Quantum unified field theory from enlarged coordinate transformation group. II

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If the relativity principle, which states that the law of propagation for light has the same form for all macroscopic observers, is extended to include quantum observers, this leads directly to the quantum unified field theory which was introduced in a previous paper. This theory appears suitable for describing all known interactions. Gravitation and electromagnetism are described by the Einstein equations $G_{\mu\nu}=\frac{1}{2}(e_{\mu\nu}-K_\mu j_\nu-K_\nu j_\mu)-RK_\mu K_\nu$, where $G_{\mu\nu}$ is the Einstein tensor, R is the Ricci scalar, $e_{\mu\nu}$ is the usual stress-energy tensor for the free electromagnetic field, and j_μ is the electromagnetic current. The vector K_μ plays a dual role. It is the electromagnetic vector potential in the covariant Lorentz gauge, and, it is also a unit timelike vector interpretable as the velocity of the observer.

I. INTRODUCTION

In a previous paper,[1] hereafter referred to as I, we obtained a quantum unified field theory by pursuing a suggestion by Einstein[2] that the diffeomorphisms somehow be extended to a larger group. The present paper is devoted to the further development of that theory. In Secs. I A–I F, we recall those developments from I which are needed so that subsequent sections may rest upon a firm mathematical foundation. A further mathematical result which will be needed subsequently is developed in Sec. I E. In Sec. II, we establish a more solid physical foundation than was recognized in I, by showing that if the relativity principle, which states that the law of propagation for light has the same form for all macroscopic observers, is extended to include quantum observers (observers who may be large, but not infinitely large, by comparison with quantum-mechanical systems), this leads to precisely the group extension of the diffeomorphisms upon which I is based.[3] In Sec. III, we recall the main physical results of I, and show that its field equations may be expressed in a form which makes it far more evident than previously that they describe gravitation and electromagnetism and also contain terms that appear suitable for describing the weak and strong interactions.

A. Noncommutativity of partial derivatives of path-dependent functionals

Any ordered set of four independent real variables x^α, where α and other indices take the values 0,1,2,3, may be regarded as coordinates of points in a four-dimensional arithmetic space σ. Let $x^\alpha(\lambda)$ be absolutely continuous functions of a real parameter λ on the interval $-\infty<\lambda<\infty$. By a path P, we mean the totality of points in σ which are identified by $x^\alpha=x^\alpha(\lambda)$ for $-\infty<\lambda\leq\Lambda$. Thus, one end point of P is the point i with coordinates $\lim_{\lambda\to-\infty}x^\alpha(\lambda)$, while the other end point is the point x with coordinates $x^\alpha(\Lambda)$. We regard i as the initial point, and regard x as the terminus, of P. The set of all paths in σ is regarded as a space of paths, and is denoted by Σ. We impose a topology on Σ by defining the (coordinate dependent) distance D between paths A and B. This distance $D(A,B)$ is defined with the aid of the usual (Euclidean) measure of distance $d(a,b)$ between points a and b. $D(A,B)$ is defined as the smallest number D such that for each point a on path A, there exists a point b on path B for which $D\geq d(a,b)$; and, for each point b on B, there exists a point a on A for which $D\geq d(a,b)$. Let F be a path-dependent functional, i.e., a rule which assigns to each path P a real number $F\{P\}$. If, for all paths A and B, the relation $\lim_{B\to A}F\{B\}=F\{A\}$ is satisfied, F is called a continuous functional. We limit our considerations to continuous functionals. Derivatives of $F\{P\}$ are defined by giving P an extension from its terminus x, while holding the rest of P completely fixed.[4] Any path may be extended in this way by extending the domain of $x^\alpha(\lambda)$ to the interval $-\infty<\lambda\leq\Lambda+\Delta\Lambda$, where $\Delta\Lambda>0$. The resulting path $P+\Delta P$ is called a path extended from P, and the totality of points in σ which are defined by $x^\alpha=x^\alpha(\lambda)$ for $\Lambda<\lambda\leq\Lambda+\Delta\Lambda$ is called an extension of P and is denoted by ΔP. We define F' by

$$F'=\lim_{\Delta\Lambda\to 0}\frac{F\{P+\Delta P\}-F\{P\}}{\Delta\Lambda}\,.$$

If the extension ΔP is chosen so that, along it, only a single coordinate x^β changes, and if the parametrization is such that on this extension $\Delta\Lambda=\Delta x^\beta$, then F' is called the partial derivative of F with respect to x^β, and denoted either by $\partial_\beta F$ or by $F_{,\beta}$. If, along ΔP, the coordinate increments Δx^β are unrestricted and independent, then F' is called the total derivative of F with respect to Λ, and denoted by $dF/d\Lambda$. It is also convenient to denote $dx^\alpha/d\lambda$, evaluated for $\lambda=\Lambda$, by $dF/d\Lambda$. If the partial derivatives and the total derivative of F are related in such a way that the chain rule for differentiation is valid, i.e., if $dF/d\Lambda=F_{,\alpha}dx^\alpha/d\Lambda$, then F is called a smooth functional. A smooth functional whose partial derivatives of all orders are also smooth is called a regular functional. We limit our considerations to regular functionals. When we wish to emphasize the path-dependent character of a functional F, we will use the notation $F\{P\}$. However, our functionals include, as a subclass, the usual onevalued functions of x^α, i.e., functionals which are "path dependent" in the trivial sense that they depend only on the terminus x of a path P; for them, we use the notation $F(x)$. From P, let two extended paths $P+\Delta P_1$ and $P+\Delta P_2$ be constructed such that the extensions ΔP_1 and ΔP_2 do not completely coincide, but such that the termini of $P+\Delta P_1$ and $P+\Delta P_2$ *do* coincide. The values of $F\{P+\Delta P_1\}$ and $F\{P+\Delta P_2\}$ are not generally equal. By letting ΔP_1 be an extension along which first only x^ν changes and then only x^μ changes, and letting ΔP_2 be an extension along which first only x^μ changes and then only x^ν changes, we see that $\partial_\mu\partial_\nu F$ equals $\partial_\nu\partial_\mu F$ for functions $F(x)$, but not generally for functionals $F\{P\}$.

