Nonstrange hyperfine splitting in the relativistic harmonic-oscillator quark model

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A mass formula relating the Δ-N and ρ-π hyperfine splittings is derived within the context of the relativistic harmonic-oscillator quark model. The spring constant, determined from the Regge trajectories, adjusts the wave-function overlap integrals so that reasonable agreement with experiment is obtained. The formula involves the square of the hadron masses and clearly indicates factors due to color, SU(6), and internal quark motion.

The relativistic harmonic-oscillator quark model[1] provides a unified calculational framework for the hadrons. The spring constant is flavor independent and taken to be $\omega = 1.05\ \text{GeV}^2$ so that the Regge trajectories are produced for both mesons and baryons. Various forms of the model have been used to investigate nucleon electromagnetic form factors,[2] $-g_A/g_V$,[3] meson decays,[4] and several other important hadronic processes.[5]

The quantum-chromodynamic picture, described by De Rújula, Georgi, and Glashow,[6] has considerably enriched the quark model. This additional feature adds short-range forces dominated by one-gluon exchange, which provides for a rich hadron spectroscopy. Kim[7] has proposed a model for a relativistic bound-state perturbation theory which employs relativistic harmonic-oscillator wave functions. This model produces a fully relativistic Fermi-Breit formula without making the usual static nonrelativistic approximation. The purpose of this note is to use the prescription of Ref. 7 to derive a formula that directly relates the Δ-N and ρ-π hyperfine splittings by calculating wave-function overlap integrals.

This note does not intend to illustrate that a one-gluon-exchange mechanism with harmonic-oscillator wave functions can produce a spectroscopy that fits well with experimental data. This task has been accomplished in a series of papers on baryons by Isgur and Karl,[8] who use a nonrelativistic harmonic-oscillator model. Meson masses have also been treated with oscillators.[9] Also, other confining potentials have been shown to work well.[10] However, a common feature of these spectroscopic models is the independent fitting for mesons and baryons. This note demonstrates that the unified picture of hadrons provided by the relativistic oscillator formalism[1,7] is capable of expressing the meson hyperfine splitting in terms of the baryon hyperfine splitting.

The reader can refer to Ref. 7 for a detailed description of the Kim model. Briefly, the matrix element due to the exchange of a massless gluon between two quarks (labeled 1 and 2) in a baryon is given by the real part of

$$M_{fi} = \left[\bar{\chi}_1\left(\frac{\gamma\cdot p_1' + m_1}{2m_1}\right)\gamma^\mu\left(\frac{\gamma\cdot p_1 + m_1}{2m_1}\right)\chi_1\right]\frac{g_{\mu\nu}}{q^2+i\epsilon}\left[\bar{\chi}_2\left(\frac{\gamma\cdot p_2' + m_2}{2m_2}\right)\gamma^\nu\left(\frac{\gamma\cdot p_2 + m_2}{2m_2}\right)\chi_2\right]\left[\bar{\chi}_3\left(\frac{\gamma\cdot p_3 + m_3}{2m_3}\right)\chi_3\right] , \tag{1}$$

where p_1, p_2, and p_3 are the initial four-momenta of the three quarks; p_1' and p_2' are the four-momenta of the first two quarks after the exchange of a massless gluon with four-momentum $q = p_1' - p_1 = -(p_2' - p_2)$; and χ_i is a static Dirac spinor for the ith quark. Baryon-wave-function overlap integrals are given in Ref. 7, where approximations are introduced to isolate the Fermi-Breit spin-spin term.

The calculation involving (1) has been performed for this note with no approximations; the spin-dependent shift in the square of the nonstrange baryon mass is

$$\delta m_B{}^2 = \tfrac{2}{3}g\frac{(\frac{4}{3}-\frac{5}{27}x)}{(1-\frac{1}{12}x)}\sum_{i<j}\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{m_i m_j} , \tag{2}$$

where $\frac{2}{3}g$ is the strength of the baryon quark-gluon coupling with g having appropriate dimensions; $m_i = m_j = m_q$ is the nonstrange-quark mass; and $x = \omega/u^2$ with $u = m_q + M/3$, where M is the unperturbed baryon mass. The spin-independent contributions are irrelevant since they cancel in forming differences of nonstrange hadron masses.

A similar calculation for the mesons gives

$$\delta m_M{}^2 = \tfrac{4}{3}g\frac{(\frac{4}{3}-\frac{1}{4}x')}{(1-\frac{1}{16}x')}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{m_1 m_2} , \tag{3}$$

where $\frac{4}{3}g$ is the strength of the meson quark-gluon coupling; $m_1 = m_2 = m_q$; and $x' = \omega/v^2$ with $v = m_q + M'/2$, M' being the unperturbed meson mass.

The ratio of the baryon and meson hyperfine splittings is

$$R = \frac{\Delta^2 - N^2}{\rho^2 - \pi^2} = (\tfrac{1}{2})_{\text{color}}[1+\delta(x,x')]_{\text{orbital}}(\tfrac{3}{2})_{\text{SU(6)}} , \tag{4}$$

where

$$\delta(x,x') = \frac{-32x + 72x' - 4xx'}{576 - 48x - 108x' + 9xx'}$$

and the hadron symbols represent the masses of the respective hadrons. A factor of $\frac{1}{2}$ is due to color, the SU(6) factor of $\frac{3}{2}$ is due to mechanical spin, and $\delta(x,x')$ is the correction due to internal quark motion. The orbital factor is determined by M, M', m_q, and ω.

The square of the unperturbed baryon mass is

$$(N^2+\Delta^2)/2=\tfrac{1}{2}[(0.939)^2+(1.232)^2]=1.200\ \text{GeV}^2\ ,$$

therefore, $M=1.10$ GeV. For the mesons,

$$(\pi^2+3\rho^2)/4=[(0.138)^2+3(0.770)^2]/4=0.45\ \text{GeV}^2\ ,$$

which gives $M'=0.67$ GeV. The nonstrange-quark mass is $m_q=0.336$ GeV, determined from the proton magnetic moment[6] in this type of quark model. Since the Regge slopes fix $\omega=1.05\ \text{GeV}^2$, all four input parameters are determined. The theoretical value for the ratio (4) is then $R_{\text{theor}}=0.98$, in reasonable agreement with the experimental value $R_{\text{expt}}=1.1$. If M, M', and g are chosen to fix $\pi(140)$, $\rho(770)$, and $N(940)$, then the theoretical value for Δ is 1200 MeV, in good agreement with the experimental value of 1232 MeV.

In summary, the unified theory of hadrons provided by the relativistic harmonic-oscillator model is successful in relating meson and baryon hyperfine splittings, where the Regge slope parameter ω plays an essential role. Note that many other models employ the usual linear mass formula for baryons. However, the baryons satisfy the well-known Gell-Mann–Okubo mass formula to within 1.5% when the squares of the masses are taken. Some authors[11] contend that a mass-squared formula for baryons is preferred when the decuplet baryons are considered along with the octet baryons. In the relativistic harmonic-oscillator model, mass-squared formulas are obtained for both mesons and baryons in the spirit of a unified picture based on the Regge plots.[1] Finally, the mass-squared shift due to the hyperfine interaction is a natural outcome of the covariance of the perturbation formula.[7]

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