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Polarization tests of one-particle-exchange mechanisms

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Since one-particle-exchange (OPE) mechanisms are predominant in all aspects of elementary-particle dynamics, a novel class of polarization tests is proposed for such mechanisms. They test whether a single particle of total angular momentum J is exchanged ("J constraints") and whether the process can be factorized into two vertices ("factorization constraints"), but the tests are independent of more detailed dynamical features such as the exact nature of the coupling at the vertices. Except for a restricted type of processes containing some low spin values, the constraints reduce the number of reaction amplitudes and offer tests of OPE which are independent of the value of J. The tests have a particularly simple form in a "magic" formalism in which the quantization directions of the particles are in the reaction plane and are rotated from the helicity directions by a "magic" angle which can be easily specified for a given s and t. The tests consist of measuring whether a certain polarization quantity vanishes or not, thus providing sensitive "null experiments" for the exploration of particle dynamics. The results are illustrated on the popular reaction $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$, which is embodied, for example, in elastic nucleon-nucleon scattering. The tests can be used either for one single-exchange mechanism or for a combination of such mechanisms (even if they involve different J exchanges), as long as they all have the same type of parity.

I. INTRODUCTION

Practically all dynamical theories and models for elementary-particle reactions are and have been some variants of a one-particle-exchange (OPE) mechanism. At the same time we know that such theories and models represent only an approximation to the actual dynamics of such reactions. The validity of such approximations and the nature of the deviations from it have therefore been in the main focus of investigations for a number of decades. Much of the recent quark dynamics is also a variation on this same theme.

It is not easy, however, to test these OPE mechansims in a way that is model independent and convincing, and provides results in an easily interpretable form. Fitting cross sections alone, especially with models containing adjustable parameters, is a highly unsatisfactory and indecisive procedure, attested to by the cumbersome history of the various Regge-pole models and their eventual failure. In some instances the OPE mechanism is expected to contribute predominantly in certain kinematic ranges (e.g., one-pion exchange to nucleon-nucleon scattering in high-angular-momentum states), but such clearcut situations are not too frequent.

The aim of this paper is, therefore, to use the polarization structure of reactions for experimental tests of OPE mechanisms. Such polarization tests, in specific instances, on an *ad hoc* basis, have been proposed in the literature previously.[1] It seems, however, very much preferable and more economical to discuss this problem in a completely general way, in terms which can then be easily applied to any reaction and in any context. As we will see, this is quite feasible.

The first task is a semantic one, namely, the analysis of what is actually meant by "OPE mechanism." This is done in Sec. II. It is found that there are three types of constraints arising from what we will call OPE. The first, discussed in Sec. III, explores the constraints of having an object of definite J in the intermediate state. We will call this the J constraint. Then, in Sec. IV, we derive the constraints due to factorizability, including J constraints at the two factorized vertices.

The third type of constraint arises from the particular form of the couplings used at the two vertices. In this paper we will *not* discuss this part of the problem, for the following reasons.

(a) The first two types of constraints hold for any OPE, and hence test the basic assumption that particle dynamics is mediated by such a mechanism. In contrast, the third type of constraints is more dependent on the specific details of the dynamics, which can be varied in an almost infinite number of ways, thus creating the kind of "slippery" models which can, it appears, never be disproven by experiments.

(b) Since there are so many different variants of OPE models, their systematic discussion goes far beyond the possible confines of one article.

(c) The existing literature is mostly devoted to the discussion of such specific models, and although the subject has by no means been exhausted, it seems more worthwhile to contribute in a direction which is significantly different from the existing literature.

Section V offers an example, namely, the reaction $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$, on which the results of the previous sections are illustrated.

Finally, Sec. VI contains a summary and conclusions.

II. WHAT IS A ONE-PARTICLE-EXCHANGE MECHANISM?

In the parlance of the last 30—40 years, OPE can mean any of a number of things. It may be the lowest-order covariant Feynman diagram used to calculate amplitudes in an M matrix, or used in an iterative way in some scattering formalism or relativistic wave equation. It may mean such a reaction amplitude but "unitarized" in one of a number of different ways. It may mean an OPE *potential* inserted in a Schrödinger equation or in any of a number of relativistic two-body equations. In the case when the particle being exchanged is known to have a short lifetime, OPE may or may not include a recognition of this through a phenomenological modification of the propagator. All this may be couched in the language of standard field theory, or of dispersion theory, or of Regge polology, or of gauge theory.

If one claims to offer tests of OPE, therefore, one must be able to specify which of the above possibilities are covered by such tests. This can perhaps be best done by describing the particular features of the OPE that are being tested. There are two such features.

(A) The interaction takes place through an intermediate state of a definite and fixed J. In this respect it does not matter whether this state is indeed a one-particle state, and, if so, whether that particle is observed on-shell in other experiments, whether such a particle has a definite mass, whether that effective mass is real or complex, etc. All that matters is that the intermediate state possesses a fixed and definite total angular momentum J. As mentioned, constraints from this condition will be referred to as J constraints.

