# Glueballs in $\pi^- p \to \phi\phi n$

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We propose a model which is able to explain the main features of the experimental data for the reaction $\pi^- p \to \phi\phi n$, starting from the assumption that two glueball resonances with $J^{PC}=2^{++}$ are produced in this reaction. The couplings of these glueball candidates to $\phi\phi$ are estimated, and come out to be of the same strength as ordinary hadronic couplings.

## I. INTRODUCTION

Quantum chromodynamics has introduced in the hadronic spectroscopy the possibility of a new type of hadron, called glueballs.[1] This is an exciting subject in the development of hadronic phenomenology. There are many reviews[2] about experimental candidates, theoretical models for the production, masses, widths, and quantum numbers of these subjects. While we do not have any quantum-field-theoretical proof of the existence of glueballs (nor of hadrons in QCD), some authors[3] have shown the nonexistence of glueballs at the classical level. These theorems[3] obviously do not forbid the existence of quantum glueballs. For us, these objects formed only by gluons must be treated as ordinary hadrons, if QCD is correct. Our main aim in this paper is to give a parametrization for a particular reaction, i.e., $\pi^- p \to \phi\phi n$, that can be used as a constraint for a microscopic model of gluon interactions producing glueballs. The choice of this reaction is related to the violation of the suppression due to the Okubo-Zweig-Iizuka (OZI) rule,[4] which forbids $(q\bar{q})$ states as possible candidates to explain the experimental results.[5] We agree with some authors[6] that the OZI-rule violation is a good condition to search for possible glueball states.

Our phenomenological analysis is completely based on the main features of the experimental results,[5] namely, peripherality and the partial-wave enhancements indicating the existence of two $J^{PC}=2^{++}$ objects, one dominantly in $S$ wave and the other dominantly in $D$ wave. Our amplitude is easily constructed taking into account a production mechanism, described by a single Regge parametrization, times a decay process and two hadronic $\phi\phi$ resonance propagators. These points are shown in detail in Sec. II. In Sec. III we present the results of our model in comparison with experimental results, and end with some conclusions.

## II. THE MODEL

The peripherality of the reaction (i.e., a great number of events for small squared momentum transfer $t_2$) and the $t_2$-channel quantum numbers suggest pion exchange (see Fig. 1).

The violation of the OZI rule hints at a glueball resonance $G_i$, as discussed in the Introduction, with $I^G J^{PC}=0^+2^{++}$ and well-defined mass $M_i$ and width $\Gamma_i$.[5] We assume that the glueball, with these quantum numbers, can be treated as a hadron, having ordinary couplings with other hadrons.

Looking at the quark diagram for the reaction $\pi^- p \to \phi\phi n$, shown in Fig. 2, we see that this reaction is OZI-rule-forbidden, but not suppressed as shown by the experimental results.[5] The violation of the OZI suppression can also be seen by the ratio[7]

$$\frac{\sigma(K^- p \to \phi K^+ K^- \Lambda)}{\sigma(K^- p \to \phi\phi\Lambda)} \simeq \frac{\sigma(\pi^- p \to \phi K^+ K^- n)}{\sigma(\pi^- p \to \phi\phi n)} \sim 5 ,$$

where all the reactions, except $\pi^- p \to \phi\phi n$, are OZI-rule-allowed, although the two ratios are the same.

Within the framework of QCD, if quarks interact with other quarks via gluons, we can expect the existence of an amplitude of the type $q_1\bar{q}_1 \to \text{gluons} \to q_2\bar{q}_2$. But the interaction among gluons in QCD can produce the new states called glueballs. The violation of the OZI rule can then be understood qualitatively by the formation of glueballs with a strong effective coupling constant to other hadrons. This fact supports our hypothesis that glueballs couple ordinarily to other hadrons, and therefore the coupling constants $g_{G_j\phi\phi}$ must be comparable to other hadronic coupling constants.

The global amplitude representing our model is given by the expression

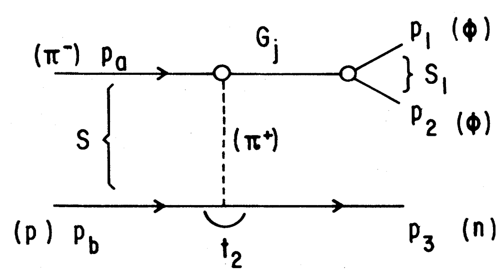


FIG. 1. Diagram representing the reaction $\pi^- p \to \phi\phi n$ with a $\pi$ exchange and a glueball or a $(\phi\phi)$ resonance in the $s_1$ channel. $s=(p_a+p_b)^2$, $s_1=(p_1+p_2)^2$, and $t_2=(p_b-p_3)^2$.

