Cosmic separation of phases

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A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

I. COSMIC SEPARATION OF PHASES

Although the evidence is divided, various theoretical arguments[1] and computer simulations[2] raise the possibility that the early universe may have undergone a first-order phase transition associated with QCD effects at a temperature of order 100 MeV. (A true second-order QCD phase transition in cosmology is implausible because in nature there is no exact chiral symmetry and no exact criterion for confinement. There may, however, have been a period in which chiral-symmetry breaking turned on continuously but abruptly. If so, the thermal history of this period can be calculated.[3]) Whether, with the actual values of the QCD parameters, the early universe would have undergone a first-order phase transition is something that probably will only be settled by a future computer simulation. In this paper, we will assume that a transition from a state of quasifree light quarks to a state of mesons and baryons occurred at $T_c \sim 100$—200 MeV and we will explore the consequences.

Recent investigations of this problem[4–6] have considered the consequences of a departure from equilibrium, and further thoughts along these lines will be discussed in an appendix to this paper. Here, however, we will assume that a first-order QCD phase transition occurred smoothly, without important departure from equilibrium. This would occur if the rate for nucleating the low-temperature phase by thermal fluctuations becomes large after relatively small supercooling. In this context, "small" supercooling means that the transition effectively occurs at a temperature at which most of the latent heat between the two phases still remains, so that phase coexistence can be established after nucleation. Such behavior seems plausible for strong interactions. Actually, in nature, first-order phase transitions are most typically mediated by impurities, not thermal fluctuations. Even if the QCD dynamics is unlucky, equilibrium will be maintained if the impurity abundance is adequate. Even without a detailed scenario, it seems plausible to assume that equilibrium is maintained to high accuracy in a QCD phase transition. As we will see, this apparently harmless assumption has surprising consequences. Some of the points that follow have been made previously by Suhonen.[7]

To fix ideas, consider first how an adiabatic first-order transition unfolds if we ignore the tiny baryon asymmetry ($10^{-8} \lesssim n_B/n_\gamma \lesssim 10^{-10}$) in the Universe. At a temperature just below T_c, bubbles of low-temperature phase appear. Unlike the nonequilibrium situation, however, the bubbles of low-temperature phase do not expand explosively. In a first-order phase transition there is a difference between the energy density of the two phases, usually called the latent heat. As the bubbles of low-temperature phase expand, they expel heat into their surroundings, heating the high-temperature phase up to T_c. At this point the pressure of the high-temperature phase prevents further expansion of the low-temperature phase. After all, T_c is the temperature at which the two phases have equal pressures and can coexist (Fig. 1).

As the Universe expands, it loses energy. Normally this results in cooling, but not here. Coexistence between the high- and low-temperature phases is possible only at T_c (ignoring the tiny quark and lepton chemical potentials), and as long as both phases are present the loss of energy does not result in cooling, but in an expansion of the bubbles of low-temperature phase at the expense of the regions of high-temperature phase. The Universe remains

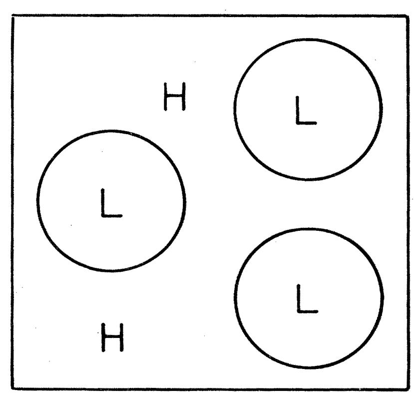

FIG. 1. Isolated expanding bubbles of low-temperature phase in the high-temperature phase.

at T_c until the full latent heat L of the transition is eliminated. If ρ is the energy density of the Universe ($\rho \sim T_c{}^4$), then the time scale for this is of order L/ρ times the cosmic expansion time $1/H_0 \sim M_{\rm Pl}/T_c{}^2$. For the QCD transition, it is plausible that $L/\rho \sim 1$.

Thus, the bubbles of low-temperature phase slowly expand. When they occupy roughly 30—50% of the total volume, they meet and percolate (Fig. 2). At this point the bubbles have a characteristic radius R_0, depending on how the transition was nucleated. The scenario below is simplest if $R_0 \lesssim 10^{-3}$ cm (the baryon diffusion length). This is a long distance by hadronic standards; it corresponds to 10^{-30} nucleation events per cubic fermi. We also will discuss a variant of the scenario which may still operate if $R_0 \gg 10^{-3}$ cm.

When bubbles collide, we must take surface tension into account. An isolated bubble is essentially spherical. When bubbles meet and percolate, however, they arrange themselves into a system of fewer, larger bubbles to minimize the surface area. We must estimate the time scale for this process.

If two bubbles of radius R collide and form a single bubble of radius $2^{1/3}R$, the energy is lowered by $\Delta E \sim \sigma R^2$, where σ is the surface tension between the two phases. The force acting is $F \sim \Delta E/R \sim \sigma R$, since energy ΔE is released in a distance R. The fluid mass that must be moved is $M \sim \rho R^3$, and it must be moved a distance of order R. A force F will move a mass M a distance R in a time $t \sim (MR/F)^{1/2}$. If $\sigma \sim T_c{}^3$ and $\rho \sim T_c{}^4$, then $t \sim T_c{}^{1/2} R^{3/2}$.

Requiring that this time be less than the expansion time $M_{\rm Pl}/T_c{}^2$ gives $R \lesssim M_{\rm Pl}{}^{2/3} T_c{}^{-5/3}$, which is of order 1—10 cm for reasonable T_c. Thus, surface tension causes the bubbles to coalesce until reaching a characteristic scale R_1 of a few centimeters.

For a brief period, this bubble coalescence is the fastest important process, but once the bubbles are close to the characteristic size R_1 the bubble coalescence effectively stops, and we must again take account of the expansion of the Universe and the corresponding removal of heat. The dilute regions of low-temperature phase grow and soon occupy more than half the volume. When the high-temperature phase fills only 30—50% of the total volume, the tables are turned; the dense regions of high-temperature phase detach into isolated, roughly spherical bubbles (Fig. 3). They have at this point a characteristic size R_1, since they form from the "holes" between expanding regions of low-temperature phase of size R_1. Further expansion results in a loss of heat and a further shrinking of the dense bubbles of high-temperature phase, until finally they disappear.

We must ask whether thermal equilibrium is maintained in this process. In fact, heat conduction is very rapid in the epoch under consideration because there are particles (photons, charged leptons, and especially neutrinos) with very long mean-free paths. Actually, sound waves also provide a very fast mechanism for equilibration of temperature. As in any fluid, sound waves rapidly establish a pressure equilibrium in the cosmic fluid. Unlike an ordinary liquid or gas, however, the cosmic fluid has the property that temperature is the only thermodynamic variable, since there is no conserved charge (except the tiny baryon and lepton excesses), so that the pressure is a definite function of temperature, and pressure equilibrium means temperature equilibrium. (In an ordinary fluid, the pressure depends on the temperature and on the density of atoms.) Therefore, thermal equilibrium is a reasonable assumption.

We have so far assumed that the original scale R_0 of bubble coalescence is small, in which case the effective bubble size R_1 is determined by surface tension and is independent of R_0. It is perfectly plausible for R_0 to be small, especially in a system with strong interactions; R_0 might even be zero if the phase transition occurs by spinodal decomposition rather than nucleation. However, it might be that R_0 is large; Hogan has shown[4] that in the QCD transition, R_0 could plausibly be as big as about 10^4 cm (less than the horizon length $M_{\rm Pl}/T^2$ by a factor of 4 ln $M_{\rm Pl}/T_c$). In that case, how might the bubbles be expected to grow?

At zero temperature, bubbles of true vacuum grow at the speed of light.[8] At nonzero temperature, but far from equilibrium, bubbles grow explosively, faster than sound, but slower than light.[9] The latter process is similar to a detonation wave in fluid mechanics.[10] In strong interactions, however, it is quite plausible that the temperature T at which nucleation occurs is very close to T_c, at the same time that the mean distance R_0 between nuclei may be

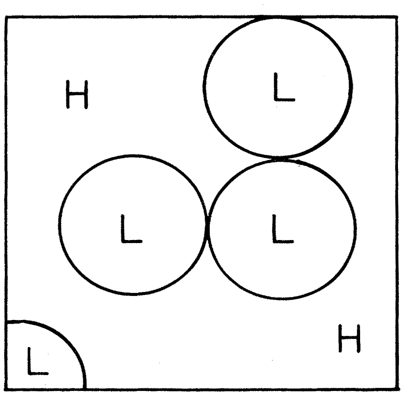

FIG. 2. The expanding bubbles meet.

