Isospin analysis of low-energy $\pi N \to \pi\pi N$ data and chiral-symmetry breaking

D. Mark Manley
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
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The amplitudes for πN single-pion production are decomposed into four independent isospin amplitudes in the $(\pi\pi)N$ representation. Reaction cross sections for the $\pi^- p$ and $\pi^+ p$ initial states below total energies of 1370 and 1470 MeV, respectively, are simultaneously fitted with near-threshold measurements of the double-differential cross section $d^2\sigma/d\Omega\, dT$ for the π^+ produced in $\pi^- p \to \pi^+\pi^- n$. The validity of soft-pion calculations for $\pi N \to \pi\pi N$ is demonstrated by showing that the threshold s-wave isospin amplitudes can be described consistently by a single value of the chiral-symmetry-breaking parameter.

I. INTRODUCTION

In recent years, several new measurements of the reaction cross sections for πN single-pion production have been obtained near threshold.[1-3] By isospin symmetry, all pion production cross sections can be expressed in terms of four isospin amplitudes and the phases between them. Unfortunately, only five charge channels are amenable to measurement whereas the isospin decomposition involves six independent parameters at a given energy. Near threshold, unitarity requires that the isospin amplitudes be almost real. At sufficiently low energy, one therefore should be able to unambiguously determine the amplitudes from the experimental data. When extrapolated to threshold, these amplitudes provide important tests of soft-pion calculations.[4] In this paper, it will be shown that the low-energy experimental data for $\pi N \to \pi\pi N$ are consistent with the assumption of isospin symmetry and with threshold predictions of soft-pion theory that describe all $\pi N \to \pi\pi N$ reactions in terms of the chiral-symmetry-breaking parameter ξ.

Section II presents low-energy parametrizations of the isospin amplitudes that satisfy the conditions of parity conservation and Bose statistics. The data included in this analysis are discussed in Sec. III. Section IV discusses the results of this analysis and compares the present determination of ξ with values from previous analyses. Finally, Sec. V summarizes the relevance of this analysis to isospin invariance and to soft-pion theory.

II. ISOSPIN AMPLITUDES

Isospin invariance of the strong interactions implies that amplitudes for different $\pi N \to \pi\pi N$ charge channels can be expressed in terms of four independent isospin amplitudes $A_{2I,I_{\pi\pi}}$. Here $I_{\pi\pi}$ is the isospin of the two final-state pions which couples with the isospin of the final-state nucleon to give a total isospin I. For the measurable charge channels, the decomposition is

$$A(\pi^- p \to \pi^0\pi^0 n) = \tfrac{2}{3}(\tfrac{1}{5})^{1/2} A_{32} + \tfrac{1}{3}\sqrt{2} A_{10} , \tag{1}$$

$$A(\pi^- p \to \pi^+\pi^- n) = \tfrac{1}{3}(\tfrac{1}{5})^{1/2} A_{32} - \tfrac{1}{3}\sqrt{2} A_{10} + \tfrac{1}{3} A_{31} - \tfrac{1}{3} A_{11} , \tag{2}$$

$$A(\pi^- p \to \pi^0\pi^- p) = -(\tfrac{1}{10})^{1/2} A_{32} + \tfrac{1}{3}(\tfrac{1}{2})^{1/2} A_{31} + \tfrac{1}{3}\sqrt{2} A_{11} , \tag{3}$$

$$A(\pi^+ p \to \pi^0\pi^+ p) = -(\tfrac{1}{10})^{1/2} A_{32} - (\tfrac{1}{2})^{1/2} A_{31} , \tag{4}$$

$$A(\pi^+ p \to \pi^+\pi^+ n) = (\tfrac{4}{5})^{1/2} A_{32} . \tag{5}$$

The low-energy isospin amplitudes can be parametrized by

$$A_{32} = a_{32} \chi_f^\dagger \vec{Q}\cdot\vec{\sigma} \chi_i , \tag{6}$$

$$A_{10} = a_{10} \chi_f^\dagger \vec{Q}\cdot\vec{\sigma} \chi_i , \tag{7}$$

$$A_{31} = a_{31} \chi_f^\dagger (\vec{q}_1 - \vec{q}_2)\cdot\vec{\sigma} \chi_i , \tag{8}$$

$$A_{11} = a_{11} \chi_f^\dagger (\vec{q}_1 - \vec{q}_2)\cdot\vec{\sigma} \chi_i , \tag{9}$$