 

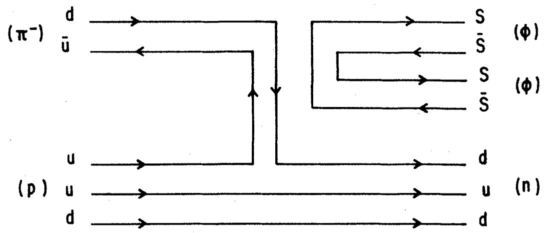


FIG. 2. Quark diagram for the reaction $\pi^- p \to \phi\phi n$.

$$A=\sum_{j=1}^{2} R(\pi^- p \to G_j n)\Phi_j(s_1)T(G_j \to \phi\phi)\ , \qquad (1)$$

where $R$ represents the production amplitude,

$$\Phi_j(s_1)=(s_1-M_j{}^2+iM_j\Gamma_j)^{-1}$$

is the resonance propagator, and $T$ represents the decay amplitude for $G_j \to \phi\phi$.

The characteristics of the subreaction $\pi^- p \to G_j n$ discussed above permit us to treat it as a high-energy $2\to 2$ reaction, well described by a standard $\pi$-exchange Reggeized amplitude:

$$R(\pi^- p \to G_j n)=g_{\pi^+pn}g_{G_j\pi^+\pi^-}\{1+\xi\exp[-i\pi\alpha_\pi(t_2)]\} \times \left[\frac{s}{s_0}\right]^{\alpha_\pi(t_2)}\Gamma(-\alpha_\pi(t_2)) \qquad (2)$$

where $(g_{\pi^+pn})^2/4\pi=14.5$, $\alpha_\pi(t_2)=0.72(t_2-m_\pi{}^2)$, $s_0=1$ GeV$^2$, and $\xi=+1$. To avoid nonessential complications we take into account the spin only in the decay amplitude.

In order to construct the decay amplitude $T(G_j \to \phi\phi)$ taking into account the spin-parity ($J^P$) of the involved particles, we have used the helicity formalism. This amplitude is given by

$$T^M_{\lambda_1\lambda_2}=\epsilon_\mu^{*\lambda_1}(\theta,\phi,p_1)\epsilon_\nu^{*\lambda_2}(\theta,\phi,p_2)C^{\alpha\beta\mu\nu}\epsilon^M_{\alpha\beta}(\vec{p}_{12}=0)\ , \qquad (3)$$

where $\lambda_1,\lambda_2$ are the helicities of particles 1 and 2 ($\phi\phi$), and $\epsilon_{\mu(\nu)}^{\lambda_1(\lambda_2)}$ and $\epsilon^M_{\alpha\beta}$ are the spin-one and spin-two wave functions, respectively, defined by[8]

$$\epsilon^{*\pm}(\theta,\phi=0,p)=\epsilon^{*\pm}(p_i) =\frac{1}{\sqrt{2}}(0;\mp\cos\theta,i,\pm\sin\theta)\ , \qquad (4)$$

$$\epsilon^0(p_i)=\frac{1}{m_i}(p_i;E_i\sin\theta,0,E_i\cos\theta)\ , \qquad (5)$$

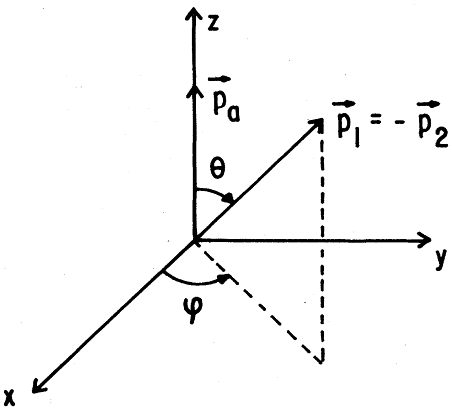


FIG. 3. The Gottfried-Jackson frame used in our calculation.

