Relation between scaling in the mean of the momentum distributions in inclusive and semi-inclusive reactions

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In view of the scaling-in-the-mean hypothesis we propose a method to derive the relation between the momentum distributions in inclusive and semi-inclusive reactions. Our formula is useful to make a discrimination among the similar p_T distribution functions. We predict that the inclusive p_L distribution function in terms of $p_L/\langle p_L\rangle$ shows energy dependence, comparing the Koba-Nielsen-Olesen scaling hypothesis and the two-component model for the multiplicity distributions.

I. INTRODUCTION

In order to investigate the characteristic features of the dominant mechanism of pion production at high energies, scaling in the mean has been proposed for the momentum distributions in pp semi-inclusive reactions. According to the experimental study of Dao *et al.*,[1] the longitudinal-momentum (p_L) and the transverse-momentum (p_T) distributions can be plotted so as to be independent of multiplicity and incident energy; this property is called scaling in the mean (SIM). Recently, Basile *et al.*[2] have reported that the scaling function of the p_T distribution in inclusive e^+e^- annihilation reactions may be universal, i.e., independent of multiplicity, energy and incident particles, both in inclusive and semi-inclusive reactions.

In the SIM hypothesis, the important theoretical problems are to deduce the scaling behavior and to decide the scaling function. Several authors have proposed different distribution functions based on information theory,[3] the thermodynamic model,[4] the quark-gluon radiation model,[5] etc. However, these model calculations so far have been applied only to inclusive or semi-inclusive reactions without taking the relation between these reactions into account. Since they reproduce quite similar distribution functions, it has been very difficult to discriminate among them. Therefore, introducing a formula to make a connection between scaling functions of those reactions, we want to propose a method to make a choice of the accurate distribution function for inclusive and semi-inclusive reactions.

One purpose of this article is to introduce a formula which makes a relation between the momentum distributions in inclusive and semi-inclusive reactions with respect to the SIM hypothesis. Next, on the basis of SIM for the semi-inclusive momentum distribution, we will examine the assumption that the same scaling law holds well in the case of the inclusive distribution and that the scaling function is the same for those reactions.[5] In this connection we study the energy and multiplicity dependence of the mean momentum values. We demonstrate that we can obtain SIM for the p_T distribution in inclusive reactions. Exact determination of the ratio of the mean momentum values between inclusive and semi-inclusive reactions in the wide energy and multiplicity regions will lead to the choice of the appropriate model according to our formula. As concerns the p_L distribution in inclusive reactions, we obtain the distribution functions in terms of $p_L/\langle p_L\rangle$ at 300 GeV/c and in the infinite-energy region in order to compare them. Since they show energy dependence, the SIM hypothesis for the inclusive p_L distribution is excluded in the strict sense. With respect to this evaluation, we compare the results obtained from multiplicity distributions based on the Koba-Nielsen-Olesen (KNO) scaling assumption and the two-component model.

In Sec. II we present the kinematical formulas to derive the relation between the momentum distributions in inclusive and semi-inclusive reactions. Analyses of the scaling hypothesis for the p_T and the p_L distribution are given in Secs. III and IV, respectively. Section V is devoted to summary and discussion.

II. KINEMATICS

According to the first study of Dao *et al.*,[1] we assume the scaling relation, independent of multiplicity, total energy, and incident channels, as

$$\frac{\langle p_L\rangle_n\langle p_T\rangle_n}{n\sigma_n}\frac{d^2\sigma_n}{dp_L dp_T}=\phi\left[\frac{p_L}{\langle p_L\rangle_n},\frac{p_T}{\langle p_T\rangle_n}\right], \quad (2.1)$$

where σ_n is the n-particle cross section and $\langle p_L\rangle_n(\langle p_T\rangle_n)$ means the average value of the magnitude of the c.m. longitudinal (transverse) momenta in semi-inclusive reactions. The average momentum values are defined by