where $a_{2I,I_{\pi\pi}}$ are reduced isospin amplitudes, χ_i and χ_f are spinors for the initial and final nucleons, $\vec{Q}$ is the momentum of the initial pion in the center-of-mass (c.m.) system, $\vec{q}_1$ and $\vec{q}_2$ are the momenta of the final-state pions in the c.m. system, and the components of $\vec{\sigma}$ are the Pauli spin matrices. The final-state pion with similar sign to the initial pion is denoted by subscript 2 and the other final-state pion is denoted by subscript 1. At threshold, when the final-state pions are at rest in the c.m. frame, A_{31} and A_{11} vanish, as required by Bose symmetry. The above parametrization has the advantage that only a single numerical integration is required for calculating reaction cross sections.

Unitarity requires that the threshold phases of a_{32} and a_{10} be equivalent ($\pm 180°$) to the πN phase shifts $\delta(P_{31})$ and $\delta(P_{11})$ at the appropriate energy. Since these phase shifts are $\lesssim 5°$ (Ref. 5), a_{32} and a_{10} were approximated by real constants at production threshold. In the absence of low-energy resonances, a_{31} and a_{11} are also expected to be approximately real near threshold.

At low energy, a_{10} is strongly influenced by the presence of the $P_{11}(1440)$ (Roper) resonance. To describe this

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influence, a_{10} was approximated by a modified Breit-Wigner form:

$$a_{10}=a'_{10}\frac{M-W_T}{M-W-\frac{i\Gamma}{2}\left[\frac{W-W_T}{M-W_T}\right]^2}\ , \tag{10}$$

where W is the total c.m. energy and W_T is the total energy at threshold. a'_{10}, M, and Γ were treated as variable parameters. Production cross sections for $\pi^- p$ reactions are well described below a total energy of 1370 MeV with a_{10} approximated by this resonance form and with a_{11} approximated by a real, constant parameter. At higher energies, a_{10} and a_{11} are influenced by other $I=\frac{1}{2}$ resonances. Production cross sections for $\pi^+ p$ reactions are well described below a total energy of 1470 MeV with a_{31} and a_{32} approximated by real, constant parameters.

III. DATA BASE

A total of 88 data were included in this analysis, 37 of which were measured within the last five years. The data base consists of 38 measurements[1,2,6,7] below 1370 MeV of $\pi^- p$ reaction cross sections, 23 measurements[3,8,9] below 1470 MeV of $\pi^+ p$ reaction cross sections, and of 27 measurements[1] at 1242 and 1262 MeV of $d^2\sigma/d\Omega\, dT$, the double-differential cross section for the π^+ produced in $\pi^- p \rightarrow \pi^+\pi^- n$. Measurements of the double-differential cross section at higher energies were not included because of the sensitivity of those measurements to angular variations associated with the $\pi\Delta$ intermediate state. To avoid double counting, measurements in Ref. 1 of the total $\pi^- p \rightarrow \pi^+\pi^- n$ cross section at 1242 and 1262 MeV were excluded from the data base.

IV. RESULTS

The data were fitted by the method of least χ^2. Normalization parameters were included in the $d^2\sigma/d\Omega\, dT$ measurements at fixed energy W as multiplicative factors of the predicted values. Normalization uncertainties of 3% were assumed, as in Ref. 1. All $d^2\sigma/d\Omega\, dT$ measurements at a given energy W were simultaneously renormalized during minimization. Both differential and total cross sections were calculated using the known physical masses of the pion and nucleon to allow for the greater phase space available near threshold whenever one of the final pions was electrically neutral. The total collection of data was fitted with $\chi^2/\nu=2.1$, where ν is the number of degrees of freedom. This high value of χ^2/ν is mainly ascribable to inconsistencies in the data set, which are discussed below. The values of a_{32} and a'_{10} obtained in this manner are 2.60 m_π^{-3} and 7.12 m_π^{-3}, respectively. These values may be compared with soft-pion calculations which predict[4]

$$a_{32}=3.05+1.21\xi\, m_\pi^{-3}\ , \tag{11}$$

$$a'_{10}=6.15-1.91\xi\, m_\pi^{-3}\ , \tag{12}$$

where ξ is the chiral-symmetry-breaking parameter. Thus, the fitted values of a_{32} and a'_{10} are consistent with a single value of ξ which is somewhat less than zero.

