PHYSICAL REVIEW D VOLUME 30, NUMBER 4 15 AUGUST 1984

New gauging of $N=8$ supergravity

C. M. Hull
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 12 January 1984)

A noncompact, 28-dimensional subgroup of E_7 that is a global symmetry of the Cremmer-Julia action is gauged. The resulting theory has $N=8$ local supersymmetry and $SU(8)\times ISO(7)^+$ gauge symmetry, where $ISO(7)^+$ is the Inönü-Wigner contraction of SO(8) about its $SO(7)^+$subgroup. The scalar potential is changed and the usual problems associated with gauging a noncompact group, such as ghosts, are avoided. In the symmetric gauge there is a manifest $SO(7)^+$, under which the gravitini transform according to the eight-dimensional spinor representation, while the noncompact generators of $ISO(7)^+$ are realized nonlinearly.

$N=8$ supergravity is the largest extended supergravity with maximum spin 2 and is the most likely to be physically relevant. Its particle content is uniquely determined and the ungauged theory, with 28 Abelian vector fields, was discovered by Cremmer and Julia.[1] de Wit and Nicolai have given a gauged version[2] in which the vector potentials become SO(8) gauge connections. It is clearly important to know how much freedom there is to change the self-coupling of the theory. That there may be several different ways of gauging the theory is suggested by the possibility of different nontrivial dimensional reductions of the 11-dimensional theory[3] and the existence of several distinct gaugings of $N=4$ supergravities.[4–6] Gates and Zwiebach[5] recently obtained three distinct gaugings of the SO(4) supergravity,[7] and it is a generalization of their remarkable results that will be applied to $N=8$ supergravity in this paper.

The ungauged $N=4$ theory,[7] with Lagrangian $\mathscr{L}_0$, has a local U(4) symmetry of the action and a global $SU(4)\times SU(1,1)$ symmetry of the equations of motion, of which the real subgroup $SO(4)\times SO(1,1)$ is a global symmetry of the action.[1] The physical scalar fields $\phi=\kappa(A+iB)$ lie in an SU(1,1)/U(1) coset space, and can be represented by an SU(1,1) matrix $\mathscr{V}$,

$$\mathscr{V}=\begin{bmatrix} u & v \\ \bar{v} & \bar{u}\end{bmatrix}=\mathscr{U}\exp\begin{bmatrix}0 & \phi \\ \bar{\phi} & 0\end{bmatrix}$$
$$=\mathscr{U}\begin{bmatrix}\dfrac{1}{(1-|y|^2)^{1/2}} & \dfrac{y}{(1-|y|^2)^{1/2}} \\ \dfrac{\bar{y}}{(1-|y|^2)^{1/2}} & \dfrac{1}{(1-|y|^2)^{1/2}}\end{bmatrix}. \quad (1)$$

Here, $\mathscr{U}$ is an element of the U(1) subgroup of SU(1,1) and could be set to unity by a suitable choice of the U(1) gauge—the symmetric gauge.[1] The inhomogeneous complex coordinate y, satisfying $|y|<1$, is related to the physical scalars by

$$y=\frac{\phi}{|\phi|}\tanh|\phi| \ . \quad (2)$$

The standard gauging of the theory[4] can be obtained by making the global SO(4) symmetry of the action local by adding minimal couplings in the usual way with coupling constant g and with the six vector fields A_μ^{IJ} ($I,J=1,\ldots,4$) acting as SO(4) connections. Supersymmetry is broken by the gauge couplings, but can be restored by modifying the supersymmetry transformation laws of the fermions by coupling-constant-dependent parts, δ_g, and by adding g-dependent terms to the Lagrangian, $\mathscr{L}_0\rightarrow\mathscr{L}=\mathscr{L}_0+\mathscr{L}_g$, where $\mathscr{L}_g$ consists of fermionic bilinear terms and a scalar potential. The theory has a local $U(4)\times SO(4)$ symmetry, the $SU(4)\times SU(1,1)$ symmetry being broken to SO(4) by the gauging.

