FIG. 3. Upper and lower bounds on the ratios of $(\nu+\bar{\nu})$ fluxes of e and μ flavors, in the upward direction, as functions of V. These bounds are shown for two values $\sin^2(2\theta_v)=0.2$ and 0.5. For $\sin^2(2\theta_v)=0.5$, the dashed curves refer to the input values of α and $\bar{\alpha}$ corresponding to 2-GeV neutrinos in the atmosphere.

is more sensitive to oscillations than (U_μ/D_μ); this is because we are comparing the flux change with the value of the original unoscillated flux which happens to be smaller for $(\nu_e+\bar{\nu}_e)$ than for $(\nu_\mu+\bar{\nu}_\mu)$. As shown in Figs. 2 and 3 the bounds approach each other and attain constant values for large V, because of matter damping.

For the interesting ratio U_e/U_μ, which is relatively free from background problems (present in the downward fluxes due to the atmospheric muons), we note the following two features. First, consider the vanishing of the sine functions in Eq. (67) when ψ_m and $\bar{\psi}_m$ are integral multiples of π. In the interesting case when both of them vanish we will have $Q_+=Q_-$, and the upper and lower bounds coincide. Such a case is approximately realized at $V\simeq 0.1$ for $\sin^2(2\theta_v)=0.5$ in Fig. 3 with both ψ's around π; thus U_e/U_μ must have a value about twice the production-point value. This is a nice example of the bounds determining the value. In general, both ψ_m and $\bar{\psi}_m$ are not simultaneously close to a multiple of π for the same value of V. For large values of ψ this effect will be hard to notice due to the presence of the factors $(1/\psi)$ multiplying $\sin\psi$ in the Q's. The second feature worth mentioning is that the reason for the maximum in the upper bound on U_e/U_μ at $V\simeq 0.8$ (see Fig. 3) is that Q_+ has a maximum there, this maximum of Q_+ is also visible as a maximum in the upper bound of (U_e/D_e) and as a minimum in the lower bound of (U_μ/D_μ) in Fig. 2.

As for the sensitivity of these bounds to the energy of the neutrino we first recall that the energy enters via the input parameters α and $\bar{\alpha}$ defined in Eq. (61). Although the curves in Figs. 2 and 3 have been computed for 5-GeV neutrinos, one may compute them easily for flux ratios of any other energy by inserting the appropriate values of α and $\bar{\alpha}$. For instance, by taking $\alpha=0.424$ and $\bar{\alpha}=0.288$ from Cowsik, Pal, and Tandon[14] corresponding to 2-GeV neutrinos, the bounds get modified only slightly as shown in Fig. 3 for the case of $\sin^2(2\theta_v)=0.5$; here, the upper and lower bounds given by the *dashed curves* for 2 GeV are nearly the same as the corresponding full curves for 5 GeV; only the upper bounds are distinguishable over a tiny range of V around 0.7. For the ratios in Fig. 2 also the changes are less than about 10% and have not been shown.

VI. CONCLUDING REMARKS AND SUMMARY

We have examined the effects of a possible ν_e-ν_μ oscillation on the fluxes of atmospheric neutrinos traveling upward and downward. Our considerations are confined to the framework of the two-component model of neutrino mixing.[15] We have ignored the effects of the geomagnetic cutoff and solar modulation on the neutrino fluxes; these effects seem to be ignorable for neutrinos of energy greater than a few GeV.[6]

It is important to comment on the range of neutrino energies relevant to our considerations. Our analysis throughout uses the two parameters θ_v, the mixing angle, and l_v, the oscillation length when matter is absent. Instead of l_v we have used the more convenient dimensionless variable $V=(l_v/l_W)$ which depends on the neutrino energy through the combination $(E/\Delta m^2)$. Thus the V-$\sin^2(2\theta_v)$ plot of Fig. 1 has the advantage that it can be viewed as a plot of E versus θ_v for a given Δm^2, or as a plot of $(1/\Delta m^2)$ versus θ_v for a fixed E. For the sake of a realistic illustration in our discussions we have converted the limits of the reactor experiment as relevant to a neutrino energy of 5 GeV.[13] Because of the linear relation between V and E it is trivial to interpret the curve (d) of Fig. 1 for any other energy.

Similarly, for demonstrating the efficacy of our bounds on the ratios of upward and downward fluxes, the input ratios α and $\bar{\alpha}$ have been taken at $E=5$ GeV from the atmospheric neutrino spectra calculated by neglecting oscillations. The bulk of the cosmic-ray neutrinos around this energy is believed to originate from the decays of the conventional particles π,μ and the strange particles. Further, the input numbers α and $\bar{\alpha}$ being only the ratios of fluxes, rather than the fluxes themselves, are subject to less uncertainty. For these reasons, the bounds displayed in Figs. 2 and 3 may serve as useful guidelines.

It should be remarked that the averaging over the path length x in Eq. (18) could also be viewed as averaging over the neutrino energy E in an appropriate range. For the downward neutrinos it is straightforward to see this since l_v is proportional to E, and the fractional spread in E would be identical to the fractional spread in x, $(\Delta E/E)=(\eta_v/x_0)$. For the upward travel, however, the corresponding energy spread is difficult to ascertain due to the complicated dependence of l_m on E. In any case, if one takes the spread in path lengths to be adequately large, one would be implicitly incorporating the effects due to a finite energy spread.

