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Non-Abelian anomaly and vector-meson decays

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We propose an extension of the old, fairly successful, phenomenological prescription for adding spin-1 fields to the chiral Lagrangian of pseudoscalar mesons to the case where the Wess-Zumino term is present. This leads to the possibility of describing within the effective-Lagrangian framework a whole class of "unnatural-parity" hadronic reactions. In particular, we study the well-known (pure hadronic) $\omega \rightarrow 3\pi$ and (radiative) $\omega \rightarrow \pi^0\gamma$ processes, and find good agreement with experiment, comparable to that for any of the current-algebra "theorems." The construction of the Lagrangian is seen to involve subtle theoretical issues. We are led to examine in more detail Witten's "trial-and-error" gauging of the Wess-Zumino term and note that the Bardeen form of the anomaly seems more suitable than the left-right-symmetric form for constructing an effective model with phenomenological spin-1 fields.

I. INTRODUCTION

A widespread hope is that QCD at low energies can be understood in terms of an effective Lagrangian constructed out of phenomenological fields. In this approach one aims to build on the successful chiral Lagrangians of the last generation by including features specific to QCD. The chiral models can be considered to be based on the "order-parameter" multiplet M_{ab} transforming like the quark-field combination $\bar{q}_{Rb}q_{La}$. The theory should confirm that M condenses in the chiral phase, $\langle M_{ab}\rangle \neq 0$. In this case it is often convenient to make a polar decomposition $M=UH$, where $U^\dagger = U^{-1}$ and $H=H^\dagger$, and approximate the "heavy" scalar fields H by a constant proportional to the unit matrix. The usual *nonet* of light pseudoscalar mesons ϕ can be related to U by

$$U=\exp\left[\frac{2i}{F_\pi}\phi\right], \tag{1.1}$$

where $F_\pi \approx 135$ MeV is the pion decay constant. Notice that the quark model suggests that one deal with nonets rather than octets. The chiral Lagrangian for pseudoscalar interactions at low energies takes the form[1]

$$\mathscr{L}_0 = -\frac{F_\pi{}^2}{8}\operatorname{Tr}(\partial_\mu U\,\partial_\mu U^\dagger) . \tag{1.2}$$

A chiral-symmetry-breaking term proportional to

$$\sum_a m_a U_{aa} + \text{H.c.} ,$$

where the m_a are the "current-algebra" quark masses, mocks up the transformation properties of the underlying quark theory and can be added to (1.2). This will then satisfactorily describe the masses of the *octet* of pseudoscalars. The mass of the ninth pseudoscalar η' cannot be understood in the framework of mocking up the *classical* transformation properties of the fundamental QCD Lagrangian. However, it can be understood by introducing a phenomenological pseudoscalar glueball field to describe[2] the *quantum* features of the U(1) axial-vector anomaly. The closely allied trace anomaly of the energy-momentum tensor can be similarly modeled[3] with the aid of a scalar glueball field and seems to provide an intriguing link with confinement physics. Thus, one gets the impression that modeling the symmetry properties of the underlying QCD theory with spin-zero fields and, especially, taking into account the quantum anomalies yields a reasonable starting point for further development. This is the point of view we shall take here. In particular, we would like to study some aspects of the Wess-Zumino term[4] which describes the *non-Abelian* anomaly[5] structure of QCD with the field U of (1.1). This term was actually found[4] a long time ago as a "leftover" piece when the chiral model was coupled to electromagnetism, but since it describes only physical processes at least as complicated as $K\bar{K}\rightarrow 3\pi$, it has until recently played no role in the discussion of purely hadronic physics.

Our study shows that the Wess-Zumino term can be generalized to encompass simpler processes than $K\bar{K}\rightarrow 3\pi$ if one includes spin-1 as well as spin-0 mesons in the effective Lagrangian. The results we get for the vector-meson ω widths agree rather well with experiment. There are a number of interesting theoretical issues which are discussed at appropriate points in the text. Section II contains a brief summary of the Wess-Zumino term and the possible hadronic physics it may describe. In Sec. III we review the old (reasonably successful) prescription for adding spin-1 mesons to the spin-0 chiral Lagrangian and suggest a generalization in the presence of the Wess-Zumino term. The formal problem of "gauging" this term as discussed by Witten[6] is reviewed in Sec. IV. Sections V and VI are devoted to a discussion of the influence on our effective Lagrangian of the well known ambiguity in the specific form of the non-Abelian anomaly in the underlying quark theory. To reproduce the $\pi^0\rightarrow\gamma\gamma$ theorem one requires Bardeen's rather than the left-right-symmetric form of the anomaly. The pure hadronic decay $\omega\rightarrow 3\pi$ is found in Sec. VII to be well described by our model. The present effective Lagrangian, used in conjunction with vector-meson dominance,[7] can also describe radiative decays of hadrons. In Sec. VIII we apply this successfully to the simple process $\omega\rightarrow\pi^0\gamma$. While the specific processes discussed in this paper have of course been treated in a similar fashion in the past,[8,9] the distinguishing feature of the present approach is the determination of all vertices directly from the coefficient of the Wess-Zumino term and Sakurai's ρ-meson coupling constant. A brief summary and directions for further work are given in Sec. IX.

II. THE WESS-ZUMINO TERM

This term[4,6,10–12] can be most compactly written using the language of differential forms.[13] From the matrix U in (1.1) we construct the two one-forms

$$\begin{aligned}\alpha&=(\partial_\mu U)U^{-1}dx^\mu\equiv(dU)U^{-1}\,,\\ \beta&=U^{-1}dU=U^{-1}\alpha U\,.\end{aligned}\tag{2.1}$$

α and β transform as nonets under the left and right U(3) flavor groups, respectively. The Wess-Zumino term in the effective action, $\Gamma_{\rm WZ}$, can be presented[6] in two alternative ways

$$\Gamma_{\rm WZ}=C\int_{M^5}\mathrm{Tr}(\alpha^5)=C\int_{M^5}\mathrm{Tr}(\beta^5)\,.\tag{2.2}$$