$$\langle p_L\rangle_n=\frac{1}{n\sigma_n}\int dp^3 p_L\frac{d^3\sigma_n}{dp^3} \quad (2.2)$$

and

$$\langle p_T\rangle_n=\frac{1}{n\sigma_n}\int dp^3 p_T\frac{d^3\sigma_n}{dp^3}\ , \quad (2.3)$$

through the normalization condition

$$\int dp^3 \frac{d^3\sigma_n}{dp^3} = n\sigma_n \ . \tag{2.4}$$

Integrating Eq. (2.1) with respect to either one of the momentum variables, we have

$$\frac{\langle p_A \rangle_n}{n\sigma_n} \frac{d\sigma_n}{dp_A} = \phi_A \left[\frac{p_A}{\langle p_A \rangle_n} \right] , \tag{2.5}$$

where A denotes L for the p_L distribution or T for the p_T distribution.

In comparison with Eq. (2.5), we obtain the inclusive p_L and p_T distribution by means of a sum of semi-inclusive cross sections

$$\frac{\langle p_A \rangle}{\langle n \rangle \sigma} \frac{d\sigma}{dp_A} = \sum_n \frac{n\sigma_n}{\langle n \rangle \sigma} \frac{\langle p_A \rangle}{\langle p_A \rangle_n} \phi_A \left[\frac{p_A}{\langle p_A \rangle_n} \right] , \tag{2.6}$$

where $\sigma = \sum_n \sigma_n$. This can be rewritten in terms of $x_A = p_A / \langle p_A \rangle$ and $R_A = \langle p_A \rangle / \langle p_A \rangle_n$ as

$$\frac{1}{\langle n \rangle \sigma} \frac{d\sigma}{dx_A} = \sum\nolimits_n \frac{n\sigma_n}{\langle n \rangle \sigma} R_A \phi_A(x_A R_A) \ . \tag{2.6$'$}$$

In the same manner we set the normalization condition

$$\int dp^3 \frac{d^3\sigma}{dp^3} = \langle n \rangle \sigma \tag{2.7}$$

and the average momentum value in inclusive reactions

$$\langle p_A \rangle = \frac{1}{\langle n \rangle \sigma} \int dp^3 p_A \frac{d^3\sigma}{dp^3} \ . \tag{2.8}$$

Equations (2.8) and (2.2) [or (2.3)] yield

$$\langle p_A \rangle = \sum_n \frac{n}{\langle n \rangle} \langle p_A \rangle_n \frac{\sigma_n}{\sigma} \ . \tag{2.9}$$

It follows that one can examine the energy dependence of Eq. (2.6) [or (2.6′)] with the help of the multiplicity distribution, the semi-inclusive momentum distribution ϕ_A, and the ratio of the mean momentum values R_A.

III. SCALING OF THE TRANSVERSE-MOMENTUM DISTRIBUTION

The investigation of the p_T distribution is worthwhile in order to visualize the mechanism of multiple pion production both in the small and the large p_T regions.[6] The experimental data of the p_T distribution in semi-inclusive reactions show the characteristic scaling behavior in the region of $p_T \lesssim 3.5\langle p_T \rangle$ independent of multiplicity and incident energy. Recently, it has been proposed that the shape of the distribution function is the same both in the case of inclusive e^+e^- annihilation and semi-inclusive pp interactions from the viewpoint of the universal production mechanism.[7] We would like to present the relation between the distribution functions of those reactions and study whether they show the same functional form or not.