Some qualifications are in order in labeling an exchanged state by definite J. An exchanged particle is necessarily off-shell spacelike (excepting massless particles in forward elastic scattering) and does not have a rest frame in which to define its spin. Such an off-shell object, if it is an "elementary field," is described by an irreducible representation of the Lorentz group. That representation reduces to the $J, J-1, J-2, \ldots, 1$ representations of the rotation group for bosons and to two J, two $J-1, \ldots,$ two $\frac{1}{2}$ representations for fermions. Various constraint equations eliminate all but J when on-shell, but off-shell these "auxiliary spins" remain.[2] However, in any covariant treatment of the OPE model the auxiliary spins will not introduce independent couplings, since covariance relates the auxiliary spin amplitudes to the J amplitudes. In noncovariant approaches auxiliary spins never appear. So for the exchange of a "particle" the complexity of the amplitudes is determined by J alone. When a composite object (e.g., two-particle intermediate state, resonance, Regge pole) is exchanged, the meaning of J is more model dependent (as a recollection of the discussion of nonsense couplings in Regge theory will confirm).

(B) The interaction is describable in a factorized form, that is, as a product of two independent parts, each containing a set of physical particles plus the exchanged particle, such that the two sets are nonoverlapping and between them contain all physical particles of the reaction. For the most common case of a four-particle reaction, each part is a three-particle vertex with two (physical) particles on-shell and the exchanged particle off-shell. In determining the number of form-factors in each vertex, J constraints for such a vertex must be taken into account.

To conclude this section, we want to emphasize a crucial feature of our tests, namely, that they can be applied not only to a single exchange mechanism but also to a simultaneous dominance of several such mechanisms, possibly, each with a different J exchange, as long as they all have the same type (e.g., natural or unnatural) of parity. In other words, we are not testing for the dominance of exchange mechanisms of a single given J, but for the dominance of exchanges of one of two *classes,* the class being characterized by the type of parity (natural or unnatural) but otherwise possibly including exchanges with a number of different J's. This feature greatly extends the domain of applicability of the tests, since if we could deal with only one particular exchange mechanism at a time, or with exchanges of only one particular J, such a process could be expected to be obscured by many other processes in most kinematic configurations.

III. J CONSTRAINTS

The J constraints can be summarized easily. In a process, as shown in Fig. 1, initial or final states in the t channel which have $|J_z|>J$ cannot play a role. Since each amplitude for the reaction connects an initial state with a given $S_z^{(i)} \equiv s_{1z}+s_{2z}$ with a final state with a given $S_2^{(f)} \equiv s_{3z}+s_{4z}$, amplitudes involving at least one state with $|S_z|>J$ must vanish. This is true in all formalisms in which the quantization directions are so chosen that no orbital angular momentum can contribute. Thus, such quantization directions must lie in the reaction plane.

The rest of this section will convert the above verbal statement into quantitative results and prescriptions for these J constraints.

To start with, we want to group the usual optimal[3] states (characterized by s_z's along some quantization

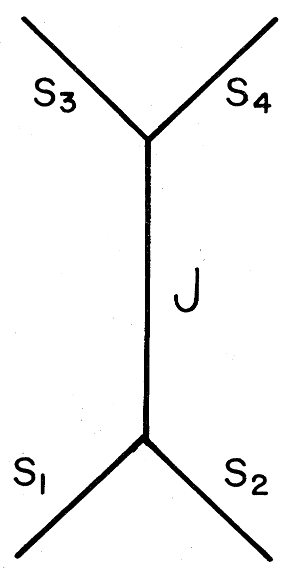

FIG. 1. Diagram of a four-particle process taking place through an intermediate state of given J.

direction) such that states with the same total S_z are together. We do this because in the J constraints only the total S_z matters and not the individual s_z's.

To do this grouping, visualize the optimal states represented by dots and arranged in a matrix form as shown in Fig. 2. It is evident from that diagram that the "multiplicity" n of a S_z set in terms of the optimal states is

$$\begin{aligned} n &= s_1+s_2-(S_z-1), \quad s_1+s_2 \geq S_z \geq s_2-s_1 , \\ n &= 2s_1+1, \quad s_2-s_1-1 \geq S_z \geq -(s_2-s_1-1) , \qquad (3.1) \\ n &= s_1+s_2+(S_z+1), \quad -(s_2-s_1) \geq S_z \geq -(s_1+s_2) . \end{aligned}$$

We assumed here that $s_1 \leq s_2$. In a similar graph for s_3 and s_4 we can assume $s_3 \leq s_4$.

The S_z sets and their multiplicities can be charted using a different type of diagram shown in Fig. 3. In it the multiplicity of each point is given by the product of the multiplicity of the row to which the point belongs and the multiplicity of the column to which the point belongs. The multiplicities of rows and columns are shown in Fig. 3 on the right and lower margins, respectively. The sum of the points, each weighted by its multiplicity, gives the total number of amplitudes for that reaction. In the example in Fig. 3 this is $5\times11\times9\times13=6435$, which is equal to the weighted sum

$$4(4.5\times10)(2.5\times6)+2\times9\times3\times(2.5\times6) \\ +2\times5\times5\times(4.5\times10)+45\times15 .$$

To ascertain now what constraints a J exchange produces, superimpose a square of side $2J$ onto the diagram in Fig. 3, concentrically with it, and count the weighted sum of the points within or on the boundary of the square. In Fig. 3 such a square for $J=4$ is indicated by dotted lines and can be seen to contain 3225 amplitudes. Thus, under $J=4$ constraint the remaining amplitudes must vanish.

The quite high numbers for the s_i's in Fig. 4 serve only to illustrate the general prescription. The practically more realistic numbers for the s_i's are 0, $\frac{1}{2}$, 1, and $\frac{3}{2}$. The diagrams for some processes with such spin values are shown in Fig. 4, together with the number of amplitudes under various constraints. N_T is the number of amplitudes without any J constraints and N_i is that number under the J constraints with $J=i$. These numbers can be read off the diagrams immediately, using the previous prescription. The $N_j^{(f)}$'s also given in Fig. 4 will be discussed in the next section.