Consider the action on the Lagrangian and supersymmetry transformation rules of this theory of the SU(1,1) transformation given by

$$E(t)=\exp\begin{bmatrix}0 & t \\ t & 0\end{bmatrix}, \quad (3)$$

where t is a real parameter. This acts on the scalar fields parametrized by $\mathscr{V}$ in (1) as

$$\mathscr{V}\rightarrow\mathscr{V}E(t)^{-1}\equiv\mathscr{V}'(t) \ , \quad (4)$$

$$u\rightarrow u'=\cosh t\, u-\sinh t\, v \ , \quad (5)$$

$$v\rightarrow v'\equiv-\sinh t\, u+\cosh t\, v \ , \quad (6)$$

while the vector fields transform as

$$A_\mu^{IJ}\rightarrow e^{t}A_{\mu+}^{IJ}+e^{-t}A_{\mu-}^{IJ} \ , \quad (7)$$

where the SO(4) (anti-) self-dual parts of the vector potentials are

$$A_{\mu\pm}^{IJ}=A_\mu^{KL}(\delta_{KL}^{IJ}\pm\tfrac{1}{2}\epsilon^{IJKL}) \ . \quad (8)$$

Since $E(t)$ is in the SO(1,1) subgroup of SU(1,1), it gives a symmetry of the ungauged Lagrangian, $\mathscr{L}_0\rightarrow\mathscr{L}_0$, but acts nontrivially on $\mathscr{L}_g$. If one rescales the coupling constant

$$g\rightarrow ge^{-t} \ , \quad (9)$$

the combination gA_μ^{IJ} appearing in the minimal couplings remains finite. Then under the action of $E(t)$ and (9),

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_g\rightarrow\mathscr{L}(t)=\mathscr{L}_0+\mathscr{L}_g(t)\ , \quad (10)$$

where $\mathscr{L}_g(0)=\mathscr{L}_g$. The coupling-constant-dependent parts of the supersymmetry transformation rules are also modified by (5)–(7), (9), $\delta_g\rightarrow\delta_g(t)$, and the theory remains supersymmetric for all values of t.

For all finite values of t, this transformation constitutes an invertible field redefinition and so the theory is equivalent to the original SO(4) gauging.[4] The Lagrangian $\mathscr{L}(t)$ is invariant under a local U(4)$\times$SO(4) symmetry but the SO(4) commutation relations and the spin-one kinetic term no longer have the canonical normalizations.[8]

Consider now the well-defined limit as t tends to infinity

$$\lim_{t\rightarrow\infty}\mathscr{L}(t)=\mathscr{L}'=\mathscr{L}_0+\mathscr{L}_g(\infty)\ , \quad (11)$$

$$\mathscr{L}(t)=\mathscr{L}'+O(e^{-2t})\ , \quad (12)$$

and the corresponding limit in the coupling-constant-dependent parts of the supersymmetry transformation rules

$$\lim_{t\rightarrow\infty}\delta_g(t)=\delta_g'\ , \quad (13)$$

$$\delta_g(t)=\delta_g'+O(e^{-2t})\ . \quad (14)$$

The limiting theory is again supersymmetric but the gauge symmetry has undergone an Inönü-Wigner contraction[9] from SO(4) to SU(2)$\times$U(1)^3 [cf. (7)], the U(1)^3 acting trivially, i.e., as central charges. It constitutes a gauging of an SU(2) subgroup of the global SU(4) symmetry and the model is related by a duality transformation[5] to one of the models obtained in Ref. 6. It has been obtained from the SO(4) gauging by a singular, noninvertible field redefinition, and so represents an inequivalent theory.

These results have previously been obtained by Gates and Zwiebach,[5] who gauged an SU(2)$\times$SU(2) subgroup of SU(4) with two coupling constants g_1, g_2, obtaining a family of models depending on the relative magnitudes of g_1+g_2 and g_1-g_2. Their parametrization of these models is related to that sketched above by

$$\tanh t=\frac{g_1-g_2}{g_1+g_2}\ . \quad (15)$$

Consider now $N=8$ supergravity in the conventions and notation of de Wit and Nicolai.[2] The ungauged theory[1] has a local SU(8) symmetry of the action and a global $E_{7(+7)}$ symmetry of the equations of motion, of which an SL(8,R) subgroup is a symmetry of the Lagrangian, $\mathscr{L}_0$. The 70 scalar fields lie in an $E_{7(+7)}/[\mathrm{SU}(8)/Z_2]$ coset space that can be parametrized by the "56-bein"

$$\mathscr{V}=\begin{bmatrix} u_{ij}{}^{IJ} & v_{ijKL} \\ \bar{v}^{klIJ} & \bar{u}^{kl}{}_{IJ}\end{bmatrix}\ , \quad (16)$$

where $i, j, k, \ldots=1,\ldots,8$ are SU(8) indices, $I, J, K, \ldots=1,\ldots,8$ are E_7 indices, and in (16) they are antisymmetric in pairs.

