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Quantum creation of universes

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A cosmological wave function describing the tunneling of the Universe from "nothing" into a de Sitter space is found in a simple minisuperspace model. The tunneling probability is proportional to $\exp(-3/8G^2\rho_v)$, where ρ_v is the vacuum energy density at an extremum of the effective potential $V(\phi)$. The tunneling is most probable to the highest maximum of $V(\phi)$.

Modern cosmology gives an evolutionary picture of the Universe. It aims to describe how the Universe has evolved to its present state from a certain initial state. The differential equations describing the evolution are derived from known laws of physics; however, there are no laws determining the initial conditions. This seems to indicate that our understanding of the Universe is bound to be incomplete: we will have to say, in effect, that the Universe is what it is because it was what is was.

I have recently suggested a cosmological model[1] in which the Universe is created by quantum tunneling from "nothing" to de Sitter space, where by "nothing" I mean a state with no classical space-time.[2] In this model the initial state of the Universe is determined by the laws of physics, and no initial or boundary conditions are required. In the present paper I would like to give a further discussion of the model. In particular, I will discuss the semiclassical wave function of the Universe and the relation to the work of Hartle and Hawking[3] and correct an error in the original version of the model.

Let me first summarize the model of Ref. 1. Consider a system of interacting gravitational and matter fields, where for simplicity the matter fields are represented by a single Higgs field ϕ with an effective potential $V(\phi)$. If $\phi=\eta$ is the true minimum of the effective potential, then we require that $V(\eta)\sim 0$, so that the cosmological constant is small today. Besides $\phi=\eta$, $V(\phi)$ can have other extrema. If $\phi=\phi_0$ is such an extremum, $V'(\phi_0)=0$, then $\phi=\phi_0=\text{const}$ is a solution of the classical field equation for ϕ, $\Box\phi+V'(\phi)=0$. The vacuum energy density at $\phi=\phi_0$ will, in general, be nonzero (and positive): $\rho_v=V(\phi_0)>0$. The model of Ref. 1 is based on a solution of the combined Einstein and scalar field equations in which $\phi=\phi_0$ and the gravitational field is described by a closed Robertson-Walker metric,

$$ds^2=dt^2-a^2(t)[d\chi^2+\sin^2\chi(d\theta^2+\sin^2\theta d\phi^2)] \tag{1}$$

The solution for the scale factor $a(t)$ is the de Sitter space

$$a(t)=H^{-1}\cosh(Ht) \ , \tag{2}$$

where

$$H=(8\pi G\rho_v/3)^{1/2} \ . \tag{3}$$

It describes a closed universe which contracts at $t<0$, then "bounces" at a minimum size $a_{\min}=H^{-1}$, and expands at $t>0$.

This behavior is similar to that of a bubble of true vacuum surrounded by a false vacuum. The radius of the bubble is given by[4-6]

$$R=(R_0{}^2+t^2)^{1/2} \ . \tag{4}$$

However, in the actual history of the bubble the $t<0$ part is absent: the bubble tunnels quantum mechanically from $R=0$ to $R=R_0$, and then evolves according to Eq. (4) with $t>0$. By analogy, it was suggested in Ref. 1 that the Universe could have emerged at the bounce point having a finite size ($a=H^{-1}$) and zero "velocity" ($\dot{a}=0$); its following evolution is described by Eq. (2) with $t>0$.

To describe the tunneling process I used the bounce solution[5] of the Euclidean field equations, which can be obtained by changing $t\rightarrow -i\tau$ in Eq. (2):

$$a(\tau)=H^{-1}\cos(H\tau) \ . \tag{5}$$

This is the well-known de Sitter instanton,[7] which describes a four-sphere of radius H^{-1}. This compact instanton does not have an asymptotic region and can be interpreted as describing a tunneling to the de Sitter space (2) from *nothing*.