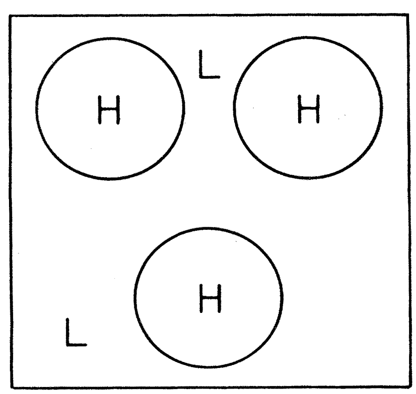

FIG. 3. Isolated shrinking bubbles of the high-temperature phase.

large. In this case the bubble growth involves conditions close to equilibrium, and an explosion or detonation does not seem likely. More likely, one will find a process similar to what Landau and Lifschitz describe as "slow combustion."[10]

In this context, slow combustion is a process in which the velocity of growth of a bubble of low-temperature phase is of order $\epsilon=(T_c-T)/T_c$, being limited by the rate at which heat transport is possible. As bubbles of low-temperature phase grow, the liberation of latent heat raises the temperature of the immediate neighborhood up to T_c (but no higher; otherwise the bubbles of low-temperature phase would have to contract instead of expanding). To avoid a local temperature higher than T_c, the bubbles of low-temperature phase can expand only at a rate determined by the ability of the liberated heat to be carried away. If heat is carried away mainly as a wave, this wave will be propagating a temperature difference of order ϵ. Since the velocity will be comparable to that of light, a wave propagating a temperature difference ϵ will carry away heat at a rate proportional to ϵ, and the velocity of bubble growth will be of order ϵ. (The possible role of neutrinos, discussed later, does not change the order of magnitude.) Of course, the heat carried off by these waves heats up the rest of the Universe.

For small ϵ, long before the low-temperature bubbles meet, the heat generated by their growth will heat the Universe up to T_c—after which the bubbles stop expanding except insofar as their growth is driven by the expansion of the Universe.

In this case, when the bubbles finally meet, surface tension plays no role; one has quasiequilibrium coexistence of phases on a scale $R_0=R_1\lesssim 10^4$ cm. As in our previous discussion, the loss of heat due to the expansion of the Universe eventually results in the detaching of isolated bubbles of the high-temperature phase, which then tend to shrink and disappear.

Thus, we can reasonably expect that for a time comparable to the Hubble time, there will be coexisting phases on a scale R_1, with $1\text{ cm}\lesssim R_1\lesssim 10^4$ cm. What observable consequences could this leave in today's universe?

There might in fact be no such consequences. The net effect of the process might be simply thermal equilibrium in the low-temperature phase below T_c. In searching for a possible observable remnant of the epoch of phase coexistence, we will explore the consequences of the tiny baryon excess in the Universe.

In the high-temperature phase, baryons exist as quasifree, almost massless quarks. In the low-temperature phase, baryons are massive particles with a minimum mass $M=938$ MeV. The density of baryons in the low-temperature phase may therefore be less than the density in the high-temperature phase by a large factor. For a more accurate estimate, assume equilibrium between the two phases with a common chemical potential μ. The baryon density in either phase is then

$$\langle B\rangle=\frac{1}{V}\langle Be^{-\mu B/T}\rangle\ , \tag{1}$$

where V is the volume and $\langle\ \rangle$ refers to a thermal average. Since the primordial μ is tiny, to good approximation

$$\langle B\rangle=-\frac{\mu}{TV}\langle B^2\rangle_0\ , \tag{2}$$

where $\langle\ \rangle_0$ is a thermal average at $\mu=0$. The ratio of the baryon densities in the high- and low-temperature phases is

$$\epsilon=\frac{\langle B\rangle^L}{\langle B\rangle^H}=\frac{\langle B^2\rangle_0^L}{\langle B^2\rangle_0^H}\ , \tag{3}$$

where $\langle\ \rangle^{L,H}$ are averages in the low- and high-temperature phases and $\langle\ \rangle_0^{L,H}$ are the averages in those phases at $\mu=0$.

Let us make a simple estimate of ϵ. Treating the low-temperature phase as an ideal gas of mesons and baryons, and assuming $T_c \ll M$ so a nonrelativistic treatment is adequate, a single baryon or antibaryon spin state of mass M contributes to $\langle B^2\rangle_0^L$ an amount

$$\Delta=\int\frac{d^3p}{(2\pi)^3}e^{-M/T_c}e^{-p^2/2MT_c}=\left[\frac{MT_c}{2\pi}\right]^{3/2}e^{-M/T_c}\ . \tag{4}$$

If the main contribution is due to the eight spin states of protons, neutrons and their antiparticles (a fair approximation if $T_c\lesssim 100$ MeV so that strange particles are suppressed), then

$$\langle B^2\rangle_0^L\simeq 8\left[M\frac{T_c}{2\pi}\right]^{3/2}e^{-M/T_c} \tag{5}$$

with an obvious generalization if T_c is larger and other particles must be included. As for the high-temperature phase, we assume that it is an ideal gas of quarks and gluons and that the u, d, and s masses can be neglected. A single quark or antiquark spin state of baryon number $\pm\frac{1}{3}$ contributes to $\langle B^2\rangle_0^H$ an amount

$$\Delta=\left[\frac{1}{3}\right]^2\int\frac{d^3p}{(2\pi)^3}\frac{e^{-|p|/T}}{1+e^{-|p|/T}}$$

$$=\frac{T^3}{12\pi^2}\zeta(3)\ , \tag{6}$$

where $\zeta(3)\cong 1.202$. The 36 spin and color states of u, d, and s quarks and antiquarks give

$$\langle B^2\rangle_0^H=3\frac{T^3}{\pi^2}\zeta(3)\ . \tag{7}$$

So with these approximations

$$\epsilon=\frac{\langle B^2\rangle_0^L}{\langle B^2\rangle_0^H}=\frac{2}{3}\frac{\sqrt{2\pi}}{\zeta(3)}\left[\frac{M}{T_c}\right]^{3/2}e^{-M/T_c}\ . \tag{8}$$

For $T_c=100$ MeV, $\epsilon=0.003$. For $T_c=200$ MeV, $\epsilon=0.15$. (For $T_c=200$ MeV, it is really necessary to include hyperons. Including all particles in the baryon octet, one gets $\epsilon=0.27$.)

Thus, this simple estimate suggests that ϵ is much less than 1, and that $\epsilon\sim 10^{-2}$ is possible, but only if T_c is relatively low. Hopefully, it will be possible eventually to determine ϵ reliably by computer simulations.

The formula (3) for ϵ assumes equilibrium between the two phases. Is this reasonable? A sufficient requirement is that R_0 is small and that thermal equilibrium existed when bubbles originally met at a scale R_0. The subsequent coalescence of bubbles into larger structures of scale R_1—due to surface tension—involves slow, gross fluid motions, probably without significant mixing of the two phases, so the relative baryon abundance at scale R_1 will be very close to what it was at scale R_0. The relative abundance will have its equilibrium value at scale R_0 if the distance d_B across which baryons can diffuse in a Hubble time is bigger than R_0. Baryons are strongly interacting, so the baryon (or quark) mean-free path at temperature T_c is of order $1/T_c$; this distance is traversed in a time of order $1/T_c$. Since the cosmic expansion time is of order $M_{\rm Pl}/T_c^2$, the baryon has time for $M_{\rm Pl}/T_c$ steps, each of mean length $1/T_c$, and can diffuse a distance

$$d_B \sim \frac{1}{T_c}\left[\frac{M_{\rm Pl}}{T_c}\right]^{1/2}$$

or about 10^{-3} cm if $T_c \sim 100$ MeV. If $R_0 \lesssim 10^{-3}$ cm, the equilibrium formula (3) for ϵ is valid.

On the other hand, we have contemplated much larger values of R_0, as big as 10^4 cm. In that case, the relative baryon abundance in the two phases cannot be determined on grounds of thermal equilibrium only. On the other hand, expanding bubbles of low-temperature phase will still tend to expel baryons, so it is conceivable that the baryon concentration would be higher in the high-temperature phase. We postpone a discussion of this point, as it is similar to our later discussion of what happens to baryons during the period when bubbles of high-temperature phase are beginning to shrink and disappear.