χ^2 was not significantly improved by introducing additional parameters to describe the isospin amplitudes. This conclusion was reached by generating solutions with a_{31} and a_{32} parametrized by linear functions of W and with a_{11} parametrized by a resonant form similar to that of a_{10}. When the entire data set was considered, six data points were fitted poorly in all solutions. These include three difficult measurements of the cross section for $\pi^- p \rightarrow \pi^0\pi^0 n$ below 1300 MeV. At energies of 1239, 1269, and 1292 MeV, the measured experimental cross sections are, respectively, 0.032 ± 0.005 mb (Ref. 2), 0.13 ± 0.02 mb (Ref. 2), and 0.32 ± 0.04 mb (Ref. 6). The corresponding theoretical values found in all solutions are 0.01 mb, 0.07 mb, and 0.2 mb. Also poorly fitted were measurements from a single experiment (Ref. 8) of the cross sections for $\pi^+ p \rightarrow \pi^0\pi^+ p$ and $\pi^+ p \rightarrow \pi^+\pi^+ n$ at

TABLE I. Parameters for isospin amplitudes. a_{32} and a'_{10} and their errors were derived from the fitted value of ξ.

ξ	-0.25 ± 0.11
a_{32}	$2.75\pm0.13\, m_\pi^{-3}$
a'_{10}	$6.63\pm0.21\, m_\pi^{-3}$
a_{31}	$-6.02\pm0.31\, m_\pi^{-3}$
a_{11}	$10.61\pm0.62\, m_\pi^{-3}$
M	1416 ± 14 MeV
Γ	287 ± 43 MeV

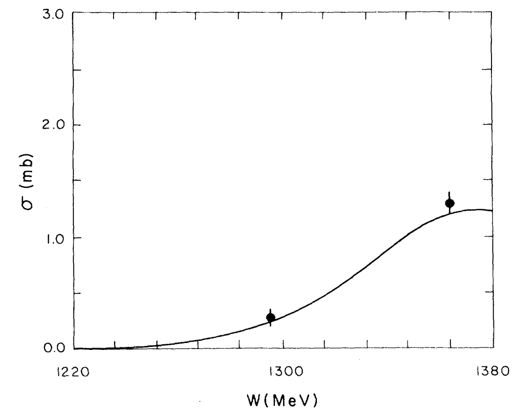

FIG. 1. Reaction cross sections for $\pi^- p \rightarrow \pi^0\pi^0 n$.

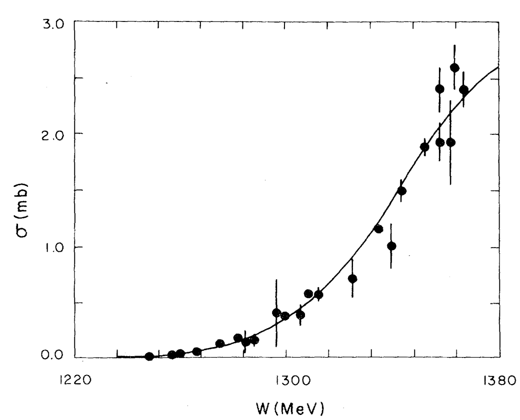

FIG. 2. Reaction cross sections for $\pi^- p \rightarrow \pi^+\pi^- n$.

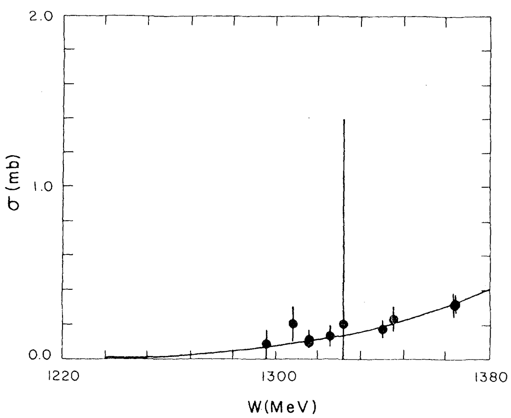

FIG. 3. Reaction cross sections for $\pi^- p \to \pi^0 \pi^- p$.

