Are $Q^2\bar{Q}^2$ states observable?

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The color-spin structure of the $Q^2\bar{Q}^2$ states suggests that three classes of the $Q^2\bar{Q}^2$ states whose decays are dominated by two vector mesons may have sufficiently narrow widths ($\sim$100–300 MeV) to be detected. The $\rho^0\rho^0$ enhancement in the $\gamma\gamma$ reaction is explained by the 0^{++} and 2^{++} $Q^2\bar{Q}^2$ states and the suppression of $\gamma\gamma\to\rho^+\rho^-$ is understood as due to cancellation between the isoscalar and the isotensor $Q^2\bar{Q}^2$ states. The VV, $V\gamma$, and $\gamma\gamma$ widths of these three classes of the $Q^2\bar{Q}^2$ states are estimated. The production of these $Q^2\bar{Q}^2$ states in J/ψ radiative decays and hadronic collisions is discussed.

I. INTRODUCTION

The hadron spectrum has always been a testing ground for the dynamics and symmetries of the strong interaction. The qualitative features of the $Q\bar{Q}$ meson and Q^3 baryon spectra can be understood in terms of colored confined quarks in the bag[1,2] or potential models.[3–5] The next group in the hierarchy of multiquark hadrons which may form a color singlet is the $Q^2\bar{Q}^2$ type of mesons. The spectroscopy of the s-wave $Q^2\bar{Q}^2$ mesons has been studied in both the MIT bag model[6,7] and the potential model.[8] The salient features of these states are the following: (1) The wave functions of the $Q^2\bar{Q}^2$ states consist of two parts. In one part, the $Q\bar{Q}$ pairs are in the color singlet representation and in another part the $Q\bar{Q}$ pairs are in the color octet representation. (2) Their decays obey the Okubo-Zweig-Iizuka rule. Most of the $Q^2\bar{Q}^2$ states can "fall apart" into two constituent color-singlet $Q\bar{Q}$ mesons, making them too broad to be observed. However, there are some exceptions whose widths are not too broad to be detected as ordinary "mass bumps."[9–11] These are the states whose decays are dominated by two $Q\bar{Q}$ vector mesons.

To exemplify these "observable" $Q^2\bar{Q}^2$ states, we reproduce in Table I the recoupling coefficients[7,12] of those $Q^2\bar{Q}^2$ states with large coupling coefficients to two singlet vector mesons. We adopt Jaffe's[7] notation for the flavor-SU(3) multiplet ($9=\bar{3}\times 3$ denotes that the quark/antiquark pair is in the flavor $\bar{3}/3$ representation and $36=6\times\bar{6}$ denotes that the quark/antiquark pair is in the flavor $6/\bar{6}$ representation). It is learned from Table I that the (0^{++},9*) states prefer to decay via two vector mesons than two pseudoscalar mesons by a factor of $(0.644/0.177)^2\cong 13.2$ (Here, we have assumed that decays from the color-octet $\underline{P}\cdot\underline{P}$ and $\underline{V}\cdot\underline{V}$ channels are suppressed by an order of α_s^2, owing to the fact that they decay with further exchange of at least one gluon.) Their masses as calculated in the MIT bag model[7] are below the threshold of two-vector-meson production. For instance, the calculated $C^0(0^{++},9^*)$ mass is 1450 MeV, which is below the $\rho\rho$ and $\omega\omega$ threshold. Hence, the widths of these (0^{++},9*) states could be narrow. $J^{PC}=2^{++}$ states are the other possible candidates for the observable ones. From Table I we see that, to first order, they decay only to two vector mesons. The fact that their calculated masses are only slightly ($\sim$100 MeV) above the two-vector-meson thresholds makes these 2^{++} states possibly narrow enough to be detectable. The s-wave decay widths of these 0^{++} and 2^{++} states into two vector mesons are calculated:[9]

$$\Gamma_{2^+}(9)=\frac{2}{3}\frac{a^2p}{8\pi}\left[1+\frac{p^2}{3}\left[\frac{1}{m_1^2}+\frac{1}{m_2^2}\right]+\frac{2}{15}\frac{p^4}{m_1^2m_2^2}\right],$$

$$\Gamma_{2^+}(36)=\tfrac{1}{2}\Gamma_{2^+}(9),$$

$$\Gamma_{0^+}(9^*)=(0.644)^2\frac{a^2p}{8\pi}\left[2+\frac{1}{4m_1^2m_2^2}(m_{0^+}{}^2-m_1{}^2-m_2{}^2)^2\right],\tag{1}$$

$$\Gamma_{0^+}(36^*)=(0.743)^2\frac{a^2p}{8\pi}\left[2+\frac{1}{4m_1^2m_2^2}(m_{0^+}{}^2-m_1{}^2-m_2{}^2)^2\right]$$

TABLE I. The recoupling coefficients of the s-wave $Q^2\bar{Q}^2$ states which decay mostly via two vector mesons. P and V are color-singlet pseudoscalar and vector $Q\bar{Q}$ mesons, $\underline{P}$ and $\underline{V}$ are color octets of the same.