Here the integral is over a five-dimensional manifold whose boundary is ordinary Minkowski space. The constant C is determined, by gauging (2.2) with electromagnetism and comparing with the current-algebra $\pi^0\rightarrow2\gamma$ amplitude,[14] to be

$$C=\frac{-iN_c}{240\pi^2}\,,\tag{2.3}$$

where $N_c=3$ is the number of colors. Equation (2.2) is not only elegant but convenient for practical calculations. For example, let us find the first term in the expansion in terms of pseudoscalars. This will come from the first term of $\alpha=(2i/F_\pi)d\phi+\cdots$. Then

$$\begin{aligned}\Gamma_{\rm WZ}&=C\left[\frac{2i}{F_\pi}\right]^5\int_{M^5}\mathrm{Tr}(d\phi)^5+\cdots\\ &=C\left[\frac{2i}{F_\pi}\right]^5\int_{M^5}d\,\mathrm{Tr}[\phi(d\phi)^4]+\cdots\\ &=C\left[\frac{2i}{F_\pi}\right]^5\int_{M^4}\mathrm{Tr}[\phi(d\phi)^4]+\cdots\\ &\equiv C\left[\frac{2i}{F_\pi}\right]^5\int d^4x\,\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(\phi\,\partial_\mu\phi\,\partial_\nu\phi\,\partial_\alpha\phi\,\partial_\beta\phi)\\ &\quad+\cdots\,,\end{aligned}\tag{2.4}$$

where M^4 is ordinary Minkowski space and the generalized Stokes' theorem was used in the next to the last step. (The relation $d^2=0$, which was used in the first step, simplifies many manipulations involving differential forms.)

Witten[6] has given an interesting motivation for the presence of $\Gamma_{\rm WZ}$ in the low-energy effective action for QCD. He observed that, without $\Gamma_{\rm WZ}$, the effective action will only allow processes for which $(-1)^{N_p}$, where N_p is the number of pseudoscalars, is conserved. On the other hand, this is not a selection rule for QCD itself. We see from (2.4) that $\Gamma_{\rm WZ}$ describes the low-energy limit for reactions such as $K\bar{K}\rightarrow3\pi$ ($\pi\pi\rightarrow3\pi$ is prohibited by G parity). Unfortunately, this is not an easy thing to test. At this point a very natural question is whether there might be some easier way to test the physics represented by $\Gamma_{\rm WZ}$. To answer this question we start by noting that Witten's criterion that a new term produce processes violating $(-1)^{N_p}$ conservation is simply another way of saying that we look for new processes whose amplitudes are (nontrivially) proportional to the Levi-Civita tensor $\epsilon_{\mu\nu\alpha\beta}$. This suggests introducing other low-mass particles, such as spin-1 vector and axial-vector mesons, and focusing on processes for which the product of an intrinsic parity number for each particle is *not* conserved. [The intrinsic parity number is taken to be $+1$ if the particle transforms as a true tensor of the appropriate rank and -1 if the particle transforms as a pseudotensor of that rank (e.g., π, γ, ρ, and A_1, have numbers -1, $+1$, $+1$, and -1).] $\pi_0\rightarrow2\gamma$ is such a process. We have in mind the study of such purely hadronic examples as $\omega\rightarrow3\pi$ and $K^*\rightarrow K\pi\pi$, for which experimental information exists. This gives us a strong motivation for investigating the proper way of adding spin-1 mesons to the chiral Lagrangian of spin-0 particles, *including* terms proportional to the Levi-Civita symbol.

III. CHIRAL LAGRANGIAN WITH SPIN-1 MESONS

There is a very well studied and phenomenologically successful traditional method[15] of adding spin-1 mesons to the spin-0 meson chiral Lagrangian. This method is ultimately based on the old "vector-meson dominance" pioneered by Sakurai.[7] Although, as we shall see, certain subtleties arise when it is applied in the presence of the Wess-Zumino term, let us first review its salient features. It is convenient to introduce left- and right-handed spin-1 mesons $A_{L\mu}$ and $A_{R\mu}$ which are related to the vector and axial-vector mesons V_μ and A_μ by

$$A_{L\mu}=\tfrac{1}{2}(V_\mu+A_\mu),\quad A_{R\mu}=\tfrac{1}{2}(V_\mu-A_\mu)\,.\tag{3.1}$$

The couplings of these mesons to (composite) matter fields are considered to be the same as if one had a gauge theory; namely, the derivative in (1.2) is replaced according to the prescription

$$\partial_\mu U\rightarrow(\partial_\mu U-igA_{\mu L}U+igUA_{\mu R})\,,\tag{3.2}$$

where g is the *phenomenological* gauge coupling constant. Its magnitude is equal to $\sqrt{2}f_{\rho\pi\pi}$, where $f_{\rho\pi\pi}$ (Sakurai's notation) is related to the experimental $\rho\rightarrow2\pi$ width by

$$\Gamma(\rho\rightarrow2\pi)=\frac{2}{3}\frac{(f_{\rho\pi\pi})^2}{4\pi}\frac{|\vec{q}_\pi|^3}{m_\rho^{\,2}}\tag{3.3}$$

($\vec{q}_\pi$ is the pion momentum in the ρ rest frame) yielding

$$\frac{(f_{\rho\pi\pi})^2}{4\pi}\simeq3.0\,.\tag{3.4}$$

In deriving (3.3) we have assumed that the $\rho\pi\pi$ vertex has a minimal momentum dependence. This actually corresponds (in the notation of the second of Refs. 15 which we are here following) to a suitable choice of the parameter ζ [see Eq. (24) and the subsequent discussion of this reference]. A strong justification for this procedure is the experimental near equality of the $\rho\pi\pi$ and $\rho N\bar{N}$ coupling constants. It should, however, be stressed that this is not a fundamental gauge theory. The way in which this phenomenological prescription differs from a fundamental gauging prescription is in the necessity for vector and axial-vector mass terms. Thus, one includes the following spin-1-meson terms

$$-\tfrac{1}{2}\,\mathrm{Tr}(F^L_{\mu\nu}F^L_{\mu\nu}+F^R_{\mu\nu}F^R_{\mu\nu})-m_\rho{}^2\,\mathrm{Tr}(A_{\mu L}A_{\mu L}+A_{\mu R}A_{\mu R})\ ,$$
$$F^L_{\mu\nu}=\partial_\mu A_{L\nu}-\partial_\nu A_{L\mu}-ig[A_{L\mu},A_{L\nu}]\ , \qquad (3.5)$$

etc., where for simplicity we have neglected SU(3) breaking. m_ρ is the mass of the ρ meson. In this model the difference between the mass of the ρ meson and the mass of the A_1 meson is supplied by a partial Higgs mechanism leading to the reasonably well-satisfied relation[16]

$$m_{A_1}{}^2-m_\rho{}^2=\frac{g^2\tilde{F}_\pi{}^2}{4}\left[\frac{m_{A_1}}{m_\rho}\right]^2 . \qquad (3.6)$$