where $i=1,2$, $m_1=m_2=m_\phi$, and we have used the phase convention of Ref. 9. Using the fact that only $M=0$ states were observed,[5] we define the tensor $\epsilon^{M=0}_{\alpha\beta}$ by[8]

$$\epsilon^0_{\alpha\beta}(\vec{p}_{12}=0)=\frac{1}{\sqrt{6}}[\epsilon^+_\alpha(\vec{p}_{12}=0)\epsilon^-_\beta(\vec{p}_{12}=0) +\epsilon^-_\alpha(\vec{p}_{12}=0)\epsilon^+_\beta(\vec{p}_{12}=0) +2\epsilon^0_\alpha(\vec{p}_{12}=0)\epsilon^0_\beta(\vec{p}_{12}=0)]\ . \qquad (6)$$

For the vertex ($2^+\to 1^- + 1^-$), the most general vertex function $C_{\alpha\beta\mu\nu}$ is given by the expression[10]

$$C_{\alpha\beta\mu\nu}=g_1g_{\alpha\mu}g_{\beta\nu}+g_2(g_{\alpha\mu}\Lambda_\nu+g_{\alpha\nu}\Lambda_\mu)\Lambda_\beta +(g_3g_{\mu\nu}+g_4\Lambda_\mu\Lambda_\nu)\Lambda_\alpha\Lambda_\beta\ . \qquad (7)$$

Here we have used the normality of the $\phi\phi$ state, $N_{\phi\phi}=+1$, the fact that we have two identical particles, and $\Lambda_\alpha=\frac{1}{2}(p_1-p_2)_\alpha$.

Thus, the decay amplitude depends upon four constants multiplying the different couplings of the vertex function $C_{\alpha\beta\mu\nu}$. We will see below how we can use some experimental constraints to determine the coupling constants $g_{G_T'\phi\phi}$ and $g_{G_T'\phi\phi}$. First, we will relate the different $g_i$ ($i=1,2,3,4$), and henceforth all calculations will be made in the Gottfried-Jackson (GJ) frame (as is shown in Fig. 3) with $\phi=0$.

Inserting (6) and (7) and the Lorentz condition $\epsilon_\mu(p)p^\mu=0$ into (3), we find

$$\begin{aligned}T^{M=0}_{\lambda_1\lambda_2}=\frac{1}{\sqrt{6}}(&g_1[(\epsilon^+\cdot\epsilon^*_{\lambda_1})(\epsilon^-\cdot\epsilon^*_{\lambda_2})+(\epsilon^-\cdot\epsilon^*_{\lambda_1})(\epsilon^+\cdot\epsilon^*_{\lambda_2})+2(\epsilon^0\cdot\epsilon^*_{\lambda_1})(\epsilon^0\cdot\epsilon^*_{\lambda_2})]\\&+g_2\{[(\epsilon^+\cdot\epsilon^*_{\lambda_1})(\epsilon^-\cdot\Lambda)+(\epsilon^-\cdot\epsilon^*_{\lambda_1})(\epsilon^+\cdot\Lambda)+2(\epsilon^0\cdot\epsilon^*_{\lambda_1})(\epsilon^0\cdot\Lambda)]\tfrac{1}{2}(\epsilon^*_{\lambda_2}\cdot p_1)\\&-[(\epsilon^+\cdot\epsilon^*_{\lambda_2})(\epsilon^-\cdot\Lambda)+(\epsilon^-\cdot\epsilon^*_{\lambda_2})(\epsilon^+\cdot\Lambda)+2(\epsilon^0\cdot\epsilon^*_{\lambda_2})(\epsilon^0\cdot\Lambda)]\tfrac{1}{2}(\epsilon^*_{\lambda_1}\cdot p_2)\}\\&+[(\epsilon^+\cdot\Lambda)(\epsilon^-\cdot\Lambda)+(\epsilon^0\cdot\Lambda)^2][2g_3(\epsilon^*_{\lambda_1}\cdot\epsilon^*_{\lambda_2})-\tfrac{1}{2}g_4(\epsilon^*_{\lambda_1}\cdot p_2)(\epsilon^*_{\lambda_2}\cdot p_1)])\ .\end{aligned} \qquad (8)$$

