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Internal structure of bosons in a composite model of particles

Han-Chao Liu*

Department of Physics, University of California Irvine, Irvine, California 92717

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The internal wave functions of gluons, horizontal bosons, weak bosons, the photon, and the Higgs boson are determined within the framework of a composite model of leptons and quarks. Some characteristic properties of the composite model are pointed out.

The basic ideas in the composite model suggested in Ref. 1 are firstly that the leptons and quarks are composite in internal space with three internal degrees of freedom denoted as A spin, B spin, and C spin, and secondly that all known symmetries are composed of and derived from the basic symmetry $SO(4)=SU(2)\times SU(2)'$. All known leptons and quarks are classified by means of definite quantum numbers and internal wave functions, based on the six "charges" A_1, A_2, B_1, B_2, C_1, and C_2 and their anticharges. They satisfy a strong constraint, namely, the mutual relations among the B_i, $\bar{B}_i$'s ($i=1,2$) should be the same as those among the A_i, $\bar{A}_i$'s and among the C_i, $\bar{C}_i$'s. Color, generation, electroweak interactions, and so on are composite properties and do not exist at the constituent level. In this composite model the usual gauge theories could remain correct at the composite level for the leptons, quarks, and bosons, since the usual symmetries can be derived from the basic symmetry. We could take the color $SU(3)^C$ as an exact local symmetry and the horizontal $SU(3)'$ and the electroweak $SU(2)'_W\times U(1)$ as broken local symmetries, since the former is related to the basic symmetry SU(2) and the latter to the basic symmetry SU(2)′ (Ref. 1). This composite model is consistent with experiment.[2]

We note that some characteristic properties of the present model are similar to those of the first version of the rishon model:[3,4]

(i) Conservation of the internal charges X_i, $\bar{X}_i$ in any process and vertex, where $X_i=A_1$, A_2, B_1, B_2, C_1, and C_2. For example,[5] consider proton decay

$$\begin{aligned} &u_Y+u_G\rightarrow \bar{d}_R+e^+ \ , \\ &(-\bar{A}_1\bar{B}_2C_1)+(-\bar{A}_2\bar{B}_1C_1)\rightarrow(\bar{A}_1\bar{B}_1C_1)+(\bar{A}_2\bar{B}_2C_1) \ , \end{aligned} \tag{1}$$

and the vertex

$$\begin{aligned} &e^-\rightarrow \nu_e+W^- \ , \\ &(-A_2B_2\bar{C}_1)\rightarrow(\bar{A}_1\bar{B}_1C_2)+(-A_1B_1A_2B_2\bar{C}_1\bar{C}_2) \ . \end{aligned}$$

(ii) Q and $B-L$ satisfy two equations

$$\begin{aligned} &-2\sum_X[n(X_2)-n(\bar{X}_2)]-\sum_X[n(X_1)-n(\bar{X}_1)]=3Q \ , \\ &-\sum_X[n(X_2)-n(\bar{X}_2)]-2\sum_X[n(X_1)-n(\bar{X}_1)] \\ &\qquad\qquad =3Q-3(B-L) \ , \end{aligned} \tag{2}$$

where X denotes A, B, and C and $n(X_i)$ denotes the number of charges X_i in a particle. From Eq. (2) one sees that the baryon number and lepton number are not conserved but their difference is conserved.

(iii) The total content of $\bar{\nu}_e$, e^-, u, $\bar{d}$ includes an equal number of X_i, $\bar{X}_i$'s [i.e., $\sum_X(X_1+\bar{X}_1+X_2+\bar{X}_2)$] generating a vanishing sum of both electric charges and $B-L$ values for each generation.

(iv) A hydrogen atom contains an equal number of X's and $\bar{X}$'s:

$$\begin{aligned} \mathrm{H}=p+e^-&=u_Y+u_G+d_R+e^- \\ &=-\bar{A}_1\bar{B}_2C_1-\bar{A}_2\bar{B}_1C_1-A_1B_1\bar{C}_1-A_2B_2\bar{C}_1 \ . \end{aligned} \tag{3}$$

Hence, at the level of the constituents, matter and antimatter may be equally abundant in the universe.

From above one sees that some good points of the rishon model are shared by the present model. On the other hand, most of the drawbacks of the rishon model as mentioned at the end of Ref. 3 seem to be avoided here. We note also that the present model succeeds in deriving color, while the rishon model fails.[6] It succeeds also in deriving three generations.