From our discussion so far, when bubbles of high-temperature phase begin to detach themselves and shrink, their radius is of order R_1 ($1-10^4$ cm), they fill roughly 50% of the total volume, and they may contain a fraction $1/(1+\epsilon)$ or about 80–99% of all the baryons in the Universe. Although baryons may thus be concentrated mostly in the high-temperature phase, the net baryon concentration is very tiny even there. The baryon-to-entropy ratio in today's world is observed to be at most 10^{-8}, and the baryon-to-entropy ratio in the high-temperature phase in the epoch under discussion is no bigger than that. In the low-temperature phase, the baryon-to-entropy ratio is less by a factor of ϵ.

What happens next? As the Universe expands, the regions of low-temperature phase tend to expand and cool. Equilibrium between the two phases is possible only at T_c, so to maintain equilibrium, heat must constantly be resupplied from the regions of high-temperature phase to regions of low-temperature phase. What is the principal mechanism for this? The high-temperature phase can lose heat by evaporation of the surface layers, or by emission of particles of very long mean-free path—neutrinos.

In an ordinary fluid, evaporation is a slow process because it is diffusion limited. For example, when a water drop evaporates, a large concentration of water vapor builds up around it, and evaporation proceeds only at the rate at which the vapor can diffuse away. There is no analogous effect in the cosmic fluid, since (as the baryon excess is tiny), there is no relevant conserved quantity analogous to the number of water molecules. Evaporation of a water drop is also limited by heat conduction, since the evaporation cools the surface layers and the evaporation rate is low at low temperatures. There is no analogous effect in the cosmic fluid, since (as we have noted), the latent heat of the transition can be efficiently carried away by sound waves.

As for neutrino emission, one might naively think that it would be very slow, proportional to G_F^2. However, neutrino emission can occur anywhere within a distance λ_ν of the surface, λ_ν being the mean-free path. In fact, λ_ν is of order $1/(G_F^2 T_c^5)$, or about 10 cm for $T_c = 100$ MeV. Since λ_ν is proportional to $1/G_F^2$, the fact that neutrino emission can occur a distance λ_ν from the surface just cancels the fact that the neutrino emission rate per unit volume is proportional to λ_ν, so there is no real suppression associated with the weakness of weak interactions.

One may therefore reasonably expect that energy losses due to surface evaporation and due to neutrinos will be comparable. Their implications are completely different, however. Evaporation involves a loss of the surface layers, including baryons and everything else. Neutrino emission from a distance λ_ν of the surface involves loss of heat without loss of baryons. If it dominates, the bubbles of high-temperature phase can shrink, developing a steadily increasing baryon excess; the baryons cannot diffuse out since λ_ν is much bigger than the baryon diffusion length d_B. In such a scenario, the high-temperature bubbles would undergo a steady process of shrinking, with the baryon excess trapped inside.

In such a scenario, the region of high-temperature phase behaves like a leaky balloon, unable to suport itself against outside pressure because of steady neutrino losses. The balloon shrinks, losing neutrinos without losing baryons; the shrinking balloon does not build up a pressure excess because excess $q\bar{q}$ pairs annihilate into neutrinos, which escape. The baryon excess is continually concentrated in a smaller volume. There is a rich element of wishful thinking here, since this picture assumes neutrino losses are the main way for the high-temperature phase to lose energy, while in fact neutrino losses and surface evaporation appear comparable. However, this scenario is worth exploring because—as we will see—it leads to fascinating consequences.

If such a scenario develops, then as the dense regions shrink, eventually the excess baryons in those regions will exert a pressure that will help stabilize those regions against further shrinkage. This is sketched in Fig. 4; the temperature at which the dense regions can coexist with the low-temperature phase outside depends on the chemical potential μ in the dense regions. Figure 4 is *not* a thermodynamic equilibrium curve (which might or might not be qualitatively similar, as we will discuss). Rather, assuming that the chemical potential in the low-temperature phase is negligible, Fig. 4 indicates the temperature T, as a function of the chemical potential μ in the dense phase, at which the two phases exert the same pressure. This corresponds to an idealized situation in which the two phases are separated by a membrane that transmits heat but no baryons, and all the baryon excess is

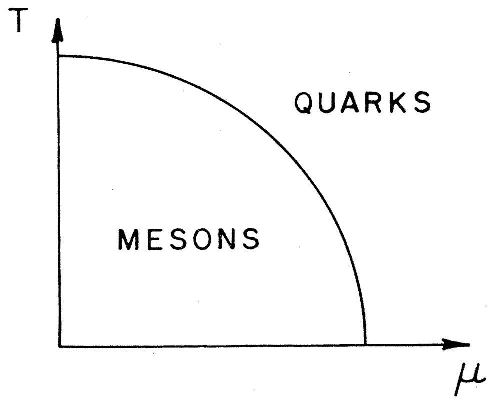

FIG. 4. A sketch of the coexistence temperature for quark matter of chemical potential μ coexisting with the meson-baryon phase of $\mu=0$. What is shown is the temperature, as a function of μ, at which the two phases exert equal pressure.

in the dense region; the problem of interest approximates that situation if diffusion of baryons is negligible and if heat is carried almost entirely by neutrinos.

As indicated in Fig. 4, once μ is not negligible, the Universe will begin to cool since the coexistence of phases now *requires* this. The baryon-to-entropy ratio in the dense regions is of order 1 when $\mu\sim T$; at this point T is comparable to but significantly less than T_c, perhaps $T\sim\frac{1}{2}T_c$.

The dense regions now occupy a tiny fraction of the total volume but contain 80—99 % of the baryon excess. How large are they? Let f be the primordial baryon to entropy ratio (so $10^{-8}\lesssim f\lesssim 10^{-10}$, experimentally). The baryon-to-photon ratio of the dense regions when they have radius R_1 is f, so this ratio becomes 1 when the radius shrinks to $r=R_1 f^{-1/3}$. If, for instance, $f\simeq 10^{-8}$, and 1 cm $\leq R_1 < 10^4$ cm, then r is between 10^{-2} and 10 cm. We have formed, in effect, lumps of hot quark matter with a density of 10^{15} g/cm^3 and a mass from 10^9—10^{18} g.

What will happen next? The most extreme possibility is that these lumps might manage to survive to the present and still exist in today's universe. Because of their extremely large inertia they would not have been incorporated in stars or planets and would be floating in interstellar space, affected significantly only by gravity. The baryons they contain would not participate in nucleosynthesis, so one could assume a closure density of baryons without disturbing the successes of big bang nucleosynthesis. In galaxy formation they would behave like the planetary mass black holes that have recently been discussed.[11] If the relative baryon abundance in the two phases was determined by thermal equilibrium, they could contain a reasonable fraction of the total mass in the Universe, and might account for the "missing mass" in the Universe[12] if $\epsilon\sim 10^{-2}$.

These lumps could survive to the present only in the form of some exotic, presently unknown state of matter, since ordinary nuclear matter is unstable at zero pressure, and ordinary atomic matter would not survive the conditions in the early universe. The lumps could plausibly survive only if there can exist a stable state of so-called "quark matter," a dense state of matter consisting of degenerate quarks at a density somewhat above ordinary nuclear matter density;[13—20] such a state would differ from ordinary nuclear matter in that the individual quark wave functions would be delocalized, spreading throughout a macroscopic volume. Of course, there is at present no experimental evidence on the existence of quark matter, and considerable theoretical uncertainty, so let us sketch the possibilities, ranging from absolutely stable to absolutely unstable quark matter.

A. Stable macroscopic quark matter

The most extreme possibility is that macroscopic quark matter is bound and stable at zero temperature and pressure. At first sight, one might think that this is excluded by the fact that ordinary nuclei do not spontaneously turn into the hypothetical dense quark state. This is not quite so, however. Observations of nuclear physics only show that in the absence of strange quarks, nuclear matter is more stable than quark matter. Addition of strangeness does not help stabilize nuclear matter, because strange baryons are heavier than nonstrange baryons. For quark matter, the story is different. (This point has been noted before in Refs. 19 and 26.) The likely Fermi momentum in quark matter is 300—350 MeV, more than the strange-quark mass, so it is energetically favored for some of the nonstrange quarks to become strange quarks, lowering the Fermi momentum and the energy.

This effect can easily be estimated in the simplest form of the bag model.[21] A single quark flavor of Fermi momentum p_F exerts a pressure $p_F{}^4/4\pi^2$. Let μ be the Fermi momentum of up quarks in quark matter of zero strangeness. For electrical neutrality, the down-quark chemical potential must then be $\mu 2^{1/3}$, and the pressure is $(1+2^{4/3})\mu^4/4\pi^2$. If we ignore the strange-quark mass, then three flavors of quarks can exert the same pressure at a common Fermi momentum $\tilde{\mu}=[\frac{1}{3}(1+2^{4/3})]^{1/4}\,\mu$ of u, d, and s quarks.