For "normal" quantum tunneling, the tunneling probability P is proportional to $\exp(-S_E)$, where S_E is the Euclidean action for the corresponding instanton.[5] For the de Sitter instanton,[7]

$$S_E = -3/8G^2\rho_v \ , \tag{6}$$

and I concluded in Ref. 1 that $P \propto \exp(3/8G^2\rho_v)$. Here, it will be argued that the correct answer is given by $P \propto \exp(-|S_E|)$. The basic reason is that the under-barrier wave function contains growing and decreasing exponentials with roughly equal coefficients, but the growing exponential, of course, dominates. Note that in the usual case of bubble nucleation the Euclidean action is positive definite, and so $|S_E| = S_E$.

The problem of determining the tunneling amplitude can be approached by solving the "Schrödinger equation" for the wave function of the Universe. In the general case, the wave function $\Psi(g_{ij}, \phi)$ is defined on a space of all possible three-geometries and scalar field configurations (superspace). The role of the Schrödinger equation for Ψ is played by the Wheeler-DeWitt equation,[8] which is a functional differential equation on superspace. Here, we shall employ a simple minisuperspace model, in which we restrict the three-geometry to be homogeneous, isotropic, and closed, so that it is described by a single scale factor a. The scalar field ϕ is restricted to a constant value at one of the extrema of the effective potential: $\phi = \phi_0$. Then the Wheeler-DeWitt equation for $\Psi(a)$ takes the form[3,8]

$$\left[a^{-p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} - \left(\frac{3\pi}{2G}\right)^2 a^2(1-H^2a^2)\right]\Psi(a) = 0 \ . \tag{7}$$

Here, the parameter p depends on one's choice of factor ordering. Variation of p affects $\Psi(a)$ only for $a < G^{1/2}$; such values of a are unimportant for our discussion, and we shall set $p=0$. Then Eq. (7) takes the form of a one-dimensional Schrödinger equation for a "particle" described by a coordinate $a(t)$, having zero energy and moving in a potential

$$U(a) = \frac{1}{2}\left(\frac{3\pi}{2G}\right)^2 a^2(1-H^2a^2) \ . \tag{8}$$

The WKB solutions of Eq. (7) in the classically allowed region ($a > H^{-1}$) are (disregarding the preexponential factor)

$$\Psi_\pm^{(1)}(a) = \exp\left(\pm i\int_{H^{-1}}^{a} p(a')da' \mp \frac{i\pi}{4}\right) \tag{9}$$

and the under-barrier ($0 < a < H^{-1}$) solutions are

$$\Psi_\pm^{(2)}(a) = \exp\left(\pm\int_a^{H^{-1}} |p(a')|da'\right) \ , \tag{10}$$

where

$$p(a) = [-2U(a)]^{1/2} \ . \tag{11}$$

Tunneling through the barrier corresponds to the choice of the "outgoing" wave for $a > H^{-1}$:

$$\Psi(a > H^{-1}) \sim \Psi_+^{(1)}(a) \ . \tag{12a}$$

Then the WKB connection formula[9] gives the under-barrier wave function of the form

$$\Psi(a < H^{-1}) \sim \Psi_+^{(2)}(a) + \frac{i}{2}\Psi_-^{(2)}(a) \ . \tag{12b}$$

Except in the immediate vicinity of $a = H^{-1}$, the second term in Eq. (12b) is negligible, and $\Psi \sim \Psi_+^{(2)}(a)$. The wave function grows exponentially towards $a=0$ (as it should). The tunneling amplitude is proportional to

$$\exp\left(-\int_0^{H^{-1}} |p(a')|da'\right) = \exp(-3/16G^2\rho_v) \ ,$$

and thus the tunneling probability is $P \propto \exp(-|S_E|)$ with S_E given by Eq. (6).

The use of the semiclassical approximation is justified if $|S_E| >> 1$ or $\rho_v << G^{-2}$. This condition is satisfied in most grand unified theories.