($\tilde{F}_\pi$ is [see (3.7)] the pion decay constant.) We note that if the vector mass term in (3.5) were not present the ordinary pion would be completely eaten up by the A_1—this underlines the crucial manner in which the present Lagrangian differs from a true gauge theory. The pseudoscalar and axial-vector fields mix with each other and one obtains the physical (tilde) quantities

$$\begin{aligned} A_\mu&=\tilde{A}_\mu+\frac{g\tilde{F}_\pi}{2m_\rho{}^2}\partial_\mu\tilde{\phi}\ ,\\ \phi&=Z^{-1}\tilde{\phi}\ ,\\ Z&=\left[1+\frac{g^2F_\pi{}^2}{4m_\rho{}^2}\right]^{-1/2} ,\\ \tilde{F}_\pi&=ZF_\pi\ . \end{aligned} \qquad (3.7)$$

How should this procedure be modified when Γ_{WZ} is present? The most satisfactory answer to this question should probably come from an attempt to derive a phenomenological meson model directly from the generating functional of QCD. At the present time this seems complicated so we shall simply generalize the prescription above in the sense that we require the effective theory to possess as much local-flavor invariance as possible. In other words, we attempt to mock up the non-Abelian *flavor* transformation properties of QCD with the phenomenological V_μ and A_μ fields. Since Γ_{WZ} is in fact an anomaly term it is of course impossible to make the effective model invariant under the full $\mathrm{U}(3)_L\times\mathrm{U}(3)_R$ group of local transformations. One could choose to gauge an anomaly-free subgroup like $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ or $\mathrm{U}(3)_V$. However $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ does not permit us to include the ω meson which is very important for phenomenological purposes. Furthermore, gauging $\mathrm{U}(3)_V$ and thus not introducing the axial-vector fields has the disadvantage that it leaves one with a set of fields which do not even allow the kinetic terms [see (3.5)] of the Lagrangian to be globally chiral symmetric. The needed generalization of Γ_{WZ} which forms the raw material for our subsequent discussion has been discussed for different purposes by Witten[6] and in the original paper[4] of Wess and Zumino. Witten's form has the advantage of being explicit, enabling one to read off various relevant vertices. We shall give a rederivation of Witten's form in Sec. IV. This will enable us to discuss some points of interest in the present connection as well as to correct some small errors in his expressions.

IV. WESS-ZUMINO TERM WITH SPIN-1 MESONS

In this section we would like to clarify some of the steps involved in Witten's gauging of the Wess-Zumino term,

$$\Gamma_{\mathrm{WZ}}(U)=C\int_{M^5}\mathrm{Tr}(\alpha^5)\ . \qquad (2.2')$$

The points that we would like to discuss can be illustrated simply by first considering the gauging of the Wess-Zumino term under electromagnetism. We thus consider the variation

$$\delta U=i\epsilon[Q,U]\ , \qquad (4.1)$$

where Q is the SU(3) charge matrix

$$Q=\begin{bmatrix}\frac{2}{3}&&\\&-\frac{1}{3}&\\&&-\frac{1}{3}\end{bmatrix} . \qquad (4.2)$$

Under the local variation (4.1), Γ_{WZ} changes by

$$\delta\Gamma_{\mathrm{WZ}}(U)=5Ci\int_{M^5}d\epsilon\,\mathrm{Tr}[Q(\alpha^4-\beta^4)]\ . \qquad (4.3)$$

From (2.1) we observe that the even powers of α and β are exact forms, in particular,

$$\begin{aligned} 0&=d\alpha-\alpha^2=d\alpha^3-\alpha^4\ ,\\ 0&=d\beta+\beta^2=d\beta^3+\beta^4\ . \end{aligned} \qquad (4.4)$$

This shows that the integrand of (4.3) is an exact form:

$$\begin{aligned}\delta\Gamma_{\mathrm{WZ}}(U)&=-5Ci\int_{M^5}d\{d\epsilon\,\mathrm{Tr}[Q(\alpha^3+\beta^3)]\}\\ &=-5Ci\int_{M^4}d\epsilon\,\mathrm{Tr}[Q(\alpha^3+\beta^3)]\ ,\end{aligned} \qquad (4.5)$$

where in the last step, Stokes' theorem was used and the integral is over Minkowski space $M^4=\partial M^5$. We now introduce the gauge field A_μ and the one-form $A=A_\mu dx^\mu$, with the transformation property (setting electric charge $e=1$)

$$\delta A=d\epsilon \qquad (4.6)$$

and we consider

$$\Gamma^{(1)}(U,A)=\Gamma_{\mathrm{WZ}}(U)+5Ci\int_{M^4}A\,\mathrm{Tr}[Q(\alpha^3+\beta^3)]\ . \qquad (4.7)$$

Thus, the variation of $\Gamma^{(1)}(U,A)$ is

$$\delta\Gamma^{(1)}(U,A)=5Ci\int_{M^4} A\,\mathrm{Tr}[Q\delta(\alpha^3+\beta^3)]\,, \tag{4.8}$$

and a straightforward evaluation, paying attention to the anticommutativity of one-forms, gives

$$\delta\Gamma^{(1)}(U,A)=+10C\int_{M^4} d\epsilon\, A\,\mathrm{Tr}[Q^2(\alpha^2-\beta^2)+Q\,d(U^{-1})Q\,dU]\,. \tag{4.9}$$