Flavor-SU (3) multiplet	J^{PC} (I)	PP	VV	$\underline{P}\cdot\underline{P}$	$\underline{V}\cdot\underline{V}$
9*	0^{++} (0)	−0.177	0.644	0.623	0.407
36*	0^{++} (0,2)	0.041	0.743	−0.646	−0.169
9	2^{++} (0)		$\sqrt{2/3}$		$-1/\sqrt{3}$
36	2^{++} (0,2)		$1/\sqrt{3}$		$\sqrt{2/3}$

with

$$p^2=\frac{1}{4m_{0^+(2^+)}^{\;2}}(m_{0^+(2^+)}^{\;2}+m_1^{\;2}-m_2^{\;2})^2-m_1^{\;2}$$

being the three-momentum squared in the center-of-mass frame. In Eq. (1), m_1 and m_2 are the masses of the two vector mesons and m_{0^+} and m_{2^+} are the masses of the 0^{++} and 2^{++} states, respectively. a is the dimensionless decay amplitude,[9] which is taken to be the same for the 0^{++} and the 2^{++} states, neglecting their mass differences. The $\gamma\gamma$, $V\gamma$, and VV widths of these "narrow" $Q^2\bar{Q}^2$ states are calculated in Sec. II. The cross sections for $\gamma\gamma\to VV$ are given in Sec. III and compared with the recent $\gamma\gamma\to\rho^0\rho^0$, $\rho^+\rho^-$, and $\rho^0\omega$ data. Section IV contains a summary and a discussion of the production of these $Q^2\bar{Q}^2$ states in J/ψ radiative decays and hadronic collisions.

II. $\gamma\gamma$, γV, AND VV WIDTHS

In Refs. 9 and 11 we showed that the $\gamma\gamma\to\rho\rho$ reaction is a preferred process to search for the four-quark states. The $\rho^0\rho^0$ enhancement in this reaction above the threshold[13–15] is interpreted[9,11] as mainly due to the three 2^{++} $Q^2\bar{Q}^2$ states at 1.65 GeV. The results are summarized as follows.

(1) They decay mainly to VV (two vector mesons) through the fall-apart mode. Other decay modes, e.g., PP (two pseudoscalar mesons), are of higher orders and are expected to be small, which agrees with the experimental findings.[13]

(2) The calculated[9,11] $\gamma\gamma\to Q^2\bar{Q}^2\to VV$ cross sections at the $Q^2\bar{Q}^2$ masses depend only on the flavor and color-spin structures of the $Q^2\bar{Q}^2$ and the vector-dominance-model (VDM) constants. They do not depend on the detailed mechanism, e.g., the parameter a. It turns out that the theoretical cross section $\sigma(\gamma\gamma\to\rho^0\rho^0)_{m_{2^+}}=94$ nb (Refs. 9 and 11) agrees fairly well with the experimental cross sections at ~ 1.65 GeV which is ~ 100 nb (see Fig. 1).

(3) The large suppression of the $\gamma\gamma\to\rho^+\rho^-$ cross sections found[16] in the mass region 1.3–2.0 GeV has been predicted[9,17] and interpreted[9,11] as due to the interference between two isoscalar 2^{++} $Q^2\bar{Q}^2$ states and an isotensor 2^{++} $Q^2\bar{Q}^2$ states. The presence of an exotic isotensor structure is taken to be the strong evidence for the existence of the $Q^2\bar{Q}^2$ mesons.

(4) The decay widths of these $Q^2\bar{Q}^2$ states depend on the dimensionless amplitude parameter a^2 which is determined by fitting the TASSO data[13] on $\gamma\gamma\to\rho^0\rho^0$. The best fit in Fig. 1 yields $a^2=45$. In view of the reasonable interpretation of the $\gamma\gamma\to\rho^0\rho^0$ and $\rho^+\rho^-$ data in terms of the $Q^2\bar{Q}^2$ states, we wish to estimate from Eq. (1) the widths of other $Q^2\bar{Q}^2$ $(0^{++},2^{++})$ states decaying to two vector mesons to check if they are indeed narrow enough to be seen. The widths of $Q^2\bar{Q}^2\to\gamma V$ and $\gamma\gamma$ can also be calculated via the vector meson dominance from which we can calculate the $\gamma\gamma\to VV$ cross sections. The $\gamma\gamma$ widths are given as

$$\Gamma^i_{\gamma\gamma,0^+}=\frac{a^2m_{0^+}}{8\pi}\left[\frac{\alpha}{4}\right]^2[b^i(0^+)]^2$$

and

$$\Gamma^i_{\gamma\gamma,2^+}=\frac{7a^2m_{2^+}}{240\pi}\left[\frac{\alpha}{4}\right]^2[b^i(2^+)]^2\,, \tag{2}$$

where

$$b^i=\sum_{j,k}\frac{4\pi}{\gamma_{V_j}\gamma_{V_k}}a^i_{V_jV_k}\times\begin{cases}1, & \text{if } V_j=V_k\\ \sqrt{2}, & \text{if } V_j\neq V_k\end{cases}\,. \tag{3}$$

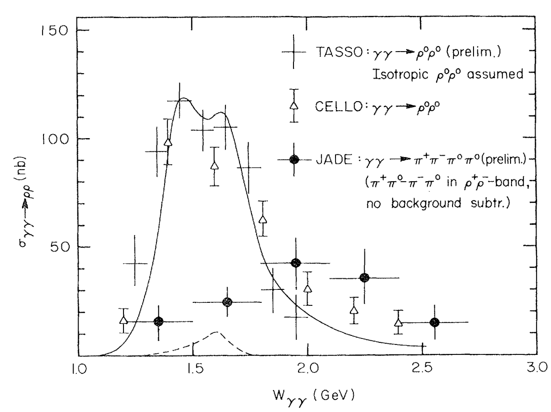

FIG. 1. The calculated $Q^2\bar{Q}^2$ contributions to the $\gamma\gamma\to\rho^0\rho^0$ cross section (solid curve) and the $\gamma\gamma\to\rho^+\rho^-$ cross section (dashed curve) in comparison with the experimental data.