We have in Eq. (2.6′) for the p_T distribution

$$\frac{1}{\langle n \rangle \sigma} \frac{d\sigma}{dx_T} = \sum_n \frac{n\sigma_n}{\langle n \rangle \sigma} R_T \phi_T(R_T x_T) , \tag{3.1}$$

in which we put $x_T = p_T / \langle p_T \rangle$ and $R_T = \langle p_T \rangle / \langle p_T \rangle_n$. It seems that the data of $\langle p_T \rangle_n$ for pp interactions at 300 GeV/c (Ref. 1), scarcely show the prong-multiplicity dependence. Hence, if we set $\langle p_T \rangle_n = \langle p_T \rangle$ in Eq. (2.9), we obtain from Eq. (3.1) with $R_T = 1$

$$\frac{1}{\langle n \rangle \sigma} \frac{d\sigma}{dx_T} = \Phi_T(x_T) = \phi_T(x_T) \ . \tag{3.2}$$

It is presumable that we can determine the functional form of the p_T distribution without noticing the multiplicity distribution. The following scaling functions of

$$\Phi_T(x_T) = \phi_T(p_T / \langle p_T \rangle_n)$$

have been proposed on the basis of the respective models:

(i) The information theory yields[3] in terms of x_T

$$\phi_T(x_T) = 4x_T \exp(-2x_T) \tag{3.3}$$

with no parameter.

(ii) The thermodynamic model gives[8]

$$\phi_T(x_T) = \frac{4a}{3\sqrt{\pi}} (a x_T)^{3/2} \exp(-a x_T) \tag{3.4}$$

with one parameter $a = 2.5$.

(iii) The gluon radiation model presents[5]

$$\phi_T(x_T) = \frac{\sqrt{\pi}\,\Gamma(1/2+\beta/2)}{2^{(\beta/2)-1}\Gamma^2(\beta/2)} z_T{}^{\beta/2} K_{\beta/2-1}(z_T) , \tag{3.5}$$

where

$$z_T = [\sqrt{\pi}\,\Gamma(\beta/2 - 1/2) x_T] / \Gamma(\beta/2)$$

and the parameter $\beta = 3.5$. It turns out from the evaluation of the second moment of the transverse momenta $M_2^T = \langle p_T{}^2 \rangle / \langle p_T \rangle^2$ that $M_2^T = 1.5$ for the information theory, $M_2^T = 1.4$ for the thermodynamic model and $M_2^T = 1.44$ for the gluon radiation model. However, we cannot distinguish among these distribution functions on the basis of experimental data presently available from pp interactions and e^+e^- annihilation reactions.[9]

Now on the basis of KNO scaling[10] let us rewrite Eq. (3.1) in terms of $z = n / \langle n \rangle$ as

$$\frac{1}{\langle n \rangle \sigma} \frac{d\sigma}{dx_T} = \int dz\, z \psi(z) R_T \phi_T(R_T x_T) , \tag{3.6}$$

where $\psi(z) = \langle n \rangle \sigma_n / \sigma$. It is noticeable that SIM and the scaling hypothesis suggested by Koba *et al.*[11] yield a formula to write $\langle p_T \rangle_n$ in terms of z. If we take

$$\langle p_T \rangle_n = a_0 + a_1 z^{-1}$$

(a_0 and a_1 are constant parameters) according to Laasanen *et al.*,[4] we get from Eq. (2.9)

$$\langle p_T \rangle = \int dz\, z \langle p_T \rangle_n \psi(z) = a_0 + a_1 , \tag{3.7}$$

and hence

$$R_T = (a_0 + a_1)/(a_0 + a_1 z^{-1})$$
$$\simeq 1 + (a_1/a_0)(1 - z^{-1})$$

for $a_0 \gg a_1$. Equation (3.6), thus, leads to the scaling formula of the inclusive p_T distribution

$$\frac{1}{\langle n\rangle\sigma}\frac{d\sigma}{dx_T}=\Phi_T(x_T)$$
$$=\int dz\left[\left[1+\frac{a_1}{a_0}\right]z-\frac{a_1}{a_0}\right]$$
$$\times\psi(z)\phi_T\left[\left[1+\frac{a_1}{a_0}-\frac{a_1}{a_0}z^{-1}\right]x_T\right], \tag{3.8}$$