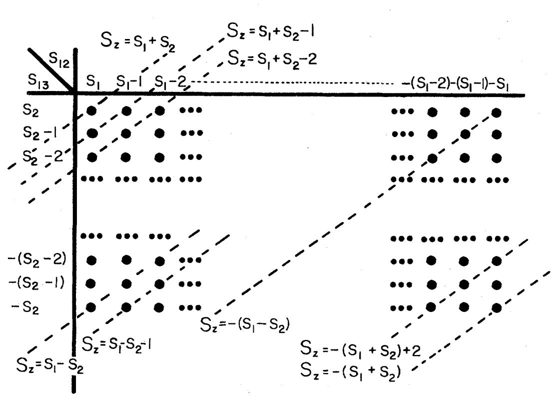

FIG. 2. Optimal states arranged for grouping into sets of given S_z. For an explanation, see the text.

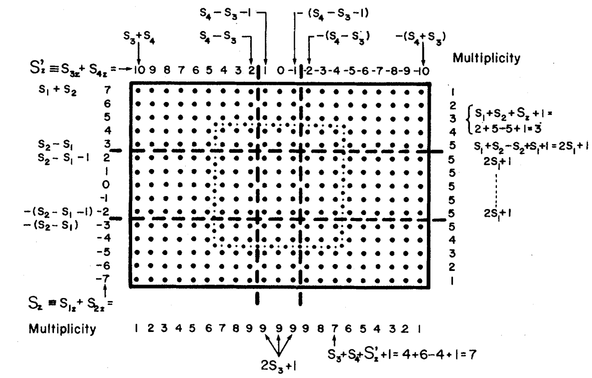

FIG. 3. Chart of the S_z sets. For an explanation, see the text. In this example $s_1=2$, $s_2=5$, $s_3=4$, and $s_4=6$.

We see from Fig. 4 that J constraints are effective even in some simple reactions (like $0+0\to\frac{1}{2}+\frac{1}{2}$), though they are more likely to have a marked effect in the case of higher spins (e.g., $\frac{1}{2}+\frac{1}{2}\to\frac{1}{2}+\frac{3}{2}$).

In Fig. 4 the reactions discussed are those with an s-channel single-J intermediate state. Appendix A shows how the argument can be carried over to the t channel.

So far we have only counted amplitudes but did not analyze *which* amplitudes and which corresponding observables will be affected by J constraints. We now turn to that question.

It is not difficult to find the answer. For reasons referred to earlier and discussed in Appendix A, we will use the "magic" planar optimal formalism for the direct channel. The amplitudes there are $D(c,a;d,b)$, where c, a, d, and b are the s_z values of the third, first, fourth, and second particles in the four-particle reaction $A+B\to C+D$. For a particular J constraint $D(c,a;d,b)=0$ if $|a-c|>J$ and/or $|b-d|>J$, that is, if at least one of $S_z^{(i)}$ and $S_z^{(f)}$ in the cross channel reaction is larger than J.

How do these constraints on amplitudes manifest themselves on observables?

In dealing with the observables, we will use the notation of Ref. 3, in which the observables for the reaction $A+B\to C+D$ are denoted by $L(uvH_p,UVH_P;\xi\omega H_q,\Xi\Omega H_Q)$, where u and v are the spin indices for particle A, the indices U and V refer to the spin of particle B, the indices ξ and ω to particle C, and the indices Ξ and Ω to particle D, and where each H can be either R (real) or I (imaginary).

Let us first consider two amplitudes $D(c_1,a_1;d_1,b_1)$ and $D(c_2,a_2;d_2,b_2)$, both of which are J forbidden. Then, as one can see from Eq. (2.30) of Ref. 3, the observables $L(a_1a_2H_p,b_1b_2H_P;c_1c_2H_q,d_1d_2H_Q)$ [of which there are, in general, eight (see Table I of Ref. 4)] all must vanish, because then all the other amplitude products appearing in the above-cited Eq. (2.30) also vanish, since at least one amplitude in each product will also be forbidden. There is, therefore, a large class of observables which van-