B. Abstract path space

Just as the x^α are regarded as coordinates of points in σ and the set of paths in σ is regarded as a space of paths Σ, another set of four independent real variables x^i may be regarded as coordinates of points in another four-dimensional arithmetic space s and the set of paths in s may be regarded as another space of paths S. Let H be a homeomorphism from Σ onto S; let $\mathscr{P}$ be the image path of P, and let $\mathscr{X}$ be the terminus of $\mathscr{P}$. Since $\mathscr{X}$ is determined by $\mathscr{P}$, and $\mathscr{P}$ is determined by P (via the homeomorphism H), it is clear that the coordinates x^i of $\mathscr{X}$ are functionals of P, i.e., $x^i=x^i\{P\}$. Similarly $x^\alpha=x^\alpha\{\mathscr{P}\}$. If the image path of each path extended from P is a path extended from $\mathscr{P}$, and if $x^i\{P\}$ and $x^\alpha\{\mathscr{P}\}$ are regular functionals, then H is called a regular homeomorphism. We limit our considerations to regular homeomorphisms.

We began by regarding a homeomorphism H as a *path transformation* (which maps each path P in Σ to a path $\mathscr{P}$ in S, and conversely). There is, however, another point of view which is more interesting and useful, and which we now adopt: We introduce an abstract path space Π in which (abstract) paths are the primary elements, and regard H as a path-dependent coordinate transformation which merely changes the arithmetic-space framework used for discussing Π. The arithmetic spaces σ and s provide equivalent frameworks for discussing Π, and the path spaces Σ and S are equivalent representations of the abstract path space Π. A path P and its image path $\mathscr{P}$ are equivalent representations of the same abstract path p in Π. The coordinates x^α and x^i provide equivalent coordinate systems for discussing Π, but the points which x^α and x^i identify in σ and x, respectively, have no meaning in Π. This is clear because a path-dependent coordinate transformation does not generally establish a one-to-one correspondence between points of σ and s, even in coordinate patches. The correspondence between x^α and x^i is generally both one-to-many and many-to-one. For example, many paths in σ with the same termini may have image paths in s with different termini, and conversely. Also, a path P in σ which is "closed," in the sense that its initial point and terminus coincide, may have an image path $\mathscr{P}$ in s which is not closed, and conversely. Thus, any assertion that an abstract path is closed (or is not closed) would have no meaning. The changed point of view which we have adopted is analogous to that in which one begins by regarding a suitable transformation $x^i=x^i(x)$ as a mapping between the points x and $\mathscr{X}$, and then recognizes that it is more interesting and useful to regard the transformation as a diffeomorphism, in which the same point of an abstract point space (a manifold) is merely relabeled with new coordinate values. Many investigators[5] have expressed skepticism that a manifold adequately describes physical space. We assume that physical space is described by the abstract path space Π. Path space possesses properties which are sufficiently close to what one conceives of intuitively as a space so that one may use it almost exactly as one conventionally uses a manifold.

C. The conservation group

A conservation law is a statement of the form $\mathscr{V}^\alpha{}_{,\alpha}=0$, where $\mathscr{V}^\alpha$ is a vector density of weight $+1$. This is a covariant statement under a path-dependent coordinate transformation relating x^α and x^i if and only if it implies and is implied by the statement $\mathscr{V}^i{}_{,i}=0$. The transformation law for a vector density of weight $+1$ is

$$\mathscr{V}^i=J\mathscr{V}^\alpha x^i{}_{,\alpha}\ ,$$

where J is the determinant of $x^\alpha{}_{,i}$. Upon differentiating with respect to x^i, and using the well-known formula[6] $\partial_\nu J=Jx^i{}_{,\alpha}\partial_\nu x^\alpha{}_{,i}$ for the derivative of a determinant, we obtain

$$\mathscr{V}^i{}_{,i}=J(\mathscr{V}^\alpha{}_{,\alpha}-\mathscr{V}^\mu x^\nu{}_{,i}[\partial_\mu,\partial_\nu]x^i)\ ,$$

where $[\partial_\mu,\partial_\nu]=\partial_\mu\partial_\nu-\partial_\nu\partial_\mu$. For arbitrary $\mathscr{V}^\mu$, we see that a conservation law is a covariant statement if and only if

$$x^\nu{}_{,i}[\partial_\mu,\partial_\nu]x^i=0\ . \qquad (1)$$

If Eq. (1) is satisfied, the path-dependent coordinate transformation relating x^α and x^i is called conservative. We note that Eq. (1) is satisfied if $x^i=x^i(x)$; thus, it is clear that each diffeomorphism is a conservative coordinate transformation, but that the converse is not true. In I, we gave an explicit proof that the conservative coordinate transformations form a group, which we call the conservation group. Finkelstein,[7] however, has noted that the group property follows immediately from the derivation given above.

D. Geometry determined on path space by the conservation group

The geometry determined on a manifold by the diffeomorphisms is Riemannian geometry, whose structure is expressed by a symmetric metric $g_{\mu\nu}(x)$. From $g_{\mu\nu}$ and its derivatives, one constructs an object $R^\alpha{}_{\beta\mu\nu}$ which is a tensor under the diffeomorphisms, and which is called the Riemann curvature tensor. It is well known that there exists a diffeomorphism to a special $x^{\overline{\alpha}}$ coordinate system in which $g_{\overline{\mu}\overline{\nu}}$ is constant, if and only if $R^\alpha{}_{\beta\mu\nu}$ vanishes. We consider the geometry which is determined in this sense,[8] on the path space Π, by the conservation group. The structure of this geometry is described by a homeomorphism h from the path space Π onto itself. This homeomorphism may, as discussed in Sec. I B, be regarded as a transformation from x^α to x^i coordinates and represented by the matrix $x^i{}_{,\alpha}$. This matrix may be thought of as a tetrad $h^i{}_\alpha$; i.e., $h^i{}_\alpha = x^i{}_{,\alpha}$, with $h_i{}^\alpha = x^\alpha{}_{,i}$. It is the tetrad which expresses the geometrical structure of the path space Π. A tetrad $h^i{}_\alpha(x)$ was first used in physics by Einstein,[9] under the name "vierbein." Our tetrad, however, is generally $h^i{}_\alpha\{P\}$. Under a transformation from x^α to $x^{\bar\alpha}$ coordinates $h^i{}_\alpha$ transforms according to $h^i{}_{\bar\nu} = h^i{}_\alpha x^\alpha{}_{,\bar\nu}$ and, under a transformation from x^i to $x^{\bar i}$ coordinates it transforms according to $h^{\bar n}{}_\alpha = x^{\bar n}{}_{,i} h^i{}_\alpha$ with similar transformation laws for $h_i{}^\alpha$. A curvature vector $C_\mu\{P\}$ which is analogous to $R^\alpha{}_{\beta\mu\nu}$ is defined by