which shows a different curvature from $\phi_T(x_T)$. Taking another formula

$$\langle p_T\rangle_n=c_0+c_1z^b ,$$

with three positive constants c_0, c_1, and b, which increases with n as suggested by the recent data from the CERN SPS $p\bar{p}$ collider at $\sqrt{s}=540$ GeV, we have in the same manner

$$\langle p_T\rangle=c_0+c_1\langle z^{1+b}\rangle . \tag{3.9}$$

The inclusive p_T distribution is

$$\Phi_T(x_T)=\int dz\,z\psi(z)R_T(z)\phi_T(x_TR_T(z)) , \tag{3.10}$$

where

$$R_T=(c_0+c_1\langle z^{1+b}\rangle)/(c_0+c_1z^b) .$$

However, it is difficult to determine these parameters by using the presently available data.[12]

The discrepancy between Φ_T and ϕ_T is to be seen in accordance with the respective models. The difference between the p_T distribution function in semi-inclusive reactions and the data ought to be amplified by summing up with n. Therefore, according to our formula, definite determination of R_T in wide energy and multiplicity regions shall play an important role for the choice of the model.

IV. SCALING OF THE LONGITUDINAL-MOMENTUM DISTRIBUTION

SIM has been proposed for the semi-inclusive p_L distribution in contrast with Feynman scaling[13] in inclusive reactions against which the data[14] show the energy dependence in the nondiffractive region. However, it is not certain that SIM of the p_L distribution is satisfied also in the case of inclusive reactions. Investigation of the validity of SIM for the inclusive distribution is necessary for the study of the universally dominant mechanism in the central region. We will examine the scaling hypothesis of the inclusive p_L distribution on the assumption of Eq. (2.5).

In order to derive the inclusive distribution we need the explicit formula of $R_L=\langle p_L\rangle/\langle p_L\rangle_n$ in the course of the actual calculations. In the fragmentation regions the mean longitudinal momenta $\langle p_L\rangle_n$ is determined by the average energy as $\langle p_L\rangle_n\simeq\sqrt{s}/n$ based on the energy-conservation sum rule, assuming $p_L\gg p_T$.[15] However, from the viewpoint of soft-gluon bremsstrahlung in the large-n region, $\langle p_L\rangle_n$ becomes compatible with $\langle p_T\rangle_n$ which shows weakly energy and multiplicity dependence. Hence, we have $\langle p_L\rangle_n$ over a wide range of n and s, superposing these contributions[16] as

$$\langle p_L\rangle_n=\alpha\frac{\sqrt{s}}{n}+\beta , \tag{4.1}$$

where α and β are the parameters, and $\sqrt{s}$ is the total c.m. energy. Equations (2.9) and (4.1) yield

$$\langle p_L\rangle=\alpha\frac{\sqrt{s}}{\langle n\rangle}+\beta . \tag{4.2}$$

Thus, we get

$$R_L(z,s)=\frac{\alpha\sqrt{s}/\langle n\rangle+\beta}{\alpha\sqrt{s}/n+\beta}$$
$$=\left[1+\frac{\beta\langle n\rangle}{\alpha\sqrt{s}}\right]\left[z^{-1}+\frac{\beta\langle n\rangle}{\alpha\sqrt{s}}\right]^{-1} . \tag{4.3}$$

It should be noted that R_L in Eq. (4.3) is energy dependent, while R_L approaches z as the energy increases.