de Wit and Nicolai[2] gauged an SO(8) subgroup of E_7 that was a global symmetry of $\mathscr{L}_0$ by adding minimal SO(8) couplings with coupling constant g and with the 28 vector fields $A_\mu{}^{IJ}=-A_\mu{}^{JI}$ as gauge connections. Supersymmetry was lost in the process and the change of the action under an infinitesimal local supersymmetry transformation was expressed in terms of the so-called T tensor[2]

$$T_i{}^{jkl}=(\bar{u}^{kl}{}_{IJ}+\bar{v}^{klIJ})(u_{im}{}^{JK}\bar{u}^{jm}{}_{KI}-v_{imJK}\bar{v}^{jmKI})\ . \quad (17)$$

If one adds to the Lagrangian coupling-constant-dependent fermionic bilinear terms and a scalar potential, $\mathscr{L}=\mathscr{L}_0+\mathscr{L}_g$,

$$\mathscr{L}_g=ge(\sqrt{2}A_1{}^{ij}\bar{\psi}_{\mu i}\sigma^{\mu\nu}\psi_{\nu j}+\tfrac{1}{6}A_2{}^i{}_{jkl}\bar{\psi}_{\mu i}\gamma^\mu\chi^{jkl}+A_3{}^{ijk,lmn}\bar{\chi}_{ijk}\chi_{lmn})+\mathrm{H.c.}+g^2e(\tfrac{3}{4}\,|A_1{}^{ij}|^2-\tfrac{1}{24}\,|A_2{}^i{}_{jkl}|^2)\ , \quad (18)$$

and adds to the supersymmetry transformations the terms

$$\delta_g\bar{\psi}_\mu{}^i=\sqrt{2}gA_1{}^{ij}\bar{\epsilon}_j\gamma_\mu\ , \quad (19)$$

$$\delta_g\bar{\chi}^{ijk}=-2gA_{2l}{}^{ijk}\epsilon^l\ , \quad (20)$$

the theory becomes fully supersymmetric, provided that the scalar-field-dependent tensors A_1, A_2, A_3 satisfy

$$\tfrac{4}{3}T_i{}^{jkl}+A_{2i}{}^{jkl}+2A_1{}^{j[k}\delta_i{}^{l]}=0\ , \quad (21)$$

$$A_{3ijk,lmn}=-\frac{\sqrt{2}}{108}\epsilon_{ijkpqr[lm}T_{n]}{}^{pqr}\ . \quad (22)$$

The consistency of Eqs. (21) and (22) follows from some nontrivial properties of the T tensor, as a result of which (21) and (22) provide definitions of A_1, A_2, A_3.

This gives a theory with local SU(8)$\times$SO(8) symmetry, the E_7 being broken to SO(8) by the gauging.

Consider now the action of the E_7 transformation given by

$$E(t)=\exp\begin{bmatrix} 0 & tX^{IJKL} \\ tX_{IJKL} & 0\end{bmatrix}\ , \quad (23)$$

under which

$$\mathscr{V}\rightarrow\mathscr{V}(t)=\mathscr{V}E(t)^{-1}\ , \quad (24)$$

$$A_\mu{}^{IJ}\rightarrow A_\mu{}^{IJ}(t)=[\exp(-tX)]^{IJKL}A_\mu{}^{KL}\ , \quad (25)$$

where X^{IJKL} is real and self-dual,

$$X^{IJKL}=\bar{X}_{IJKL}=\frac{\eta}{24}\epsilon^{IJKLMNPQ}X_{MNPQ} \quad (26)$$