Here $a^i_{V_jV_k}$ is the product of the recoupling coefficient for the color-singlet VV in Table I in the $Q^2\bar{Q}^2$ state i and the coefficient for V_jV_k in the flavor representation. The $\sqrt{2}$ factor in Eq. (4) corrects for the nonidentical particle effect. γ_{V_j} and γ_{V_k} are the VDM coupling constants for those vector mesons which are commensurate with the quark flavors in $Q^2\bar{Q}^2$. The γ's are taken to be

$$\frac{\gamma_\rho^2}{4\pi}=0.61,\quad \frac{\gamma_\omega^2}{4\pi}=5.49,\quad \frac{\gamma_\phi^2}{4\pi}=4.3\ . \tag{4}$$

The γV widths are calculated in a similar fashion,

$$\Gamma^i_{\gamma V,0^+}=\frac{p}{4\pi}\left[\frac{\alpha}{4}\right][C^i(0^+)]^2\ ,$$
$$\Gamma^i_{\gamma V,2^+}=\frac{p}{8\pi}\left[\frac{\alpha}{4}\right][C^i(2^+)]^2\left[\frac{2}{3}+\frac{1}{5}\frac{p^2}{m_V^2}\right], \tag{5}$$

where

$$C^i=\frac{(4\pi)^{1/2}}{\gamma_{V_1}}a^i_{V_1V_2}\times\begin{cases}1, & \text{if } V_1=V_2\\ 1/\sqrt{2}, & \text{if } V_1\neq V_2\end{cases}, \tag{6}$$

with V_1 being the neutral vector meson which couples to the photon and V_2 being the vector meson in the final state. The $1/\sqrt{2}$ factor is a correction for the case when V_1 and V_2 are not identical particles.

For those $(0^{++},9^*)$ states which are below the threshold of the corresponding VV decay channels, we shall consider the multipseudoscalar decay via the VV channel. Their decay widths are calculated by folding in the Breit-Wigner factors for the vector mesons:

$$\begin{aligned}\Gamma(0^{++}(9^*)\to\rho\rho\to4\pi)&=\tfrac{3}{4}(0.644)^2\Gamma_{0^{++}\to\rho\rho}\\&=\frac{3(0.644)^2a^2}{4(2\pi)^3}\int_{4m_\pi^2}^{(m-2m_\pi)^2}dm_1^2\int_{4m_\pi^2}^{(m-m_1)^2}dm_2^2F_{\rm BW}(m_1,m_\rho)F_{\rm BW}(m_2,m_\rho)\\&\quad\times p\left[2+\frac{1}{4m_1^2m_2^2}(m^2-m_1^2-m_2^2)^2\right],\end{aligned} \tag{7}$$

where $F_{\rm BW}(m_1,m_\rho)$ and $F_{\rm BW}(m_2,m_\rho)$ are the Breit-Wigner factors, i.e.,

$$F_{\rm BW}(m_1,m_\rho)=\frac{m_\rho\Gamma_\rho(m_1)}{(m_1^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2(m_1)}\ , \tag{8}$$

and m is the mass of the 0^{++} state;

$$\begin{aligned}\Gamma(0^{++}(9^*)\to\omega\omega\to6\pi)&=\tfrac{1}{4}(0.644)^2\Gamma_{0^{++}\to\omega\omega}\\&=\frac{(0.644)^2a^2}{4(2\pi)^3}\int_{9m_\pi^2}^{(m-3m_\pi)^2}dm_1^2\int_{9m_\pi^2}^{(m-m_1)^2}dm_2^2F_{\rm BW}(m_1,m_\omega)F_{\rm BW}(m_2,m_\omega)\\&\quad\times p\left[2+\frac{1}{4m_1^2m_2^2}(m^2-m_1^2-m_2^2)^2\right],\end{aligned} \tag{9}$$

where $F_{\rm BW}(m_1,m_\omega)$ and $F_{\rm BW}(m_2,m_\omega)$ are the Breit-Wigner factors for the ω meson with $\Gamma_\omega=0.01$ GeV;

$$\begin{aligned}\Gamma(0^{++}(9^*)\to\rho K^*\to3\pi K)&=\frac{3(0.644)^2a^2}{4(2\pi)^3}\int_{(m_K+m_\pi)^2}^{(m-m_K-m_\pi)^2}dm_1^2\int_{4m_\pi^2}^{(m-m_1)^2}dm_2^2F_{\rm BW}(m_1,m_{K^*})F_{\rm BW}(m_2,m_\rho)\\&\quad\times p\left[2+\frac{1}{4m_1^2m_2^2}(m^2-m_1^2-m_2^2)^2\right],\end{aligned} \tag{10}$$

where $F_{\rm BW}(m_1,m_{K^*})$ is the Breit-Wigner factor for the K^* meson; and

$$\begin{aligned}\Gamma(0^{++}(9^*)\to\omega K^*\to4\pi K)&=\frac{(0.644)^2a^2}{4(2\pi)^3}\int_{(m_K+m_\pi)^2}^{(m-m_K-m_\pi)^2}dm_1^2\int_{9m_\pi^2}^{(m-m_1)^2}dm_2^2F_{\rm BW}(m_1,m_{K^*})F_{\rm BW}(m_2,m_\omega)\\&\quad\times p\left[2+\frac{1}{4m_1^2m_2^2}(m^2-m_1^2-m_2^2)^2\right].\end{aligned} \tag{11}$$

TABLE II. The masses, the full widths, the decay modes (VV, γV, and $\gamma\gamma$) and the corresponding partial widths are listed for those $Q^2\bar{Q}^2$ states which decay primarily via two vector mesons.