At this point, we cannot simply go to the next step in the procedure by replacing $d\epsilon$ by A, since $A^2=0$. Instead, we observe that the quantity inside the trace is a closed form. It can actually be written as an exact form:

$$\alpha^2-\beta^2=d(\alpha+\beta)\,, \tag{4.10}$$

$$Q\,d(U^{-1})Q\,dU=d[aQU^{-1}Q\,dU-bQ\,d(U^{-1})QU]\,. \tag{4.11}$$

In the last expression a and b are arbitrary constants satisfying $a+b=1$. Witten's choice at this step corresponds to taking $a=0$, $b=1$. However, as we shall see, parity invariance requires $a=b=\frac{1}{2}$. We can now proceed by integrating by parts and bringing the differential d to act on A,

$$\delta\Gamma^{(1)}(U,A)=10C\int_{M^4} d\epsilon\, dA\,\mathrm{Tr}[Q^2(\alpha+\beta)+aQU^{-1}Q\,dU-bQ\,d(U^{-1})QU]\,, \tag{4.12}$$

and thus, consider

$$\Gamma^{(2)}(U,A)=\Gamma^{(1)}(U,A)-10C\int_{\partial M^5} A\,dA\,\mathrm{Tr}[Q^2(\alpha+\beta)+aQU^{-1}Q\,dU-bQ\,d(U^{-1})QU]\,. \tag{4.13}$$

This expression is now gauge invariant and is the desired gauging of the Wess-Zumino term. We can write its last term in a more transparent form by noting that

$$\mathrm{Tr}[aQU^{-1}Q\,dU-bQ\,d(U^{-1})QU]=\tfrac{1}{2}\mathrm{Tr}[QU^{-1}Q\,dU-QUQ\,d(U^{-1})]+\frac{r}{2}d[\,\mathrm{Tr}(QU^{-1}QU)]\,, \tag{4.14}$$

where we used $a+b=1$ and $r=a-b$ is an arbitrary constant. Therefore, from (4.7), (4.13), and (4.14),

$$\Gamma^{(2)}(U,A)=\Gamma_{\mathrm{WZ}}(U)+5Ci\int_{\partial M^5} A\,\mathrm{Tr}[Q(\alpha^3+\beta^3)]$$
$$-10C\int_{\partial M^5} A\,dA\,\mathrm{Tr}[Q^2(\alpha+\beta)+\tfrac{1}{2}QU^{-1}Q\,dU-\tfrac{1}{2}QUQ\,d(U^{-1})]-5Cr\int_{\partial M^5} dA\,dA\,\mathrm{Tr}(QU^{-1}QU)\,. \tag{4.15}$$

The last term of this expression involving the arbitrary constant r is gauge invariant. It may be kept or it may be dropped without affecting the gauge invariance of $\Gamma^{(2)}$. On the other hand, since

$$dA\,dA=d^4x\tfrac{1}{4}F_{\mu\nu}F_{\alpha\beta}\epsilon^{\mu\nu\alpha\beta}\,,$$

this term is not parity invariant and we must choose $r=a-b=0$, for the parity invariance of electromagnetism.

As we shall see, a similar situation also occurs in the gauging of non-Abelian subgroups of $U(3)_L\times U(3)_R$. In this case, parity invariance may or may not be relevant depending on the specific application that one has in mind. Nevertheless, gauging of the Wess-Zumino term according to the trial-and-error procedure will always generate, in addition to a term whose variation reproduces the anomalies, a further term which is *gauge invariant* and is multiplied by an arbitrary coefficient. It should therefore be dropped for the purpose of representing the anomalies in an "irreducible" way. Such terms may always be reintroduced later, if they are needed for reasons other than the anomalies.

Let us now discuss the gauging of an arbitrary subgroup of $U(3)_L\times U(3)_R$. Introducing the group parameters

$$\epsilon_L=\sum_a \epsilon_L^a\frac{\lambda^a}{2}\,,\qquad \epsilon_R=\sum_a \epsilon_R^a\frac{\lambda^a}{2}\,, \tag{4.16}$$

where some of the ϵ_L^a or ϵ_R^a may be zero depending on the subgroup of interest and where $\lambda^a/2$ are the generators of U(3), we have the following transformation properties:

$$U\to e^{i\epsilon_L}Ue^{-i\epsilon_R}\,,$$
$$\delta A_L=d\epsilon_L+i[\epsilon_L,A_L]\,, \tag{4.17}$$
$$\delta A_R=d\epsilon_R+i[\epsilon_R,A_R]\,.$$

Trial-and-error gauging of the Wess-Zumino term proceeds along the same lines and we simply quote the final result:

$$\begin{aligned}\Gamma_{WZ}(U,A_L,A_R)=&\Gamma_{WZ}(U)+5Ci\int_{M^4}\mathrm{Tr}(A_L\alpha^3+A_R\beta^3)-5C\int_{M^4}\mathrm{Tr}[(dA_LA_L+A_LdA_L)\alpha+(dA_RA_R+A_RdA_R)\beta]\\&+5C\int_{M^4}\mathrm{Tr}[dA_LdU\,A_RU^{-1}-dA_Rd(U^{-1})A_LU]+5C\int_{M^4}\mathrm{Tr}(A_RU^{-1}A_LU\beta^2-A_LUA_RU^{-1}\alpha^2)\\&+\frac{5C}{2}\int_{M^4}\mathrm{Tr}[(A_L\alpha)^2-(A_R\beta)^2]+5Ci\int_{M^4}\mathrm{Tr}(A_L{}^3\alpha+A_R{}^3\beta)\\&+5Ci\int_{M^4}\mathrm{Tr}[(dA_R\,A_R+A_RdA_R)U^{-1}A_LU-(dA_LA_L+A_LdA_L)UA_RU^{-1}]\\&+5Ci\int_{M^4}\mathrm{Tr}(A_LUA_RU^{-1}A_L\alpha+A_RU^{-1}A_LUA_R\beta)\\&+5C\int_{M^4}\mathrm{Tr}[A_R{}^3U^{-1}A_LU-A_L{}^3UA_RU^{-1}+\tfrac{1}{2}(UA_RU^{-1}A_L)^2]-5Cr\int_{M^4}\mathrm{Tr}(F_LUF_RU^{-1})\ .\end{aligned}\tag{4.18}$$