$$C_\mu = h_i{}^\nu (h^i{}_{\mu,\nu} - h^i{}_{\nu,\mu}) , \tag{2}$$

or, equivalently, by $C_\mu = C_i h^i{}_\mu$, where $C_i = h^j{}_\nu (h_j{}^\nu{}_{,i} - h_i{}^\nu{}_{,j})$. Under a transformation from x^α to $x^{\bar\alpha}$ coordinates, C_μ transforms as a vector (and C_i as a scalar) if and only if the transformation is conservative. We showed in I that the geometrical interpretation of C_μ is this: *There exists a conservative coordinate transformation from* x^α *to a special* $x^{\bar\alpha}$ *coordinate system in which* $h^i{}_{\bar\mu}$ *is constant, if and only if* C_μ *vanishes.* An equivalent interpretation is that *there exists a conservative coordinate transformation from* x^i *to a special* $x^{\bar i}$ *coordinate system in which* $h^{\bar i}{}_\mu$ *is constant, if and only if* C_μ *vanishes.*

E. Criterion for determining whether two tetrads are related by a conservative coordinate transformation

Any two tetrads $h^i{}_\mu$ and $h^i{}_{\bar\mu}$ are related by a path-dependent coordinate transformation between x^α and $x^{\bar\alpha}$ coordinates. The relation is $h^i{}_\mu = h^i{}_{\bar\alpha} x^{\bar\alpha}{}_{,\mu}$, where $x^{\bar\alpha}{}_{,\mu}$ is defined by $x^{\bar\alpha}{}_{,\mu} = h_i{}^{\bar\alpha} h^i{}_\mu$. Let $C_{\bar\mu} = h_i{}^{\bar\nu}(h^i{}_{\bar\mu,\bar\nu} - h^i{}_{\bar\nu,\bar\mu})$, as required for consistency with Eq. (2). We noted in I that the relation between C_μ and $C_{\bar\mu}$ is

$$C_\mu = C_{\bar\alpha} x^{\bar\alpha}{}_{,\mu} - x^\nu{}_{,\bar\alpha}[\partial_\mu, \partial_\nu] x^{\bar\alpha} .$$

Upon multiplying by $h_i{}^\mu$, we obtain

$$C_i = \bar C_i - h_i{}^\mu x^\nu{}_{,\bar\alpha}[\partial_\mu, \partial_\nu] x^{\bar\alpha} , \tag{3}$$

where $\bar C_i = C_{\bar\mu} h_i{}^{\bar\mu}$. It is clear from Eq. (3) that $h^i{}_\mu$ and $h^i{}_{\bar\mu}$ *are related by a conservative coordinate transformation if and only if* $C_i = \bar C_i$. This criterion is especially useful, because C_i involves only $h^i{}_\mu$, and $\bar C_i$ involves only $h^i{}_{\bar\mu}$.

F. Introduction of a metric

A symmetric metric $g_{\mu\nu}\{P\}$ is defined by the relation $g_{\mu\nu} = g_{ij} h^i{}_\mu h^j{}_\nu$, where $g_{ij} = g^{ij} = \text{diag}(-1,1,1,1)$. That we have not introduced g_{ij} in an *ad hoc* manner may be seen by recalling the well-known[10] correspondence in which the Levi-Civita symbols of two-dimensional complex "spin space" induce a metric with space-time signature onto four-dimensional real space. (This also suggests that nature chooses four dimensions, rather than any other number of dimensions, for reasons of parsimony; with any other choice, a metric could be introduced only in an *ad hoc* way.) We define $g^{\mu\nu}$ by $g^{\mu\nu} g_{\nu\alpha} = \delta^\mu{}_\alpha$, where $\delta^\mu{}_\alpha$ is the usual Kronecker δ. Latin indices are raised and lowered by using g^{ij} and g_{ij}, just as greek indices are raised and lowered by using $g^{\mu\nu}$ and $g_{\mu\nu}$. The transformation law for $g_{\mu\nu}$ under a coordinate transformation from x^α to $x^{\bar\alpha}$ is the usual one, i.e., $g_{\mu\nu} = g_{\bar\rho\bar\sigma} x^{\bar\rho}{}_{,\mu} x^{\bar\sigma}{}_{,\nu}$. The symmetry between coordinate transformations on greek and latin indices, however, is broken by the introduction of a metric. Indeed, it is clear that a transformation from x^i to $x^{\bar i}$ coordinates leaves $g_{\mu\nu}$ invariant if and only if $g_{ij} = g_{\bar n\bar m} x^{\bar n}{}_{,i} x^{\bar m}{}_{,j}$, where $g_{\bar i\bar j} = g_{ij}$. Coordinate transformations on latin indices which satisfy this condition will be called frame transformations, and the symbol $L^{\bar n}{}_i$ used instead of $x^{\bar n}{}_{,i}$. It is well known that the homogeneous Lorentz transformations form a six-parameter Lie group; i.e., that $L^{\bar n}{}_i$ depends upon six parameters, $\alpha_1, \ldots, \alpha_6$. If the α's are constant, then $L^{\bar n}{}_i$ is constant, and the frame transformation is called global. If the α's are functions $\alpha(x)$, then $L^{\bar n}{}_i$ is $L^{\bar n}{}_i(x)$, and the frame transformation is called local. If the α's are functionals $\alpha\{P\}$, then $L^{\bar n}{}_i$ is $L^{\bar n}{}_i\{P\}$, and the frame transformation is called path dependent. If the α's satisfy conditions such that $L^i{}_{\bar n}(L^{\bar n}{}_{i,j} - L^{\bar n}{}_{j,i}) = 0$, then the frame transformation is called conservative.

II. QUANTUM OBSERVERS

Newtonian mechanics may be regarded as a relativity theory which recognizes the equivalence of all observers who differ only in the fact that they are moving with respect to each other with constant velocity. In Newtonian relativity, however, the measurements of such observers are related by Galilean coordinate transformations. Einstein's replacement of the Galilean group with the Lorentz group led from a three-dimensional Euclidean manifold and absolute time to a four-dimensional Minkowski manifold, and from Newtonian theory to special relativity. Einstein's recognition of the need to include, on an equivalent basis, observers who are accelerated with respect to one another caused him to extend the group from the Lorentz group to the diffeomorphisms. It was this step which led from the Minkowski manifold to a Riemannian manifold with space-time signature, and, from special relativity to general relativity. Subsequently, Einstein[2] suggested that a unified field theory be obtained by somehow extending the diffeomorphisms to a larger group. We pursued this suggestion, in I, by introducing the conservation group. It is necessary to ask, however, whether the introduction of the conservation group is justified by the kind of compelling physical motivation which caused Einstein to introduce the diffeomorphisms. After all, there do exist observers who are accelerated with respect to one another—and, evidence of their equivalence is provided by the proportionality of inertial and gravitational mass. But, is there a need for some still more general class of observers?