$Q^2\bar{Q}^2$	Mass (MeV)	Full width Γ (MeV)	Decay mode	Partial widths (MeV)
0^{++}, $C^0(9^*)$	1450	193.8	$4\pi(\rho\rho)$	190.3
			$6\pi(\omega\omega)$	2.9
			$\gamma\rho^0$	0.58
			$\gamma\omega$	0.06
			$\gamma\gamma$	1.9×10^{-3}
0^{++}, $C_K(9^*)$	1600	79.6	$3\pi K(K^*\rho)$	73.8
			$4\pi K(K^*\omega)$	5.8
			γK^*	0.68
0^{++}, $C_\pi^s(9^*)$	1800	104.9	$K^*\bar{K}^*$	69.2
			$\phi\rho$	34.9
			$\gamma\phi$	0.08
			$\gamma\rho$	0.12
			$\gamma\gamma$	1.69×10^{-3}
0^{++}, $C^s(9^*)$	1800	80.7	$K^*\bar{K}^*$	69.2
			$\omega\phi$	11.3
			$\gamma\phi$	0.08
			$\gamma\omega$	0.12
			$\gamma\gamma$	0.18×10^{-3}
2^{++}, $C^0(9)$	1650	357.8	$\rho\rho$	272.7
			$\omega\omega$	84.6
			$\gamma\rho$	0.46
			$\gamma\omega$	0.05
			$\gamma\gamma$	0.82×10^{-3}
2^{++}, $C_K(9)$	1800	230.8	$K^*\rho$	172.8
			$K^*\omega$	57.6
			γK^*	0.39
2^{++}, $C^s(9)$	1950	257.1	$K^*\bar{K}^*$	133.2
			$\phi\omega$	123.8
			$\gamma\omega$	0.09
			$\gamma\phi$	0.05
			$\gamma\gamma$	0.06×10^{-3}
2^{++}, $C_\pi^s(9)$	1950	260.7	$K^*\bar{K}^*$	133.2
			$\rho\phi$	126.9
			$\gamma\phi$	0.49
			$\gamma\rho$	0.09
			$\gamma\gamma$	0.58×10^{-3}
2^{++}, $E_{\pi\pi}(36)$	1650	182.4	$\rho\rho$	181.8
			$\gamma\rho^0$	0.62
			$\gamma\gamma$	1.4×10^{-3}
2^{++}, $C_\pi(36)$	1650	91.4	$\rho\omega$	90.0
			$\gamma\omega$	0.46
			$\gamma\rho$	0.05
			$\gamma\gamma$	0.45×10^{-3}
2^{++}, $C^0(36)$	1650	172.6	$\rho\rho$	45.5
			$\omega\omega$	126.9
			$\gamma\rho$	0.07
			$\gamma\omega$	0.08
			$\gamma\gamma$	0.31×10^{-3}

TABLE II. (*Continued*).

$Q^2\bar{Q}^2$	Mass (MeV)	Full width Γ (MeV)	Decay mode	Partial widths (MeV)
2^{++}, $C_K(36)$	1800	115.2	$K^*\rho$	28.8
			$K^*\omega$	86.4
			γK^*	0.16×10^{-3}
2^{++}, $E_{\pi K}(36)$	1800	115.5	$K^*\rho$	115.2
			γK^*	0.32
2^{++}, $C^s(36)$	1950	128.6	$K^*\bar{K}^*$	66.6
			$\omega\phi$	61.9
			$\gamma\phi$	0.03
			$\gamma\omega$	0.05
			$\gamma\gamma$	0.04×10^{-3}
2^{++}, $C^s_\pi(36)$	1950	130.2	$K^*\bar{K}^*$	66.5
			$\rho\phi$	63.4
			$\gamma\rho$	0.05
			$\gamma\phi$	0.22
			$\gamma\gamma$	0.34×10^{-3}
2^{++}, $E_{KK}(36)$	1950	133.0	$K^*\bar{K}^*$	133.0
2^{++}, $C^s_K(36)$	2100	148.0	$K^*\phi$	147.9
			γK^*	0.09
2^{++}, $C^{ss}(36)$	2250(2160)	326(230)	$\phi\phi$	325.8(230)
			$\gamma\phi$	0.18(0.17)
			$\gamma\gamma$	0.02×10^{-3}

The calculated VV, γV, and $\gamma\gamma$ widths from Eqs. (1), (2), (5), and (7)—(11) for the $Q^2\bar{Q}^2$ states with dominant VV decays are listed in Table II together with their masses. Here, we use the predicted masses from the MIT bag model[7] as a guide to estimate the widths. Whenever an experimental mass is known, it should be used as an input to obtain a more accurate estimate of the width. For example, the predicted mass of $2^{++}C^{SS}(36)$ is 2250 MeV which yields a width of 362 MeV, whereas, using the experimental mass of 2160 MeV, the width is 230 MeV. It is seen from Table II that there are three kinds of $Q^2\bar{Q}^2$ states whose widths are not very large.

(1) Those $(0^{++},9^*)$ states which are below the threshold of their VV decay channels tend to have narrow widths, such as $(0^{++},C^0(9^*))$ and $(0^{++},C_K(9^*))$.