To summarize, we started essentially with the expression (18) for the averaged probability for flavor oscillation. The uncertainty (η_v or η_m) in the neutrino path length plays an important role in the present formulation. Using the path-averaged formulas we explored the consequences of laying down criteria for detecting oscillation effects in the atmospheric neutrinos traveling (i) downward through the near vacuum and (ii) upward through the Earth's medium. Criteria for ignoring the oscillation effects have also been examined. The limitations implied by all these criteria (which should be viewed only as *necessary* conditions) have been illustrated, for 10% tolerance limits, in the V-$\sin^2(2\theta_v)$ plot of Fig. 1.

By comparing the allowed or forbidden zones of Fig. 1 with the limits set by the Gösgen reactor experiment[12] we have the following remarks. First, the travel length in the atmosphere is too short to reveal the effects of oscillations in the downward fluxes. Second, if the parameters characterizing the neutrino oscillations lie in the region between curves (d) and (g) of Fig. 1, then oscillation effects may be important in the upward fluxes; here matter corrections to the oscillation effects also may be significant. However, if the present limit provided by the Gösgen experiment is improved by an order of magnitude (i.e., if $\Delta m^2 \lesssim 10^{-3}$ eV^2), then even the upward neutrino fluxes in the few-GeV range will be immune to oscillations. Lastly, the up-down flux ratios (for which only bounds were shown in Fig. 2) can deviate substantially from unity only if the oscillation parameters are situated in the region between curves (d) and (g) of Fig. 1; the interesting ratio U_e/U_μ can also reflect these effects.

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[1]For reviews, see, for instance, S. M. Bilenky and B. Pontecorvo, Phys. Rep. 41C, 225 (1978); V. Barger, in *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981), p. 334.

[2]P. H. Frampton and S. L. Glashow, Phys. Rev. D 25, 1982 (1982).

[3]S. Pakvasa, in *Proceedings of the 1980 DUMAND Symposium, Honolulu,* edited by V. J. Stenger (Hawaii DUMAND Center, Honolulu, 1981), Vol. II, p. 45.

[4]L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).

[5]V. Barger, K. Whisnant, S. Pakvasa, and R. J. N. Phillips, Phys. Rev. D 22, 2718 (1980).

[6]For a recent calculation, and earlier references, see T. K. Gaisser, T. Stanev, S. A. Bludman, and H. Lee, Phys. Rev. Lett. 51, 223 (1983); A. Dar, *ibid.* 51, 227 (1983).

[7]For a relaxation of this assumption and the ensuing consequences, see G. V. Dass and K. V. L. Sarma, Phys. Rev. D 28, 49 (1983).

[8]It would be difficult in practice to ascertain the arrival angle of the cosmic-ray neutrino with better precision. Consider an upward ν_μ of energy E undergoing inelastic scattering in the detector. If the final μ has an energy E' and an emission angle θ, then the momentum transfer is $Q^2 \simeq EE'\theta^2$. However, we can use the fact that $\langle Q^2 \rangle / E \simeq 0.17$ GeV is independent of energy; see, e.g., the review of B. P. Roe, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California,* edited by W. T. Kirk (SLAC, Stanford, 1975), p. 551. Thus we estimate $\theta_{\rm rms} \simeq 11°$ when $E' \simeq 5$ GeV. Our choice of 20° for the angular spread is therefore reasonable.

[9]Increasing η_v would proportionately raise the curve (a) because the criterion (35) involves η_v only through the combination (η_v / V).

[10]G. V. Dass and K. V. L. Sarma, in *18th International Cosmic Ray Conference, Bangalore, India, 1983, Conference Papers,* edited by N. Durgaprasad *et al.* (Tata Institute of Fundamental Research, Bombay, 1983), Vol. 7, p. 96.

[11]In defining the criteria (38) and (41), we have followed the convention $\theta_v < \pi/4$ so that $\cos(2\theta_v)$ is positive; see Ref. 5.

[12]J. L. Vuilleumier *et al.*, Phys. Lett. 114B, 298 (1982).

[13]The choice of optimal energy E is obviously important. Estimates of neutrino fluxes excluding oscillations rely on the assumed characteristics of (i) hadron-nucleus interactions of primary and secondary cosmic rays with the atmospheric nuclei, and (ii) the subsequent semileptonic decay of the produced hadrons. At low energies ($\lesssim 1$ GeV), where the fluxes are abundant, the flux calculations will have to include the effects of geomagnetic cutoff which depends on the location of the experimental site (see Ref. 6), relatively large emission angles of the charged leptons produced by neutrinos, etc. On the other hand, for atmospheric neutrino fluxes at higher energies ($\gtrsim 10$ GeV), one has to contend with smaller event rates due to the rapid fall-off of the cosmic ray spectrum with energy. One may therefore choose an intermediate value for the neutrino energy, say, 5 GeV, so that the estimated fluxes at the production level are reliable and the expected event rates are also reasonable.

[14]R. Cowsik, Y. Pal, and S. N. Tandon, Proc. Indian Acad. Sci. 63A, 217 (1966); J. L. Osborne, S. S. Said, and A. W. Wolfendale, Proc. Phys. Soc. 86, 93 (1965); L. V. Volkova, Yad. Fiz. 31, 1510 (1980) [Sov. J. Nucl. Phys. 31, 784 (1980)].

[15]The validity of the two-component model can in principle be checked from the equality of the upward and downward total fluxes of $(\nu_e + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu)$. For a study of the oscillation effects on the cosmic-ray neutrino spectra in a three-component model, with maximal mixing, see, R. Silberberg and M. M. Shapiro, in *17th International Cosmic Ray Conference, Paris, 1981, Conference Papers,* Centre d Études Nucleaires, Saclay, (1981), Vol. 7, p. 180.