General relativity makes use of a classical observer who can observe the motion of a physical system without disturbing the system. This violates the fundamental principles of quantum theory. Most discussions of observation in quantum theory make use of a "macroscopic" classical observer—one who can "stand outside" a quantum-mechanical system and act upon the system without being acted upon by the system. This is unsatisfactory, because there exist no observers who are infinitely large by comparison with quantum-mechanical systems. One solution of this problem was given by Everett.[11] Briefly, he considers a quantum observer's memory to have quantum states that are correlated with the states of what he has observed. Each observer can then consider himself a macroscopic observer (since his different states are independent) and still treat other observers as part of his quantum-mechanical universe. The uncertainty principle does not limit the precision with which he can do this, because the four operators which represent his coordinates commute. Let Ω and $\bar{\Omega}$ be two observers, each of whom considers himself a macroscopic observer, while treating the other as part of a quantum-mechanical universe. Let x^{α} and $x^{\bar{\alpha}}$ be coordinates which are assigned by Ω and $\bar{\Omega}$, respectively, to events in their universes. The assumption that physical space-time is a manifold rests squarely upon the assertion that it is possible to establish a one-to-one correspondence between x^{α} and $x^{\bar{\alpha}}$, at least in coordinate patches. Einstein challenged the validity of Newton's absolute time on the ground that no operational method had been, or could be, given for its measurement. In this spirit, we challenge the validity of the assumption that space-time is a manifold on the ground that no operational method has been, or can be, given for establishing a one-to-one correspondence between the coordinates x^{α} and $x^{\bar{\alpha}}$. Our two observers are free to exchange information so that, for example, Ω can possess a complete description of the procedure which $\bar{\Omega}$ uses in assigning coordinates to events. If Ω could also state with certainty (as in general relativity) that $\bar{\Omega}$'s world line is a particular path P, then he could write a function uniquely specifying $\bar{\Omega}$'s coordinates $x^{\bar{\alpha}}$ in terms of his own coordinates x^{α}. Thus, $x^{\bar{\alpha}}$ could be regarded as a functional of the path P (which has the terminus x^{α}, since the observer in general relativity is a tetrad at the event being investigated), i.e.,

$$x^{\bar{\alpha}}=x^{\bar{\alpha}}\{P\} \ . \tag{4}$$

However, Ω has described $\bar{\Omega}$'s world line as completely as nature permits when he states that *all* paths occur with equal probability amplitude, in the sense that the probability amplitude for a path P is $N e^{iL\{P\}/\hbar}$, where L is Ω's Lagrangian for $\bar{\Omega}$, N is a normalization factor (the same for all paths), and $\hbar$ is the usual quantum of angular momentum. Therefore, Ω can only state that the probability amplitude for $\bar{\Omega}$'s coordinate numbers $x^{\bar{\alpha}}$ corresponding to his own coordinate numbers x^{α} is

$$\Psi(x^{\bar{\alpha}},x^{\alpha})=\sum_{P} N e^{iL\{P\}/\hbar} \ , \tag{5}$$

where $\sum_P$ denotes the democratic sum with equal weight of contributions due to all paths P with terminus x^{α} for which Eq. (4) yields the value $x^{\bar{\alpha}}$. As a world-point mapping, $\Psi(x^{\bar{\alpha}},x^{\alpha})$ is both one-to-many and many-to-one; hence, nonunique in both directions, as are our conservative coordinate transformations. As the size of observer $\bar{\Omega}$ increases without limit, we find that the competing alternatives in Eq. (5) interfere destructively on all but the classically allowed path. Thus, Eq. (4) goes over to $x^{\bar{\alpha}}=x^{\bar{\alpha}}(x)$ in the macroscopic limit. This just means that the group of all quantum transformations, defined by Eqs. (4) and (5), contains the diffeomorphisms as a proper subgroup, as does our conservation group.

We are now in a position which permits us to show that the inclusion of quantum observers, on an equivalent basis, requires the extension of the diffeomorphisms to the conservation group. We could simply say, "Experience shows that if Ω observes that a certain quantity is conserved, then $\bar{\Omega}$ also observes that the same quantity is conserved." On the other hand, it is absolutely essential that such a fundamental principle as the covariance law be derivable from the simplest possible basic assumption. We therefore return to the assumption which led Einstein to special relativity: that the equation which describes the propagation of light (the wave equation) has the same form for all observers. It is well known that in general relativity the wave equation may be written in the form $(\sqrt{-g}g^{\mu\nu}\Phi_{,\nu})_{,\mu}=0$, where g is the determinant of $g_{\mu\nu}$, and, that this form, which does not involve "covariant derivatives" or Christoffel symbols, is nevertheless covariant under the diffeomorphisms. We note that this general relativistic statement of the wave equation is already in the form of a conservation law: $\mathscr{V}^{\mu}{}_{,\mu}=0$, where $\mathscr{V}^{\mu}$ is the vector density of weight $+1$ which is defined by $\mathscr{V}^{\mu}=\sqrt{-g}g^{\mu\nu}\Phi_{,\nu}$. The discussion given in Sec. I C now suffices to show that the conservation group is the largest group of coordinate transformations under which the equation for the propagation of light is covariant.

III. PHYSICS AS A MANIFESTATION OF PATH-SPACE GEOMETRY

A. Quantization of the path-space geometry

We define a scalar $L=C^{\mu}C_{\mu}$, which is invariant under conservative coordinate transformations on greek indices as well as conservative frame transformations on latin indices. Because L is generally $L\{P\}$, rather than $L(x)$, it is easy and natural to quantize the geometry of the path space Π by using essentially the path-integral method. Quantum geometry says that all paths occur with equal probability amplitude in the following sense: The probability amplitude for a path P is[12] $N e^{iL\{P\}/\hbar}$. We use infinitesimal extensions from the terminus x of a given path P in almost exactly the same way that one conventionally uses infinitesimal displacements from a point. Since the tetrad is generally $h^{i}{}_{\mu}\{P\}$, it is clear that the behavior of $h^{i}{}_{\mu}$ in the "extension neighborhood" of x is governed by a "probability amplitude to transit from x to $x+\Delta x$." This amplitude or "propagator" $\langle x \mid x+\Delta x \rangle$ is[12] the democratic sum with equal weight of contributions due to every path extension from x to $x+\Delta x$; thus,

$$\langle x \mid x+\Delta x \rangle = N\int e^{iL\{P\}/\hbar}\mathscr{D}x \ , \tag{6}$$

where $\mathscr{D}x$ is the "volume element" for the sum over these finite path extensions.