($\eta=\pm1$ is an arbitrary duality phase, which it will be convenient to choose here as $+1$) and $X^{IJ,KL}$ is to be regarded as a 28$\times$28 matrix, labeled by antisymmetric index pairs. Since $E(t)$ is in the real SL(8,R) subgroup of E_7, it constitutes a symmetry of the ungauged action $\mathscr{L}_0\rightarrow\mathscr{L}_0$, but modifies $\mathscr{L}_g$. For all finite values of t this yields a theory which is field-redefinition equivalent to the de Wit-Nicolai theory. If one now attempts to take the limit $t\rightarrow\infty$, in analogy with the $N=4$ construction, one finds that for many choices of the four-form X^{IJKL} that a limit does not exist. This suggests that the construction might only work for a four-form with rather special properties.

Consider then

$$X^{IJKL}=Y^{IJKL}+\frac{\eta}{24}\epsilon^{IJKLMNPQ}Y^{MNPQ}\ , \tag{27}$$

where

$$Y^{IJKL}=\tfrac{1}{2}(\delta^{IJKL}_{1234}+\delta^{IJKL}_{1256}+\delta^{IJKL}_{1278}+\delta^{IJKL}_{1375}+\delta^{IJKL}_{1368}+\delta^{IJKL}_{1458}+\delta^{IJKL}_{1467})\ . \tag{28}$$

This four-form gives the associator of the octonions and is closely related to the torsion that parallelizes S^7 (Refs. 10 and 11). It is invariant under the SO(7)$^+$ subgroup of SO(8) (Ref. 11) [i.e., the stabilizer of a right-handed SO(8) spinor] and giving the scalar fields an expectation value proportional to X^{IJKL} in the de Wit-Nicolai theory gives a spontaneous symmetry breaking of SO(8) to SO(7)$^+$ (Ref. 11)

$$(X^{IJKL}-3\delta^{IJ}_{KL})(X^{KLMN}+\delta^{KL}_{MN})=0 \tag{29}$$

and, as a 28×28 matrix, X has 21 eigenvalues of -1 and 7 eigenvalues of $+3$. Introducing the projector P_+ onto the 21-dimensional eigenspace

$$P_+^{IJ,KL}=\tfrac{3}{4}(\delta^{IJ}_{KL}-\tfrac{1}{3}X^{IJKL}) \tag{30}$$

and

$$P_-^{IJKL}=\delta^{IJ}_{KL}-P_+^{IJKL}=\tfrac{1}{4}(\delta^{IJ}_{KL}+X^{IJKL})\ . \tag{31}$$

P_+ projects the generators of SO(8) onto those of SO(7)$^+$ (Ref. 11). Then

$$[\exp(-tX)]^{IJKL}=e^{t}P_+^{IJKL}+e^{-3t}P_-^{IJKL}\ , \tag{32}$$

so that (25) becomes

$$A_\mu^{IJ}(t)=e^{t}A_{\mu+}^{IJ}+e^{-3t}A_{\mu-}^{IJ}\ , \tag{33}$$

where

$$A_{\mu\pm}^{IJ}=P_\pm^{IJ,KL}A_\mu{}^{KL}\ . \tag{34}$$

Rescaling the coupling constant

$$g\to ge^{-t} \tag{35}$$

to preserve the finiteness of gA_μ^{IJ}, one obtains a one-parameter family of Lagrangians $\mathcal{L}(t)=\mathcal{L}_0+\mathcal{L}_g(t)$. For all finite values of t, the theory is field-redefinition equivalent to de Wit-Nicolai theory ($t=0$) and so is supersymmetric with supersymmetry transformations $\delta_Q=\delta_0+\delta_g(t)$, and has an SO(8) gauge invariance. An SO(8) generator L_{IJ} acts on the scalar fields and vector-field strength 56-vector as an E_7 matrix $D(L,t)$,

$$\delta\mathcal{V}=-\mathcal{V}D(L,t)\ , \tag{36}$$

$$\delta\begin{bmatrix}F_{1\mu\nu}{}^{IJ}\\ F_{2\mu\nu}{}^{IJ}\end{bmatrix}=D(L,t)\begin{bmatrix}F_{1\mu\nu}{}^{IJ}\\ F_{2\mu\nu}{}^{IJ}\end{bmatrix}\ , \tag{37}$$