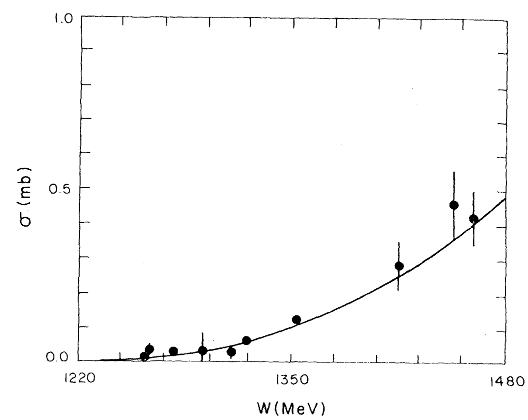

FIG. 5. Reaction cross sections for $\pi^+ p \to \pi^+ \pi^+ n$.

1428 MeV. For the first reaction, the experimental and theoretical cross sections are, respectively, 0.80 ± 0.05 mb and 0.94 mb. For the second reaction, the experimental and theoretical cross sections are, respectively, 0.18 ± 0.02 mb and 0.26 mb. The final point with a large contribution to χ^2 is a measurement of $d^2\sigma/d\Omega\, dT$ at $W = 1262$ MeV, $T = 30.5$ MeV, and $\cos\theta = 0.849$. The experimental value is 0.158 ± 0.016 μb/sr MeV (Ref. 1), whereas the theoretical value is 0.21 μb/sr MeV.

In the hope of improving the stability of the results, the data base was pruned by eliminating the above six points. Since the data base is fairly small, only those points that are clearly inconsistent were removed. The pruned data set was then fitted as before except that ξ was searched directly assuming a_{32} and a'_{10} to be given by Eqs. (11) and (12). The pruned data set was fitted with $\chi^2/\nu = 1.3$. In view of this much better χ^2/ν, the remainder of this paper is focused on the reduced data base and the pruned solution, which is summarized in Table I. The predicted values of $d^2\sigma/d\Omega\, dT$ at 1242 MeV were renormalized by the factor 1.03 ± 0.05 and those at 1262 MeV were renormalized by 0.98 ± 0.05. Parameter errors were estimated, as usual, by the change in any parameter that increases χ^2 from its minimum value by 1. To reflect the fact that the cross section measurements often contain normalization errors (on the order of 5%) which were neglected or underestimated by the experimentalists, these estimates have been conservatively tripled for all errors tabulated in this paper.

The values obtained for M and Γ are well within the accepted range[10] for the mass and width of the Roper resonance, which is the first resonance that can influence a_{10}. As a consequence of the mass of the resonance being outside the fit region, there is a strong correlation in M and Γ. When a'_{10} and a_{32} were varied independently, a'_{10} was found to be strongly correlated with M and Γ, which implies that, by itself, a'_{10} is less reliable than a_{32} for determining ξ. Error correlations are small for the other parameters.

Although reaction cross sections determine the relative signs a'_{10} and a_{32} and of a_{11} and a_{31}, the relative signs of a'_{10} and a_{11} are determined entirely from interference effects in the double-differential cross sections. Models of pion production in which one-pion-exchange contributions dominate near threshold predict $a_{11}/a_{31} \simeq -2$ (Ref. 11). This prediction is in reasonable agreement with the present result, $a_{11}/a_{31} = -1.8 \pm 0.1$.

The present effort determines $\xi = -0.25 \pm 0.11$. This value may be compared with the results of two earlier analyses. Bjork *et al.*[1] found $\xi = 0.05 \pm 0.26$ from a

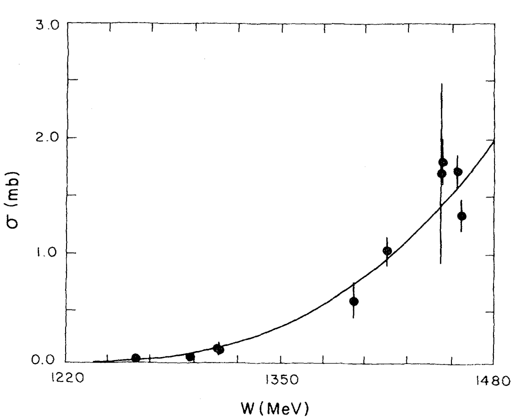

FIG. 4. Reaction cross sections for $\pi^+ p \to \pi^0 \pi^+ p$.

TABLE II. Results of the fit for $d^2\sigma/d\Omega\, dT$, the double-differential cross section for the π^+ produced in $\pi^- p \to \pi^+ \pi^- n$ at 1242 MeV. T is the kinetic energy of the π^+ in the c.m. system and θ is its scattering angle. Units of the double-differential cross section are μb/sr MeV.