B. Macroscopic limit of the quantum geometry

A "classically allowed" path extension ΔP_c receives what Misner, Thorne, and Wheeler[12] call "preference without preference" over other path extensions from x to $x+\Delta x$. This path extension and path extensions that differ from it so little that $\delta L=L\{P+\Delta P\} -L\{P+\Delta P_c\}$ is only of order $\hbar$ and less give contributions to the probability amplitude $\langle x \mid x+\Delta x \rangle$ in Eq. (6) that interfere constructively. In contrast, destructive interference effectively wipes out the contribution that comes from path extensions that differ more from ΔP_c. Thus, there are quantum fluctuations in the geometrical structure expressed by $h^i{}_\mu$, but they are fluctuations of limited magnitude. The smallness of $\hbar$ ensures that the scale of these is unnoticeable at everyday distances. For a "skeleton path extension," defined by giving $x_n = x^\alpha(\lambda_n)$ at $\lambda_n = \Lambda + n\Delta\lambda$, we see by analogy with Misner, Thorne, and Wheeler[12] that the volume element in Eq. (6) is equal (up to a multiplicative constant) to $\sqrt{-g}\,d^4x$, where d^4x denotes $dx^0dx^1dx^2dx^3$. This permits us to use infinitesimal path extensions in considering the macroscopic limit of our quantum geometry. We find from Eq. (6) that the macroscopic limit is described by field equations which flow from the variational principle $\delta\int\sqrt{-g}\,L\,d^4x = 0$, where the 16 components of $h^i{}_\mu$ are varied independently. We showed in I that the resulting field equations are

$$C_{\mu|\nu} - g_{\mu\nu}C^\alpha{}_{|\alpha} + \tfrac{1}{2}g_{\mu\nu}C^\alpha C_\alpha = 0 , \tag{7}$$

where $C_{\mu|\nu} = C_{\mu,\nu} - C_\alpha L^\alpha{}_{\mu\nu}$ and $L^\alpha{}_{\mu\nu} = h_i{}^\alpha h^i{}_{\mu,\nu}$. Since $L^\alpha{}_{\mu\nu}$ transforms as an affine connection under *all* coordinate transformations from x^α to $x^{\bar\alpha}$, it follows that our field equations are covariant under conservative coordinate transformations. We note that they are also covariant under global, but *not* local or conservative frame transformations (from x^i to $x^{\bar i}$). Since L is invariant under conservative frame transformations (latin) as well as under conservative coordinate transformations (greek), it is clear that the quantum theory expressed by Eq. (6) is invariant under both types of transformation. It may therefore appear surprising that the corresponding macroscopic theory is not covariant under conservative frame transformations. It seems clear, however, that this is just an example of a new type of dynamical symmetry breaking. In the quantum theory, there is democracy among Lorentz frames which are connected by conservative frame transformations; but certain classically allowed Lorentz frames which are connected by global frame transformations receive preference without preference over other Lorentz frames.

C. First integral of the field equations

If we multiply Eq. (7) by $h_i{}^\mu h_j{}^\nu$, we obtain

$$C_{i,j} - g_{ij}C^k{}_{,k} + \tfrac{1}{2}g_{ij}C^kC_k = 0 . \tag{8}$$

The antisymmetric part of Eq. (8) is $C_{i,j} - C_{j,i} = 0$, which just implies that C_i is a gradient, i.e., that $C_i = C_{,i}$, where C is a path-independent function of the latin coordinates. It follows that the C_i are path-independent functions of the latin coordinates, as are their partial derivatives. For distinct values of i and j, Eq. (8) becomes $C_{i,j} = 0$. Thus, we see that the function C_i can depend only upon the single coordinate x^i; similarly, C^i can depend only upon x^i. The trace of Eq. (8) is

$$3C^k{}_{,k} - 2C^kC_k = 0 . \tag{9}$$

Upon raising the index i in Eq. (8) and using Eq. (9) to eliminate $C^k{}_{,k}$, we obtain

$$C^i{}_{,j} - \tfrac{1}{6}\delta^i{}_jC^kC_k = 0 . \tag{10}$$

If we set i and j equal to the same value N (no summation on N), we find from Eq. (10) that, for all N,

$$C^N{}_{,N} = \tfrac{1}{6}C^kC_k . \tag{11}$$

It follows from Eq. (11) that $C^0{}_{,0} = C^1{}_{,1} = C^2{}_{,2} = C^3{}_{,3}$. But, $C^0{}_{,0}$ can depend only upon x^0; $C^1{}_{,1}$ only upon x^1, etc. Thus, it is clear that $C^N{}_{,N}$ is a constant (same constant for all N). Hence, Eq. (10) may be written as

$$C^i{}_{,j} = \tfrac{1}{6}\delta^i{}_jL , \tag{12}$$

where $L = C^kC_k$ is a constant. We may integrate Eq. (12) to obtain

$$C^i = \tfrac{1}{6}Lx^i + b^i , \tag{13}$$

where b^i=const. It follows from Eq. (13) that

$$L = g_{ij}(\tfrac{1}{6}Lx^i + b^i)(\tfrac{1}{6}Lx^j + b^j) . \tag{14}$$

Upon differentiating Eq. (14) with respect to x^N, and using the fact that L is constant, we obtain

$$L(Lx^N + 6b^N) = 0 . \tag{15}$$

If $Lx^N + 6b^N = 0$, then the constancy of L and b^N implies that L vanishes. Thus, we see from Eq. (15) that $L = 0$; i.e., that C^kC_k vanishes, so that C_i must either vanish or be lightlike. In either case, we find from Eq. (10) that $C^i{}_{,j} = 0$; hence, C^i must be constant. Our conclusion is that each tetrad which satisfies the field equations gives a curvature vector which either vanishes or is lightlike, and that the latin components of this curvature vector are constant.