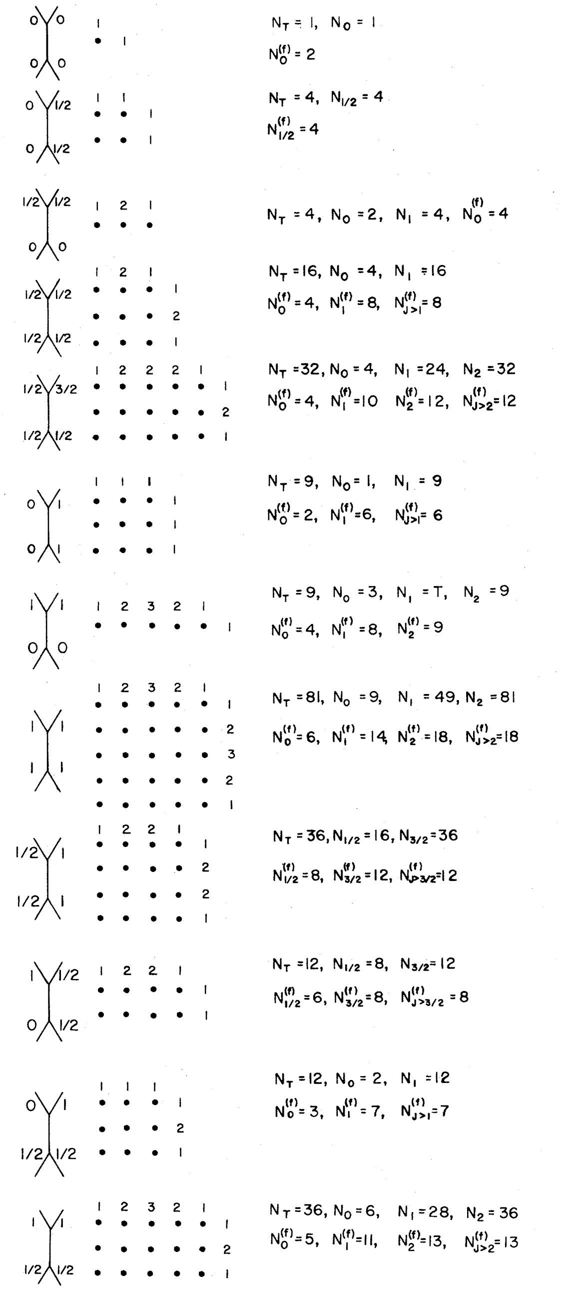

FIG. 4. J-constraint diagrams for some simple reactions and the number of amplitudes under various contraints. For an explanation, see the text.

ish under a particular J constraint, with arguments corresponding to pairs of forbidden amplitudes, as explained above.

But there are also other observables that vanish. Consider two amplitudes $D(c_1,a_1;d_1,b_1)$ and $D(c_2,a_2;d_2,b_2)$, such that $c_1+c_2+a_1+a_2=2J+\alpha$, where j is the parameter in the J constraint we are considering, and α is a positive quantity. It is evident that at least one of these two amplitudes will be J-forbidden, while the other may or may not be. Even in this case, however, all amplitude products in the above-cited Eq. (2.30) of Ref. 3 vanish and so do therefore the observables with the corresponding arguments.

The above results are formulated in terms of the direct (unaveraged) optimal observables in which the polarization states of all four particles are specified. In the experimentally more accessible or at least more traditional observables we either average over one or several of the arguments (corresponding to the use of unpolarized particles) or we take differences (i.e., measure the asymmetries of various sorts). The J constraints will manifest themselves also in these observables. Consider, for example, a reaction $A+B \to C+D$, in which A is a spin-$\frac{1}{2}$ particle. Consider now the amplitudes $D(c_1,\frac{1}{2};d_1,b_1)$ and $D(c_2,\frac{1}{2};d_2,b_2)$, and the corresponding observables

$$L(\tfrac{1}{2}\tfrac{1}{2},b_1b_2H_p;c_1c_2H_q,d_1d_2H_Q)\equiv L_+ \ .$$

If we do not want to measure the polarization of A or want to measure only asymmetries with respect to it, we have to consider also a second observable

$$L(-\tfrac{1}{2}-\tfrac{1}{2},b_1b_2H_P;c_1c_2H_q,d_1d_2H_q)\equiv L_-$$

and the associated two amplitudes $D(c_1,-\frac{1}{2};d_1,b_1)$ and $D(c_2,-\frac{1}{2};d_2,d_2)$, and then measure $L_+ + L_- \equiv \Sigma$ or $L_+ - L_- \equiv \pi$.

If both of the observables L_+ and L_- are J allowed, these observables are not suitable for OPE tests. If both observables are J-forbidden, they serve as OPE tests, and so do Σ and π. But we have OPE tests also if, say, L_+ is J-allowed and L_- is J-forbidden, since in that case $\Sigma=\pi$, a result that in general one would obtain only as an extremely rare "dynamical accident."

These general considerations will be illustrated on a specific example in Sec. V.

IV. FACTORIZATION CONSTRAINTS

The discussion of the factorization constraints will be carried out in the following four steps.

(A) Constraints due to factorization into two three-legged vertices, without regard to possible J constraints on the two vertices themselves.

(B) J constraints on one such three-legged vertex.

(C) Factorization constraints with J constraints on each three-legged vertex.

(D) Comparisons and summary.

We will now proceed to these four steps.

(A) To determine the constraints due to the factorizability of the vertices in one-particle-exchange processes, we must first consider the structure of the vertices separately. A simple three-particle vertex, say, the lower vertex in Fig. 1, has at most $(2s_1+1)(2s_2+1)(2s_3+1)$ amplitudes. Then the number of three-point amplitudes involved in the overall factorizable four-particle process with a J intermediate state is at most

$$N_f \equiv [(2s_1+1)(2s_2+1)+(2s_3+1)(2s_4+1)](2J+1) \ . \tag{4.1}$$

The overall reaction has, in general,

$$N \equiv (2s_1+1)(2s_2+1)(2s_3+1)(2s_4+1) \tag{4.2}$$

amplitudes. So this first constraint of factorization will reduce the number of independent amplitudes, providing $N_f < N$, which is equivalent to

$$\frac{1}{2J+1} > \frac{1}{(2s_1+1)(2s_2+1)} + \frac{1}{(2s_3+1)(2s_4+1)} \ . \tag{4.3}$$

As an example, consider a process involving $\frac{1}{2}+\frac{1}{2} \rightarrow \frac{1}{2}+\frac{1}{2}$. If $J=0$, then the inequality in Eq. (4.3) is satisfied and $N_f=8$ while $N=16$. On the other hand, for $J=1$ the inequality does not hold and so no reduction ensues. An example for this situation is nucleon-nucleon elastic scattering with one-pion exchange, except that in that case the constraints of additional symmetries need to be considered also.