We note that the isotopic spin of a quark or lepton is not given by the "I_3" of Ref. 1. However, it can be defined by the Gell-Mann–Nishijma relation $I_3=Q-(B-L)/2$, where Q is defined in Ref. 1 and $B-L$ by Eq. (2) above.

The internal structure of the gluons can be obtained by means of three different methods. In the first method, the color octet of gluons ϕ_i^j is built from the color triplet of quarks and antiquarks as described in Fig. 1. The positions

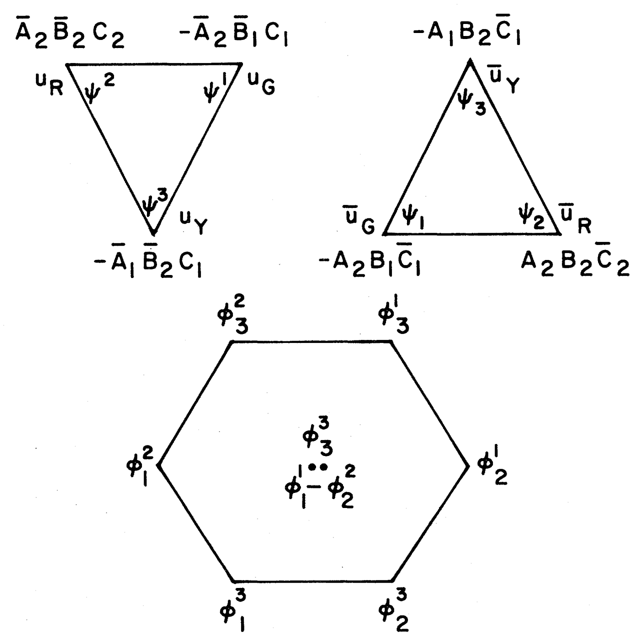

FIG. 1. The relation of the color octet of gluons $\phi_i^j=\psi_i\psi^j$ with a color triplet of quarks $(u_G,u_R,u_Y)=(\psi^1,\psi^2,\psi^3)$ and a color triplet of antiquarks $(\bar{u}_G,\bar{u}_R,\bar{u}_Y)=(\psi_1,\psi_2,\psi_3)$.

of the quark triplets on the vertices of the triangles in Fig. 1 are determined by the relations, using (u_R, u_Y, u_G) as an example,

$$U_+ u_Y = u_R, \quad \bar{u}_R U_+ = \bar{u}_Y \ , \tag{4}$$

$$V_+ u_G = u_Y, \quad \bar{u}_Y V_+ = \bar{u}_G \ , \tag{5}$$

$$T_+ u_R = u_G, \quad \bar{u}_G T_+ = \bar{u}_R \ , \tag{6}$$

where U_+, V_+, and T_+ are three generators of the color $SU(3)^C$ group.[1] For compatibility with Eqs. (4)–(6) and with the usual conventions in Fig. 1, we change slightly the labels of the wave functions in Table I of Ref. 1. In our paper the wave function of a particle (an antiparticle) in Table I of Ref. 1 is used as our wave function of the corresponding antiparticle (particle). Thus, we obtain, as usual, and tabulate in Table I the normalized wave functions of the eight gluons, relating to the u quarks. Similarly, the wave functions of the gluons relating to other quarks are obtained. For example, we have

$$\phi_3^2 = A_1 \bar{A}_2 B_1 \bar{B}_1 \bar{C}_1 C_2 \quad \text{(relating to the } d \text{ quarks)} \ , \tag{7}$$

$$\phi_3^2 = \bar{A}_2 A_1 B_1 \bar{B}_1 \bar{C}_1 C_2 \quad \text{(relating to the } c \text{ quarks)} \ , \tag{8}$$

$$\phi_3^2 = -A_1 \bar{A}_2 \bar{B}_2 B_2 C_2 \bar{C}_1 \quad \text{(relating to the } s \text{ quarks)} \ , \tag{9}$$

$$\phi_3^2 = -A_1 \bar{A}_2 \bar{B}_2 B_2 \bar{C}_1 C_2 \quad \text{(relating to the } t \text{ quarks)} \ , \tag{10}$$

$$\phi_3^2 = \bar{A}_2 A_1 B_1 \bar{B}_1 C_2 \bar{C}_1 \quad \text{(relating to the } b \text{ quarks)} \ , \tag{11}$$

where $A_1 \bar{A}_2$ $(\bar{A}_2 A_1)$ reflects the fact that A_1 $(\bar{A}_2)$ is inside a particle and $\bar{A}_2$ (A_1) is inside an antiparticle. The actual wave function of the gluon ϕ_3^2 should be a superposition of those in Eqs. (7)–(11) and in Table I.