The four independent amplitudes are obtained in a straightforward calculation of the above scalar products. They are (omitting the label $M=0$ from now on)

$$T_{+-}=\sqrt{\tfrac{3}{8}}g_1\sin^2\theta\ , \tag{9}$$

$$T_{++}=\frac{1}{\sqrt{6}}(g_1+2g_3\,|\,\vec{p}_1\,|^2)\mathscr{P}_2(\cos\theta)\ , \tag{10}$$

$$T_{+0}=\frac{\sqrt{3}}{2}\frac{E_1}{m_\phi}(-g_1+g_2\,|\,\vec{p}_1\,|^2)\sin\theta\cos\theta\ , \tag{11}$$

$$T_{00}=\frac{1}{\sqrt{6}}\frac{1}{{m_\phi}^2}[2E_1{}^2(-g_1+2g_2\,|\,\vec{p}_1\,|^2) +2g_3(\,|\,\vec{p}_1\,|^4+E_1{}^2\,|\,\vec{p}_1\,|^2) -2g_4E_1{}^2\,|\,\vec{p}_1\,|^4]\mathscr{P}_2(\cos\theta)\ , \tag{12}$$

where $\mathscr{P}_2(\cos\theta)=(3\cos^2\theta-1)/2$.

We can now obtain the partial-wave amplitudes using the well-known[8] formulas

$$T_{L\mathscr{s}}^{JM}=\sum_{\lambda_1\lambda_2}\left[\frac{2L+1}{4\pi}\right]^{1/2}C^{\mathscr{s}_1\mathscr{s}_2\mathscr{s}}_{\lambda_1-\lambda_2\lambda}C^{L\mathscr{s}J}_{0\lambda\lambda} \times 2\pi\int_{-1}^{+1}d^J_{M\lambda}(\theta)T^M_{\lambda_1\lambda_2}(\theta)d(\cos\theta)\ . \tag{13}$$

Inserting (9)–(12) into (13) and taking into account the symmetrization of these amplitudes

$$\widetilde{T}_{L\mathscr{s}}^{JM}=[1+(-1)^{L+\mathscr{s}}]T_{L\mathscr{s}}^{JM}\ ,$$

we find

$$\widetilde{T}\,^{20}_{20}(s_1)=\beta_{20}(a_{20}g_1+b_{20}g_2+c_{20}g_3+d_{20}g_4)\ , \tag{14}$$

$$\widetilde{T}\,^{20}_{02}(s_1)=\beta_{02}(a_{02}g_1+b_{02}g_2+c_{02}g_3+d_{02}g_4)\ , \tag{15}$$

$$\widetilde{T}\,^{20}_{22}(s_1)=\beta_{22}(a_{22}g_1+b_{22}g_2+c_{22}g_3+d_{22}g_4)\ , \tag{16}$$

$$\widetilde{T}\,^{20}_{42}(s_1)=\beta_{42}(a_{42}g_1+b_{42}g_2+c_{42}g_3+d_{42}g_4)\ . \tag{17}$$

The $a_{L\mathscr{s}}$, $b_{L\mathscr{s}}$, $c_{L\mathscr{s}}$, $d_{L\mathscr{s}}$, and $\beta_{L\mathscr{s}}$ are given in Table I, in terms of the invariants.

Using now the experimental results,[5] i.e., only the waves $J^PL\mathscr{s}M=2^+020$ and $2^+220$ give significant contributions to the states $G_T$ and $G_T'$, respectively, we can obtain constraints between the $g$'$\mathscr{s}$. We choose to relate $g_2$, $g_3$, and $g_4$ to $g_1$ setting $\widetilde{T}_{20}=\widetilde{T}_{22}=\widetilde{T}_{42}=0$ for the $G_T$ state, and $\widetilde{T}_{02}=\widetilde{T}_{20}=\widetilde{T}_{42}=0$ for the $G_T'$ state.