Now we postulate KNO scaling for the multiplicity distribution.[17] The inclusive p_L distribution is given in terms of $x_L=p_L/\langle p_L\rangle$ by

$$\frac{\langle p_L\rangle}{\langle n\rangle\sigma}\frac{d\sigma}{dp_L}=\Phi_L(x_L,s)$$
$$=\int dz\,z\psi(z)R_L(z,s)\phi_L(R_Lx_L) , \tag{4.4}$$

on the assumption of KNO scaling, SIM, and the relation (4.3). We will show in Fig. 1 the theoretical estimates of the distribution function $\phi_L(x_L,s)$ at 300 GeV/c, and in the infinite-energy region, making use of the best-fitted function of Dao *et al.*,[1]

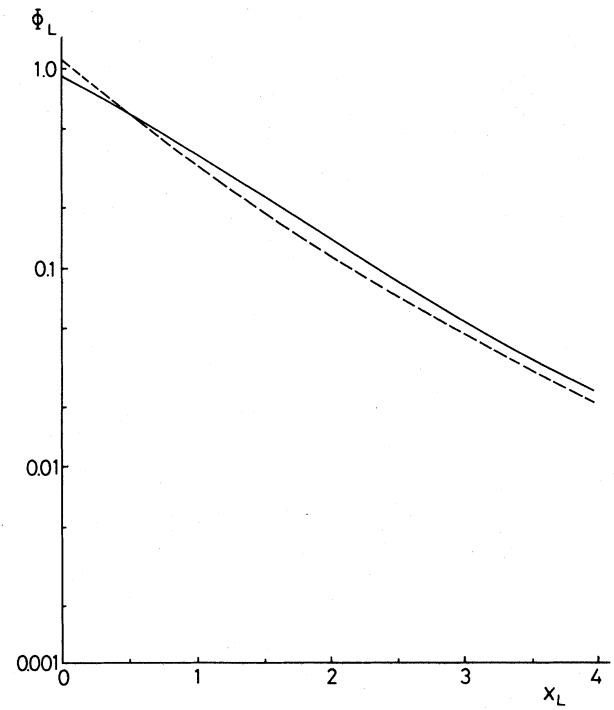

FIG. 1. Plot of $\Phi_L=(\langle p_L\rangle d\sigma/dp_L)/(\langle n\rangle\sigma)$ vs $x_L=p_L/\langle p_L\rangle$. The solid curve represents our calculation of Eq. (4.4) at 300 GeV/c. The dashed curve is the prediction of Eq. (4.4) for $s\rightarrow\infty$.

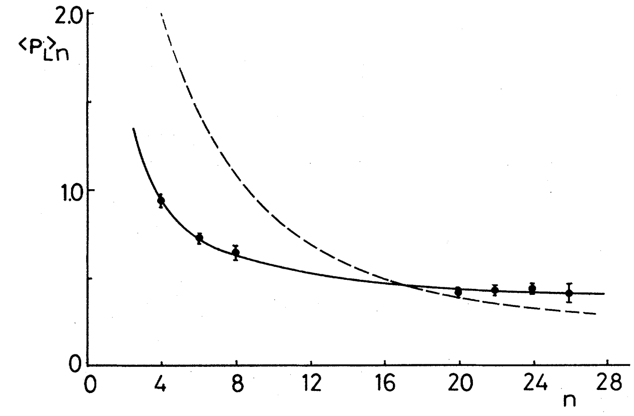

FIG. 2. $\langle p_L \rangle_n$ vs n, charged multiplicity, for $pp \to \pi^-$ reactions. The data points are from Ref. 1. The solid curve shows our parameter fit with $\alpha=0.1$ and $\beta=0.32$ in Eq. (4.1) which is compared with the result of the relation $\langle p_L \rangle_n = \sqrt{s}/(3n)$ (dashed curve).

$$\phi_L\left[\frac{p_L}{\langle p_L\rangle_n}\right]=0.91\exp\left[-0.83\left[\frac{p_L}{\langle p_L\rangle_n}\right]-0.03\left[\frac{p_L}{\langle p_L\rangle_n}\right]^2\right], \quad (4.5)$$

Slattery's fit[18]

$$\psi(z)=(1.895z+16.85z^3-3.32z^5+0.166z^7)\times\exp(-3.04z), \quad (4.6)$$

and the well-fitted parameters $\alpha=0.1$ and $\beta=0.32$ in Eq. (4.1) (see Fig. 2).