D. Solutions of the field equations which can be transformed into one another

Let $h^i{}_\mu$ and $h^{\bar i}{}_{\bar\mu}$ be two tetrads which satisfy the field equations, and let C_i and $\bar C_{\bar i}$ be the latin components of the curvature vectors which correspond to them, respectively. From the results of Sec. I E, we see that $h^i{}_\mu$ and $h^{\bar i}{}_{\bar\mu}$ are related by a conservative coordinate transformation, if and only if the constants C_i and $\bar C_{\bar i}$ are equal. But the field equations [Eq. (7)] are covariant under global frame transformations as well as conservative coordinate transformations. Hence, it is useful to have a criterion for determining whether $h^i{}_\mu$ and $h^{\bar i}{}_{\bar\mu}$ can be transformed into one another through the combined action of a conservative coordinate transformation and a global frame transformation. Clearly, this is possible if C_i and $\bar C_{\bar i}$ are both zero. It is also possible if C_i and $\bar C_{\bar i}$ are both nonzero, because any two constant lightlike vectors are related by a global frame transformation (perhaps including a time inversion). It is not possible if one of C_i and $\bar C_{\bar i}$ is zero while the other is nonzero.

E. Some convenient notation

A semicolon is used to denote the usual "covariant derivative" (i.e., covariant under diffeomorphisms) with respect to the Christoffel symbol $\Gamma^\alpha{}_{\mu\nu}$, defined by

$$\Gamma^{\alpha}{}_{\mu\nu}=\tfrac{1}{2}g^{\alpha\beta}(g_{\beta\mu,\nu}+g_{\beta\nu,\mu}-g_{\mu\nu,\beta}) .$$

The Ricci rotation coefficient[13] γ_{ijk} is defined by

$$\gamma_{\alpha\mu\nu}=h^{i}{}_{\alpha}h_{i\mu;\nu}=\gamma_{ijk}h^{i}{}_{\alpha}h^{j}{}_{\mu}h^{k}{}_{\nu} .$$

In I, we showed that

$$\gamma_{\alpha\mu\nu}=\tfrac{1}{2}(h_{i\alpha}F^{i}{}_{\mu\nu}+h_{i\mu}F^{i}{}_{\nu\alpha}+h_{i\nu}F^{i}{}_{\mu\alpha}) , \tag{16}$$

where $F^{i}{}_{\mu\nu}=h^{i}{}_{\mu,\nu}-h^{i}{}_{\nu,\mu}$. It is convenient to denote contraction on latin indices by a dot between adjacent terms, e.g.,

$$h^{i}{}_{\alpha}h_{j}{}^{\beta}F_{i\mu\nu}F^{j}{}_{\rho\sigma}=h_{\alpha}\cdot F_{\mu\nu}F_{\rho\sigma}\cdot h^{\beta} .$$

The suppression of latin indices via the "dot" notation is very useful in reducing the cluttered appearance of certain expressions, thus facilitating their interpretation. The Riemann curvature tensor (i.e., tensor under diffeomorphisms) is defined in the usual way by

$$R^{\alpha}{}_{\beta\mu\nu}=\Gamma^{\alpha}{}_{\beta\nu,\mu}-\Gamma^{\alpha}{}_{\beta\mu,\nu}+\Gamma^{\alpha}{}_{\gamma\mu}\Gamma^{\gamma}{}_{\beta\nu}-\Gamma^{\alpha}{}_{\gamma\nu}\Gamma^{\gamma}{}_{\beta\mu} ,$$

while the Ricci tensor $R_{\mu\nu}$, Ricci scalar R, and Einstein tensor $G_{\mu\nu}$ (under diffeomorphisms) are defined, as usual, by $R_{\mu\nu}=R^{\alpha}{}_{\mu\alpha\nu}$, $R=R^{\alpha}{}_{\alpha}$, and $G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$. In the conventional development of Riemannian geometry, $G_{\mu\nu}$ is symmetric; however, we noted in I that $G_{\mu\nu}$ is symmetric when $g_{\mu\nu}$ is $g_{\mu\nu}(x)$, but not generally when $g_{\mu\nu}$ is $g_{\mu\nu}\{P\}$. We denote the symmetric part of $G_{\mu\nu}$ by $G_{\underline{\mu\nu}}$.

F. Transformation of the field equations to electromagnetic coordinates

By using Noether's theorem, we showed in I that the six currents

$$J_{ij}{}^{\nu}=C_{i}h_{j}{}^{\nu}-C_{j}h_{i}{}^{\nu}$$

satisfy the conservation laws $J_{ij}{}^{\nu}{}_{;\nu}=0$, and that the antisymmetric part of the field equations is expressed by these conservation laws. We also showed that the symmetric part of the field equations may be written in terms of $G_{\underline{\mu\nu}}$. The result is

$$\begin{aligned}G_{\underline{\mu\nu}}=&-\tfrac{1}{2}(E_{\mu\nu}+h_{\mu}\cdot J_{\nu}+h_{\nu}\cdot J_{\mu}+h^{\alpha}\cdot F_{\nu\beta}F_{\mu\alpha}\cdot h^{\beta})\\&-\tfrac{1}{4}(h_{\mu}\cdot F_{\alpha\beta}F^{\alpha\beta}\cdot h_{\nu}-g_{\mu\nu}h_{\alpha}\cdot F_{\sigma\beta}F^{\sigma\alpha}\cdot h^{\beta})\\&+\tfrac{1}{2}h^{\alpha}\cdot([\partial_{\alpha},\partial_{\mu}]h_{\nu}+[\partial_{\alpha},\partial_{\nu}]h_{\mu}-g_{\mu\nu}[\partial_{\alpha},\partial_{\beta}]h^{\beta}) ,\end{aligned} \tag{17}$$

where $E_{\mu\nu}$ is defined, in obvious analogy to the usual electromagnetic stress-energy tensor, by

$$E_{\mu\nu}=F_{\mu\alpha}\cdot F_{\nu}{}^{\alpha}-\tfrac{1}{4}g_{\mu\nu}F^{\alpha\beta}\cdot F_{\alpha\beta} , \tag{18}$$

and, J_{μ} is defined, in similar analogy to the electromagnetic current, by

$$J^{i}{}_{\mu}=F^{i}{}_{\mu}{}^{\nu}{}_{;\nu} . \tag{19}$$

The third line of Eq. (17) vanishes when $h^{i}{}_{\mu}$ is $h^{i}{}_{\mu}(x)$, and, everywhere we look in the first two lines, we see terms which are suggestive of electromagnetism. Indeed, we are confronted with an embarrassing richness of "electromagnetic fields" $F^{0}{}_{\mu\nu}$, $F^{1}{}_{\mu\nu}$, $F^{2}{}_{\mu\nu}$, and $F^{3}{}_{\mu\nu}$. We encountered a similar situation, in a previous paper,[4] when we considered how a path-dependent coordinate transformation from x^{i} to x^{α} coordinates transforms the special relativistic equation of motion for a free particle

