Dynamical generation of fermion masses

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We study the self-consistency condition of fermion-mass generation based on the Nambu—Jona-Lasinio approach in an arbitrary gauge and show that the mass so generated is a gauge invariant. We also calculate the mass difference between the up and down quarks.

I. INTRODUCTION

In the lexicon of renormalizable field theory, the fermion mass is destined to play a subtle but very important role. In the old days, we simply put in the fermion mass by hand. Unlike the vector boson, putting in a fermion mass does not disturb or destroy the manifest renormalizability of the theory. The successful QED is such a theory where m_e is put in by hand. It has, in fact, met the experimental test to such an embarrassing degree that, indeed, it could in turn be used to probe the hadronic content of the theory.

But in our drive to grand unification, it is natural to assume the fermions to be fundamentally massless and to seek for the theory to generate the masses dynamically. Setting the fermion masses to zero also gains for us some chiral symmetry. This is useful in protecting the fermions from developing large masses due to higher-order radiative corrections in a grand unified theory. But this chiral invariance may only be formal. It is well known that fermion field theories can encounter the Adler-Bell-Jackiw anomaly which destroys the chiral invariance. For non-Abelian gauge theories, the non-Abelian chiral anomalies even threaten the renormalizability of the theory, unless the fermion content has been arranged so that the total non-Abelian chiral anomaly vanishes.

In grand unified theories (GUT's), this is usually arranged. The fermion mass is then finally generated through their Yukawa coupling to Higgs fields,

$$-h\bar{\psi}\psi\phi \ ,$$

which upon symmetry breaking becomes ($\phi\rightarrow\phi+v$)

$$-hv\bar{\psi}\psi-h\bar{\psi}\psi\phi \ .$$

The mass, so obtained, is the "current" mass with renormalization-group transformation properties that any tree Lagrangian mass parameter should have.

Such a mechanism has often been used to study and analyze the observed fermion-mass spectrum. In order to fit the spectrum, it is found that the Yukawa coupling constant for the electron is of order 10^{-5}, while the h for the u and d quarks is of order 10^{-4}. These extremely small coupling constants are to be compared with the gauge coupling constants in the theory, being of order 10^{-1}. Perhaps in the fundamental Lagrangian these h's should really be zero. In that case, we must attribute the fermion masses (at least of the first generation) to dynamical mass generation.[1]

Recently, Chang and Chang[2] have developed a new approach to calculate the dynamically generated fermion mass in QCD based on the Nambu—Jona-Lasinio[3] (NJL) mechanism. Whereas the original NJL mechanism was proposed in the context of an unrenormalizable field theory, Chang and Chang made a renormalization-group analysis of the NJL gap equation (i.e., the self-consistency condition) and found that it is indeed a renormalization-group invariant for QCD. They have found that the mass of the quark so generated is, to two-loop renormalization-group accuracy,

$$M=\Lambda_c^{(2)}e^{1/6} \ , \tag{1.1}$$

where $\Lambda_c^{(2)}$ is the two-loop-invariant cutoff in QCD. This mass M is a renormalization-group (RG) invariant.

In this paper we will further study this approach. In Sec. II, we will extend their study to include an investigation of the role the two-loop constants could play in their mass determination. The answer is that the two-loop constants will not affect (1.1). In Sec. III, we will extend their analysis to an arbitrary gauge, and show that, even in an arbitrary gauge, (1.1) obtains.