It is interesting to compare those results with the prediction based on the two-component model,[19] since the two-component model (TCM) of the multiplicity distribution has been proposed as an alternative to the KNO scaling function.

The multiplicity distribution of the TCM is

$$P_n=\frac{\sigma_n}{\sigma}=\left[\frac{\sigma^\pi}{\sigma}\right]\frac{\langle n_-\rangle_\pi^{n_-}}{n_-!}\exp(-\langle n_-\rangle_\pi)+\frac{\sigma_n^D}{\sigma}, \quad (4.7)$$

where n_- means the number of negative charged particles, accordingly

$$\sum_n nP_n=\langle n\rangle=\left[\frac{\sigma^\pi}{\sigma}\right]\langle n\rangle_\pi+\left[\frac{\sigma^D}{\sigma}\right]\langle n\rangle_D, \quad (4.8)$$

where $\sigma^D=\sum_n\sigma_n^D$, $\sigma=\sigma^\pi+\sigma^D$, and $\langle n\rangle_\pi$ ($\langle n\rangle_D$) denote the mean charged-particle number of the pionization (diffraction) component. At 300 GeV/c the data of pp interaction[20] provide $\langle n\rangle\simeq 8.8$, $\langle n\rangle_D\simeq 4.5$, and $\langle n_-\rangle_\pi\simeq 4.9$, since $\sigma^D\simeq 7.6$ mb and $\sigma\simeq 39$ mb.

The TCM for $\langle p_L\rangle_n$ indicates that the diffraction component of $\langle p_L\rangle_n$ comes from the approximate energy-conservation sum rule and the independent-production mechanism of $\langle p_L\rangle_n$ gives rise to the isotropic n-independent component $\langle p_L\rangle_n\simeq\langle p_T\rangle$ in the central region. Postulating charge symmetry for secondary pions and assuming the energy carried away by the incident particles to be 40% of $\sqrt{s}$, we get

$$\langle p_L\rangle_n=\left[\frac{\sigma^D}{\sigma}\right]\left[\frac{2}{3}\right]\frac{0.6\sqrt{s}}{n}+\langle p_T\rangle \simeq 0.08\frac{\sqrt{s}}{n}+0.35 \quad (4.9)$$

(in GeV units), which is consistent with Eq. (4.1).

We obtain from Eqs. (2.9) and (4.9)

$$R_L(z,s)=\left[1+4.4\frac{\langle n\rangle}{\sqrt{s}}\right]\left[z^{-1}+4.4\frac{\langle n\rangle}{\sqrt{s}}\right]^{-1}. \quad (4.10)$$

Equations (4.5), (4.7), and (4.10) lead to the p_L distribution function in terms of x_L:

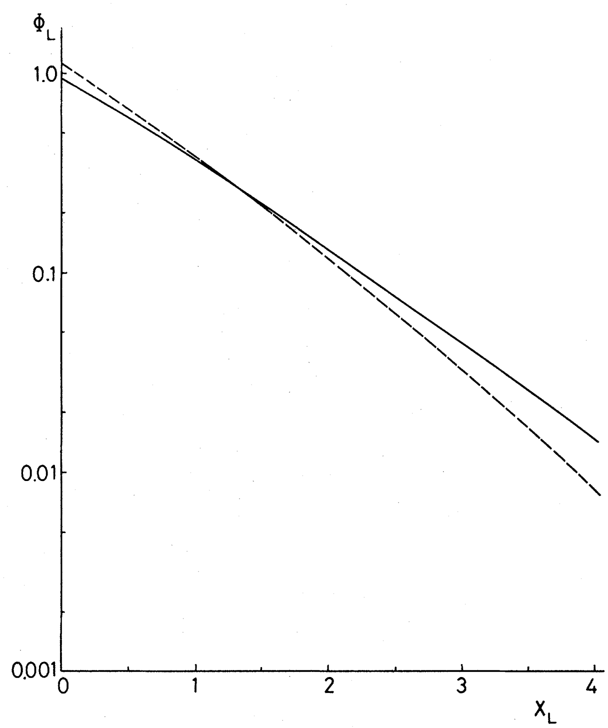

FIG. 3. Plot of $\Phi_L=(\langle P_L\rangle d\sigma/dp_L)/(\langle n\rangle\sigma)$ vs $x_L=p_L/\langle p_L\rangle$. The solid curve represents our calculation with use of the TCM (4.11) at 300 GeV/c. The dashed curve is the prediction of Eq. (4.11) for $s\to\infty$, i.e., the fit of Eq. (4.13) with $\gamma=0.8$.

$$\Phi_L(x_L,s)=\frac{2}{\langle n\rangle}\sum_{n_-=1}^{\infty}(n_-+1)\left[\left[\frac{\sigma^\pi}{\sigma}\right]\frac{\langle n_-\rangle_\pi^{n_-}}{n_-!}\exp(-\langle n_-\rangle_\pi)+\frac{\sigma_n^D}{\sigma}\right]R_L\phi_L(R_Lx_L). \quad (4.11)$$

We will present the result of Eq. (4.11) at 300 GeV/c in Fig. 3. In the infinite-energy region

$$\langle n \rangle P_n \rightarrow \gamma \delta \left[z - \frac{1}{\gamma} \right] + (1-\gamma)\delta(z) , \quad (4.12)$$

where $\gamma = 1 - (\sigma^D/\sigma) \simeq 0.8$. Consequently, we arrive at

$$\Phi_L(x_L) = \left[\frac{1}{\gamma} \right] \phi_L \left[\frac{x_L}{\gamma} \right] \quad (4.13)$$

which is plotted in Fig. 3. These are different from the results evaluated by means of the KNO scaling function.

It turns out from Eq. (4.13) that the independent emission model with $\gamma = 1$ yields

$$\Phi_L(x_L) = \phi_L(x_L) . \quad (4.14)$$

This model suggests that there exists the same scaling function both for inclusive and semi-inclusive reactions in the infinite-energy region.

V. SUMMARY AND DISCUSSION

We have presented a way to derive the inclusive momentum distribution from the semi-inclusive distribution by means of the SIM hypothesis. It should be noted that we can make a discrimination among the different models for the p_T distribution, with the help of definite determination of $R_T = \langle p_T \rangle / \langle p_T \rangle_n$. If $R_T = 1$ in wide n and s regions, then one must demonstrate the n-independent production mechanism for the semi-inclusive p_T distribution. If $R_T \neq 1$, we get the different scaling functions, i.e., $\Phi_T(x_T) \neq \phi_T(x_T)$ in inclusive and semi-inclusive reactions. The inclusive distribution function derived from ϕ_T is useful in making a choice of the appropriate model, since the discrepancy between the model calculation and the data can be amplified by summing up with n. However, even the recent data of the CERN SPS $p\bar{p}$ collider at 540 GeV are not enough to determine definitely the dependence of R_T on n and s.

We predict for the p_L distribution that the inclusive SIM function shows the energy dependence, whereas SIM holds well in semi-inclusive reactions. In view of the SIM assumption, the inclusive p_L distribution function at 300 GeV/c and at infinite energy are determined in two distinct ways, i.e., the KNO scaling hypothesis and the TCM. If the fractional contribution of pionization is around 30% of all at $\sqrt{s} = 540$ GeV as proposed by Lim and Phua,[21] we obtain the distribution (4.13) with $\gamma \simeq 0.3$ at this energy. We can expect an apparent difference between the p_L distribution functions predicted by KNO and the TCM. These models concerning the multiplicity distribution may be examined through our results obtained in this article, if we can compare them with the data at the high energy. The experimental data of the inclusive p_L and the p_T distributions in the wide energy regions are eagerly awaited with respect to the SIM hypothesis and the choice of a suitable model.

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