The next step consists in replacing the obtained values for $g_2$, $g_3$, and $g_4$ into expression (18) to calculate $g_1$:

$$M\Gamma=\frac{|\,\vec{p}_1\,|}{8\pi\sqrt{s_1}}\sum_{L,\mathscr{s}}\frac{|\,T_{L\mathscr{s}}^{JM}\,|^2}{4\pi}M_N{}^2\ . \tag{18}$$

In expression (18) $M_N{}^2$ (squared nucleon mass) has been introduced so that $|\,T_{L\mathscr{s}}^{JM}\,|^2$ becomes dimensionless.[11] We have, for each state $G_j$,

$$\Gamma_{G_T}=\frac{M_N{}^2}{16\pi}\left[\frac{x_1}{s_1}\right]^{1/2}\left[\frac{(g_{G_T\phi\phi})^2}{4\pi}\right] \times\beta_{02}{}^2(a_{02}+b_{02}\gamma_2+c_{02}\gamma_3+d_{02}\gamma_4)^2 \tag{19a}$$

and

$$\Gamma_{G_T'}=\frac{M_N{}^2}{16\pi}\left[\frac{x_1}{s_1}\right]^{1/2}\left[\frac{(g_{G_T'\phi\phi})^2}{4\pi}\right] \times\beta_{22}{}^2(a_{22}+b_{22}\gamma_2'+c_{22}\gamma_3'+d_{22}\gamma_4')^2\ , \tag{19b}$$

where $x_1=s_1-4m_\phi{}^2$ and

$$\gamma_2=\frac{4}{x_1}\left[\frac{6m_\phi{}^3-\sqrt{s_1}(s_1+4m_\phi{}^2)}{\sqrt{s_1}(-s_1+4m_\phi\sqrt{s_1}-m_\phi{}^2)}\right]\ , \tag{20}$$

$$\gamma_3=\frac{2}{x_1}\left[\frac{x_1+4m_\phi\sqrt{s_1}}{4m_\phi\sqrt{s_1}-s_1-m_\phi{}^2}\right]\ , \tag{21}$$

$$\gamma_4=\frac{112}{x_1{}^2}\left[\frac{m_\phi{}^2}{s_1}\right]\left[\frac{x_1+4m_\phi\sqrt{s_1}}{s_1-4m_\phi\sqrt{s_1}+m_\phi{}^2}\right]\ , \tag{22}$$

$$\gamma_2'=\frac{4}{x_1}\left[\frac{9m_\phi{}^3+\sqrt{s_1}(s_1-4m_\phi\sqrt{s_1}-m_\phi{}^2)}{\sqrt{s_1}(s_1-2m_\phi\sqrt{s_1}-m_\phi{}^2)}\right]\ , \tag{23}$$

$$\gamma_3'=\frac{2}{x_1}\left[\frac{6m_\phi\sqrt{s_1}-s_1-6m_\phi{}^2}{s_1-2m_\phi\sqrt{s_1}-m_\phi{}^2}\right]\ , \tag{24}$$

TABLE I. Value of the coefficients of Eqs. (14)–(17) in terms of the invariants, where $x_1\equiv s_1-4m_\phi{}^2$.

| $L$ $\mathscr{s}$ | $a_{L\mathscr{s}}$ | $b_{L\mathscr{s}}$ | $c_{L\mathscr{s}}$ | $d_{L\mathscr{s}}$ | $\beta_{L\mathscr{s}}$ |
|---|---|---|---|---|---|
| 2 0 | $s_1+4m_\phi{}^2$ | $-s_1x_1/2$ | $x_1(6m_\phi{}^2-s_1)$ | $s_1x_1{}^2/16$ | $\frac{2}{3m_\phi{}^2}\left[\frac{\pi}{10}\right]^{1/2}$ |
| 0 2 | $2(14m_\phi{}^2-s_1-6m_\phi{}^2\sqrt{s_1})$ | $(s_1+3m_\phi\sqrt{s_1})x_1$ | $s_1x_1$ | $-s_1x_1{}^2/8$ | $\frac{1}{15m_\phi{}^2}\sqrt{\pi}$ |
| 2 2 | $2(10m_\phi{}^2+s_1+3m_\phi\sqrt{s_1})$ | $-x_1(2s_1+3m_\phi\sqrt{s_1})/2$ | $-s_1x_1$ | $s_1x_1{}^2/8$ | $\frac{2}{3m_\phi{}^2}\left[\frac{\pi}{70}\right]^{1/2}$ |
| 4 2 | $4m_\phi\sqrt{s_1}-x_1$ | $x_1(s_1-2m_\phi\sqrt{s_1})/2$ | $s_1x_1/2$ | $s_1x_1{}^2/16$ | $\frac{4}{5m_\phi{}^2}\left[\frac{\pi}{14}\right]^{1/2}$ |

$$\gamma'_4 = \frac{80}{x_1^2}\left[\frac{m_\phi^2}{s_1}\right]\left[\frac{8m_\phi^2 - s_1 - 2m_\phi\sqrt{s_1}}{s_1 - 2m_\phi\sqrt{s_1} - m_\phi^2}\right] . \quad (25)$$