This agrees with Witten's expression except that he inadvertently omitted the term

$$5Ci\int_{M^4}\mathrm{Tr}(A_L{}^3\alpha+A_R{}^3\beta)$$

and chose the arbitrary constant r to be 1. The last term which involves r is manifestly gauge invariant (the field strength two-forms F_L and F_R are defined as $F_L=dA_L-iA_L{}^2$, $F_R=dA_R-iA_R{}^2$) and should be dropped to give an irreducible gauging of Γ_{WZ}. If the gauge group is such that one can define a parity operation,

$$\begin{aligned}&A_L^\mu(\vec{x})\leftrightarrow-A_{R\mu}(-\vec{x})\ ,\\&U(\vec{x})\leftrightarrow U^{-1}(-\vec{x})\ ,\end{aligned}\tag{4.19}$$

all the terms in (4.18) will be parity invariant with the exception of the last term which will change sign under parity. This provides another reason to put $r=0$.

The variation of $\Gamma_{WZ}(U,A_L,A_R)$ under a gauge transformation gives, of course, the well-known anomaly expression

$$\begin{aligned}&\delta\Gamma_{WZ}(U,A_L,A_R)\\&\quad=-10Ci\int\mathrm{Tr}\left[\epsilon_L\left[(dA_L)^2-\frac{i}{2}dA_L{}^3\right]-(L\to R)\right]\ .\end{aligned}\tag{4.20}$$

V. CONSISTENCY CHECK FOR THE EFFECTIVE LAGRANGIAN

We notice that the expression for $\Gamma_{WZ}(U,A_L,A_R)$ in (4.18) is manifestly invariant under *global* $U_L(3)\times U_R(3)$ transformations. Hence, it is a logical first candidate for an effective term which mocks up the non-Abelian anomalies and is constructed from phenomenological meson fields. The gauge coupling constant g in (3.2) should be introduced into (4.18) by the rescaling $A_{L,R}\to g\,A_{L,R}$ and the phenomenological fields should be identified as in (3.1).

There is a possible immediate objection to this approach in that one expects[17] the equations of motion for a *fundamental* theory with some gauged anomalies to be inconsistent. This is simply seen by comparing the anomaly equation $D_\mu J_\mu=G$ with the covariant derivative of the field equation $D_\mu F_{\mu\nu}=J_\nu$. These two equations appear to be consistent with each other only if the anomaly G vanishes. However, when mass terms for the spin-1 fields are added to the Lagrangian the mutual consistency of these equations is better thought of as a complicated equation for the fields which may in fact be related to the mechanism for chiral-symmetry breakdown in QCD. In any event, it is almost certain that the low-energy effective Lagrangian for QCD involves many more new fields and interactions so one should not worry too much about the equations of motion. The effective Lagrangian is, after all, being used as a handy mnemonic to read off the relevant vertices. While it is true that one obtains the baryons "for free" as solitons from (1.2), the usual bosons of arbitrary spin are not expected to arise in this way if one takes the results of the $1/N_c$ approach[18] as a guide.

We would like our *hadronic* effective Lagrangian to display the usual feature that the neutral vector mesons dominate the electromagnetic form factors at low energies. This amounts to coupling vector mesons to the photon A_μ by the term[19]

$$\begin{aligned}\mathcal{L}_{EM}=&\frac{\sqrt{2}e}{g}A_\mu\left[m_\rho{}^2\rho_\mu^0+\tfrac{1}{3}m_\omega{}^2\omega_\mu-\frac{\sqrt{2}}{3}m_\phi{}^2\phi_\mu\right]\\&+O(A_\mu{}^2)\ ,\end{aligned}\tag{5.1}$$

and considering that other hadrons interact with A_μ through their couplings to the vector mesons. In (5.1) we have made the nonet ansatz

$$\begin{aligned}&\rho^0\sim\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})\ ,\\&\omega\sim\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\ ,\\&\phi\sim s\bar{s}\ .\end{aligned}\tag{5.2}$$

This vector-meson-dominance approach agrees quite well with experiment.

One can now imagine computing the amplitude for the famous reaction $\pi^0\to2\gamma$ in two ways. First, one could use (4.15) to reproduce the current-algebra theorem. From our present point of view, however, we would like to calculate this process from $\Gamma_{WZ}(U,A_L,A_R)$ together with (5.1), employing vector-meson dominance. The relevant vertex, as noted[8] many years ago, is a trilinear interaction among π^0, ρ^0, and ω and the diagram is shown in Fig. 1. This vertex is completely predicted as a piece of $\Gamma_{\mathrm{WZ}}(U,A_L,A_R)$ in the present approach. Defining $g_{VV\phi}$ by the coefficient of the appropriate term in the effective Lagrangian

$$\mathcal{L}_{VV\phi}=-g_{VV\phi}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\partial^\mu V^\nu\partial^\alpha V^\beta\tilde{\phi}) \quad (\epsilon_{0123}=+1)\,, \tag{5.3}$$

we identify, after some algebra and with use of (3.7) to describe the pion—axial-vector-meson mixing

$$g_{VV\phi}=\frac{3g^2}{16\pi^2\tilde{F}_\pi}\left[1-\frac{g^2\tilde{F}_\pi^{\,2}}{6m_\rho^{\,2}}\right]. \tag{5.4}$$

A straightforward calculation of Fig. 1 yields

$$\Gamma(\pi^0\to\gamma\gamma)=\frac{8\pi}{9}\frac{\alpha^2(g_{VV\phi})^2m_\pi^{\,3}}{g^4}\,, \tag{5.5}$$

where m_π is the pion mass and $\alpha\simeq\frac{1}{137}$ in this formula. The current-algebra result is

$$\Gamma_{\mathrm{CA}}(\pi^0\to\gamma\gamma)=\frac{\alpha^2}{32\pi^3}\frac{m_\pi^{\,3}}{\tilde{F}_\pi^{\,2}}\simeq 7.1\ \mathrm{eV}\,, \tag{5.6}$$

which nicely agrees with the experimental value of 7.9 eV. Comparing (5.5) and (5.6) shows that to reproduce the theorem one requires

$$g_{VV\phi}=\frac{3g^2}{16\pi^2\tilde{F}_\pi}\,. \tag{5.7}$$

Unfortunately, this is about 1.6 times larger than (5.4) so the $\Gamma(\pi^0\to\gamma\gamma)$ predicted from (4.18) is considerably smaller than the experimental value. Clearly the simplest first candidate for an anomalous term involving the phenomenological spin-1 fields is not a suitable one.