(B) The counting of three-particle amplitudes above does not take into account the J constraints on the three-particle vertices. If we consider a real decay processs in which $J \rightarrow s_1+s_2$, then $s_{1z}+s_{2z}$ must not exceed J, as we have previously noted. To count the number of nonzero amplitudes, we can use the same enumeration procedure as in the four-particle process. It is easier, however, to observe that for each s_{12} value, where $\vec{S}_{12}=\vec{s}_1+\vec{s}_2$, the number of allowed $(S_{12})_z$ values is either $(2S_{12}+1)$ or $(2J+1)$, whichever is *smaller*. Then the total number of nonzero amplitudes

$$N_{Js_1s_2} = \sum_{S_{12}=|s_1-s_2|}^{s_1+s_2} (2S_{\min}+1) \ , \tag{4.4}$$

where $S_{\min}$ is S_{12} or J, whichever is smaller. We can write the result of this summation for three different cases:

$$N_{Js_1s_2} = (2s_1+1)(2s_2+1), \quad J \geq s_1+s_2 \ ,$$
$$N_{Js_1s_2} = (2J+1)(s_1+s_2-J) + (J+s_2-s_1+1)(J-s_2+s_1+1) \ , \quad s_1+s_2 \geq J \geq s_2-s_1 \ , \tag{4.5}$$
$$N_{Js_1s_2} = (2J+1)(2s_1+1), \quad s_2-s_1 \geq J \ ,$$

where we chose $s_1 \leq s_2$ for specificity. These numbers are considerably smaller than the product of the multiplicities of the legs. Useful examples are tabulated in Table I. Note that as long as all those legs are on-shell, the numbers obtained are independent of the choice of the "decaying" particle.

(C) With the numbers obtained from Eq. (4.5) we see that the total number of nonzero three-particle amplitudes that contribute to the four-particle process with a J intermediate state is

$$N_J^{(f)} = N_{Js_1s_2} + N_{Js_3s_4} \ . \tag{4.6}$$

Generally, this number is smaller than the number of J-constrained four-particle amplitudes, which is seen to be

$$N_J = N_{Js_1s_2} \times N_{Js_3s_4} \ . \tag{4.7}$$

Examples are included in Fig. 4.

(D) We can see from the examples and from the above general formulas that there are two different cases. In the first case, including the reactions $0+\frac{1}{2} \rightarrow 0+\frac{1}{2}$, $0+0 \rightarrow X+Y$, and $X+Y \rightarrow 0+0$, where X and Y are arbitrary, we have J-constraint tests for low values of J but not for higher J's. In the second case, including all other reactions, the quantity $N_J^{(f)}$, as a function of J, "saturates" below the value of N_T, thus providing tests of *any* J exchange, and hence for exchange in general, that is, for whether the reaction proceeds through a one-particle-exchange mechanism at all. As one goes toward higher spins, the difference (and even the ratio) between N_T and $N_J^{(f)}$ becomes increasingly large.

TABLE I. The number of amplitudes for various three-legged vertices. For the definition of the notation, see the text.

s_1	s_2	J	$N_{Js_1s_2}$	$(2s_1+1)(2s_2+1)(2J+1)$
0	0	0	1	1
0	$\frac{1}{2}$	$\frac{1}{2}$	2	4
0	0	1	1	3
0	1	1	3	9
1	$\frac{1}{2}$	$\frac{1}{2}$	4	12
0	$\frac{1}{2}$	$\frac{3}{2}$	2	8
0	$\frac{3}{2}$	$\frac{3}{2}$	4	16
1	$\frac{1}{2}$	$\frac{3}{2}$	6	24
1	$\frac{3}{2}$	$\frac{3}{2}$	10	48
1	1	1	7	27
0	1	2	3	15
1	1	2	9	45
$\frac{1}{2}$	$\frac{1}{2}$	2	4	20
$\frac{1}{2}$	$\frac{3}{2}$	2	8	40
$\frac{1}{2}$	$\frac{1}{2}$	3	4	28
$\frac{1}{2}$	$\frac{3}{2}$	3	8	56
1	1	3	9	63
1	1	$\frac{5}{2}$	9	54
1	1	$\frac{7}{2}$	9	72
0	$\frac{1}{2}$	$\frac{5}{2}$	2	12
1	$\frac{1}{2}$	$\frac{5}{2}$	6	36
0	1	3	3	21

In summary, factorization reduces the number of independent quantities on which the J-constrained four-particle amplitudes depend ($N_J^{(f)}$), thereby reducing the number of *independent* four-particle amplitudes. The total number of nonzero four-particle amplitudes N_J, however, is not reduced, but these N_J amplitudes no longer form an independent set. It should be noted that the factorization constraints constitute *nonlinear* constraints on the J-constrained four-particle amplitudes. The form that these nonlinear constraints take can be expressed simply as

$$\pm D_J(c,a;d,b) D_J(c',a';d',b') = D_J(c,a;d',b') D_J(c',a';d,b) \ , \tag{4.8}$$

where the $D_J(c,a;d,b)$ is an amplitude for $A+B \rightarrow C+D$ with spin projections a, b, c, and d along some quantization axes, and the exchanged particle is a spin-J intermediate state in the crossed channel $\bar{D}+B\to C+\bar{A}$, with $D_J(c,a;d,b)$ defined through the crossed channel angular momentum decomposition of the optimal amplitude in the helicity basis,

$$D(c,a;d,b)=\sum_J D_J(c,a;d,b)d^J_{c-a,d-b}(\theta_t)\ . \tag{4.9}$$

V. AN APPLICATION

The results of the previous sections will now be illustrated on a specific reaction, namely, on $\frac{1}{2}+\frac{1}{2}\to\frac{1}{2}+\frac{1}{2}$. This reaction was chosen for a number of reasons: It is a relatively simple process, yet OPE constraints are well manifested on it. It is also an experimentally well accessible and, in fact, quite well explored reaction, for example, in the form of proton-proton elastic scattering about which large sets[5] of polarization data are available, some of which have been phenomenologically analyzed.[6] Finally, the question of OPE is of considerable interest for that reaction.