Introducing the branching ratio $\eta_i = \Gamma_i/\Gamma_{\text{total}}$ for each channel $i$ considered in the decay process, and inserting into (18) and (19) the values of the parameters: $M_{G_T} = 2.18$ GeV, $M_{G'_T} = 2.35$ GeV, $\Gamma_{G_T} = 0.28$ GeV, and $\Gamma_{G'_T} = 0.32$ GeV, we obtain

$$g_{G_T\phi\phi} = 13.7(\eta_{G_T\to\phi\phi})^{1/2} , \quad (26)$$

$$g_{G'_T\phi\phi} = 2.8(\eta_{G'_T\to\phi\phi})^{1/2} . \quad (27)$$

We do not have experimental values for $\eta_{G_T(G_T')\to\phi\phi}$ however, using some naive arguments to be discussed below, we obtain what we think is a reasonable bound for $\eta_{G_T(G'_T)}$.

The number of possible channels for a decay of $J^{PC} = 2^{++}$ objects, taking into account that the glueball is a flavor singlet, is considerable. If the flavor independence is confirmed, we can expect at least $G_j$ decays into other pairs of nonstrange vector mesons of the same $SU_F(3)$ nonet $(\rho\rho, \omega\omega)$. Then neglecting the different factors coming from phase space, we get $\eta_{G_T(G'_T)} \leq \frac{1}{3}$. Consequently, from (26) and (27)

$$g_{G_T\phi\phi} \lesssim 7.9 , \quad (28)$$

$$g_{G'_T\phi\phi} \lesssim 1.6 , \quad (29)$$

and we can see that these values are comparable to ordinary hadronic coupling constants.

On the other hand, if by some as-yet-unknown dynamical reason the $\phi\phi$ channel is favored $[\eta_{G_T(G'_T)\to\phi\phi} \simeq 1]$ we can understand this apparent violation of the flavor independence as indicating that glueball production in processes described by an allowed diagram is strongly suppressed (even if it has the right quantum numbers) with respect to the production of such states in processes described by forbidden diagrams which are experimentally seen not to be suppressed.

We believe that this point is very important to the study of the glueball phenomenology. A possible partial confirmation of this statement is the recent study of the reaction $\pi^- p \to \pi^+\pi^- n$,[12] indicating that the $M_{\pi^+\pi^-}$ spectrum does not show any structure in the glueball mass range.

Finally our amplitude is obtained by replacing (2), (15), and (16) in (1),

$$|A|^2 = \frac{1}{4\pi} |R(\pi^- p \to Gn)|^2$$
$$\times [\, |\tilde{T}^{20}_{02}(G_T \to \phi\phi)\Phi_{G_T}|^2$$
$$+ r^2 |\tilde{T}^{20}_{22}(G'_T \to \phi\phi)\Phi_{G'_T}|^2 ] , \quad (30)$$

where $r \equiv g_{G'_T\pi\pi}/g_{G_T\pi\pi}$.

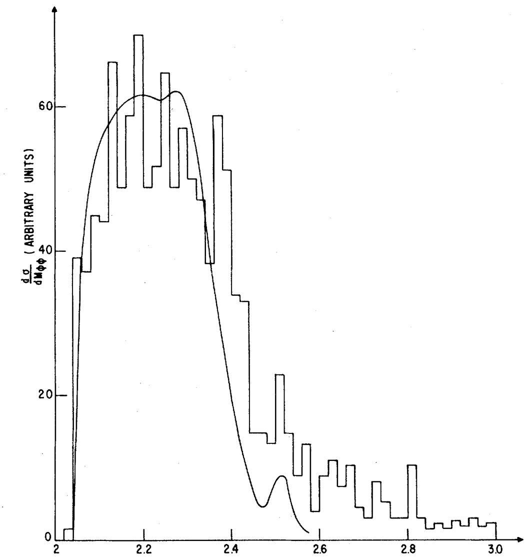


FIG. 4. Total-invariant-mass distribution obtained from Eq. (31), as described in the text, in comparison with the data from Ref. 5(a) and 5(b).