where

$$D(L,t)=E(t)^{-1}D(L')E(t)\ , \tag{38}$$

where $D(L)=D(L,0)$ is the matrix giving the SO(8) action in the de Wit-Nicolai model

$$D(L)=\begin{bmatrix}L_{[I}{}^{[K}\delta_{J]}{}^{L]} & 0\\ 0 & L^{[M}{}_{[P}\delta^{N]}{}_{Q]}\end{bmatrix}\ , \tag{39}$$

and $D(L')$ is obtained by replacing $L_I{}^J$ in (39) with $L'_{IJ}=(P_{+IJKL}+e^{-4t}P_{-IJKL})L_{KL}$.

It can be shown that the Lagrangian $\mathcal{L}(t)$ satisfies

$$\mathcal{L}(t)=\mathcal{L}'+O(e^{-4t})\ , \tag{40}$$

where $\mathcal{L}'$ is independent of t, and the supersymmetry transformation laws $\delta_Q(t)=\delta_0+\delta_g(t)$, which have t dependence through the t dependence of the terms $A_1{}^{ij}$, $A_{2i}{}^{jkl}$, satisfy

$$\begin{aligned}\delta_Q(t)&=\delta'_Q+O(e^{-4t})\\ &=\delta_0+\delta'_g+O(e^{-4t})\ ,\end{aligned} \tag{41}$$

where δ'_Q is independent of t. Although there are, in principle, terms that grow exponentially with t in (40) and (41), these vanish identically as a result of certain special properties of the tensor X_{IJKL}. Then the limit of the theory as $t\to\infty$ exists and, since for all finite values of t, the theory is supersymmetric,

$$\delta_Q(t)\mathcal{L}(t)=(\text{total divergence})$$

it follows that

$$\delta'_Q\mathcal{L}'=(\text{total divergence})+O(e^{-4t})\ ,$$

which can only hold for all t if the limiting form of the theory is supersymmetric.

A similar argument can be used to demonstrate gauge invariance. For any particular gauge generator L,

$$D(L,t)=D'(L)+O(e^{-4t}) \tag{42}$$

where

$$\begin{aligned}D'(L)&=R\begin{bmatrix}\underline{L_+}+P_+\underline{L_-}P_- & 0\\ 0 & \underline{L_+}+P_-\underline{L_-}P_+\end{bmatrix}R^{-1}\\ &=\begin{bmatrix}\underline{L_+}+\frac{1}{2}\underline{L_-} & Z\\ Z & \underline{L_+}+\frac{1}{2}\underline{L_-}\end{bmatrix}\ ,\end{aligned} \tag{43}$$

where the diagonalizing similarity transformation is given by

$$R=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{44}$$

and

$$L_\pm^{IJ}+P_\pm^{IJKL}L^{KL}\ , \tag{45}$$

and for any 8×8 matrix $\Lambda_I{}^J$,

$$\underline{\Lambda}\equiv\Lambda_{[I}{}^{[K}\delta_{J]}{}^{L]}\ . \tag{46}$$

The four-form Z^{IJKL} is

$$Z^{IJKL}=-\tfrac{1}{4}L^{[I}{}_M X^{JKL]M}\ . \tag{47}$$

Since for all t, $D(L,t)$ is an element of the E_7 algebra generating a symmetry of $\mathscr{L}(t)$, it follows that $D'(L)$ must also be in E_7, and when acting on the spin-zero and spin-one fields as in (36), (37), must give a local symmetry of $\mathscr{L}'$.

The commutation relations of the gauge transformations are, from (43),

$$[D'(L_1),D'(L_2)]=D'([L_1,L_2]) \tag{48}$$

where, in terms of the projections (45),

$$[L_{+I}{}^J,L_{+K}{}^L]=4L_{+[I}{}^{[L}\delta_{J]}{}^{K]} , \tag{49}$$

$$[L_{+I}{}^J,L_{-K}{}^L]=4L_{-[I}{}^{[L}\delta_{J]}{}^{K]} , \tag{50}$$

$$[L_{-I}{}^J,L_{-K}{}^L]=0 . \tag{51}$$

(49) gives the usual commutation relations for $SO(7)^+$, but as a result of the vanishing of (51) the full algebra is no longer that of SO(8) but is an Inönü-Wigner contraction[9] of SO(8) about its $SO(7)^+$ subgroup, giving $ISO(7)^+$, which is isomorphic to the group of motions of Euclidean 7-space. The 21 L_+ generate SO(7) "rotations," while the L_- give 7 "translations."