II. FORMALISM

Consider the QCD Lagrangian with vanishing fermion mass. Let it be written as

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_{\rm int} \ , \tag{2.1}$$

where by $\mathscr{L}_0$ we mean the massless kinetic terms for the fermion and gauge fields. Following NJL, the Lagrangian (2.1) is to be rewritten as

$$\mathscr{L}=(\mathscr{L}_0-M\bar{\psi}\psi)+(\mathscr{L}_{\rm int}+M\bar{\psi}\psi) \tag{2.2}$$

and the perturbation theory is to be taken around the nonperturbative vacuum ($M\neq0$). To do this new perturbation theory, Chang and Chang took the crucial step in introducing the intermediate Lagrangian

$$\mathscr{L}'=(\mathscr{L}_0-M\bar{\psi}\psi)+(\mathscr{L}_{\rm int}+\delta M\bar{\psi}\psi) \ . \tag{2.3}$$

The δM is treated just like a counterterm in usual Lagrangian field theory, except that it is used to fix the two-point proper Green's function to be

$$\Gamma_r^{(2)}(p)=(\gamma\cdot p-iM)\tilde{Z}_2^{-1} \tag{2.4}$$

for $p^2 \ll M^2$. The M in (2.4) is the same M as in (2.3).

To one loop in λ ($\equiv g_r^2/16\pi^2$), for example, they found (in the $\alpha=0$ gauge)

$$\delta M=-3\lambda C_f M\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right] \tag{2.5}$$

and to all orders in λ, but accurate to one-loop RG accuracy, they found

$$\delta M=M\left\{1-\left[1+\frac{\lambda b}{2}\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]\right]^{6C_f/b}\right\}, \tag{2.6}$$

where ($\ln\mu\equiv t$)

$$\frac{d}{dt}\lambda=-b\lambda^2 . \tag{2.7}$$

So far, this result is but a simple exercise in renormalization-group theory. To make contact with NJL, they then impose the self-consistency condition

$$\delta M=M . \tag{2.8}$$

Upon noting that Eq. (2.6) may be written as

$$\delta M=M\left[1-\left[\frac{\lambda}{\lambda_0}\right]^{6C_f/b}\right] \tag{2.9}$$

with (one-loop RG accuracy)

$$\frac{1}{\lambda_0}=\frac{1}{\lambda}+\frac{b}{2}\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right], \tag{2.10}$$

the NJL self-consistency condition (or, in superconducting terminology, the gap equation) will be solved by

$$\frac{1}{\lambda}+\frac{b}{2}\left[\ln\frac{M^2}{\mu^2}-\frac{1}{3}\right]=0 . \tag{2.11}$$

In terms of $\Lambda_c^{(1)}$, the one-loop cutoff, defined by

$$\frac{1}{\lambda}+b\ln\frac{\Lambda_c^{(1)}}{\mu}=0 , \tag{2.12}$$

the NJL dynamically generated mass, accurate to one-loop RG accuracy, is then given by

$$M=\Lambda_c^{(1)}e^{1/6} . \tag{2.13}$$

Another way to look at their result is to look at (2.3) as defining a massive QCD with

$$m_r=M-\delta M \tag{2.14}$$

as the mass parameter. Therefore, the RG equation for m_r is fixed by the usual modified minimal-subtraction renormalization to be (to two-loop accuracy, say)

$$\frac{d}{dt}m_r=-(h_1\lambda+h_2\lambda^2)m_r \tag{2.15}$$

with [f=number of flavors, $C_2=N$ for SU(N) group]

$$h_1=6C_f \quad (T_aT_a\equiv C_f\mathbb{1}) , \tag{2.16}$$

$$h_2=3C_f^2+\tfrac{97}{3}C_fC_2-\tfrac{10}{3}C_ff , \tag{2.17}$$

and the corresponding equation for λ reads

$$\frac{d}{dt}\lambda=-b\lambda^2-c\lambda^3 \tag{2.18}$$

with

$$b=\tfrac{22}{3}C_2-\tfrac{4}{3}f , \tag{2.19}$$

$$c=\tfrac{68}{3}C_2^2-4C_ff-\tfrac{20}{3}C_2f . \tag{2.20}$$

In general, to two-loop RG accuracy, Eq. (2.15) may be solved by

$$m_r=m_0\left[\frac{\lambda}{\lambda_0^{(2)}}\right]^{h_1/b}\left[\frac{b+c\lambda}{b+c\lambda_0'}\right]^{h_2/c-h_1/b}, \tag{2.21}$$