The average quark kinetic energy is proportional to μ, so (with a common pressure in the two cases) it is smaller in the three-flavor case by a factor

$$\tilde{\mu}/(\tfrac{1}{3}\mu+\tfrac{2}{3}2^{1/3}\mu)=[3/(1+2^{4/3})]^{3/4}\simeq 0.89 \ .$$

In equilibrium, the energy per quark equals the chemical potential, so the energy per quark in strange-quark matter is less than the energy per quark in zero strangeness quark matter by this factor of 0.89; in this idealization, strange-quark matter is more tightly bound than nonstrange-quark matter by about 100 MeV per baryon. The strange-quark mass will reduce this effect, but it is still plausible that strange quarks lower the energy per baryon of quark matter by 50—70 MeV per baryon. This is somewhat paradoxical, because in nuclear matter strange quarks have the opposite effect.

No observational argument seems to exclude the possibility that quark matter with up and down quarks only is unbound, but by less than 50—70 MeV per baryon, and that quark matter with strangeness is bound and stable. Ordinary nuclei will not convert to the dense, strange state because of the difficulty in creating strange quarks by weak interactions.

If quark matter with strangeness is bound, then energetically it can grow indefinitely by absorbing nucleons. One might at first think that quark matter would be dangerous in contact with ordinary matter, but this is not the case. Ordinary nuclei (except iron) are also energetically capable of undergoing nuclear reactions, with a big release of energy; this is prevented at low temperature by Coulomb barriers, because the nuclei are positively charged. A nugget of quark matter will have a net positive charge on the surface (not in the bulk) for the same reason that nuclei are positively charged. For instance, in the quark matter model of Freedman and McLerran[19] (their model I, extrapolated to zero pressure) the ratio of up-to-down-to-strange-quark abundance is roughly 2:3:1. This means that the up and down quark chemical potentials μ_u and μ_d are in the ratio $\mu_u/\mu_d=(\frac{2}{3})^{1/3}$, so that $\mu_d-\mu_u \simeq 50$ MeV.

To balance the chemical potential difference, there will be an almost equal electrostatic potential of $+50$ MeV at the surface of (and throughout) a nugget of quark matter;[22] this potential will be more than adequate to repel positively charged nuclei at ordinary temperatures or at reasonably low velocities. Quark matter is in this respect no more dangerous than oxygen.

Electrostatic potentials, however, would not prevent quark matter from absorbing neutrons. (This is somewhat retarded, however, by the need to equilibrate strangeness, if nonstrange quark matter is unbound.) If quark matter is stable, it is probably necessary to assume that ordinary neutron stars are really quark stars.[23] However, there are conventional models of neutron stars in which these stars have a large quark core (see, for instance, Ref. 24), and it has been claimed[25] that the macroscopic properties of quark stars are hard to distinguish from those of neutron stars. This point is discussed further in the Appendix.

In the models of McLerran and Freedman, quark matter with strangeness is unbound by about 25 MeV per baryon (I am here extrapolating their data to zero pressure); without strangeness, it would be unbound by about 100 MeV per baryon. As they note, however, the energy per baryon of quark matter is quite sensitive to the assumed QCD parameters, with an uncertainty of order 100 MeV per baryon.

In short, there does not seem to be any conclusive theoretical or experimental argument ruling out the possibility that there exists a stable state of quark matter with large strangeness. One experiment that might shed some light on this question would be the search for the H—a hypothetical $\Lambda\Lambda$ bound state[26] whose binding energy might be 100—200 MeV.

B. Stable drops

If quark matter, even with strangeness, is unstable in bulk, it is possible that it would have been stable in the absence of electromagnetism, and is unstable only because the constraint of electrical neutrality forces an inconvenient ratio of quark abundance. (This is true of ordinary nuclear matter, which with equal numbers of neutrons and protons would be bound as far as QCD effects are concerned, and is unbound only because electromagnetic effects prevent a nucleus from having large electric charge.) If electromagnetism is the only reason that bulk quark matter with strangeness is unbound, then such matter will probably be stable in droplets small enough that the electrostatic energy is not too large. (In the conventional case the corresponding droplets are ordinary nuclei.) The possibility of metastable (but not truly stable) droplets of quark matter has been considered previously as a possible explanation for Centauro events.[27]

C. Quark matter unstable because of weak interactions

It is possible that quark matter is unstable (both in bulk and in drops) but only because of weak interactions. For instance, in model I of Freedman and McLerran, quark matter (at zero pressure) has an energy per baryon of about 965 MeV and the quantum numbers of a roughly equal mix of neutrinos and Λ's. Since 965 MeV is above the neutron mass, quark matter is unstable in this model; but since 965 MeV is well below the average of the neutron and Λ masses, quark matter can decay in this model only with the help of weak interactions. Bulk quark matter would be relatively long-lived under these conditions, with evaporation of neutrons at the surface creating a strangeness excess that must be equilibrated by weak decays or else by diffusion of strangeness. The stability against strong interactions of quark drops of varying strangeness has been discussed previously.[28]

D. Quark matter unstable against strong interactions

Finally, there is the possibility that even with equilibrium strangeness content, quark matter is unstable against decay by strong interactions; if so it would probably decay rather quickly.

It is unfortunate that at present there is no reliable way to choose between these possibilities. There is, in addition, another major uncertainty. Even if lumps of quark matter form in the manner that has been conjectured at temperatures of order T_c, and even if there is a stable ground state at low temperatures that the lumps could logically settle into, they may evaporate at intermediate temperatures before settling into their stable ground states.

At temperatures near T_c, the evaporation of baryons from the dense lumps is suppressed because of the relatively low diffusion length d_B of the outgoing baryons and because of the relatively low equilibrium vapor pressure of baryons in the low-temperature phase, proportional to e^{-M/T_c}. (Even if the full equilibrium pressure of baryons would leak out of the dense regions to a distance d_B, this would be a minor loss of baryons.) At very low temperatures, evaporation of baryons from quark matter will be suppressed because there simply is not available enough energy to power the evaporation. (At low temperatures the thermal excitation energy of a degenerate Fermi gas is of order T^2/p_F, and less if a superconducting gap forms. This means that by the time neutrinos decouple at $T_D \sim 1-2$ MeV, the thermal excitation energy of quark matter is perhaps a tenth of an MeV per baryon, far too little to enable quark matter to evaporate if it is bound by, say, 10 MeV per baryon. Under such conditions quark matter is thermodynamically stable.)

However, at temperatures below T_c and well above T_D, quark matter has a large quark chemical potential and hence a relatively large vapor pressure and is not stable thermodynamically, in contact with the dilute low-temperature phase. It will tend to evaporate, but the evaporation will be limited by the time scales for equilibration of heat, baryon number, and strangeness. (For instance, the evaporation of a lump of quark matter will involve mainly the emission of neutrons at the surface, creating a strangeness excess at the surface; the time scale for diffusion or weak decay of strange quarks will be one of the considerations limiting the evaporation rate.) A serious study of the evaporation process between T_c and T_D will not be attempted here.

However, a naive estimate, similar to our previous estimate of the baryon diffusion length at T_c, is that at $T \simeq 10$ MeV, the diffusion scale is about 10^{-2} cm, so that quark lumps at the lower end of our estimated range (10^{-2}–10 cm) are in danger of evaporating before they become thermodynamically stable at temperatures somewhat below T_c.

The scenario proposed here is subject to many uncertainties. Various QCD uncertainties have been noted. Even more serious is the lack of a clear reason for neutrino losses to be the main way that bubbles of low-temperature phase lose energy. If the QCD situation is favorable and the scenario can somehow be made to work, it would have one major advantage. In many particle physics explanations of the dark matter, one must postulate a neutral particle with a lucky combination of mass and abundance, and it is a coincidence that this particle has a mass abundance comparable to that of baryons. In the scenario suggested here, the luminous and dark matter have the same origin—baryons—and the ratio of luminous to dark matter is computable, in principle, as a QCD effect—if the scenario somehow works.

If it turns out that quark lumps of size estimated here are responsible for the dark matter, what is their flux at the earth? The density of dark matter at our location seems to be[13] about 10^{-24}g/cm^3. Lumps of quark matter of radius 10^{-2}–10 cm weigh about 2×10^9–2×10^{18} g, so the number abundance is about 5×10^{-34}–5×10^{-43}/cm^3. They would have typical galactic velocities of order 2×10^7 cm/sec, corresponding to a flux 10^{-26}–10^{-35}/cm^2/sec. With the cross sectional area of the earth 8×10^{17} cm^2, the rate of collisions is 10^{-8}–10^{-17}/sec or between almost one per year (if the radius is 10^{-2} cm) and almost one per 10^9 yr (if the radius is 10 cm).