Let us first explore the J constraints. As Fig. 4 shows, $N_0=4$ while $N_1=N_T=16$, so that the only J constraint for this reaction is for $J=0$, which shows that if the reaction is mediated entirely by the exchange of one pion and/or one of each of some other scalar or pseudoscalar particles, then, when only Lorentz invariance is considered, only four of the 16 amplitudes are nonzero.

In the cross channel, and hence also in the direct channel for the "magic" formalism (see Appendix A), the amplitudes that must vanish under $J=0$ constraint are

$$\begin{aligned} D(-+;\text{anything})&=D(+-;\text{anything})\\ &=D(\text{anything};+-)\\ &=D(\text{anything};-+)\\ &=0\ . \end{aligned} \tag{5.1}$$

In simpler terms, only the following four amplitudes remain nonzero:

$$\begin{aligned} &D(++;++),\ \ D(++,--)\ ,\\ &D(--,++),\ \ D(--;--)\ . \end{aligned} \tag{5.2}$$

It is easy to ascertain that once the additional constraints of parity conservation, time-reversal invariance, and identical particles are also imposed, and the original 16 amplitudes have been reduced to 5, the above 4 amplitudes are reduced to 2, namely, to a and c in the final notation of Ref. 7.

All that now remains is to read off from Tables V or VI of Ref. 7 the observables that vanish if only a and c are nonzero. In this case, the constraints are very severe. For example, out of the 26 observables listed in Table VI of Ref. 7, only 10 are nonzero, namely, σ_0, C_{LL}, K_{LL}, D_{LL}, $(0,z;z,0)$ H_{SSSS}, D_{NN}, D_{SS}, H_{LSN}, and H_{LNS}, and even among them we have the relationships

$$\begin{aligned} &\sigma_0=-D_{LL}=-H_{SSSS}=2(|a|^2+|c|^2)\ ,\\ &C_{LL}=-K_{LL}=-(0,z;z,0)=-2(|a|^2-|c|^2)\ ,\\ &D_{NN}=-D_{SS}=4\mathrm{Re}ac^*\ ,\\ &H_{LSN}=-H_{LNS}=4\mathrm{Im}ac^*\ . \end{aligned} \tag{5.3}$$

So much for the J constraints. Let us now turn to the factorization constraints.

The factorization constraints are expressed by Eq. (4.8). In our example, for $J=0$, we get

$$\begin{aligned} &D(++,++)D(--;--)\\ &\qquad=D(++,--)D(--;++)\ . \end{aligned} \tag{5.4}$$

When we impose the additional symmetries also, we get

$$a^2=c^2\ , \tag{5.5}$$

so now we have only one independent amplitude if we combine this with the J constraints for $J=0$. We get, instead of Eq. (5.3),

$$\sigma_0=-D_{LL}=-H_{SSSS}=\pm D_{NN}=\mp D_{SS}=4|a|^2\ , \tag{5.6}$$

and all other observables in Table VI of Ref. 7 vanish.

For $J\geq 1$ we have a whole set of 36 relationships arising from Eq. (4.8). With the additional symmetries these relations yield altogether three different relations,

$$a_J=\pm c_J,\ \ d_J=\mp e_J,\ \ a_Je_J=\pm {b_J}^2\ , \tag{5.7}$$

where the signs depend on the intrinsic parity of the spin-J exchange and the subscript J indicates that these are the $D_J(c,a;d,b)$ amplitudes of Eq. (4.9). Because a_J and c_J are both multiplied by $d^J_{\infty}(\theta)$ to obtain a, and c, respectively, we obtain

$$a=\pm c \tag{5.8}$$

for (natural/unnatural) parity exchange. The b_J, d_J, and e_J are each multiplied by different angular functions so that no simple relation remains between d and e. Then, substituting $a=\pm c$ into the observables of Table VI of Ref. 7, one obtains, for OPE constraints when $J\geq 1$,

$$\begin{aligned} &C_{LL}=K_{LL}=(0,z;z,0),\ \ C_{SS}=\mp K_{SS}\ ,\\ &H_{SNL}=\pm H_{SLN},\ \ H_{NSL}=\mp H_{NLS}\ , \end{aligned} \tag{5.9}$$

while

$$C_{NN}=K_{NN},\ \ C_{SL}=K_{SL},\ \ H_{SSN}=H_{SNS} \tag{5.10}$$

for $a=+c$ and

$$P=H_{NSS},\ \ D_{LS}=-H_{SSSL} \tag{5.11}$$

for $a=-c$.

It might be mentioned that in special situations and using particular kinds of couplings, one can of course obtain our results in other ways. For example, for the reaction involving four spin-$\frac{1}{2}$ particles, scalar-meson exchange in the usual coupling predicts only the identity operator while the pseudoscalar exchange in the usual coupling gives the operator $(\sigma_1\cdot r)(\sigma_2\cdot r)$ at long distances. These results correspond to the $a=\pm c$ result obtained above. The advantage of our method is, however, that it does not depend on the exact form of the coupling and holds for all J's of the exchanged particle.