where m_0, $\lambda_0^{(2)}$, and λ_0' are RG invariants. In particular, we choose to write ($\ln\mu\equiv t$)

$$\frac{1}{\lambda_0^{(2)}}=\frac{1}{\lambda}-\frac{c}{b}\ln\left[\frac{1+b/c\lambda}{1+b/c\lambda_0^{(2)}}\right]+\frac{b}{2}\left[\ln\frac{m_0^2}{\mu^2}-\frac{1}{3}\right], \tag{2.22}$$

$$\frac{1}{\lambda_0'}=\frac{1}{\lambda_0^{(2)}}-ba' . \tag{2.23}$$

So far this appears to be a formal exercise in the renormalization-group theory. In claiming that (2.21) is a solution of (2.15), what we mean is that λ is the function of t as given by (2.22) and m_r is a function of t through its dependence on λ. $\lambda_0^{(2)}$, λ_0', and m_0 are independent of t.

Suppose now we proceed to calculate the two-point Green's function in massive QCD (which we shall refer to as the old theory). In general, we will find (at $p\to 0$)

$$\Gamma_r^{(2)}(\text{old})=\tilde{Z}_2^{-1}(\gamma\cdot p-i\mathcal{M}) \tag{2.24}$$

with $\mathcal{M}$ a RG invariant. By explicit calculation it can be verified that, indeed, $\mathcal{M}$ is given by

$$\mathcal{M}=m_r\left[\frac{\lambda}{\lambda_0^{(2)}}\right]^{-h_1/b}\left[\frac{b+c\lambda}{b+c\lambda_0'}\right]^{-h_2/c+h_1/b}, \tag{2.25}$$

where, in using the right-hand side, we are to continually re-express λ_0 in terms of λ, μ, m_r, achieving finally a complete perturbative expansion for M in λ, μ, m_r.

Now what about the limit $m_r\to 0$? Equation (2.21) tells us that there is a trivial way to achieve the limit, viz. by taking $m_0=0$. Perturbatively then, the full two-point function is also zero. But there is a nontrivial way to achieve $m_r=0$. That is, for $m_0\neq 0$, we look for

$$\frac{1}{\lambda_0^{(2)}}=0 . \tag{2.26}$$

In that case, the $\mathcal{M}$ of the two-point function is nonvanishing. In fact, it is exactly m_0. Contact is finally made with the earlier approach when we realize that $m_0=M$.

The alert reader will find that our Eq. (2.21) is a generalization of Eq. (3.5) of Ref. 2. We now have $\lambda_0^{(2)}$ and

λ_0', where previously there was only λ_0. This generalization is to allow for the fact that in a realistic calculation, we must sooner or later include not just the two-loop logarithms, but also the two-loop constants. How would the inclusion of such two-loop constants affect the determination of M?

A complete two-loop calculation, including constants, has been carried out by one of us (Li) with the result that $(\alpha=0)$

$$\Sigma \equiv M - \delta M = M[1+\lambda C_f(3L-1)+\lambda^2 D(3L-1)^2 + \lambda^2 E(3L+1) - \lambda^2 F + O(\lambda^3)] , \quad (2.27)$$

where

$$D = \tfrac{1}{2}C_f^2 - \frac{b}{12}C_f ,$$
$$E = \tfrac{1}{2}C_f^2 + \tfrac{97}{18}C_f C_2 - \tfrac{5}{9}C_f f , \quad (2.28)$$
$$F = 32.0675 C_f^2 - 6.84185 C_f C_2 + 1.73209 C_f f .$$