(2) The $(2^{++},36)$ states. Their masses are roughly 100—200 MeV above the VV threshold. Their widths turn out to be comparable to those of the ordinary hadrons.

(3) The $(2^{++},9)$ states. Their recoupling coefficients to VV are $\sqrt{2}$ times larger than those of the $(2^{++},36)$ states. Hence, their widths are twice of those of the $(2^{++},36)$ states.

There are other states which also decay to two vector mesons predominantly, e.g., the $(0^{++},36^*)$ states. If these states are indeed farther above the VV thresholds, they tend to be very broad. For example, $(0^{++},C^0(36^*))$ in the MIT-bag calculation has a mass of 1800 MeV and its dominant decay channels are $\rho\rho$ and $\omega\omega$. Using Eq. (1), we obtain $\Gamma=2.43$ GeV which is even larger than its mass. These states with such broad widths are not likely to be resonances. They are probably artifacts of the bag boundary conditions.[18] On the other hand, if for some reason they are closer to the threshold, they could probably be seen along with the other narrow $Q^2\bar{Q}^2$ states. It is seen from Table II that the γV widths are in the MeV range and the $\gamma\gamma$ widths are in the keV range.

III. $\gamma\gamma\to VV$ CROSS SECTIONS

Data from TASSO[13] and CELLO[15] revealed a large cross section of $\gamma\gamma\to\rho^0\rho^0\to\pi^+\pi^-\pi^+\pi^-$ even below the threshold of $\rho^0\rho^0$. While we have interpreted[9,11] the major part of the cross section above the $\rho\rho$ threshold as originated from the 2^{++} $Q^2\bar{Q}^2$ states which decay to $2\pi^+2\pi^-$ via $\rho^0\rho^0$, the large contribution below the $\rho^0\rho^0$ threshold has to come from somewhere else. One obvious contribution comes from the $0^{++}C^0(9^*)$ state at 1.45 GeV which, from the tabulated $2\pi^+2\pi^-$ width of $\Gamma_{2\pi^+2\pi^-}=\frac{1}{3}\Gamma_{4\pi}=63.4$ MeV and the $\Gamma_{\gamma\gamma}$ in Table II, yields a cross section of 18.6 nb at peak. This is not enough to explain the experimental result at this energy. Motivated by the TASSO observation[13] that there is a sizable contribution from 0^{++} below 1.7 GeV and above

1.7 GeV the main contribution comes from 2^{++}, we consider that the $0^{++}\,C^0(36^*)$ be located below the $\rho\rho$ threshold, instead of at 1800 MeV as predicted in the MIT bag model, so that it contributes to the $\rho^0\rho^0$ cross section in addition to the $0^{++}C^0(9^*)$ state.

To compare with the experimental data, we evaluate the $\gamma\gamma\to 2\pi^+2\pi^-$ cross sections in the following manner.

When the reaction takes place via the $\rho^0\rho^0$ channel, the cross section can be written[9,11] as

$$\sigma=\frac{1}{16W}\left[\frac{7}{3}\frac{\Gamma_{2^{++}\to\rho\rho}(W)}{a^2}|A_{2^+}|^2+2\frac{\Gamma_{0^{++}\to\rho\rho}(W)}{a^2}|A_{0^+}|^2\right], \tag{12}$$

where W is the total energy of the two photons in the center-of-mass frame. A_{0^+} and A_{2^+} are the 0^{++} and 2^{++} amplitudes,

$$A_{0^+}=\frac{\alpha}{4}a^2\left[(\tfrac{1}{2}\times 0.644)^2\frac{\left[\frac{4\pi}{\gamma_\rho^2}-\frac{4\pi}{\gamma_\omega^2}\right]}{W-m_{0^+,9^*}+\frac{i}{2}\Gamma_{0^+,9^*}(W)}+\frac{\left[\frac{1}{2\sqrt{3}}\times 0.743\right]^2\left[\frac{4\pi}{\gamma_\rho^2}+3\frac{4\pi}{\gamma_\omega^2}\right]}{W-m_{0^+,36^*}+\frac{i}{2}\Gamma^{I=0}_{0^+,36^*}(W)}+\frac{\left[(\sqrt{2/3}\times 0.743)^2\frac{4\pi}{\gamma_\rho^2}\right]}{W-M_{0^+,36^*}+\frac{i}{2}\Gamma^{I=2}_{0^+,36^*}(W)}\right], \tag{13}$$

$$A_{2^+}=\frac{\alpha}{4}a^2\left[\frac{\frac{1}{6}\left[\frac{4\pi}{\gamma_\rho^2}-\frac{4\pi}{\gamma_\omega^2}\right]}{W-m_{2^+,9}+\frac{i}{2}\Gamma_{2^+,9}(W)}+\frac{\frac{1}{36}\left[\frac{4\pi}{\gamma_\rho^2}+3\frac{4\pi}{\gamma_\omega^2}\right]}{W-m_{2^+,36}+\frac{i}{2}\Gamma_{2^+,36}(W)}+\frac{\frac{2}{9}\frac{4\pi}{\gamma_\rho^2}}{W-m_{2^+,36}+\frac{i}{2}\Gamma_{2^+,36}(W)}\right]. \tag{14}$$