Now we note that the anomaly term in (4.18) was set up in such a way as to reproduce the left-right-symmetric variation $\delta\Gamma_{\mathrm{WZ}}$ in (4.20). It is well known that the choice of the anomalous variation is not unique but depends on a choice of counterterms. For example, in ordinary electromagnetism one could choose to have an anomaly in the vector current, in the axial-vector current, or even in some

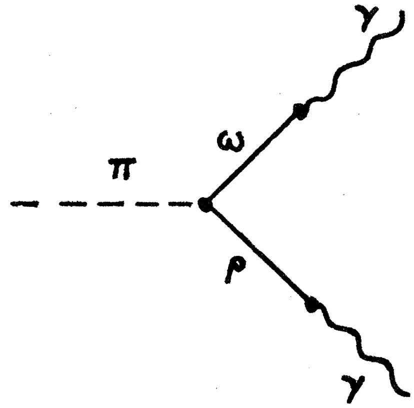

FIG. 1. Feynman diagram for $\pi^0\to\gamma\gamma$.

linear combination of them. To be able to renormalize the theory it is important to choose counterterms in such a way as to keep the vector current anomaly free. Now the anomalous variation (4.20), though very symmetrical, corresponds to a situation in which both the axial-vector and vector currents are anomalous. Although in this case there is evidently no requirement of renormalizability (for $\mathcal{L}_{\mathrm{eff}}$) the fact that phenomenology implies that the photon couples through the vector *mesons* at low energies strongly suggests that we try to mock up an anomalous variation which leaves the vector currents anomaly free. Such an anomalous variation was in fact the one chosen by Bardeen[5] in his original discussion of non-Abelian anomalies. In Sec. VI we shall discuss the additional terms needed in the effective Lagrangian for reproducing Bardeen's form of the anomaly. We shall see that this leads to the formula (5.7) for $g_{VV\phi}$, which perfectly agrees with the current-algebra theorem.

VI. BARDEEN'S FORM OF THE ANOMALY

We would like to find the extra term (counterterm) Γ_c which should be subtracted from (4.18) to arrive at Bardeen's anomalous variation. The answer to this question can be simply stated as follows. Given an action Γ whose anomalous variation is

$$\delta\Gamma=-10iC\int\mathrm{Tr}\left[\epsilon_L\left[(dA_L)^2-\frac{i}{2}dA_L^{\,3}\right]-(L\to R)\right], \tag{4.20$'$}$$

we consider the counterterm

$$\Gamma_c=5Ci\int_{M^4}\mathrm{Tr}[(dA_R\,A_R+A_R dA_R)A_L-(L\leftrightarrow R)] +5C\int_{M^4}\mathrm{Tr}[A_R^{\,3}A_L-A_L^{\,3}A_R+\tfrac{1}{2}(A_RA_L)^2]\,. \tag{6.1}$$

It is then a simple matter of algebra to show that

$$\begin{aligned}\delta(\Gamma-\Gamma_c) &=30Ci\int_{M^4}\mathrm{Tr}(\epsilon_R-\epsilon_L)\\ &\quad\times[\tfrac{1}{4}F_V^{\,2}+\tfrac{1}{12}F_A^{\,2}\\ &\qquad-\tfrac{1}{6}(F_V\,A^2+AF_V\,A+A^2F_V)-\tfrac{1}{6}A^4]\,,\end{aligned} \tag{6.2}$$

where

$$\begin{aligned}F_V&=dV+\frac{i}{2}(V^2+A^2)\,,\\ F_A&=dA+\frac{i}{2}(VA+AV)\,,\end{aligned} \tag{6.3}$$

and V and A are the vector and axial-vector fields defined in (3.1). [Note the slightly unconventional factor $\frac{1}{2}$ in (3.1).] Equation (6.2) is the anomaly expressed in Bardeen's form. Notice that it vanishes for vector transformations where $\epsilon_L=\epsilon_R$. The counterterm (6.1) is also the one considered by Bardeen who in fact first derived the anomaly in its left-right-symmetric form.

It is interesting to note that Γ_c in (6.1) can be gotten from $\Gamma_{\mathrm{WZ}}(U,A_L,A_R)$ in (4.18) by setting U equal to 1:

$$\Gamma_c=\Gamma_{\mathrm{WZ}}(1,A_L,A_R)\ . \tag{6.4}$$

The effective term which reproduces the Bardeen form of the anomaly is thus,

$$\Gamma'_{\mathrm{WZ}}(U,A_L,A_R)=\Gamma_{\mathrm{WZ}}(U,A_L,A_R)-\Gamma_{\mathrm{WZ}}(1,A_L,A_R)\ . \tag{6.5}$$

This clearly shows that Γ' vanishes when $U=1$ which was the boundary condition used by Wess and Zumino[4] in their original paper.

When the $g_{VV\phi}$ coupling constant defined in (5.3) is computed from Γ'_{WZ} one finds that the counterterm Γ_c gives a contribution which exactly cancels the second term of (5.4) leading to $g_{VV\phi}$ of (5.7), and hence the $\pi^0\rightarrow 2\gamma$ current-algebra theorem. Hence, we will use (6.5) as the preferred anomalous term for our effective Lagrangian. In this way, with the inclusion of Γ_c we will obtain some predictions for hadronic processes which agree well with experiment.