VI. SUMMARY AND CONCLUSIONS

We have seen that one can construct a novel set of polarization tests for OPE mechanisms, which tests are independent of the specific details of the various mechanisms and which experimentally do not involve an extensive program of measurements. Indeed, they involve only the measurement of one or a few relatively easy polarization quantities, defined in the "magic" system in which the quantization direction of the particles is in the reaction plane and is rotated from the helicity direction by a certain given "magic" angle which varies with s and t. Such polarization tests can contribute decisively to the exploration of elementary-particle dynamics.

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APPENDIX A: CROSSING RELATIONS AND PLANAR AMPLITUDES

The J constraints and factorization constraints of Secs. III and IV have been applied to the amplitudes for which the spin-J particle is an intermediate state, that is, in the t-channel physical region. Hence, the statement that a particular set of amplitudes is zero is true for the t channel or crossed process. Of course the basis in which the amplitudes vanish must be specified. A natural basis is the t-channel helicity basis, for which there can be no "z component" of orbital angular momentum for initial or final state, in which case $|S_{1z}+S_{2z}|$ is limited by J. The same result will obtain for any basis in which the initial quantization axis is rotated in the scattering plane from the helicity direction, and independently for the final quantization axis.

Given that a certain number of t-channel amplitudes vanish, crossing guarantees that the number of independent s-channel amplitudes is correspondingly reduced. The simple question naturally arises: Is there a basis in which the same number of s-channel amplitudes vanish? To answer that question we first cross s-channel helicity amplitudes for $A+B\rightarrow C+D$ to t-channel helicity amplitudes for $\bar{D}+B\rightarrow C+\bar{A}$ in the s-channel physical region, that is,

$$G(c',d';a',b') = \sum_{a,b,c,d} d^{s_A}_{aa'}(-\chi_A)d^{s_B}_{bb'}(-\chi_B)d^{s_C}_{cc'}(-\chi_C) \times d^{s_D}_{dd'}(-\chi_D)D^h(c,a;d,b)\ , \tag{A1}$$

where $a,\dots,d$ are helicities for particles $A,\dots,D$. Furthermore $D^h(c,\dots)$ are s-channel helicity amplitudes, and a', b', c', and d' are helicities for $\bar{A}$, B, C, and $\bar{D}$ in that t channel for which $G(c',\dots)$ are helicity amplitudes. The phase convention used here treats B and D as "type-2" helicity states[7] in the s channel and B and $\bar{A}$ are type 2 in the t channel. The crossing angles are given by complicated functions of the kinematic variables s and t and of the masses.[8]

Now Eq. (A1) has the form of a planar (type-2) optimal amplitude in the s channel, that is, an amplitude in which the spin-quantization axis for particle A is obtained by a counter-clockwise rotation through χ_A about the normal to the scattering plane, and correspondingly for the other particles.[7] Direct comparison of Eq. (A1) with Eqs. (2.15) and (3.7) of Ref. 7 shows that the relation is precisely

$$D^{P^2}(c',a';d'b'\,|\,\chi_C,\chi_A{:}\chi_D,\chi_B)=G(c',d';a',b')\ . \tag{A2}$$

Hence an s-channel amplitude with *spin-quantization axes given by the crossing angles,* and components of spin along those axes given by a' for $A,\dots,d'$ for D, is the same as the continued t-channel helicity amplitude with helicity a' for $\bar{A},b'$ for B,c' for C, and d' for $\bar{D}$. Thus if J constraints force certain G's to vanish, the equivalent s-channel planar amplitudes will also vanish. This very useful planar basis will be referred to as the "magic" planar basis.

For the case of four equal masses, the χ's are given by[8]

$$\eta_i\cos\chi_i=\frac{-st}{[s(s-4m^2)t(t-4m^2)]^{1/2}}$$
$$i=A,B,C,D\ , \tag{A3}$$
$$\eta_a=\eta_D=+1=-\eta_B=-\eta_C\ ,$$

where s and t are the usual kinematic variables. This formula can be expressed in terms of the center-of-mass scattering angle as

$$\cos\chi_A=\left[1+\frac{4m^2}{s}\cot^2\frac{\theta}{2}\right]^{-1/2}, \tag{A4}$$

so that χ_A varies from $\pi/2$ down to 0 as θ varies from 0 to π.

APPENDIX B: SCALAR EXCHANGE IN A "MAGIC" BASIS

To illustrate how the "magic" basis for spin quantization makes J constraints manifest we consider the example of spin-0 exchange in $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$ in a covariant formalism. We assume no symmetries other than Lorentz invariance. Labeling the particles $A+B\rightarrow C+D$ as in Appendix A, the general form of the scattering amplitude for spin-0 exchange in the helicity basis will be expressible in factorized form as

$$D^h(c,a;d,b)=[\bar{u}_c(p_c)(\alpha_s+\alpha_p\gamma_5)u_a(p_a)] \times[\bar{u}_d(p_d)(\beta_s+\beta_p\gamma_5)u_b(p_b)]\ , \tag{B1}$$

where α_s, α_p, β_s, and β_p are scalar complex functions of kinematic variables and the subscripts $a,\dots,d$ on the Dirac spinors label the helicities. Because parity conservation is not assumed, both scalar and pseudoscalar couplings appear. To utilize the factorized form, let

$$D^h(c,a;d,b)=\left[\frac{2m}{E+m}\right]^2\Gamma_{ca}\Gamma'_{db}\ , \tag{B2}$$