These collisions would apparently not be as spectacular as one might think. Quark lumps would not initiate nuclear reactions (for reasons discussed earlier). Lumps of the radius we have estimated would plunge all the way through the earth, displacing everything they met in a tube of 0.01–10 cm radius, since their mass per unit area (10^{13}–10^{16} g/cm^2) is much greater than that of the earth (5×10^9 g/cm^2). Such events might be detectable if they occur as often as once a year or once a decade, but they apparently would not be conspicuous.

If quark matter is not produced cosmologically, there might still be a quark matter flux in cosmic rays. This is discussed in an appendix.

We have tried in this paper to explain dark matter purely on the basis of known physics. Departing from this rigorous rule, one intriguing variant of the scenario we have considered is to suppose that the primordial baryon-to-photon ratio n_B/n_γ was much bigger than today's value of 10^{-8}–10^{-10}.

Suppose, for instance, that primordially $n_B/n_\gamma \sim 1$. In this case, the primordial cosmic fluid would behave rather like a normal fluid, with baryon number conservation playing the role that conservation of the number of molecules plays in an ordinary fluid. Pressure equilibrium would be maintained in the cosmic fluid by sound waves, but this would not preserve temperature equilibrium since (as in water or air) temperature gradients could be compensated for by gradients in the baryon concentration, while keeping a constant pressure. In such a situation, heat transport is likely to be by diffusion (as in any normal fluid near equilibrium), and since neutrino mean-free paths are enormous, heat transport would be via neutrinos.

If so, shrinking bubbles of high-temperature phase would lose heat without losing baryons, and quark matter would form. In fact, if $n_B/n_\gamma \sim 1$ primordially, one would form quark matter lumps 10^8 times heavier than we previously estimated (so $m \sim 10^{17}$–10^{26} g), since the high-temperature bubbles would not need to shrink very much before the quarks would become degenerate. In such a picture, one would of course need a mechanism for generating a large entropy prior to nucleosynthesis. One possible picture is the idea[29] that the decay of a 10^4 GeV gravitino generates a large entropy and reignites nucleosynthesis well after the QCD epoch.

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APPENDIX A: PRESENT-DAY PRODUCTION OF QUARK MATTER

This paper has really considered two independent ideas: the possibility that three-flavor quark matter is stable, and the possibility that it was manufactured in the big bang. It is possible that quark matter is stable, but was not produced cosmologically. If so, is there any hope of observing quark matter today? This question naturally leads one to ask whether there exists a mechanism for producing quark matter in today's universe.

There is another good reason for asking this question. It has been claimed that the Centauro cosmic-ray events may be evidence for a quark-matter component in cosmic rays. (The Centauros are events[31] in which the primary seems to fragment into hundreds of baryons and almost nothing else. This is a plausible fate[27] for a small drop of quark matter that strikes the atmosphere at high energy. The Centauro primaries seem to penetrate much further in the atmosphere than a conventional heavy nucleus would.)

If the Centauro events are real and are to be explained in terms of quark matter, then we must explain a flux of quark-matter droplets with roughly 10^3 baryons per drop and roughly 10^3–10^4 GeV per baryon. The actual flux is small, of order $10^{-2}/\mathrm{m}^2/\mathrm{yr}$. One cannot account for this flux by synthesizing quark matter in the big bang, since drops of quark matter produced in the big bang, regardless of their velocities at that time, would have long since red-shifted to low velocities.

To discuss contemporary production of quark matter, we must bear in mind the following: regardless of whether quark matter is stable at zero pressure and temperature, it is certainly the ground state of a large assembly of baryons if the external pressure is high enough. At high pressure (or density), the strong interactions are asymptotically free, and the ground state of a dense collection of quarks can be found perturbatively.

We are assuming in this paper that at zero external pressure (and temperature), quark matter is stable if, and only if, the strangeness is large enough.

If strange quarks are not available in reasonable numbers—which is true except in the big bang—the only way to create quark matter is to compress protons and neutrons to a pressure at which two-flavor quark matter becomes stable. Such high pressures are not attained in ordinary stars or planets. In addition, a small nucleus of quark matter that might be present in an ordinary star or planet would not grow spontaneously, since Coulomb barriers would prevent it form absorbing charged nuclei.

But there is one conspicuous astrophysical environment in which quark matter can readily form and grow. This is the neutron star. It may be that when neutron stars form they already contain a small nucleus of quark matter of cosmological or cosmic-ray origin. Even if no nucleus of quark matter is present to begin with, it is very likely that the pressure at the center of a neutron star is so large that in that region quark matter will form spontaneously even without strangeness. (If three-flavor quark matter is stable at zero pressure, then two-flavor quark matter is relatively close to being stable even at zero pressure and is probably stable under the high pressure in the center of a neutron star.) One way or another, neutron stars almost surely contain a quark-matter component.

Once a quark-matter component is present in a neutron star, two things will rapidly happen. It will quickly develop the equilibrium strangeness content via weak interactions, such as $ud \rightarrow us$. The energy will be lowered as strange quarks are created one by one until equilibrium is reached. Even more important, the quark-matter component can rapidly grow in a neutron star by absorbing free neutrons, since there is no Coulomb barrier. The entire star will turn into quark matter, except perhaps for a thin outer crust in which there are no free neutrons.

This outer crust, consisting of nuclei and degenerate electrons, is rather thin and dilute. It is usually estimated to have a thickness of a few tenths of a kilometer (compared to the 10-km radius of the neutron star) and a density of at most 4×10^{11} g/cm^3 (10^{-3} of the average density of the star). Actually, if quark matter is stable at zero pressure, this conventional estimate is only an upper bound on the thickness of the "normal" outer crust. One can envisage a star that consists entirely of quark matter, with no crust, the surface density of the star being simply the density of quark matter at zero pressure, or about 3–4×10^{14} g/cm^3.

It is estimated that about 10^{-2} of the mass in a typical galaxy such as ours is in the form of neutron stars (corresponding to 10^9–10^{10} neutron stars in the galaxy). Thus, if quark matter is stable, it is a fairly abundant component of our galaxy. What are the prospects that some of this neutron star material is ejected from its place of origin and is incident on earth? This would depend on violent events in which part of the contents of a neutron star are expelled into the galaxy.

For instance, Gilden and Shapiro[32] recently made a numerical study of a head-on collision of two neutron stars at moderately relativistic velocities. They found that about 13% of the mass was ejected from the star system. If quark matter is stable, this 13% would probably mostly escape as lumps of quark matter of various sizes—the spectrum of sizes being difficult to estimate. Since this mass loss occurs in violent conditions, some part of the outgoing matter might be accelerated to relativistic velocities, perhaps furnishing a candidate for Centauros.

Head-on neutron star collisions may be rare events. However, neutron star collisions of *some* kind are almost certainly not rare. Of less than 10^3 known pulsars, there seems to be one known binary neutron star system, the binary pulsar PSR 1913 + 16. The gravitational radiation from this system gives it a lifetime of 10^8 yr; in that time—short compared to the age of the Galaxy—the two neutron stars will spiral inward and collide. Naively extrapolating from one known example out of 10^3, it is likely that 10^6–10^7 of the 10^9–10^{10} neutron stars in our galaxy have undergone this fate.

The decay of a binary system seems less promising for mass ejection than a head-on collision. However, Clark and Eardley have speculated[33] that also in this case, during mass transfer between the two stars, some mass will be ejected to infinity by hydrodynamic effects or neutrino pressure; in fact, they suggest that this could be as much as 10^{-1} solar masses. With 10^6 binary systems ejecting $10^{-1}M_\odot$ each, there could be $10^5 M_\odot$ of quark matter free in our galaxy, or 10^{-6} of all the luminous galaxy. This figure is probably an extreme upper bound.

Actually, to explain Centauro events the flux required is rather small. The Centauro flux, if real, is about $10^{-2}/\mathrm{m}^2/\mathrm{yr}$. If the Centauro primaries are relativistic particles of baryon number 10^3, the baryon density in Centauro primaries is about 10^{-21} baryons/cm^3. The average universal abundance of baryons in luminous form seems to be about $10^{-7}/cm^3$, so on this interpretation Centauro primaries are 10^{-14} of all baryons if they are uniformly distributed in the Universe; if Centauro primaries are trapped in galaxies by magnetic fields they are 10^{-20} of all baryons.