Here, I should mention an alternative approach to the definition of the wave function of the Universe. Hartle and Hawking[3] have suggested that $\Psi(g, \phi)$ is given by a path integral over all compact *Euclidean* four-geometries and scalar field histories bounded by the configuration (g, ϕ):

$$\Psi(g, \phi) = \int [dg_{\mu\nu}][d\phi]\exp(-S_E[g_{\mu\nu}, \phi]) \ . \tag{13}$$

This definition seems to be very similar to ours. Indeed, a compact four-geometry bounded by g can be thought of as interpolating between a point ("nothing") and the three-geometry g. However, the wave function for a de Sitter universe obtained by Hartle and Hawking and by Moss and Wright[10] using Eq. (13) is different from the one obtained here. They find $\Psi(a < H^{-1}) \sim \Psi_-^{(2)}(a)$ and $\Psi(a > H^{-1}) \sim \Psi_+^{(1)}(a) + \Psi_-^{(1)}(a)$. This wave function corresponds to a "particle" bouncing off the potential barrier at $a = H^{-1}$; under the barrier $\Psi(a)$ is exponentially suppressed. It describes a contracting and reexpanding universe. In fact, this could be anticipated, since, by construction, the wave function (13) is real. Thus, the Hartle-Hawking approach automatically gives a time-symmetric picture of the universe: a contracting and reexpanding universe in the case of a de Sitter space and an oscillating universe in more complicated minisuperspace models.[3]

The "creation-from-nothing" picture can also be formulated in terms of path integrals. One can define $\Psi(g, \phi)$ as

$$\Psi(g, \phi) = \int [dg_{\mu\nu}][d\phi]\exp(iS[g_{\mu\nu}, \phi]) \ , \tag{14}$$

where the integration is over compact Lorentzian four-geometries and scalar field histories bounded by the three-geometry g with the field configuration ϕ and lying to the past of (g, ϕ). (This corresponds to using Teitelboim's causal propagator[11] for the gravitational field.) In the sum over histories (14) one has to allow four-geometries with integrable singularities (and finite action), since nonsingular compact Lorentzian manifolds do not exist. A similar approach to the problem of topology change in quantum gravity has been discussed in Ref. 12. Alternatively, one can assume that space-time ceases to be a Lorentzian manifold on scales smaller than $G^{1/2}$. As long as $\rho_v << G^{-2}$, our results are not sensitive to the modifications of general relativity on Planck scales.

In the semiclassical approximation, the dominant contribution to (14) is given by the classical trajectory and its neighborhood. Since "creation from nothing" is a quantum tunneling process, no classical trajectory exists in the classically forbidden region under the barrier. For example, in our simple minisuperspace model, no classical trajectory passes through a three-sphere of radius $a < H^{-1}$. To find the under-barrier semiclassical wave function, one has to analytically continue to the integration over Euclidean space-times (this is similar to what one does in the path-integral approach to nonrelativistic quantum mechanics[13]). Then the path integral is dominated by the classical solution of the Euclidean field equations, which, in our case, is the de Sitter instanton (5). With this prescription, the wave function obtained from Eq. (14) is given by Eq. (12).

Needless to say, there is a host of problems, both technical and conceptual, with quantum gravity. The interpretation of the wave function of the Universe is one of them. Since Ψ can have only a probabilistic interpretation, we are faced with the problem of having only one copy of the Universe. We have found that the tunneling probability is

$$P \propto \exp(-3/8G^2\rho_v) \quad , \tag{15}$$

where $\rho_v = V(\phi_0)$ and ϕ_0 is an extremum of the effective potential. This equation suggests that of all such extrema, the tunneling is "most probable" to the highest maximum of $V(\phi)$, $\phi = \phi_{max}$. If one assumes the existence of an observer who can do a statistical survey of all nucleating universes, he will find that the most of the universes nucleate with $\phi = \phi_{max}$. Our best guess seems to be that we live in a "typical" universe which has started with $\phi = \phi_{max}$. It may happen, however, that typical universes are not suitable for life, and then we have to invoke the anthropic principle and conclude that we live in one of the rare universes which nucleated at $\phi \neq \phi_{max}$. If the effective potential is sufficiently flat near $\phi = \phi_0$, then the newly born universe can evolve along the lines of the new inflationary scenario,[14] as discussed in Refs. 1 and 15.

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where τ is the proper time on the wall.

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