Upon expanding (2.21) and comparing with (2.27), we find

$$a' = \frac{bF}{ch_1 - h_2 b} . \quad (2.29)$$

Note that with our generalization, the solution (2.26) remains valid even in the presence of two-loop constants. It is interesting to note, nevertheless, that there is an alternate solution with

$$\frac{1}{\lambda_0'} = 0 , \quad (2.30)$$

which leads to the complementary solution

$$M_2 = \Lambda_c^{(2)} e^{a'+1/6} . \quad (2.31)$$

For the case of three generations and SU(3),

$$a' = -0.435\,355 . \quad (2.32)$$

III. GAUGE INDEPENDENCE

So far everything has been calculated in the $\alpha=0$ gauge. In dynamical symmetry breaking, a particularly difficult problem has been to establish the gauge independence of the dynamical mass so generated. In this section we will now exhibit the problem in a general $\alpha\neq 0$ gauge and show how the dynamically generated mass can indeed be gauge invariant.

Consider the Lagrangian (2.3), and continue to treat δM as a counterterm, used to fix $\Gamma_r^{(2)}$ to be (2.4) even when $\alpha \neq 0$. To one loop, we then find

$$\delta M = -3\lambda C_f M(L-\tfrac{1}{3}) + \frac{\alpha}{2}\lambda C_f , \quad (3.1)$$

and

$$\tilde{Z}_2 = 1 + \alpha\lambda C_f (L - \tfrac{1}{2}) . \quad (3.2)$$

At this level, it is hard to see how the α-dependent terms will disappear upon inclusion of higher-order terms.

But, upon including two-loop terms, we find

$$\Sigma = M\left[1+\lambda C_f\left[3L-1-\frac{\alpha}{2}\right] + D\lambda^2(3L-1)^2 + \lambda^2(E+E_1\alpha+E_2\alpha^2)(3L-1) + \lambda^2(F+\alpha F_1+\alpha^2 F_2) + O(\lambda^3)\right] , \quad (3.3)$$

where D,E,F are given in (2.28), and

$$E_1 = \tfrac{1}{2}C_f^2 + \tfrac{1}{4}C_f C_2 ,$$
$$E_2 = \tfrac{1}{12}C_f C_2 , \quad (3.4)$$
$$F_1 = 7.369\,53 C_f^2 + 8.256\,02 C_f C_2 ,$$
$$F_2 = -16.3273 C_f^2 - 3.885\,38 C_f C_2 .$$

Following the earlier work of Ref. 2, we now check on the RG properties of Σ, using again the fact that M is an RG invariant. Following our remark which follows Eq. (2.14), it is no surprise that we find

$$\frac{1}{\Sigma}\frac{d}{dt}\Sigma = -(h_1\lambda + h_2\lambda^2) , \quad (3.5)$$

and the right-hand side of (3.5) is in fact gauge invariant.

Based on this fact, we find that the solution to (3.5) must be of the form

$$\Sigma(\alpha\neq 0,\lambda,M,\mu) = Z_a\left[\alpha,\lambda,\frac{M}{\mu}\right]\Sigma(\alpha=0,\lambda,M,\mu) , \quad (3.6)$$

where $\Sigma(\alpha=0,\lambda,M,\mu)$ is the series we had before. Here Z_a is given by the series expansion

$$Z_a = 1 - \tfrac{1}{2}\alpha\lambda C_f + \lambda^2 C_f C_2 \frac{\alpha(3+\alpha)}{12}(3L-1) + \lambda^2 C_f C_2 (8.256\,02\alpha - 3.885\,38\alpha^2) + \lambda^2 C_f^2 (7.369\,53\alpha - 16.3273\alpha^2) + O(\lambda^3) \quad (3.7)$$

and

$$\frac{d}{dt}Z_a = 0 . \quad (3.8)$$

We can express Z_a in a renormalization-group-invariant form

$$Z_a = e^{-(\alpha_0/2)\lambda_a C_f} , \quad (3.9)$$

where

$$\frac{1}{\lambda_a} = \frac{1}{\lambda} - b(t-a) , \quad (3.10)$$

$$a = \ln M + \frac{16.5120 C_2 + 14.7391 C_f}{b} + \frac{(\tfrac{26}{3}C_2 - 2f)(3.885\,38 C_2 + 16.2023 C_f)}{bC_2} . \quad (3.11)$$

Unlike $1/\lambda_0$, $1/\lambda_a$ is not zero.