The widths $\Gamma_{2^{++}\to\rho\rho}(W)$, $\Gamma_{2^+,9}(W)$, and $\Gamma_{2^+,36}(W)$ are proportional to $\Gamma_{2^+}(9)$ and $\Gamma_{2^+}(36)$ defined in Eq. (1), i.e.,

$$\begin{aligned}\Gamma_{2^{++}\to\rho\rho}(W)&=\tfrac{3}{2}\Gamma_{2^+}(9)\,,\\ \Gamma_{2^+,9}(W)&=\Gamma_{2^+}(9)\,,\\ \Gamma_{2^+,36}(W)&=\Gamma_{2^+}(36)\,,\end{aligned} \tag{15}$$

except that p in $\Gamma_{2^+}(9)$ and $\Gamma_{2^+}(36)$ goes as

$$p^2=\frac{1}{4W^2}(W^2+m_1^2-m_2^2)^2-m_1^2\,, \tag{16}$$

instead of as in Eq. (1), so that they are functions of W. The width $\Gamma_{0^{++}\to\rho\rho}(W)$ is defined similarly to $\Gamma_{0^{++}\to\rho\rho}$ in Eq. (7), except that the mass m is replaced by W. Accordingly, $\Gamma_{0^+,9^*}(W)$ and $\Gamma_{0^+,36^*}(W)$ are defined as

$$\Gamma_{0^+,9^*}(W)=(0.644)^2[\tfrac{3}{4}\Gamma_{0^{++}\to\rho\rho}(W)\;\tfrac{1}{4}\Gamma_{0^{++}\to\omega\omega}(W)+\Gamma_c]\,, \tag{17}$$

$$\Gamma_{0^+,36^*}(W)=(0.743)^2[\tfrac{1}{4}\Gamma_{0^{++}\to\rho\rho}(W)+\tfrac{3}{4}\Gamma_{0^{++}\to\omega\omega}(W)+\Gamma_c]\,. \tag{18}$$

Here we added a constant width Γ_c to simulate effects which we have neglected in our fall-apart mechanism, e.g., decays of other channels such as PP, mixture with other 0^{++} states with large widths, final-state interactions,[13] etc. We leave Γ_c as a parameter to fit the $\gamma\gamma\to\rho^0\rho^0$ cross section below the threshold. Figure 1 gives the result with $\Gamma_c=130$ MeV. The masses of these 0^{++} and 2^{++} states are allowed to vary in view of the fact that 9 (9*) and 36 (36*) have different widths and $C^0(9)^{I=0}$ and $C^0(36)^{I=0}$ will mix with other $I=0$ states and with each other through the annihilation channel. Their masses are varied to fit the $\rho^0\rho^0$ data. It is found that the $\rho^0\rho^0$ cross sections above the threshold are sensitive to the mass of $E^{I=2}_{\pi\pi}(36,2^{++})$ and below the threshold they are sensitive to the mass of $E^{I=2}_{\pi\pi}(36^*,0^{++})$, respectively, due to their large contributions. A mass of 1.7 [1.45] GeV for $E^{I=2}_{\pi\pi}(36,2^{++})$ [$E^{I=2}_{\pi\pi}(36^*,0^{++})$] is found satisfactory to fit the $\rho^0\rho^0$ data (Fig. 1). Masses of $C^0(9,2^{++})$, $C^0(36,2^{++})$, $C^0(9^*,0^{++})$, and $C^0(36^*,0^{++})$ are less certain. Variation with a range of $\sim$100 MeV away from the $E_{\pi\pi}(36,2^{++})$ and $E_{\pi\pi}(36^*,0^{++})$ would not change the $\rho^0\rho^0$ result qualitatively. They would, however, change the qualitative features of the $\rho^+\rho^-$ data. Since the existing $\rho^+\rho^-$ data are not good enough to distinguish different fits, we report in Fig. 1 the results with $m^{I=0}_{2^{++}}=1.65$ GeV, $m_{0^+,9^*}=1.4$ GeV, and $m^{I=0}_{0^+,36^*}=1.45$ GeV. We see from Fig. 1 that the theoretical $\rho^0\rho^0$ cross sections below the threshold are improved substantially due to the inclusion of the $0^{++}(36^*)$ states. For the $\rho^+\rho^-$ cross sections, we find from Fig. 1 that they are smaller than the upper limits set by the JADE results.[16] This is a direct consequence of the destructive interference between the $I=0$ and $I=2$ parts of the amplitudes for both 2^{++} and the 0^{++} $Q^2\bar{Q}^2$ states.