T (MeV)	$\cos\theta$	Expt. (Ref. 1)	Fitted
11.0	−0.106	0.0705±0.0129	0.0495
15.5	−0.148	0.0732±0.0236	0.0414
15.4	0.112	0.0479±0.0135	0.0517
6.5	0.193	0.0744±0.0113	0.0539
11.0	0.194	0.0716±0.0131	0.0607
6.4	0.493	0.0595±0.0082	0.0614
10.9	0.505	0.0855±0.0094	0.0722
15.3	0.474	0.0730±0.0100	0.0659
6.6	0.784	0.0845±0.0127	0.0700
10.9	0.822	0.0864±0.0104	0.0839
15.2	0.840	0.0778±0.0109	0.0804

TABLE III. Results of the fit for $d^2\sigma/d\Omega\, dT$, the double-differential cross section for the π^+ produced in $\pi^- p \to \pi^+\pi^- n$ at 1262 MeV. T is the kinetic energy of the π^+ in the c.m. system and θ is its scattering angle. Units of the double-differential cross section are μb/sr MeV.

T (MeV)	$\cos\theta$	Expt. (Ref. 1)	Fitted
24.5	−0.356	0.159±0.026	0.100
31.1	−0.357	0.096±0.026	0.079
17.4	−0.057	0.172±0.019	0.136
24.7	0.045	0.183±0.027	0.144
31.0	0.043	0.105±0.012	0.120
10.4	0.141	0.146±0.018	0.133
6.0	0.288	0.128±0.015	0.111
10.3	0.442	0.154±0.015	0.152
17.6	0.447	0.208±0.025	0.185
24.6	0.446	0.185±0.023	0.189
32.1	0.406	0.128±0.014	0.147
3.3	0.665	0.096±0.016	0.092
10.5	0.748	0.162±0.018	0.173
17.5	0.819	0.210±0.017	0.221
24.6	0.849	0.203±0.015	0.234

straightforward threshold analysis of seven measurements of the total cross section for $\pi^- p \to \pi^+\pi^- n$ between 1242 and 1353 MeV. These data were also included in the present analysis although differential rather than total cross sections were fitted at the two lower energies. In another analysis, Aaron *et al.*[12] found $\xi = -0.2 \pm 0.3$ from a K-matrix fit to the $PS11(\epsilon N)$ amplitude below 1400 MeV determined from various isobar-model analyses. This fit was constrained by the total cross sections of Ref. 1. The chiral-symmetry-breaking parameter determined by the present analysis is consistent with these earlier determinations but possesses an error about a third as large. The small error in the present value can be attributed to the comparatively much larger data base and relatively small number of parameters used in this analysis.

The present work may also be compared with the previous isospin analysis of Kravtsov *et al.*[3] These authors found the threshold ratio of s-wave amplitudes to be $a'_{10}/a_{32} = 0.8 \pm 0.4$, in disagreement with the present result, $a'_{10}/a_{32} = 2.4 \pm 0.2$. The difference is mainly due to their poor determination of a'_{10} since the accurate $\pi^- p \to \pi^+\pi^- n$ measurements of Ref. 1 were not available at the time of their analysis. Consequently, these authors were unsuccessful in describing all five channels for $\pi N \to \pi\pi N$ in terms of a single chiral-symmetry-breaking parameter.

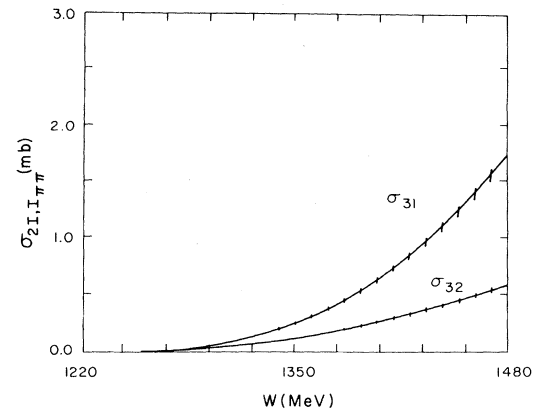

FIG. 7. Isospin cross sections σ_{32} and σ_{31}. Hatched areas indicate the range of uncertainties in their determinations.