Equation (3.6) exhibits the full gauge dependence of the self-consistency condition. We need to obtain the solution

$$\Sigma(\alpha,\lambda,M,\mu)=0 \tag{3.12}$$

in general for $\alpha\neq 0$. But (3.6) tells us that it is sufficient to solve for the $\alpha=0$ gauge, since if $\Sigma(\alpha=0,\lambda,M,\mu)$ vanishes, $\Sigma(\alpha,\lambda,M,\mu)$ will also vanish, at least for a range of α close enough to zero. Therefore, in (2.3), when M is given by (1.1), the two-point proper Green's function will still be given by

$$\Gamma_r^{(2)}(p)=(\gamma\cdot p-iM)\widetilde{Z}_2^{-1}$$

with, of course, a gauge-dependent Z_2.

IV. UP-DOWN MASS DIFFERENCE

In QCD, we can only generate the same mass for the up and the down quarks. In order to study the origin of the mass difference between the up and down quarks, we should consider including the effects of QED. The theory that we study thus is SU(3)×U(1). The procedure for mass generation will be the same as in Sec. II. For simplicity, we will from now on work exclusively in the $\alpha=0$ gauge.

First, consider the mass determination to one-loop RG accuracy. We have

$$\frac{d}{dt}\lambda_3=-b\lambda_3^2\,, \tag{4.1}$$

$$\frac{d}{dt}\lambda_1=b'\lambda_1^2\,, \tag{4.2}$$

$$\frac{d}{dt}\Sigma_Q=-(h_1\lambda_3+6Q^2\lambda_1)\Sigma_Q\,, \tag{4.3}$$

where Σ_Q refers to the self-consistency condition for the quark with charge Qe and $\lambda_3\equiv g_3^2/16\pi^2$, $\lambda_1\equiv e^2/16\pi^2$. The solution to (4.3) is

$$\Sigma_Q=M_Q\left[\frac{\lambda_3}{\lambda_{30}}\right]^{h_1/b}\left[\frac{\lambda_1}{\lambda_{10}}\right]^{-6/b'} \tag{4.4}$$

and can be satisfied by

$$\frac{1}{\lambda_{30}}=0\,, \tag{4.5}$$

and the situation is the same as when QED was turned off. To one-loop RG accuracy, the degeneracy between up and down quarks is not lifted.

We proceed now to two loops. Here, because λ_1 is so much smaller than λ_3, we have kept only terms to first order in λ_1, as a perturbation to order λ_1 of the previous SU(3) result. The self-consistency condition now reads

$$\Sigma_Q=M_Q\left[1+\lambda_3C_f(3L-1)+\lambda_3^2(3L-1)^2\left[\frac{1}{2}C_f^2-\frac{b}{12}C_f\right]+\lambda_3^2(3L-1)\frac{h_2}{6}+Q^2\lambda_1(3L-1)\right.$$
$$\left.+Q^2\lambda_1\lambda_3C_f(3L-1)^2+\frac{Q^2}{6}\lambda_1\lambda_3h_{13}(3L-1)+Q^2\lambda_1\lambda_3C_fd_{13}+O(\lambda_1^2,\lambda_3^3)\right]. \tag{4.6}$$

To perform the RG summation, we are to take[4]

$$\frac{d}{dt}\lambda_3=-b\lambda_3^2-c\lambda_3^3-c_{31}\lambda_3^2\lambda_1\,, \tag{4.7}$$

$$\frac{d}{dt}\lambda_1=0\,. \tag{4.8}$$

This last approximation is to make the summation a straightforward one and is entirely consistent with keeping only order $\lambda_1\lambda_3^n$ terms in the perturbation series in (4.6). The equation for Σ_Q then reads

$$\frac{d}{dt}\Sigma_Q=-(h_1\lambda_3+Q^2h_{13}\lambda_1\lambda_3+h_2\lambda_3^2+6Q^2\lambda_1)\Sigma_Q\,. \tag{4.9}$$