As a further check of the $Q^2\bar{Q}^2$ picture, we consider the cross sections $\sigma(\gamma\gamma\rightarrow\omega\omega,\rho^0\omega,\rho^0\phi,K^*\bar{K}^*,\rho^0J$, etc.) via the corresponding 2^{++} $Q^2\bar{Q}^2$ states. Due to the cancellation between $C^0(9)^{I=0}$ and $C^0(36)^{I=0}$, $\sigma_{\omega\omega}$ are two orders of magnitude smaller than $\sigma_{\rho^0\rho^0}$, $\sigma_{K^{*+}K^{*-}}$, and $\sigma_{K^{*0}\bar{K}^{*0}}$ are also small due to the interferences between $C^s(9)$ and $C^s(36)$, and between $C^s_\pi(9)$ and $C^s_\pi(36)$. $\sigma(\gamma\gamma\rightarrow C_\pi(36)\rightarrow\rho^0\omega)$, $\sigma(\gamma\gamma\rightarrow C^s_\pi(9),C^s_\pi(36)\rightarrow\rho^0\phi)$, and $\sigma(\gamma\gamma\rightarrow\rho^0J)$, on the other hand, could be reasonably large, since there are no cancellations. But there are other complications which we need to be concerned about. Since these 2^{++} $Q^2\bar{Q}^2$ states (i.e., C_π, C^s_π, C^c_π) decay to two nonidentical particles, other things being equal their widths are a factor of 2 smaller than those of the $Q^2\bar{Q}^2$ states (i.e., C^0, $E_{\pi\pi}$) which decay to two identical vector mesons (see Table II). This makes their cross sections more susceptible to the decay modes which we have neglected so far, i.e., decays other than the fall-apart mode. For example, adding $0.15\ \Gamma_{2^{++},E_{\pi\pi}}=23$ MeV to the total widths of $2^{++}C^0$, C_π, and $E_{\pi\pi}$ lowers the 2^{++} $Q^2\bar{Q}^2$ contribution to $\sigma(\gamma\gamma\rightarrow\rho^0\rho^0)$ by 22% only, yet it lowers the $\sigma(\gamma\gamma\rightarrow\rho^0\omega)$ by 41%. Furthermore, C_π is expected to mix with other 2^{++} $I=1$ $Q\bar{Q}$ states, especially the radially excited states of A_2 which is expected to lie around 1.8 GeV and with a width of ~200—400 MeV. A few percent mixture will lower the $\sigma_{\rho^0\omega}$ substantially, if this 2^{++} $I=1$ $Q\bar{Q}$ state has little $\rho^0\omega$ branching ratio. This is in contrast with the case of $\sigma_{\rho^0\rho^0}$, where the major contributors $E_{\pi\pi}(0^{++},2^{++})$, being isotensor, do not mix with other $Q\bar{Q}$ states, so that $\sigma_{\rho^0\rho^0}$ is fairly insensitive to the mixtures of C^0's and other $I=0$ $Q\bar{Q}$ states. For definiteness, we present in Fig. 2 a calculated result of $\sigma(\gamma\gamma\rightarrow C_\pi\rightarrow\rho^0\omega)$ assuming a 20% mixture with a nearby 2^{++} $I=1$ $Q\bar{Q}$ state with $\Gamma=300$ MeV. We see from Fig. 2 that $\sigma(\gamma\gamma\rightarrow\rho^0\omega)$ in this case is below the recent preliminary experimental upper limit[16] measured by the JADE group. $\sigma(\gamma\gamma\rightarrow\rho^0\omega)$ is also sensitive to the mass of C_π. If we take the mass of C_π to be at 1.75 GeV instead of at 1.65 GeV, the peak cross section $\sigma_{\rho^0\omega}$ will be ~10 nb instead of 17 nb shown in Fig. 2. The latter result uses 1.65 GeV as the input mass for C_π.

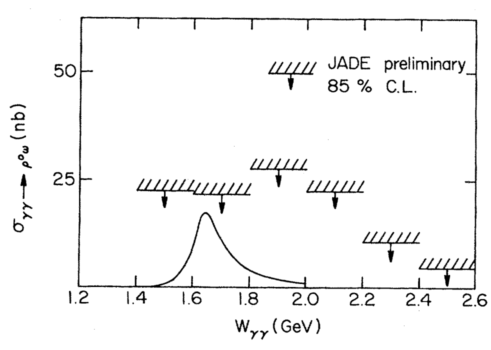

FIG. 2. The calculated $Q^2\bar{Q}^2$ contribution to the $\gamma\gamma\rightarrow\rho^0\omega$ cross section in comparison with the preliminary upper bounds from the JADE experiment.

IV. CONCLUSION AND DISCUSSION

To conclude, we find that three classes of the $Q^2\bar{Q}^2$ states, $(0^{++},9^*)$, $(2^{++},9)$ and $(2^{++},36)$, and possibly four if $(0^{++},36^*)$ lie close to the VV thresholds, may have widths narrow enough to be observed as mass bumps. The common feature of these states is that they decay via two vector mesons predominantly. In the $\gamma\gamma\rightarrow\rho^0\rho^0$ reaction, the structure above $\rho\rho$ threshold is well explained by three 2^{++} $Q^2\bar{Q}^2$ states $C^0(9)$, $C^0(36)$, and $E_{\pi\pi}(36)$; the structure below the threshold can be explained by three 0^{++} $Q^2\bar{Q}^2$ states $C^0(9^*)$, $C^0(36^*)$, and $E_{\pi\pi}(36^*)$. The substantial contribution of $\sigma_{\rho^0\rho^0}$ comes from the isotensor $E_{\pi\pi}(36)$ and $E_{\pi\pi}(36^*)$. The strong suppression of $\sigma(\gamma\gamma\rightarrow\rho^+\rho^-)$ in the mass region 1.3—2.0 GeV[16] is understood as due to the cancellation between the $I=0$ and the $I=2$ states. The required presence of the exotic isotensor states to explain both the $\rho^0\rho^0$ and $\rho^+\rho^-$ data is taken as model-independent evidence for the existence of the $Q^2\bar{Q}^2$ mesons. The recent preliminary upper limit of ~25 nb for $\sigma(\gamma\gamma\rightarrow\rho^0\omega)$ between 1.4 and 1.8 GeV can be accommodated by the 2^{++} $I=1$ $Q^2\bar{Q}^2$ state $C_\pi(36)$ with ~10—20 % mixture with the other 2^{++} $I=1$ $Q\bar{Q}$ states. $\sigma(\gamma\rightarrow\omega\omega)$ is predicted to be small which agrees with the late experimental findings in the relevant mass region.[16] Also $\sigma(\gamma\gamma\rightarrow K^{*+}K^{*+},K^{*0}\bar{K}^{*0})$ are predicted to be small which should be checked experimentally.