The theory, then, constitutes a gauging of the 28-dimensional, noncompact $ISO(7)^+$ symmetry of the Cremmer-Julia action $\mathscr{L}_0$, acting through (43) as (36) and (37) (with $t=\infty$).

Minimal couplings are added so that the Yang-Mills field strength is

$$\begin{aligned} F_{\mu\nu}{}^{IJ}=2\partial_{[\mu}A^{IJ}{}_{\nu]}-2gA_{[\mu+}{}^{IK}A^{KJ}{}_{\nu]+} \\ -2gA_{[\mu+}{}^{IK}A_{\nu-]}{}^{KJ} \\ -2gA_{[\mu-}{}^{IK}A^{KJ}{}_{\nu]+} , \end{aligned} \tag{52}$$

and the covariant derivative of the 56-bein gains a gauge convariantization

$$(D_\mu \mathscr{V})\mathscr{V}^{-1}\rightarrow(D_\mu \mathscr{V})\mathscr{V}^{-1}-g(\mathscr{V}D'(A_\mu)\mathscr{V}^{-1} , \tag{53}$$

so that the SU(8) connection is

$$\begin{aligned} \mathscr{B}_{\mu i}{}^j=-\tfrac{2}{3}(\bar{u}^{jk}{}_{IJ}\partial_\mu u_{ik}{}^{IJ}-\bar{v}^{jkIJ}\partial_\mu v_{ikIJ}) \\ +\tfrac{4}{3}gA_\mu{}^{IJ}[M^{IJKL}(u_{ik}{}^{KM}\bar{u}^{jk}{}_{LM}-v_{ikKM}\bar{v}^{jkLM}) , \\ -N_{IJ}{}^{KLMN}(v_{ikKL}\bar{u}^{jk}MN \\ -u_{ik}{}^{KL}\bar{v}^{jkKL})] , \end{aligned} \tag{54}$$

where

$$\begin{aligned} M_{IJKL}&=P_{+IJKL}+\tfrac{1}{2}P_{-IJKL} \\ &=\tfrac{1}{8}(7\delta_{IJ}^{KL}-X_{IJKL}) , \end{aligned} \tag{55}$$

$$\begin{aligned} N_{IJ}{}^{KLMN}&=\tfrac{1}{2}P_-^{IJ[K}{}_{[P}\delta^{L]}{}_{Q]}(P_-^{PQMN}-P_+^{PQMN}) \\ &=-\tfrac{1}{4}\delta_I{}^{[K}X^{LMN]J} . \end{aligned} \tag{56}$$

The scalar kinetic term is

$$-\tfrac{1}{96}e\mathscr{A}_\mu{}^{ijkl}\mathscr{A}^\mu{}_{ijkl} ,$$

where

$$\begin{aligned} \mathscr{A}_{\mu ijkl}=-2\sqrt{2}(u_{ij}{}^{IJ}\partial_\mu v_{klIJ}-v_{ijIJ}\partial_\mu u_{kl}{}^{IJ}) \\ +4\sqrt{2}gA_{\mu IJ}[M^{IJKL}(-u_{ij}{}^{KM}v_{klLM}+v_{ijKM}u_{kl}{}^{LM}) \\ +N_{IJ}{}^{KLMN}(u_{ij}{}^{KL}u_{kl}{}^{MN} \\ -v_{ijKL}v_{klMN})] . \end{aligned} \tag{57}$$