$$\frac{d^{2}x^{i}}{ds^{2}}=0 , \tag{20}$$

where $ds^{2}=g_{ij}dx^{i}dx^{j}$. We showed[4] that the image equation under the transformation is

$$\frac{d^{2}x^{\alpha}}{ds^{2}}+\Gamma^{\alpha}{}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}=\mathscr{F}^{\alpha}{}_{\nu}\frac{dx^{\nu}}{ds} , \tag{21}$$

where $ds^{2}=g_{\mu\nu}dx^{\mu}dx^{\nu}$, $\mathscr{F}_{\mu\nu}=V_{i}F^{i}{}_{\mu\nu}$, and $V^{i}=dx^{i}/ds$ is the (constant) first integral of Eq. (20). If $h^{i}{}_{\mu}$ is $h^{i}{}_{\mu}(x)$, then $\mathscr{F}_{\mu\nu}$ satisfies the Maxwell equations $\mathscr{F}_{\mu\nu,\beta}+\mathscr{F}_{\beta\mu,\nu}+\mathscr{F}_{\nu\beta,\mu}=0$. Nevertheless, $\mathscr{F}_{\mu\nu}$ cannot be regarded as the electromagnetic field, because the relation $V^{i}=dx^{i}/ds=h^{i}{}_{\mu}dx^{\mu}/ds$ implies that V_{i} depends upon dx^{μ}/ds. Although $\mathscr{F}_{\mu\nu}$ is a linear combination of the $F^{i}{}_{\mu\nu}$ with coefficients V_{i} which are constant along the world line of a particle, it is unsatisfactory that the values of these coefficients should depend upon dx^{μ}/ds. This would imply that the electromagnetic field experienced by a particle depends upon the velocity of the particle (in disagreement with experiment). We have, however, considered[14,15] a tetrad $h^{i}{}_{\mu}(x)$ such that the four antisymmetric tensors $F^{i}{}_{\mu\nu}$ are constant multiples of one another; i.e., such that

$$F^{i}{}_{\mu\nu}=K^{i}f_{\mu\nu} , \tag{22}$$

where K^{i}=const. Such a tetrad is of the form

$$h^{i}{}_{\mu}=\theta^{i}{}_{,\mu}+K^{i}A_{\mu} , \tag{23}$$

where θ^{i} is $\theta^{i}(x)$, A_{μ} is $A_{\mu}(x)$, $f_{\mu\nu}=A_{\mu,\nu}-A_{\nu,\mu}$, and the determinant of $\theta^{i}{}_{,\mu}$ is nonzero. We have noted[15] that, for such a tetrad, Eq. (21) reduces to

$$\frac{d^{2}x^{\alpha}}{ds^{2}}+\Gamma^{\alpha}{}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}=V_{i}K^{i}f^{\alpha}{}_{\nu}\frac{dx^{\nu}}{ds} , \tag{24}$$

which is the equation of motion for a particle with charge-to-mass ratio $V_{i}K^{i}$. We have also noted, in I, that for such a tetrad, similar simplifications occur in the symmetric part of the field equations. Equations (17), (18), and (19) become

$$\begin{aligned}G_{\mu\nu}=&-\tfrac{1}{2}K^{i}K_{i}e_{\mu\nu}-\tfrac{1}{2}(K_{\mu}j_{\nu}+K_{\nu}j_{\mu}+C_{\mu}C_{\nu})\\&-\tfrac{1}{4}f^{\alpha\beta}f_{\alpha\beta}K_{\mu}K_{\nu} ,\end{aligned} \tag{25}$$

where $K_{\mu}=K_{i}h^{i}{}_{\mu}$,

$$e_{\mu\nu}=f_{\mu\alpha}f_{\nu}{}^{\alpha}-\tfrac{1}{4}g_{\mu\nu}f^{\alpha\beta}f_{\alpha\beta} , \tag{26}$$

and

$$j_{\mu}=f_{\mu}{}^{\nu}{}_{;\nu} . \tag{27}$$

In I, however, we merely noted that these simplifications occur *if* the tetrad can be transformed into the form specified in Eq. (23) via a conservative coordinate transformation plus global frame transformation. We are now in a position to show that this *can* be done for any tetrad which satisfies the field equations. We have seen, in Sec. III D, that all solutions of the field equations give either $C_i=0$, or C_i constant and lightlike. We now show that, in either case, we may transform the tetrad into the form given in Eq. (23). Moreover, we shall see that if $C_i=0$, then we may choose among existing transformations which lead to Eq. (23) with K^i timelike, spacelike, or lightlike. Similarly, we shall see that if C_i is constant and lightlike, then we may choose between existing transformations which lead to Eq. (23) with K^i spacelike or lightlike. We proceed by exhibiting specific examples of solutions to the field equations in each of these cases. When these have been exhibited, the existence of the transformations leading to these cases follows immediately from the results of Sec. III D. From Eqs. (2) and (22), we find that

$$C_i=K^j f_{ij}\ , \tag{28}$$

where

$$f_{ij}=h_i{}^\mu h_j{}^\nu f_{\mu\nu}\ . \tag{29}$$

If we multiply Eq. (29) by K^i, we obtain

$$K^i C_i=0\ . \tag{30}$$

A special case of Eq. (23) is

$$h^i{}_\mu=\delta^i{}_\mu+K^i A_\mu\ . \tag{31}$$

By using Eq. (31), we easily verify that

$$h_i{}^\nu=\delta^\nu{}_i-\frac{\delta^\nu{}_n\delta^\alpha{}_i K^n A_\alpha}{1+\delta^\beta{}_m K^m A_\beta}\ . \tag{32}$$

From Eqs. (28), (29), (31), and (32), we find that

$$C_i=\frac{\delta^\mu{}_i\delta^\nu{}_j K^j f_{\mu\nu}}{1+\delta^\beta{}_m K^m A_\beta}\ . \tag{33}$$

If the constant frame-vector K^i is timelike, then there exists a global frame transformation to a frame in which

$$K^i=(K^0,K^1,K^2,K^3)=(K^0,0,0,0)\ .$$

In this frame, we find from Eq. (30) that C_0 vanishes; hence C_i is either zero or spacelike. But, as we have seen in Sec. III C, it follows from the field equations that C_i is zero or lightlike. Thus, we see that if K^i is timelike, then C_i must be zero. We now use Eq. (31) for exhibiting the specific examples mentioned above. For $C_i=0$, let $K^i=(1,0,0,0)$, and let $A_\mu=(A_0,A_1,A_2,A_3)=(0,0,x^1,0)$; alternatively, let $K^i=(0,1,0,0)$ or $K^i=(1,1,0,0)$, and let $A_\mu=(0,0,x^3,0)$. For C_i constant and lightlike, let $K^i=(0,1,0,0)$ and let $A_\mu=(0,e^{x^0+x^3}-1,0,0)$; alternatively, let $K^i=(1,1,0,0)$, and let $A_\mu=(e^{x^1}-1,0,0,0)$. That the tetrads in these examples yield the stated values for C_i is easily verified with the use of Eq. (33).