Some of these four quark states can be produced by other means. Since these states have $\underline{V}\cdot\underline{V}$ (color-octet-vector—color-octet-vector) parts in their wave functions besides the VV parts, those isoscalar $Q^2\bar{Q}^2$ states can couple to two gluons in much the same way as the coupling to two photons, an therefore are expected to be produced in J/ψ radiative decays and hadronic collisions. Recent data from $J/\psi\rightarrow\gamma+2\pi^+2\pi^-$ shows a $\rho^0\rho^0$ enhancement near the $\rho\rho$ threshold.[19] The branching ratio is measured[19] to be

$$B(J/\psi\rightarrow\gamma\rho^0\rho^0)=(1.25\pm0.35\pm0.40)\times10^{-3}$$

and its mass is determined to be 1650±50 MeV with a width of 200±100 MeV. This has been interpreted[20] as two isoscalar 2^{++} $Q^2\bar{Q}^2$ states $C^0(9)$ and $C^0(36)$, since they have the same mass and width. It is believed to be different from $\theta(1640)$ which decays to $\eta\eta$ (Ref. 21) and $K\bar{K}$ (Ref. 22) since $\theta\rightarrow K\bar{K}$ was not found in $\gamma\gamma$ reactions.[23,24] In order to distinguish between $\theta(1640)$ and the $\rho^0\rho^0$ structure in J/ψ radiative decay and identify the latter as the $Q^2\bar{Q}^2$ states, we have proposed[20] to measure the angular correlation between γ and ρ^0 in $e^+e^-\rightarrow J/\psi\rightarrow\gamma\rho^0\rho^0$ in order to check if the ratios of the helicity amplitudes are the same as those measured in $e^+e^-\rightarrow J/\psi\rightarrow\gamma\eta\eta$.[21] We have also proposed[20] to measure the branching ratio of the decay mode $J/\psi\rightarrow\gamma\omega\omega$. If the $\rho^0\rho^0$ structure in J/ψ radiative decay is due to the $I=0$ $Q^2\bar{Q}^2$ states, we predict the $\omega\omega$ branching ratio to be 2.8 times that of $\rho^0\rho^0$. On the other hand, if the $\rho^0\rho^0$ structure is the θ meson, a glueball candidate, we expect the $\omega\omega$ branching ratio to be the same as that of $\rho^0\rho^0$. In case these proposed experiments favor the $Q^2\bar{Q}^2$ interpretation, it would settle the $Q^2\bar{Q}^2$ assignment to the $\rho^0\rho^0$ structure in $\gamma\gamma$ reactions and rule out the possibility of

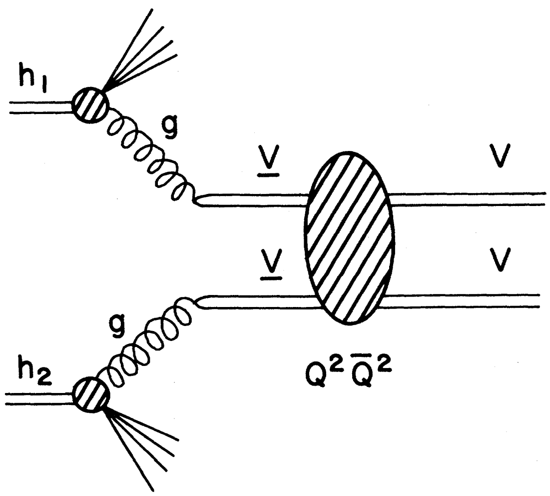

FIG. 3. Production of $Q^2\bar{Q}^2$ mesons in hadronic collisions, a Drell-Yan-type mechanism.

the threshold effect.[11]

Those $I=0$ $Q^2\bar{Q}^2$ states can also be produced in hadronic collisions. The Drell-Yan-type production mechanism is shown in Fig. 3. There are several candidates for the $Q^2\bar{Q}^2$ states produced in this process. There is evidence for the $\rho\rho$ enhancement at $M \sim 1700$ MeV from $p\bar{p} \to 3\pi^+ 3\pi^- \pi^0$ at 5.7 GeV/c.[25] This could be due to the same $Q^2\bar{Q}^2$ states observed in $\gamma\gamma \to \rho^0\rho^0$ and $J/\psi \to \gamma\rho^0\rho^0$ at this mass. The $2^{++}\phi\phi$ resonance at 2.16 GeV produced in pp[26] and πp collisions[27] has been interpreted[28] as the 2^{++} $C^{ss}(36)$ $s^2\bar{s}^2$ state. The calculated mass,[7] width,[28] and the theoretical cross sections[28] (from Fig. 3) in pp reaction at $p_L = 400$ GeV/c and πp reaction at $p_L = 100$ GeV/c are all in agreement with the experimental data. The correlated J/ψ pair produced around 7 GeV in πN collisions[29] at 150 and 280 GeV/c has been considered[30] as a candidate for the 2^{++} $c^2\bar{c}^2$ meson. The calculated cross sections at these energies and the longitudinal and transverse momentum distributions are consistent with data.[30]

With more data accumulating from the various reactions (e.g., $\gamma\gamma$ reactions, radiative decays of J/ψ, and hadronic collisions), we are hopeful of learning more on the structure and the dynamics of the $Q^2\bar{Q}^2$ four-quark mesons, and thereby furnishing their existence with a firmer ground.

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