In the present analysis, the effects of isospin-breaking final-state Coulomb interactions were neglected. These effects are expected to be negligible except at energies near threshold and are difficult to properly calculate. It is, however, reasonable to ask what effects these interactions would have on the present results. Final-state Coulomb interactions should enhance the cross section for $\pi^- p \to \pi^+\pi^- n$ (which mainly determines a'_{10}) and suppress the cross section for $\pi^+ p \to \pi^+\pi^+ n$ (which mainly determines a_{32}). If ξ were exactly zero, as required by Weinberg's phenomenological πN Lagrangian,[13] then Eqs. (11) and (12) imply that any analysis which neglects Coulomb interactions would determine the "uncorrected" value of ξ to be slightly negative.

The pruned data and results of the fit are shown for $\pi^- p \to \pi^0\pi^0 n$ in Fig. 1, for $\pi^- p \to \pi^+\pi^- n$ in Fig. 2, for $\pi^- p \to \pi^0\pi^- p$ in Fig. 3, for $\pi^+ p \to \pi^0\pi^+ p$ in Fig. 4, and for $\pi^+ p \to \pi^+\pi^+ n$ in Fig. 5. Table II presents the results

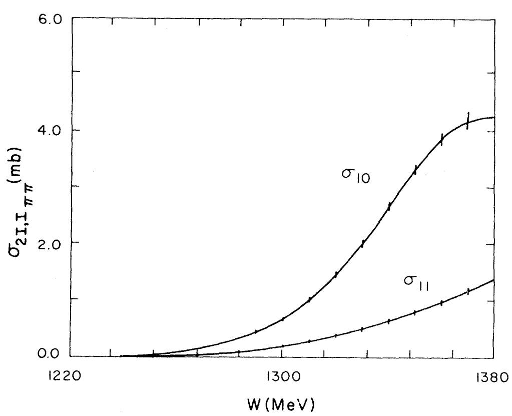

FIG. 6. Isospin cross sections σ_{10} and σ_{11}. Hatched areas indicate the range of uncertainties in their determinations.

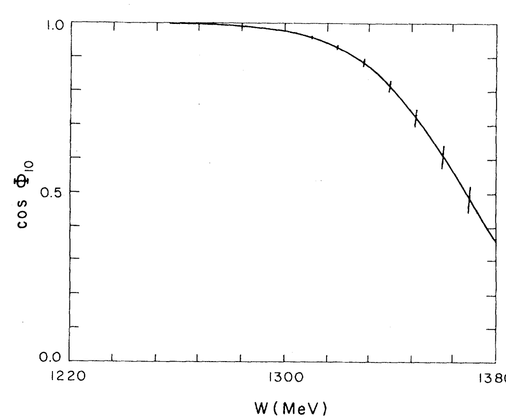

FIG. 8. $\cos\Phi_{10}$, where Φ_{10} is the phase of a_{10}. The hatched area indicates the range of uncertainty in its determination.

of the fit for the double-differential cross sections at 1242 MeV and Table III presents the results for those at 1262 MeV. The present work also allows a determination of isospin cross sections $\sigma_{2I,I_{\pi\pi}}$. Cross sections for $I=\frac{1}{2}$ are shown in Fig. 6 and those for $I=\frac{3}{2}$ are shown in Fig. 7. When a_{11} was parametrized with a resonance form in an attempt to fit π^-p reaction cross sections above 1370 MeV, it was found that the data required a rapidly growing σ_{11} cross section that reached about 10 mb at 1500 MeV. This behavior is attributable mainly to the influence of the D_{13} (1520) resonance. The influence of the Roper resonance is apparent in Fig. 8 which displays $\cos\Phi_{10}$, where Φ_{10} is the phase of a_{10}.

V. SUMMARY AND CONCLUSIONS

The first resonance that can influence a_{10} is the Roper resonance and its effects on the cross sections for $\pi^-p\to\pi^0\pi^0n$ and $\pi^-p\to\pi^+\pi^-n$ are significant at energies just above threshold. For those reactions, it is imperative that comparisons with soft-pion calculations be carried out by threshold extrapolations such as those in this work.

The success of this analysis in describing low-energy single-pion production with a minimum of parameters and within the context of isospin symmetry constitutes a validation of isospin invariance. For the first time, it has been demonstrated that the threshold behavior of single-pion production is consistent with a single value of the chiral-symmetry-breaking parameter. The value determined for this parameter is close to zero, the value required by Weinberg's phenomenological πN Lagrangian[13] and expected from quantum chromodynamics.[12] We may conclude that soft-pion calculations are indeed valid for $\pi N\to\pi\pi N$.

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