To explain Centauros, a typical galaxy with $10^{11}M_\odot$ of luminous matter must convert $10^{-3}M_\odot$ or $10^{-9}M_\odot$ of baryons into Centauro primaries. Of course, the Centauro primaries are highly relativistic (10^3–10^4 GeV/baryon), and there may be a much bigger, yet unobserved flux of lower-energy particles of the same composition as Centauro primaries. Even so, it would appear that the mass requirement for explaining Centauros is quite moderate; all observed Centauros might even be the product of a single energetic neutron star collision in this history of the Galaxy. To develop a detailed theory of Centauros along these lines, one would have to understand the mechanisms of mass ejection from neutron stars, the fraction of ejected matter which escapes in drops of order 10^3 baryons, and the fraction of those drops which are accelerated to relativistic velocities.

Finally, one may worry that it violates basic observations about neutron stars to suppose that neutron stars are made of quark matter except for a thin outer crust. That this is not so can be seen from work by Fechner and Joss,[25] who considered an equation of state for which quark matter (at zero pressure) was slightly unbound and showed that the bulk properties of quark stars were very similar to those of neutron stars.

This point can be made in the following terms. Simplifying to ignore quark masses (which are important in vacuum, but not in neutron stars), the relation between pressure p and density ρ of quark matter is roughly $p=\frac{1}{3}(\rho-4B)$, where B is a phenomenological constant—essentially the MIT bag constant. The density of zero-pressure quark matter is just $4B$. With this form of the pressure-density relation, a simple numerical integration of the standard equations of stellar structure shows that the maximum mass of a stable quark star is

$$\bar{M}=0.0258/(G^{3/2}B^{1/2}) , \tag{A1}$$

where G is Newton's constant. The corresponding radius and critical density are

$$R=0.095/G^{1/2}B^{1/2} , \quad \rho(0)=19.2B . \tag{A2}$$

A typical value of B is $B_0=56$ MeV/cm^3=(145 MeV)^4. In conventional units, (A1) and (A2) give

$$\bar{M}=2.00M_\odot\left[\frac{B_0}{B}\right]^{1/2}, \quad R=11.1\text{ km}\left[\frac{B_0}{B}\right]^{1/2}, \quad \rho(0)=1.91\times10^{15}\frac{\text{g}}{\text{cm}^3}\left[\frac{B}{B_0}\right], \tag{A3}$$

for the stable quark star of maximum mass. The surface density is $4B=(4.0\times10^{14}\text{ g/cm}^3)(B/B_0)$.

TABLE I. The table shows the mass and radius of a quark star of various central densities, for $B=B_0=(145\text{ MeV})^4$. The central density is given in units of $\rho^*=4B=4\times10^{14}$ g/cm^3, which is the density of quark matter at zero pressure. The maximum central density of a stable star is 4.8 ρ^*. The radius and mass are again given in kilometers and solar masses, respectively. If $B\neq B_0$, then R and M must be multiplied by $(B_0/B)^{1/2}$.

$\rho(0)/\rho^*$	1.5	2.0	3.0	4.1	4.8
R(km)	10.5	12.0	12.3	11.2	11.1
$M/M_\odot$	1.11	1.58	1.89	1.98	2.00

If there is an outer crust of normal matter, it is only the radius that changes appreciably (by a few percent). The values of (11) are within the range of conventional neutron star models.[24] For further details of the radius and central density of quark stars for various mass, see Table I.

APPENDIX B: GRAVITATIONAL SIGNAL FROM THE QCD EPOCH

In this paper, we have assumed that the QCD phase transition occurred in near-equilibrium conditions. It is possible, instead, that this transition occurred explosively, with the nucleation of the low-temperature phase triggering detonation waves[9]—in which much of the latent heat would go into acceleration of bubble walls—as opposed to the process of slow combustion that we have discussed. The collision of detonation waves would leave a rather complicated universe on scales less than the horizon scale at the QCD epoch. What observable consequences might remain in today's world?

Various possibilities have been discussed in the literature.[4–6] Here we will consider the possibility that gravitational waves produced in the violent bubble collisions would be detectable today.

As we will see, these waves would have very long wavelengths in today's universe. Long-wavelength gravitational waves can be detected by Doppler tracking of spacecraft[34] or by pulsar timing measurements.[35] The two methods are similar in principle, but the latter method is sensitive at the longest wavelength and is therefore (for reasons we will see) most suited to exploring the conditions that prevailed during the QCD epoch.

To understand how pulsar timing can be used to detect gravitational waves, consider a pulsar which (prior to the passage of a gravitational wave) is separated from the earth by a distance L in the z direction. If a weak gravitational wave with $h_{zz}=h\cos(k\cdot x-\omega t)$ passes by, the light travel time from earth to pulsar is not precisely L, but is

$$\tilde{L}(t)=L+\tfrac{1}{2}h\int_0^L dz\cos[k_z z-\omega(t-z)] . \tag{B1}$$

If $k_zL\lesssim1$, then the time-dependent part of $\tilde{L}$ is of order hL. If $k_zL\gg1$, then the time-dependent part of $\tilde{L}$ is of order $h\lambda$, where $\lambda=2\pi/k$.

The time dependence of $\tilde{L}$ causes a correction to the arrival time of the pulsar pulses. A random gravitational wave background would show up as a residual "noise" in the pulse arrival times which could not be removed by fitting for various physical effects that are relevant (the pulsar period and phase, its spin-down rate, its velocity, and numerous solar system effects).

Limits on pulsar timing noise have already been used to set cosmologically interesting bounds on gravitational waves of period 1—10 yr.[36] If it turns out someday that a gravitational wave background is really the cause of part of the pulsar timing noise, then—as originally noted by Detweiler—general relativity predicts definite correlations in the timing noise of different pulsars; observations of such correlations would be convincing evidence that gravitational waves are really being detected. What makes this subject exciting is the fact that the newly discovered millisecond pulsar[37] is far more stable than previously known pulsars, and offers the hope[38] of a dramatic improvement in the bounds on a cosmological background of gravitational waves.

An experiment conducted for a period τ is sensitive only to gravitational waves of period $\omega \lesssim 2\pi/\tau$. (The technical reason for this perhaps obvious fact is that the influence on timing measurements of the first few time derivatives of $\tilde{L}$ can be absorbed in a fit for the unknown pulsar phase, period, and spin-down rate.) In practice, for normal experiments τ is at most a few years or decades, while pulsars are thousands of light years away. So $k_z L \gg 1$, and the time-dependent part of $\tilde{L}$ is of order $h\lambda$. This modifies the pulse arrival time by $\delta t = h\lambda/c$. With $\lambda \lesssim c\tau$ the timing fluctuations due to gravitational waves in an experiment of duration τ are of order

$$\delta t \sim h\tau \ . \tag{B2}$$

For δt to be detectable, it must be greater than the timing noise Δt due to other causes; the smallest detectable h is $h \sim \Delta t/\tau$.

The remarkable fact is then that (for τ less than the light travel time to the pulsar) one can measure smaller and smaller h by studying the pulsar for a longer and longer τ. But at long periods one may reasonably expect a *larger* amplitude h. The reason for this is that in general relativity the energy density of a gravitational plane wave of amplitude h and period P is

$$\rho = \frac{1}{8\pi G}\left[\frac{2\pi}{P}\right]^2 h^2 \ . \tag{B3}$$

This means that gravitational waves of period P and energy density

$$\tilde{\rho}(P) = \frac{1}{8\pi G}(2\pi\Delta t)^2 \frac{1}{P^4} \tag{B4}$$

are detectable by means of pulsar timing observations.

For the millisecond pulsar, a timing stability $\Delta t \sim 10^{-6}$ sec has been demonstrated,[31] and it is plausible that $\Delta t \sim 10^{-7}$ sec may be attainable.[39] If so, at a period $P = 1$ yr$\cong 3\times 10^7$ sec, a detectable gravitational wave amplitude is $h \gtrsim 3\times 10^{15}$. This corresponds to an energy density of about 10^{-37} g/cm^3 or 10^{-8} times closure density. More detailed and precise theoretical treatments can be found in Ref. 35.

The only general theoretical limit on a cosmological background of gravitational waves comes from considerations of nucleosynthesis. This argument was first given by Carr[35] and Zeldovich and Novikov.[40] Let Ω_g and Ω_γ be the fraction of closure density in gravitational waves and photons, respectively. The ratio Ω_g/Ω_γ is approximately conserved in time, since gravitons and photons red-shift in the same way. At the time of nucleosynthesis one must have $\Omega_g \lesssim \Omega_\gamma$; otherwise the extra cosmic expansion due to gravitational waves would spoil conventional nucleosynthesis calculations, the effect being similar to the effect of including extra neutrino types. Since today $\Omega_\gamma \sim 10^{-5}$, one has a bound $\Omega_g \lesssim 10^{-5}$.