The solution is

$$\Sigma_Q=M_Q\left[\frac{\lambda_3}{\lambda_{30}}\right]^{(h_1+h_{13}\lambda_1Q^2)/b}\left[\frac{b+c\lambda_3}{b+c\lambda_{30}}\right]^{h_2/c-(h_1+h_{13}Q^2\lambda_1)/b}e^{6Q^2\lambda_1(t-t_0)} \tag{4.10}$$

with

$$\frac{1}{\lambda_{30}}=\frac{1}{\lambda_3}+\frac{c}{b}\ln\left[\frac{b+c\lambda_3}{\lambda_3}\frac{\lambda_{30}}{b+c\lambda_{30}}\right]-bt+a_3\,. \tag{4.11}$$

Upon expanding Σ_Q as a power series and comparing with (4.6), we find

$$a_3=b\left[\ln M_Q-\tfrac{1}{6}+Q^2\lambda_1\frac{d_{13}}{6}\right]. \tag{4.12}$$

In (4.6), d_{13} represents the genuine two-loop constant of the self-energy graphs involving both gluon and photon exchange. Explicit calculation gives

$$d_{13}=64.1350\,. \tag{4.13}$$

From the solution

$$\frac{1}{\lambda_{30}}=0 \tag{4.14}$$

we find

$$M_Q=\Lambda_c^{(2)}e^{(1-Q^2\lambda_1 d_{13})/6} \tag{4.15}$$

$$=M_0 e^{-Q^2\lambda_1 d_{13}/6}\simeq M_0\left[1-\frac{Q^2\lambda_1}{6}d_{13}\right], \tag{4.16}$$

where M_0 is the mass generated in QCD. From here, we can immediately find

$$M_u-M_d=-M_0\frac{e^2}{288\pi^2}d_{13}, \tag{4.17}$$

which is of the right sign and also of the right order of magnitude, but we hesitate to claim any physical significance since the meaning of M_u,M_d itself is not completely clear. Also, until we have understood the generation problem, it would be dangerous to apply this to the higher generations where $m_c>m_S$ and $m_t>m_b$. If we argue that the heavier generations decouple, then this result is of some significance.

V. CONCLUSION

In this paper we have analyzed further the work described in Ref. 2 and have been able to clarify some important issues. We have shown how, doing a better calculation, including two-loop constants will not destabilize the solution for M. We have also shown how the RG approach of Ref. 2, in fact, can give a gauge-invariant determination of dynamical mass. Finally, we have also made an attempt to go beyond QCD to include QED effects and found an encouraging M_u-M_d of the right sign and order of magnitude.

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APPENDIX

In this appendix, we list for reference the entire calculation up to the two-loop level, in the $\overline{\text{MS}}$ scheme. We start with the tree Lagrangian, in $n=4-\epsilon$ dimensions, with 't Hooft scale μ,

$$\mathscr{L}_t=-\bar\psi\gamma_\mu(\partial_\mu-i\bar g T_a A_\mu^a)\psi-\tfrac{1}{4}(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+\bar g f^{abc}A_\mu^b A_\nu^c)^2-M\bar\psi\psi+\delta M\bar\psi\psi+\mathscr{L}_{\rm GF}+\mathscr{L}_{\rm ghost},$$

where

$$\bar g\equiv\left[\frac{\mu^2 e^\gamma}{4\pi}\right]^{\epsilon/4} g_r,\quad \gamma=0.577\ldots=\text{Euler's constant},$$

and introduce the counterterms

$$\mathscr{L}_c=\text{usual } m=0 \text{ counterterms } -(Z_m Z_2-1)(M-\delta M)\bar\psi\psi$$