Since the vector and scalar kinetic terms are just gauge covariantizations of those of the Cremmer-Julia theory,[1] the usual problems associated with the gauging of noncompact groups, such as ghosts or nonpropagating degrees of freedom, are avoided. The change of the minimal couplings under supersymmetry gives a net change of the action under an infinitesimal local supersymmetry that can be parametrized by a new "T tensor"

$$\begin{aligned} T_i'{}^{jkl}=(\bar{u}^{kl}{}_{IJ}+\bar{v}^{klIJ}) \\ \times[M_{IJKL}(u_{im}{}^{KM}\bar{u}^{jm}{}_{LM}-v_{imKM}v^{jmLM}) \\ +N_{IJ}{}^{KLMN}(v_{imKL}\bar{u}^{jm}{}_{MN}-u_{im}{}^{KL}\bar{v}^{jmMN})] , \end{aligned} \tag{58}$$

where M_{IJKL}, $N_{IJ}{}^{KLMN}$ are defined in (55) and (56). Then supersymmetry of the theory is restored by adding terms (18) to the action and (19) and (20) to the supersymmetry transformation rules, where the tensors A_1, A_2, A_3 now have a new functional dependence on the scalar fields given by (21) and (22) but with the T tensor in these equations replaced by $T_i'{}^{jkl}$ in (58). Again, the consistency of these equations can be demonstrated.

The theory has $N=8$ local supersymmetry and $SU(8)\times ISO(7)^+$ gauge symmetry. On going to the symmetric gauge,[1] the diagonal $SO(7)^+$ subgroup is manifest, with the gravitini, for example, transforming according to the eight-dimensional spinor representation of $SO(7)^+$. The noncompact generators of $ISO(7)^+$ are then realized nonlinearly.

In the symmetric gauge, the scalar potential $V(\phi_{IJKL})$ is given in the $SO(7)^+$ singlet direction in scalar space, where $\phi_{IJKL}=2\sqrt{2}sX_{IJKL}$, by

$$V(s)=-\tfrac{35}{8}g^2e^{2s} \tag{59}$$

which has the same shape as the potential of the Freedman-Schwarz model.[6] There are thus no $SO(7)^+$-invariant critical points of the potential.

I would like to thank D. Z. Freedman, S. J. Gates, and N. P. Warner for many helpful discussions. The author was supported by a NATO Fellowship.

Note added. I have recently found a number of other noncompact gaugings of $N=8$ supergravity. The Yang-Mills groups are SO(7,1), SO(6,2), SO(5,3), and SO(4,4), together with the Inönü-Wigner contractions of SO(8) about the subgroups $SO(6)\times SO(2)$, $SO(5)\times SO(3)$, and $SO(4)\times SO(4)$. [The contraction about SO(7) gives ISO(7).] The theories with gauge groups SO(8), SO(4,4), and the contraction of SO(8) about $SO(4)\times SO(4)$ can be truncated to give the three gauged $N=4$ supergravities, discussed above.

[1]E. Cremmer and B. Julia, Nucl. Phys. **B159**, 141 (1979).
[2]B. de Wit and H. Nicolai, Phys. Lett. **108B**, 285 (1981); Nucl. Phys. **B208**, 323 (1982).
[3]E. Witten, Nucl. Phys. **B186**, 412 (1982); M. J. Duff and C. N. Pope, in *Supergravity '82*, edited by S. Ferrara, J. G. Taylor, and P. van Nieuwenhuizen (World Scientific, Singapore, 1983); F. Englert, Phys. Lett. **119B**, 339 (1982).
[4]A. Das, M. Fischler, and M. Rocek, Phys. Rev. D **16**, 3427 (1977).
[5]S. J. Gates and B. Zwiebach, Phys. Lett. **123B**, 200 (1983).
[6]D. Z. Freedman and J. Schwarz, Nucl. Phys. **B137**, 333 (1978).
[7]A. Das, Phys. Rev. D **15**, 2805 (1977); E. Cremmer and J. Scherk, Nucl. Phys. **B127**, 259 (1977).
[8]B. Zwiebach, Nucl. Phys. **B238**, 367 (1984).
[9]E. Inönü and E. P. Wigner, Proc. Nat. Acad. Sci. U.S.A. **39**, 510 (1953).
[10]L. Castellani and N. P. Warner, Phys. Lett. **130B**, 46 (1983); F. Englert, M. Rooman, and P. Spindel, *ibid.* **130B**, 50 (1983).
[11]N. P. Warner, Phys. Lett. **128B**, 169 (1983); B. de Wit and H. Nicolai, Nucl. Phys. **B231**, 506 (1984).