If a cosmological background of gravitational radiation is discovered, it may have been produced in the conventional matter or radiation-dominated eras, or it may have been produced even earlier, perhaps in the "inflationary universe." Starobinsky[41] has calculated the gravitational radiation produced in a transition from a de Sitter inflationary phase to a radiation-dominated era. Observation of the spectrum computed by Starobinsky, with its $1/\omega$ frequency dependence, would be an extraordinary confirmation of the inflationary universe. We will have nothing to add here to Starobinsky's beautiful calculation, some of whose implications have been discussed by Rubakov, Sazhin, and Veraskin.[42] Here we will consider only the case of gravitational radiation produced in the radiation-dominated era.

In a model-independent sense, what can one learn by observations sensitive to waves of period P and amplitude $h = (10^{-7}\ \text{sec})/P$? If gravitational waves are generated during the radiation-dominated era at a time when the temperature of the Universe is T, their wavelength when they are produced would be no longer than the horizon scale at that epoch. In order of magnitude this scale is M_{Pl}/T^2; a more precise value is

$$\lambda_0 = \frac{M_{\text{Pl}}}{T^2}\left[\frac{3}{4\pi\alpha N(T)}\right]^{1/2} , \tag{B5}$$

where $\alpha = \pi^2/45$ is the Stefan-Boltzmann constant and $N(T)$ is the effective number of particle spin states at temperature T. Let T^* be the current cosmic temperature, $T^* = 2.7\,^\circ$K. Because of the cosmic expansion, the wavelength of a wave produced at temperature T is bigger today by a factor of T/T^* than it was when the wave was generated. The maximum possible wavelength today of waves generated when the temperature was T is thus

$$\bar{\lambda}(T) = \frac{1}{T^*}\frac{M_{\text{Pl}}}{T}\left[\frac{3}{4\pi\alpha N(T)}\right]^{1/2}$$
$$= (0.088\ \text{cm})\left[\frac{M_{\text{Pl}}}{T}\right]\frac{1}{\sqrt{N(T)}} \ . \tag{B6}$$

For $T = 100$ MeV (a plausible QCD temperature) and $N = 70$ (a reasonable value just before the QCD transition) we get

$$\bar{\lambda} = 1.28\times 10^{18}\ \text{cm} = 1.37\ \text{light years} \ . \tag{B7}$$

We see, then, that gravitational waves produced in the QCD epoch would have periods as long as about one year.

[However, the fact that $\bar{\lambda}$ of Eq. (B7) is miniscule by galactic standards means that the QCD transition could influence galaxy formation only indirectly, if at all.[4-6]]

If a gravitational wave is created at temperature T with amplitude h_0, its amplitude today is $h = h_0(T^*/T)$. (This follows from standard formulas for propagation of gravitational waves in the expanding universe. See, for instance, pp. 584 and 585 of Ref. 43.) With $T^* = 2.7\,^\circ\mathrm{K}$ and $T = 100$ MeV, $h = 2.3\times10^{-12}h_0$. From our previous discussion, we know that for waves of period 1.37 yr, the detectable range is crudely (and perhaps optimistically) $h \geq 10^{-7}$ sec/(1.37 yr)$=2.3\times10^{-15}$. So a gravitational wave created in the QCD epoch with primordial wavelength equal to the horizon scale and primordial amplitude $h_0 \geq 10^{-3}$ may be detectable. This potential sensitivity is truly remarkable.

In general, the wavelength of waves created at the QCD epoch may not be the maximum $\bar{\lambda}$ allowed by the horizon scale. If the actual wavelength is λ, the sensitivity is 10^{-7} sec/λ instead of 10^{-7} sec/$\bar{\lambda}$, and a detectable primordial amplitude could be

$$h_0 \geq 10^{-3}\left[\frac{\bar{\lambda}}{\lambda}\right] . \tag{B8}$$

Let us now estimate—under reasonable but optimistic assumptions—the amplitude of gravitational waves that might be generated in the QCD phase transition.

A body of mass M and radius R has a gravitational field of order GM/R. If two such bodies collide at moderately relativistic velocities, and are brought to a halt or scattered through a large angle, the gravitational waves generated have amplitude of order GM/R.

In our case the colliding objects are expanding bubbles of low-temperature phase. We suppose these bubbles are expanding explosively, as a detonation wave, with the latent heat going into the bubble walls, whose velocities are moderately relativistic.[7] The latent heat is a large fraction of the cosmic energy density ρ. When a bubble has radius R its expanding walls have energy $(4\pi/3)R^3\rho$ so $GM/R = (4\pi/3)G\rho R^2$.

The horizon radius is $R_H = (\sqrt{8\pi}/3)G\rho$, so if we set $R = R_H$, then $GM/R \sim 1$. In such a case, abundant gravitational waves would be produced; however, if the expanding bubble walls have $GM/R \sim 1$, then their collisions would be likely to convert much of the mass of the Universe into black holes. A gross contradiction with measurements of the total mass density of today's universe would arise if more than about 10^{-7} of the total mass is converted into black holes during the QCD epoch.

Let us suppose that the bubbles when they meet have a typical radius R that is less than the horizon scale R_H; let $\epsilon = R/R_H$. Then $GM/R \sim \epsilon^2$. Since the expanding bubbles fill all space when they collide, they produce an all-pervasive gravitational wave signal of mean primordial amplitude $h_0 \sim \epsilon^2$ and wavelength R. Since $\lambda/\bar{\lambda} = R/R_H = \epsilon$, we see from Eq. (B8) that this signal may be detectable if $h_0 \gtrsim 10^{-3}\epsilon^{-1}$. In other words, we require $\epsilon \geq 1/10$; $\epsilon = 1/10$ corresponds to waves of one month period in today's universe.

What value of ϵ is plausible? Here we must make assumptions about how the phase transition was nucleated. In particle physics it is often assumed that phase transitions are nucleated by thermal fluctuations. In practice, experience shows that except in very pure, homogeneous samples, phase transitions are often nucleated by various forms of impurities and inhomogeneities of nonthermal origin.

Hogan has recently estimated[4] the plausible range of ϵ for the case in which the QCD transition is nucleated by thermal fluctuations. He finds $\epsilon \leq 10^{-2}$. The origin of this estimate is as follows: If $S(T)$ is the free energy for a bubble that forms at temperature T, then the nucleation rate is of order $T^4 e^{-S(T)/T}$. There is at least one bubble created per space-time horizon volume if $T^4 e^{-S(T)/T} \geq (T^2/M_{\mathrm{Pl}})^4$, or if $S(T)/T \geq 4\ln M_{\mathrm{Pl}}/T \simeq 180$. If $S(T)/T = 180$, then (without very lucky fine tuning) $dS/dT \gtrsim 10^2$. This means that if the first bubble forms in a typical horizon volume at temperature T_1, then many will have formed by temperature

$$T_2 = \left[1 - \frac{1}{10^2}\right]T_1 .$$

From T_1 to T_2 a bubble can expand only 10^{-2} horizon lengths, and this is then a reasonable upper bound on ϵ. More detail can be found in Hogan's paper.

Although this upper limit on ϵ is temptingly close to the $\epsilon = 10^{-1}$ that can give a detectable gravitational wave signal, we must note that the ratio of signal to sensitivity scales like ϵ^3. If the QCD transition was nucleated thermally, it would not be likely that the gravitational wave signal is detectable with 1984 technology.

What if the transition was nucleated by impurities? In this case the mean spacing between bubbles has nothing to do with free energies of nucleation and is simply the spacing r between the relevant impurities. Impurities of $r > R_H$ are too few to play a role, while if $r \ll R_H/10$, the resulting gravitational wave signal is again too small, since we need $\epsilon = r/R_H \geq 1/10$. The interesting case is $10^{-1}R_H \lesssim r \lesssim R_H$.

An example of a type of impurity that could nucleate the QCD transition is a magnetic monopole. The core of a monopole is a region of strongly broken chiral symmetry,[44] and this region could easily serve as a nucleus for the low-temperature QCD phase, rather as in weakly coupled gauge theories.[45] However, it would be quite a fluke for the mean monopole spacing to be between $10^{-1}R_H$ and R_H during the epoch considered.