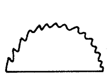
with

$$Z_m=1-\frac{6\lambda C_f}{\epsilon}+\frac{\lambda^2}{\epsilon^2}(18C_f{}^2+3b)+\frac{\lambda^2}{\epsilon}(-\tfrac{3}{2}C_f{}^2-\tfrac{97}{6}C_fC_2+\tfrac{5}{3}C_ff),$$

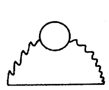
$$Z_2=1-\frac{2\alpha\lambda C_f}{\epsilon}+\frac{\lambda^2}{\epsilon^2}[-\alpha(\alpha+3)C_fC_2-2\alpha^2C_f{}^2]+\frac{\lambda^2}{\epsilon}\left[\left[\frac{25}{4}+2\alpha+\frac{\alpha^2}{4}\right]C_fC_2-\tfrac{3}{2}C_f{}^2-C_ff\right].$$

Then

$$\Gamma_r=\gamma\cdot p-iM$$

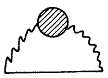


$$+\left[\frac{M^2}{\mu^2}\right]^{-\epsilon/2}\left[\gamma\cdot p(\lambda C_f)\left[\frac{2\alpha}{\epsilon}+\frac{\alpha}{2}\right]-iM(\lambda C_f)\left[\frac{6+2\alpha}{\epsilon}+1+\alpha\right]\right]$$



$$+\lambda^2\left[\frac{M^2}{\mu^2}\right]^{-\epsilon}\left[1+\frac{\epsilon^2\pi^2}{24}\right]\left[\gamma\cdot p(C_ff)\left[-\frac{1}{\epsilon}+A_f\right]-iM(C_ff)\left[-\frac{4}{\epsilon^2}-\frac{4}{\epsilon}+B_f\right]\right.$$

gauge+ghost

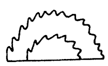


$$+\gamma\cdot p(C_fC_2)\left[\frac{13-3\alpha}{4\epsilon}+\frac{27}{8}+\frac{\alpha}{16}+\frac{3\alpha^2}{16}\right]$$

$$-iM(C_fC_2)\left[\frac{13-3\alpha}{\epsilon^2}+\frac{41+4\alpha+3\alpha^2}{4\epsilon}+\frac{85}{8}-3\alpha+\frac{3\alpha^2}{2}+\tfrac{13}{12}\pi^2-\frac{\pi^2\alpha}{4}\right]$$

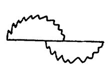
$$+\gamma\cdot p(C_fC_2)\left[\frac{3\alpha(\alpha+1)}{\epsilon^2}+\frac{9+17\alpha+8\alpha^2}{4\epsilon}+A_t+\alpha A_{t1}+\alpha^2A_{t2}\right]$$

$$-iM(C_fC_2)\left[\frac{9+12\alpha+3\alpha^2}{\epsilon^2}+\frac{15+5\alpha+2\alpha^2}{\epsilon}+B_t+\alpha B_{t1}+\alpha^2B_{t2}\right]$$



$$+\gamma\cdot p(C_f{}^2)\left[-\frac{2\alpha^2}{\epsilon^2}-\alpha\frac{(24+3\alpha)}{2\epsilon}+A_s+\alpha A_{s1}+\alpha^2A_{s2}\right]$$