where equal masses m for external lines will be assumed for simplicity and the values for Γ_{ca} will be

$$\begin{aligned}\Gamma_{++}&=\Gamma_{--}=\alpha_s\cos\frac{\theta}{2}\ ,\\ \Gamma_{+-}&=\left[\frac{E}{m}\alpha_s-\frac{p}{m}\alpha_p\right]\sin\frac{\theta}{2}\ ,\\ \Gamma_{-+}&=\left[\frac{E}{m}\alpha_s-\frac{p}{m}\alpha_p\right]\sin\frac{\theta}{2}\ ,\end{aligned} \tag{B3}$$

where E, p, and θ are center-of-mass energy, momentum, and scattering angle, respectively. Analogous expressions can be obtained for Γ'_{ab}.

Next we are interested in amplitudes for spin-quantization axes rotated away from the helicity directions but in the reaction plane. These are the planar optimal amplitudes.[7] Such amplitudes are obtained by applying rotation operators to each leg of the reaction separately, so that the planar amplitudes will also factorize and we may consider the Γ and Γ' rotations separately. Let $\tilde{\Gamma}$ and $\tilde{\Gamma}'$ be the corresponding planar factors, that is,

$$\tilde{\Gamma}_{c'a'}=\sum_{c,a}d^{1/2}_{a'a}(-\beta_A)d^{1/2}_{a'a}(-\beta_c)\Gamma_{ca}\ , \tag{B4}$$

where β_A and β_C are the clockwise rotation angles to the new quantization axes, and similarly for $\tilde{\Gamma}'$. A straightforward calculation, using Eq. (B3) for Γ, then yields

$$\tilde{\Gamma}_{\pm\mp}=\pm\alpha_s\left[\sin\left[\frac{\beta_A-\beta_C}{2}\right]\cos\frac{\theta}{2}-\frac{E}{m}\cos\left[\frac{\beta_A-\beta_C}{2}\right]\sin\frac{\theta}{2}\right]-\frac{p}{m}\alpha_p\cos\left[\frac{\beta_A+\beta_C}{2}\right]\sin\frac{\theta}{2}\ , \tag{B5}$$

$$\tilde{\Gamma}_{\pm\pm}=\alpha_s\left[\cos\left[\frac{\beta_A-\beta_C}{2}\right]\cos\frac{\theta}{2}+\frac{E}{m}\sin\left[\frac{\beta_A-\beta_C}{2}\right]\sin\frac{\theta}{2}\right]\mp\frac{p}{m}\alpha_p\sin\left[\frac{\beta_A+\beta_C}{2}\right]\sin\frac{\theta}{2}\ . \tag{B6}$$

Now, to choose planar angles for which the $J=0$ constraint is manifest, we must satisfy $\tilde{\Gamma}_{+-}=0=\tilde{\Gamma}_{-+}$. From Eq. (B5) we see that, because of the sign change in the α_s term, the α_s and α_p factors must vanish separately. For the α_p term that requires

$$\frac{\beta_A+\beta_C}{2}=\pm\frac{\pi}{2}\ \text{ or }\ \beta_C=\pm\pi-\beta_A\ . \tag{B7}$$

Then, to annihilate the α_s factor, we must have

$$\mp(\cos\beta_A)\cos\frac{\theta}{2}-(\sin\beta_A)\frac{E}{m}\sin\frac{\theta}{2}=0\ , \tag{B8}$$

the solution of which is

$$\cos^2\beta_A=\left[1+\frac{4m^2}{s}\cot^2\frac{\theta}{2}\right]^{-1}\ . \tag{B9}$$

This is precisely the equation for the crossing angle χ_A in Eq. (A4). So $\beta_A=\chi_A$ gives the magic basis for which $\tilde{\Gamma}_{+-}=0$. Finally, the remaining Γ's become

$$\tilde{\Gamma}_{\pm\pm}=\alpha_s\left[1+\frac{s}{4m^2}\tan^2\frac{\theta}{2}\right]^{1/2}\cos\frac{\theta}{2}\mp\frac{p}{m}\alpha_p\sin\frac{\theta}{2}\ . \tag{B10}$$

[1]For one discussion of such tests, see A. C. Irving and R. P. Worden, Phys. Rep. 34C, 117 (1977).

[2]See, for example, M. D. Scadron, Phys. Rev. 165, 1640 (1968); M. D. Scadron and H. F Jones, *ibid.* 173, 1734 (1968); H. W. Fearing, G. R. Goldstein, and M. J. Moravcsik, Phys. Rev. D 29, 2612 (1984).

[3]G. R. Goldstein and M. J. Moravcsik, Ann. Phys. (N.Y.) 98, 128 (1976).

[4]M. J Moravcsik, Phys. Rev. D 22, 135 (1980).

[5]Beside the extensive collection of older data below 500 MeV, there are more recent but also extensive sets of data up to 6 GeV/c by groups at LAMPF, TRIUMF, SIN, Saclay, and Argonne ZGS.

[6]Up to about 500 MeV such analyses were carried out in terms of partial waves. The most recent and higher-energy analyses have been in terms of amplitudes, see, for example, E. Aprile *et al.*, Phys. Rev. Lett. 46, 1047 (1980); G. R. Goldstein and M. J. Moravcsik, Phys. Lett. 102B, 189 (1981); Ref. 7; N. Ghahramany, G. R. Goldstein, and M. J. Moravcsik, Phys. Rev. D 28, 1086 (1983).

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