Far more promising is the possibility that cosmic strings could serve as nucleation sites. Unlike monopoles (or any other pointlike particles), strings have the property that the number of strings per horizon volume is roughly constant as the Universe expands.[46] (Just this property makes strings plausible as agents of galaxy formation.) It is quite natural for the mean separation between strings to be comparable to, but a bit less than, the horizon scale. Whether strings arising in a given context would actually serve as nuclei for the QCD phase transition is somewhat model dependent, but if so, there is a good chance of a detectable gravitational signal.

One last possibility is that the cosmic fluid prior to the QCD epoch might not be uniform, but could have a low amplitude of turbulent motion. Such turbulent motion might be, for instance, a remnant of bubble collisions in the earlier $SU(2)\times U(1)$ transitions. (Such turbulence might be large initially, but could be largely damped by neutrinos prior to the QCD epoch.) In a fluid in a turbulent state, the temperature and pressure are not uniform. Preexisting turbulence could play a crucial role in nucleating the QCD transition since the regions of most extreme temperature depression would act as nucleation sites for the QCD transition. Under such conditions, to predict the mean spacing between bubbles one would need to know the detailed spectrum of the fluid motion, so it is very hard to estimate the possible gravitational signal.

Let us conclude this discussion of the gravitational signal from the QCD transition with a comment of a model independent sort. It has been suggested[4–6] that the QCD transition produces black holes which constitute the "dark matter" in today's universe.

For this to be so, the QCD transition must convert about 10^{-8} of the mass into black holes. Black holes can possibly be formed if bubbles of low-temperature phase are so far apart when they nucleate that by the time they collide, the mass in colliding bubble walls is adequate to form black holes.

If the bubbles expand to radius R before colliding, the colliding walls carry a mass $M \sim T^4 R^3$, since the energy density is of order T^4. For colliding objects of mass M and radius R to form a black hole, one needs $GM/R \gtrsim 1$, or in this case, $R \gtrsim 1/G^{1/2}T^2$, which is the horizon scale.

Nucleated bubbles cannot be *farther* apart when they form than the horizon scale, or a cosmological disaster would ensue.[47] If we hope to form black holes, we must suppose $R \sim 1/G^{1/2}T^2$. In this case, the mass of the black holes is $M \sim T^4 R^3 \sim 1/G^{3/2}T^2$, which is of order a solar mass for $T \sim 100$ MeV.

We do not want a *typical* horizon volume to react into a black hole, because this would lead to by far an excessive mass density in today's universe. Rather, it must be that a given horizon volume has a 10^{-8} probability to turn into a black hole. Where a black hole forms, the metric perturbation is of order 1. If a black hole was produced in 10^{-8} of all horizon volumes, then even if nothing happened elsewhere, the root mean square metric perturbation in primordial gravitational waves is of order $(h_0{}^2)^{1/2} \sim (10^{-8})^{1/2} \sim 10^{-4}$. This is not quite detectable, because we saw earlier that for a detectable signal today, the mean primordial gravitational wave signal at the QCD horizon scale should be $h_0 \gtrsim 10^{-3}$. However, if black holes formed in 10^{-8} of all horizon volumes, a metric disturbance that is large but not large enough to make a black hole must have been the typical occurrence. If black holes of QCD genesis are the explanation of the dark matter in the Universe, the gravitational radiation produced in their formation is almost certainly detectable, but it is impossible to be more quantitative without better understanding of how the QCD transition occurs.

As for the possibility that the nucleation scale R in the QCD transition was comparable to the horizon scale, this depends on how the QCD transition was nucleated. If it was nucleated thermally, then as we discussed earlier, the nucleation scale was almost certainly at least about 10^2 times less than the horizon scale, and black hole formation seems quite unlikely. Of the scenarios we considered earlier for nonthermal nucleation of the transition, the only one that is promising for making black holes is the possibility that the transition was nucleated by cosmic strings (or walls bounded by strings); it is natural for the mean string separation to be comparable to but slightly less than the horizon scale.

This completes our discussion of gravitational wave generation in the QCD transition. Of course, there are many other possible sources of a cosmological gravitational wave background. We have already noted Starobinsky's calculation in the context of the inflationary universe model. There are many galactic processes that might possibly generate long period gravitational waves.[40] Without claiming to be comprehensive, we will here consider several scenarios which have been considered in the literature and which are of interest as possible sources of gravitational waves.

Stecker and Shafi[48] suggested that axion walls bounded by strings detach themselves at $T \sim 100$ MeV. At this time their radius is of order the horizon scale, and they begin oscillating at the speed of light with a period of order 10^{-4}–10^{-5} sec.

The oscillating walls decay by emitting gravitational radiation; this process takes 10^4–10^5 sec, and is completed when the temperature of the Universe is of order 10 keV. Most of the radiation is emitted near that temperature. Although these gravitational waves were generated with a period of order 10^{-4}–10^{-5} sec, the period today would be larger by a factor of (10 keV)/(2.7 °K) and would be of order 10^3–10^4 sec.

To estimate the amplitudes of these waves, let us note that in the model of Stecker and Shafi, the walls were on the verge of dominating the mass in the Universe when they disappeared into gravitational waves. This means that, when produced, the waves have energy comparable to the energy in photons (which are a large component of the energy content of the Universe in that epoch). Since gravitons and photons red-shift in the same way, the energy content in gravitational waves would still be comparable today to the energy content of the blackbody radiation, which is about 4×10^{-5} of closure density. The model of Stecker and Shafi therefore corresponds to 10^{-4}–10^{-5} of closure density in gravitational waves of period 10^3–10^4 sec. The period of these waves is too short to be favorable for detection by pulsar timing measurements, and detection in that way appears farfetched. However, this signal might ultimately become detectable as a result of future developments in the related area of Doppler tracking of spacecraft.[34]

Another scenario that leads to an interesting gravitational wave flux is the idea[46] that topologically stable strings served as seeds for galaxy formation. In this case, the strings are still oscillating and generating gravitational waves in today's universe.

In such a context, Turok[49] recently estimated the gravitational wave flux in today's universe would have a typical period $P \sim 5000$ yr and a typical amplitude $h \sim 2\times 10^{-11}$. Such a flux can definitely be detected by pulsar timing measurements if one is patient enough. Over a 5000-yr period, the gravitational wave would cause a discrepancy in the pulse arrival time of about $(2\times10^{-11})\times(5000\ \text{yr})\simeq3$ sec, while an effect of 10^{-7} sec might be detectable with 1984 technology for pulsar observation.

If one is not so patient, how long will it take to observe this signal? If the measurements are conducted not for 5000 yr, but for a smaller time t, the amplitude of the delay in pulse arrival time due to the gravitational wave is not 3 sec, but (3 sec)$\cos(t/T+\varphi)$, where φ is an unknown phase and $T=5000$ yr. Expanding this in powers of t/T for $t\ll T$, the terms of order $(t/T)^0$, $(t/T)^1$, and $(t/T)^2$ can be absorbed in the fact that one does not know *a priori* the phase, period, and spin-down rate of the pulsar. The first term from which a gravitational wave signal can be extracted is the term of order $(t/T)^3$. The gravitational wave signal is hence of order (3 sec)$(1/3!)(t/T)^3$, and requiring this to be at least 10^{-7} sec, one finds it would take about 25 yr to detect the effects of cosmic strings in this way, in the absence of technological improvements.

If the spin-down rate of the binary pulsar were known *a priori*, one would obtain a gravitational wave signal of order (3 sec)$(1/2!)(t/T)^2$, which would be detectable within a few years. Bertotti, Carr, and Rees[35] pointed out that the orbital motion of the binary pulsar is a clock whose behavior can be predicted (if one believes general relativity) and used to search for gravitational waves of period up to 10^4 yr, but on the basis of their numbers, the known binary pulsar PSR 1913 + 16 could not be used to detect a signal of the magnitude estimated by Turok.

Evidently, a great many mechanisms might generate a cosmic background of long period gravitational waves. If such a background is found, its frequency dependence will play an important role in determining whether the signal was generated in the inflationary universe, in the QCD phase transition, by oscillating strings, or in some other way.

Note added in proof. N. Turok and A. Vilenkin have independently pointed out (private communications) that the time required to detect gravitational waves from cosmic strings is much less than the 25 years estimated hre. The estimate here corresponds to gravitational waves that are *currently* being generated, but red-shifted waves emitted at *large z are more favorable.* The spectrum of gravitational waves emitted by strings at large z was computed by Vilenkin.[50] The question of detection of these waves via the millesecond pulsar has been discussed independently by Hogan and Rees.[51]

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