An intriguing feature of Γ_c in (6.1) is that it breaks the *global* $\mathrm{U}_L(3)\times\mathrm{U}_R(3)$ chiral symmetry down to $\mathrm{U}_V(3)\times\mathrm{U}_A(1)$. This can be seen by noting that Γ_c is obtained from the manifestly globally chiral-symmetric form $\Gamma_{\mathrm{WZ}}(U,A_L,A_R)$ by setting $U=1$ which clearly breaks the chiral symmetry. Whereas, usually one attempts to formulate the effective theory in such a way that the global chiral symmetry is present at the Lagrangian level and only (spontaneously) broken on the physical states, in the present case we are forced to introduce some explicit breaking terms. This breaking clearly does not affect any of the usual current-algebra theorems (which involve amplitudes independent of $\epsilon_{\mu\nu\alpha\beta}$) and therefore cannot be criticized on phenomenological grounds. It is not difficult to see that any effective action which leads to Bardeen's form of the anomaly violates *global* chiral symmetry since Bardeen's variation, unlike the left-right-symmetric one in (4.20), cannot be expressed as a total differential.

VII. $\omega\rightarrow 3\pi$

The $\omega\rightarrow 3\pi$ decay is the most well known purely hadronic process for which the product of intrinsic parity numbers, as defined in Sec. II, is not conserved. It does not seem to have been previously treated in the effective-chiral-Lagrangian framework although it has been the subject of numerous discussions by a variety of techniques. An early approach by Gell-Mann, Sharp, and Wagner[8] (GSW) computed this process according to the $\omega\rho\pi$ pole diagram which also enters into our model. In those days the $\omega\rho\pi$ vertex strength was considered an arbitrary parameter, whereas the present approach anchors its value in terms of the strength of the Wess-Zumino term and Sakurai's coupling constant. In the GSW model, which was motivated by dispersion theory, there was no reason to neglect a possible four-point contact term. A later current-algebra calculation by Ali and Hussain[4] included the contact term as a parameter but did not directly predict its value. In the present model this term is also predicted as a piece of the gauged Wess-Zumino term (6.5). The contribution of the contact term turns out actually to be quite small compared to the pole term, thereby giving a justification for the GSW model. It is worthwhile to remark that the effective Lagrangian based on the gauged Wess-Zumino term, while expressing the same physics as many earlier models, is intended to give a convenient, systematic and compact framework for discussing all processes related to the non-Abelian anomalies. In such a framework the strengths of all vertices will be determined by the coefficients of the Wess-Zumino term and the phenomenological $\rho\pi\pi$ coupling constant g.

The needed Feynman diagrams for $\omega\rightarrow 3\pi$ are shown in Fig. 2. In addition to the $\omega\rho\pi$ vertex described by (5.3) and (5.7) we use the minimal $\rho\pi\pi$ interaction term derived from (1.2) with the substitution indicated by (3.2):

$$\mathcal{L}_{V\phi\phi}=\frac{ig}{2}\,\mathrm{Tr}(V_\mu\tilde{\phi}\overleftrightarrow{\partial}^{\mu}\tilde{\phi})\ , \tag{7.1}$$

as well as the contact term derived, after some algebra, from (6.5)

$$\mathcal{L}_{V\phi\phi\phi}=ih\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(V^{\mu}\partial^{\nu}\tilde{\phi}\,\partial^{\alpha}\tilde{\phi}\,\partial^{\beta}\tilde{\phi})\ ,$$
$$h=\frac{-g}{2\pi^2\tilde{F}_\pi^{\,3}}\left[1-\frac{3}{4}\left[\frac{g^2\tilde{F}_\pi^{\,2}}{m_\rho^{\,2}}\right]+\frac{3}{32}\left[\frac{g^2\tilde{F}_\pi^{\,2}}{m_\rho^{\,2}}\right]^2\right]. \tag{7.2}$$

It might be comforting to the reader who attempts to check (7.2) [and (5.7)] for us to mention that we have also checked these expressions by using the series expansion method[4] of Wess and Zumino.

The amplitude for

$$\omega_\mu(p)\rightarrow\pi^+(q^+)+\pi^0(q^0)+\pi^-(q^-)$$

is computed to be

$$\mathcal{M}_\mu=i\epsilon^{\mu\nu\alpha\beta}q_\nu^{\,0}q_\alpha^{\,-}q_\beta^{\,+}F\ , \tag{7.3}$$

$$F=-3h+\frac{3g^3}{8\pi^2\tilde{F}_\pi}\sum_{a=+,-,0}\frac{1}{(p-q^a)^2+m_\rho^{\,2}}\ . \tag{7.3a}$$

This is related to the experimental width by

$$\Gamma(\omega\rightarrow\pi^+\pi^0\pi^-)$$
$$=\frac{m_\omega}{192\pi^3}\int\int dE^+dE^-[(\vec{q}^{\,-})^2(\vec{q}^{\,+})^2-(\vec{q}^{\,+}\cdot\vec{q}^{\,-})^2]F^2\ , \tag{7.4}$$

where E^+ and E^- are the π^+ and π^- energies. A numerical evaluation of the integral (7.4) yields the prediction

$$\Gamma(\omega\rightarrow\pi^+\pi^-\pi^0)=7.6\ \mathrm{MeV}\ . \tag{7.5}$$

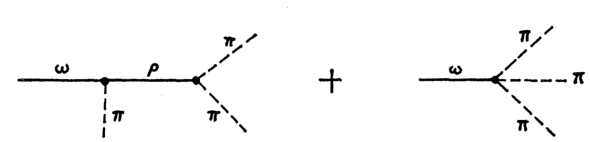

FIG. 2. Feynman diagrams for $\omega\rightarrow 3\pi$.