## III. RESULTS AND CONCLUSIONS

To obtain our results we have used expression (30) in the differential cross sections defined by

$$\frac{d\sigma}{dM_{\phi\phi}dt_2} = 2M_{\phi\phi}[2^{10}\pi^4\lambda(s, m_\pi^2, m_p^2)]^{-1}$$
$$\times \frac{\lambda^{1/2}(s_1, m_\phi^2, m_\phi^2)}{s_1} |A|^2 , \quad (31)$$

where the terms in front of $|A|^2$ come from phase-space and flux factors, and

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) .$$

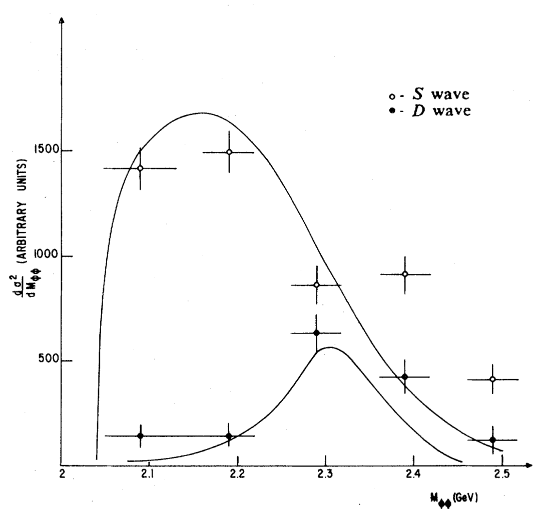


FIG. 5. $M_{\phi\phi}$ distributions for $S$ wave and $D$ wave, respectively, in comparison with the data from Ref. 5(a) and 5(b).

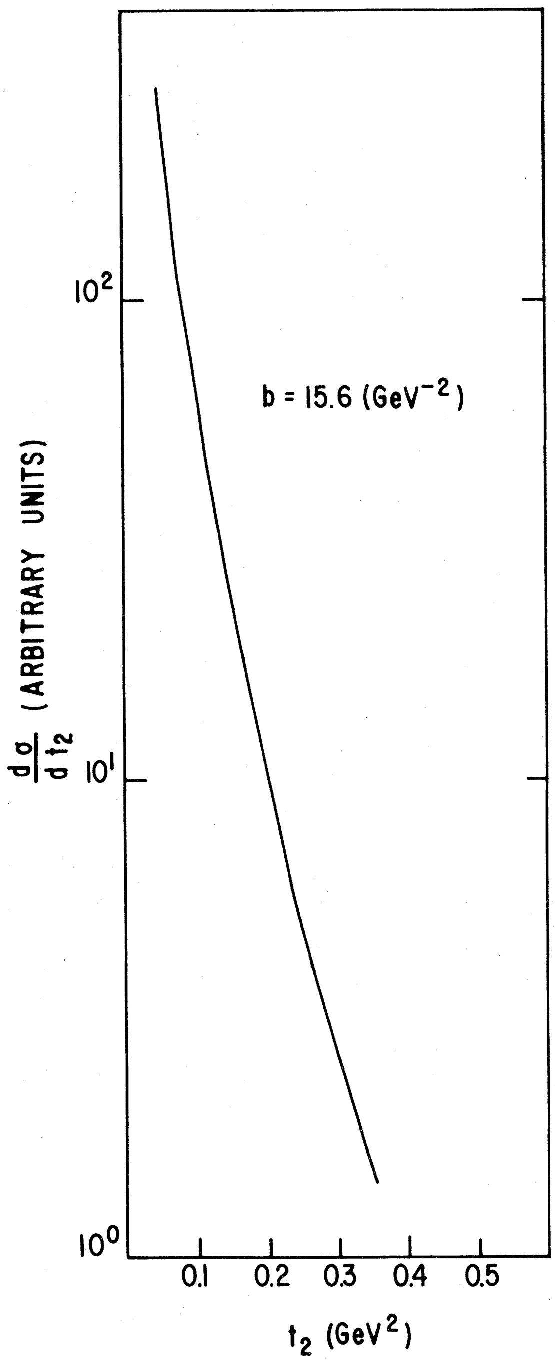


FIG. 6. $d\sigma/dt_2$ distribution obtained from Eq. (31), as described in the text.