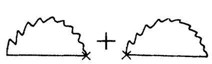
$$-iM(C_f{}^2)\left[\frac{18-2\alpha^2}{\epsilon^2}-\frac{21+6\alpha+\alpha^2}{\epsilon}+B_s+\alpha B_{s1}+\alpha^2B_{s2}\right]$$



$$+\gamma\cdot p(C_f{}^2-\tfrac{1}{2}C_fC_2)\left[\frac{4\alpha^2}{\epsilon^2}-\frac{3-5\alpha^2}{2\epsilon}+A_c+\alpha A_{c1}+\alpha^2A_{c2}\right]$$

$$-iM(C_f{}^2-\tfrac{1}{2}C_fC_2)\left[\frac{4\alpha(\alpha+3)}{\epsilon^2}+\frac{(-9+2\alpha+3\alpha^2)}{\epsilon}+B_c+\alpha B_{c1}+\alpha^2B_{c2}\right]\Bigg]$$

$$+\lambda^2\left[\frac{M^2}{\mu^2}\right]^{-\epsilon/2}\left[1+\frac{\epsilon\pi^2}{48}\right]$$



$$\times\Bigg[\gamma\cdot p[(\alpha+3)C_fC_2+4\alpha C_f{}^2]\left[-\frac{2\alpha}{\epsilon^2}-\frac{\alpha}{2\epsilon}-\frac{\alpha}{8}\right]$$

$$-iM[(\alpha+3)C_fC_2+4\alpha C_f{}^2]\left[\frac{-6-2\alpha}{\epsilon^2}-\frac{1+\alpha}{\epsilon}-\frac{1+\alpha}{2}\right]$$

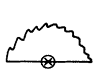
$$-iM[(\alpha-\tfrac{13}{3})C_fC_2+\tfrac{4}{3}C_ff]\left[\frac{6}{\epsilon^2}+\frac{1}{\epsilon}+\frac{1}{2}\right]$$

$$+\gamma\cdot pC_f{}^2\left[\frac{4\alpha^2}{\epsilon^2}+\frac{\alpha(\alpha+12)}{\epsilon}+\frac{\alpha(\alpha+3)}{4}\right]$$

$$-iMC_f{}^2\left[\frac{-36+4\alpha}{\epsilon^2}+\frac{30+8\alpha+2\alpha^2}{\epsilon}+3+4\alpha+3\alpha^2\right]\Bigg]$$

$$+i(Z_mZ_2-1)(M-\delta M)$$

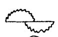
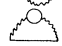
$$+i\delta M$$



$$+\gamma\cdot p(\lambda C_f)\left[-2\alpha\frac{\delta M}{M}\right]-iM\lambda C_f\left[\frac{M^2}{\mu^2}\right]^{-\epsilon/2}\left[\frac{-6-2\alpha}{\epsilon}+5+\alpha\right]\frac{\delta M}{M}\ ,$$

where all the two-loop constants can be found in Table I.

TABLE I. Coefficients for the fermion self-energy.

Label	A	A_1	A_2	B	B_1	B_2
 s	0	$-11+\pi^2$	$-\frac{3}{8}-\pi^2$	$-\frac{29}{2}+\frac{3\pi^2}{2}$	-7	$-\frac{1}{2}-\frac{\pi^2}{6}$
 t	12.265 3	3.855 33	−3.523 58	−12.354 2	0.379 222	2.741 48
 c	16.555 1	−2.050 47	0.603 112	−23.816 8	−6.300 44	7.070 79
 f	4.134 27	0	0	1.068 85	0	0

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[1]References on GUT, QCD, and dynamical symmetry breaking are so long that we will simply refer to the general review by P. Langacker, Phys. Rep. **72**, 185 (1981); W. Marciano and H. Pagels, *ibid.* **36C**, 137 (1978); G. 't Hooft, in *Recent Developments in Gauge Theories,* edited by G. 't Hooft *et al.* (Plenum, New York, 1980); and in *Proceedings of the International Conference on High Energy Physics, Lisbon, 1981,* edited by J. Dias de Deus and J. Soffer (European Physical Society, Erice, 1982).

[2]L. N. Chang and N. P. Chang, Phys. Rev. D **29**, 312 (1984).

[3]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[4]T. Goldman and D. Ross, Phys. Lett. **84B**, 208 (1979); Nucl. Phys. **B171**, 273 (1980).