This corresponds to the parameter choices $m_\pi = 140$ MeV, $m_\rho = 769$ MeV, $m_\omega = 782$ MeV, $m_\pi/\tilde{F}_\pi = 1.05$, and g given by (3.4). The experimental value is $\Gamma(\omega \to \pi^+\pi^-\pi^0) = 8.9 \pm 0.3$ MeV so the prediction (7.5) is quite reasonable—about 17% too low. For comparison the current-algebra prediction for $\pi^0 \to 2\gamma$ in (5.6) is about 11% too low. As a rule of thumb current-algebra predictions are expected to be trusted to about 20%. Thus, it would be premature to consider corrections to (7.3) involving, for example, a more complicated momentum dependence at the $\rho\pi\pi$ vertex. We note that the second term of (7.3a) involving the sum of three different propagators is fairly constant over the entire phase-space region. It dominates the first contact term. Actually, ignoring the contact term makes the prediction almost perfect, but we have no reason to regard this as significant. For comparison the effective model based on (4.18) leading to a left-right-symmetric form of the anomaly would give a prediction $\Gamma(\omega \to \pi^+\pi^-\pi^0) = 2.6$ MeV. This bad result can, as in the discussion of Sec. V, be traced to too small a value for $g_{VV\phi}$. (The contact term in this case is also very small.) This situation confirms the importance of Bardeen's counterterm (6.1) in the effective Lagrangian.

VIII. $\omega \to \pi^0\gamma$

The phenomenological Lagrangian under study implies many predictions for both purely hadronic and radiative processes involving spin-1 mesons. For the radiative processes we have in mind the procedure outlined in Sec. V, based on vector-meson dominance. The simplest radiative reaction is the decay of a vector meson into a pseudoscalar and a photon. There are about a dozen of these processes. Here we shall compute $\omega \to \pi^0\gamma$ which is the one with the largest width and for which there is very good experimental information. In the present model, which is essentially identical to the GSW model for this process, the Feynman diagram shown in Fig. 3 involves the $\omega\rho\pi$ vertex again. A straightforward calculation gives the formula for the width,

$$\Gamma(\omega \to \pi^0\gamma) = \frac{3}{64\pi^4} \frac{\alpha g^2}{\tilde{F}_\pi^{\,2}} |\vec{q}_\pi|^3 = 0.80 \text{ MeV} . \tag{8.1}$$

This is in good agreement with the experimental value $\Gamma(\omega \to \pi^0\gamma) = 0.86 \pm 0.05$ MeV. Other radiative processes such as $\rho^+ \to \pi^+\gamma$, $K^{0*} \to K^0\gamma$, $\eta' \to \rho^0\gamma$, etc. can be related to (8.1) by simple phase-space and Clebsch-Gordan factors corresponding to the Okubo-Zweig-Iizuka rule (which is implicit in our model). The situation has been recently reviewed by O'Donnell.[20] There appear to be some nontrivial SU(3)-symmetry-breaking effects; we hope to return to this problem elsewhere.

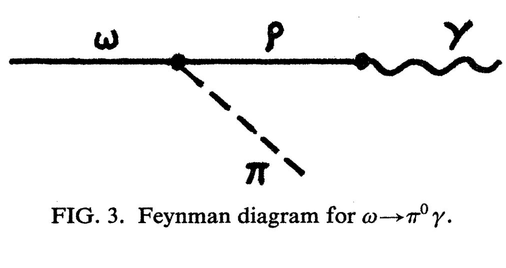

FIG. 3. Feynman diagram for $\omega \to \pi^0\gamma$.

The radiative meson decay in conjunction with vector-meson dominance may lead to some other interesting predictions.

In a recent paper[21] Freund and Zee have compared the calculation of $\gamma \to 3\pi$ using current algebra [essentially (4.15)] with the calculation based on the GSW model. For mutual consistency they found the well known relation[22] $g^2\tilde{F}_\pi^{\,2}/m_\rho^{\,2} = 2$ (1.67 experimentally). In our model, because of the presence of the contact term, this formula would be slightly modified. We should remark, however, that the implied extrapolations to zero momentum involved in the use of the current-algebra formula for radiative decays with three or more π mesons seems a bit hard to justify insofar as the very important ρ-meson form-factor effects are being neglected.

IX. SUMMARY AND DISCUSSION

In this paper we have attempted to generalize the old phenomenological prescription for introducing spin-1 meson fields into the chiral Lagrangian of spin-0 mesons, including the Wess-Zumino term. The resulting Lagrangian makes a very large number of predictions. We have examined in detail the processes $\pi^0 \to \gamma\gamma$ (as a check), $\omega \to \pi^0\gamma$, and $\omega \to 3\pi$. The results agree rather well with experiment, comparable to any current-algebra predictions. It appears that the underlying reason for this good agreement is the connection of these processes with the non-Abelian anomaly which has a specific structural form in field theories. While these processes have of course been computed with similar results a number of times in the last twenty years, the distinguishing feature of the present approach is the determination of all vertices directly from the coefficient of the Wess-Zumino term and Sakurai's ρ-meson coupling constant. An advantage of working in the Lagrangian framework is that the various assumptions being made are more clearly related to the symmetry structure of the underlying theory.

It would be desirable to try to derive the effective Lagrangian by making approximations directly on the generating functional of QCD. While this would be unlikely to change the phenomenological consequences of our model it may shed some light on the theoretical questions we have discussed in the text. In particular, the explicit chiral-symmetry breakdown $U(3)_L \times U(3)_R \to U(3)_V \times U_A(1)$ which appears to be forced upon us may get clarified.

Finally, it seems very interesting to investigate in more detail the rich variety of processes contained in this model. We hope to report further results elsewhere.

Note added. After this paper was submitted for publication three papers have appeared which also deal with the gauging of the Wess-Zumino term. K. C. Chou, H. Y. Guo, and K. Wu [Phys. Lett. **134B**, 67 (1984)] found an interesting condition for the anomaly-free gauging of the Wess-Zumino term and their final result, for the Wess-Zumino action agrees with our (4.18) with $r=0$. We disagree with their statement that Witten's formula [rather than our (4.15)] is correct for the U(1) gauging. They do not obtain our formula (6.5) for the effective action which corresponds to the Bardeen form of the anomaly. Similarly, H. Kawai and S. Tye [Cornell report, 1984 (unpublished)] obtained the same action using the methods of Ref. 11. Finally N. K. Pak and P. Rossi [CERN report, 1984 (unpublished)] gave an explicit integration of the Wess-Zumino consistency condition and also discussed the anomaly in Bardeen's form. Their result also agrees with our (4.18) with $r=0$ as well as our (6.1). Our Eq. (4.18), it should be recalled, also gives Witten's (non-parity-invariant) result when we set $r=1$.

ACKNOWLEDGMENTS

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