In order to give the curves presented in Figs. 4 to 6, we have integrated (31) in the limits given below. As we have discussed in Sec. II, we set

$$\eta_{G_T \to \phi\phi} = \eta_{G'_T \to \phi\phi}, \quad r = g_{G_T'\pi\pi}/g_{G_T\pi\pi} = 0.6$$

and all masses and widths as given in Sec. II.

*$d\sigma/dM_{\phi\phi}$, total-invariant-mass ($M_{\phi\phi}$) distribution.* Figure 4 shows this distribution obtained from expression (31), integrated in $t_2$ between the limits $-1 \le t_2 \le 0$. We remark that the region of the two states $G_T$ and $G'_T$ is well described by the model, while for masses $M_{\phi\phi} \gtrsim 2.4$ GeV the comparison is not so good. The model with two hadronic resonances near the threshold is not able to saturate the spectrum at high masses. The little enhancement observed between 2.4 and 2.6 GeV comes from spin factors in $\gamma'_i(s_1)$.

*$d\sigma^L/dM_{\phi\phi}$, partial-wave distributions for $M_{\phi\phi}$.* Figure 5 presents the distributions in invariant mass $M_{\phi\phi}$ for each partial wave given by the experimental results of Refs. 5(a) and 5(b), obtained from (31) with the same set of parameters used to obtain Fig. 4. We remark that the $S$ wave presents the same shortcomings observed in Fig. 4. Perhaps the shortcomings, in both cases, may be caused by the fact that we have neglected the $S$ wave for the $G'_T$ state.

*$d\sigma/dt_2$, squared-momentum-transfer ($t_2$) distribution.* We present in Fig. 6 the distribution $d\sigma/dt_2$, showing the peripheral character of the studied reaction. The slope obtained by our model is $b = 15.6$ GeV$^{-2}$, calculated for $0.1 \lesssim t_2 \lesssim 0.3$ GeV$^2$ and $2.04 \le M_{\phi\phi} \le 3.0$ GeV, while the experimental results give $b = 9.4 \pm 0.7$ GeV$^{-2}$ from Refs. 5(a) and 5(b), and $b = 12 \pm 2$ GeV$^{-2}$ from Ref. 5(c). It is clear that if we slightly vary the value of $\alpha'$ we can obtain a slope closer to the experimental values, but it is not our aim to present a perfect fit to the experimental results.

Our curves presented in Fig. 4—6 do not have absolute normalization. If we had more accurate values for cross sections and branching ratios we could predict the value of the coupling constants $g_{G_T(G'_T)\pi\pi}$.

Let us now make a few general comments. The good agreement with the experimental data shows that our aim in this paper has been attained. We think that our amplitude can be used by the experimentalists for the best determination of the involved parameters. Of course it would be extremely desirable to have other channels, such as $\omega\omega$ and $\rho\rho$ observed, in order to clarify the question about branching ratios and the flavorless assumption for glueball decay. Among the candidates (see reviews in Ref. 2) for glueball states, we believe, in agreement with the authors of Refs. 5(a) and 5(b), that the reaction $\pi^- p \to \phi\phi n$ with $\phi\phi$ states is a good place to search for these objects because of the violation of the OZI suppression. Other experiments also show some structures in the $M_{\phi\phi}$ spectrum in $\phi\phi$ inclusive production in $\pi$Be and $pp$ interactions.[13]

We once again stress the fact that we have started the construction of our model, taking into account the peripheral nature of the data being in disagreement with other authors[14] who use a central mechanism for studying this process. However, an interesting question is the possibility of centrally producing these objects,[15,2(i)] since this could throw light on the coupling of a Pomeron to a glueball, thus making possible a test of the old conjecture of a glueball-Pomeron identity.[16] This brings to mind the related question of which place a glueball would occupy in the standard Regge phenomenology. As the glueball is as good a hadron as any other quark-made hadron, we may ask, where is the glueball Regge trajectory? The mechanism related to this problem, i.e., the double-Pomeron exchange (as $\gamma\gamma$ interactions) also permits the important test of a flavorless assumption for glueball decay. We call attention to the fact that this subject is in a certain sense related to our proposition that establishes a glueball suppression in allowed diagrams as discussed in the end of Sec. II.

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