Having established the possibility of transforming any tetrad which satisfies the field equations into the form given in Eq. (23), we note that only for K^i timelike and C_i zero does Eq. (25) include the electromagnetic stress-energy tensor $e_{\mu\nu}$ in a manner which is consistent with the conventional interpretation of general relativity. For K^i lightlike, the coefficient of $e_{\mu\nu}$ in Eq. (25) vanishes. For K^i spacelike, the coefficient of $e_{\mu\nu}$ is negative, corresponding to the wrong sign for the gravitational constant. We have also noted previously[15] that only K^i timelike guarantees that G_{00} is non-negative, as required by the weak energy condition.[16] As Synge[17] and Hawking[16] have emphasized, G_{00} must be everywhere non-negative in any theory which gives a realistic description of macroscopic physics. This is the general relativistic analog of the Newtonian requirement that the density in Poisson's equation shall be everywhere non-negative. Stated covariantly, the requirement is that the eigenvalue corresponding to the timelike eigenvector of $G_{\mu\nu}$ shall be everywhere non-negative.

For $K^i=(1,0,0,0)$ and $C_i=0$, Eq. (25) becomes

$$G_{\mu\nu}=\tfrac{1}{2}(e_{\mu\nu}-K_\mu j_\nu-K_\nu j_\mu)-RK_\mu K_\nu\ . \tag{34}$$

In obtaining Eq. (34) from (25), we have used the fact (established in I) that if C_α, $F^1{}_{\mu\nu}$, $F^2{}_{\mu\nu}$, $F^3{}_{\mu\nu}$, and $[\partial_\mu,\partial_\nu]h^i{}_\alpha$ all vanish, then $R=\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}$. From Eq. (23) and the definition $K_\mu=K_i h^i{}_\mu$, we obtain $K_\mu=-A_\mu-\theta^0{}_{,\mu}$. Clearly, K_μ is the vector potential for $f_{\mu\nu}$ in a different gauge from A_μ. It is easily seen by using Eqs. (16) and (22) that $K^\mu{}_{;\mu}=0$. Thus, we see that K_μ automatically satisfies the covariant Lorentz gauge condition. We also note that K_μ is a unit timelike vector, interpretable as a velocity.

G. The weak and strong interactions

In I we suggested the following interpretation, which includes all known interactions.

(a) Gravitation is described by the metric $g_{\mu\nu}$ as in general relativity.

(b) The timelike vector of the tetrad is identified as the vector potential whose curl is the antisymmetric tensor (under diffeomorphisms) which represents the electromagnetic field in Maxwell's theory.

(c) The three spacelike vectors of the tetrad are identified as vector potentials whose curls represent the weak field.

(d) The strong interaction is described by terms such as $h^{\alpha\cdot}[\partial_\alpha,\partial_\mu]h_\nu$ in Eq. (17), which vanish when $h^i{}_\mu$ is $h^i{}_\mu(x)$. An argument which supports this identification, based upon considerations of gauge symmetry, is given in I.

In obtaining Eq. (34) from (17), we have used a conservative coordinate transformation plus global gauge transformation to transform away the weak and strong fields (analogous to the familiar manner in which a gravitational field which is not "permanent," such as the Coriolis force, can be transformed away by a suitable transformation). We shall now see, however, that these fields cannot be transformed away by a freely falling, nonrotating, observer.

The orthodox interpretation, which we adopt, is that $h^i{}_\mu$ describes an observer frame. The vector $h^0{}_\mu$ is the (timelike) velocity vector of an observer carrying a spatial frame described by the triad $h^1{}_\mu,h^2{}_\mu,h^3{}_\mu$. It is well known that the condition for a freely falling observer frame is that the tetrad be carried by Fermi transport;[17] i.e., that the Ricci rotation coefficient γ_{0j0} shall vanish. This condition is satisfied by the tetrad which led to Eq. (34). However, no tetrad of the form given in Eq. (23) can describe a freely falling, *nonrotating,* observer frame (except trivially), in the case where K^i is timelike and C_i is zero. It is well known that the condition for a freely falling, nonrotating, observer frame is that the tetrad be carried by Fermi-Walker transport;[17] i.e., that the Ricci rotation coefficient γ_{ij0} shall vanish. For K^i timelike and C_i zero, the vanishing of γ_{ij0} implies that $f_{\mu\nu}$ vanishes and that $g_{\mu\nu}$ describes a flat Riemann space. This just means that a freely falling, nonrotating, observer who transforms away the weak- and strong-interaction terms in Eq. (17) also transforms away all material aspects of the Universe—including all material aspects of himself.

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[2]A. Einstein, in *Albert Einstein: Philosopher-Scientist,* edited by P. A. Schilpp (Harper, New York, 1949), Vol. I, p. 89.

[3]Much of the material presented in Sec. II was discussed by the author at the Second New Orleans Conference on Quantum Theory and Gravitation, Loyola University, 1983. The proceedings of this conference are to be published in the Int. J. Theor. Phys.

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[5]See, e.g., footnote 3 of Ref. 1.

[6]C. E. Weatherburn, *Riemannian Geometry and the Tensor Calculus* (Cambridge University Press, Cambridge, England, 1966), p. 3.

[7]D. Finkelstein (private communication).

[8]We would say more generally, "The unquantized geometry which is determined by our group in the sense of Klein's Erlanger program," but this could cause some confusion. Roughly speaking, Klein's program states that a group of transformations on a space determines a geometry on the space, and vice versa; however, mathematicians appear to differ somewhat concerning the precise modern interpretation of Klein's program. See, e.g., F. Klein, Math. Ann. **43**, 63 (1893); H. Weyl, *The Theory of Groups and Quantum Mechanics* (Dover, New York, 1931), p. 112; R. S. Millman, Am. Math. Monthly **84**, 338 (1977), and references therein.

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