In the second method the structure of the color octet of gluons is obtained directly by inverting both Eqs. (4)–(6) and the following relations:

$$U_- u_R = u_Y, \quad \bar{u}_Y U_- = \bar{u}_R \ , \tag{12}$$

$$V_- u_Y = u_G, \quad \bar{u}_G V_- = \bar{u}_Y \ , \tag{13}$$

$$T_- u_G = u_R, \quad \bar{u}_R T_- = \bar{u}_G \ , \tag{14}$$

$$2U_3 u_Y = -u_Y \ , \tag{15}$$

$$(2T_3 - 2V_3) u_R = -u_R, \quad (2T_3 - 2V_3) u_G = 2u_G \ . \tag{16}$$

All these relations are proved by using the composite structure of the color-octet generators given in Eq. (3) of Ref. 1. From Eq. (4) we have

$$U_+ = -\bar{A}_2 A_1 \bar{B}_2 B_2 C_2 \bar{C}_1 = \phi_3^2 \ . \tag{17}$$

Similarly from Eqs. (5), (6), and (12)–(16) we have

$$V_+ = \phi_1^3, \quad T_+ = \phi_2^1, \quad 2U_3 = (\tfrac{2}{3})^{1/2} \phi_3^3 \ , \tag{18}$$

$$U_- = \phi_2^3, \quad V_- = \phi_3^1, \quad T_- = \phi_1^2 \ , \tag{19}$$

$$2(T_3 - V_3) = \frac{2\sqrt{2}}{3} (\phi_1^1 - \phi_2^2) \ . \tag{20}$$

Eqs. (18)–(20) are true for all kinds of quarks. Therefore, the generators of the composite color $SU(3)^C$ can be considered as the wave functions of the gluons.

In the third method the gluon structure is obtained without using the quark wave functions but only using the composite structure of the generators. If A_2 is inside the antiparticle or $\bar{A}_1$ inside the particle, we have from $\tau_{A-} A_2 = A_1$ and $(-\bar{A}_1)\tau_{A-} = \bar{A}_2$

$$\tau_{A-} = \bar{A}_2 A_1 \ . \tag{21}$$

If A_2 is inside the particle or $\bar{A}_1$ is inside the antiparticle we have

$$\tau_{A-} = A_1 \bar{A}_2 \ . \tag{22}$$

Similarly, we have for $\bar{A}_2$ inside the antiparticle and A_1 inside the particle

$$\tau_{A+} = -\bar{A}_2 A_1 \ , \tag{23}$$

and for $\bar{A}_2$ inside the particle and A_1 inside the antiparticle

$$\tau_{A+} = -\bar{A}_1 A_2 \ . \tag{24}$$

Similarly

$$\tau_{A3} = A_2 \bar{A}_2, \quad \tau_{A3} = -\bar{A}_2 A_2 \ , \tag{25}$$

$$\tau_{A3} = -\bar{A}_1 A_1, \quad \tau_{A3} = -A_1 \bar{A}_1 \ . \tag{26}$$

One obtains, independently, Eqs. (18)–(20) by substituting Eqs. (21)–(26) in the composite expressions of the generators [see Eq. (3) of Ref. 1].

TABLE I. Normalized wave functions of the eight gluons relating to the u_R, u_G, u_Y quarks and the eight horizontal bosons relating to the u_R, t_R, c_R quarks.

	Gluons	Horizontal bosons
ϕ_3^2	$-\bar{A}_2 A_1 \bar{B}_2 B_2 C_2 \bar{C}_1$	$-\bar{A}_2 \bar{A}_1 \bar{B}_2 B_2 C_2 C_1$
ϕ_3^1	$\bar{A}_2 A_1 \bar{B}_1 B_2 C_1 \bar{C}_1$	$\bar{A}_2 \bar{A}_1 B_1 B_2 \bar{C}_1 C_1$
ϕ_2^1	$-\bar{A}_2 A_2 \bar{B}_1 B_2 C_1 \bar{C}_2$	$-\bar{A}_2 A_2 B_1 B_2 \bar{C}_1 \bar{C}_2$
ϕ_2^3	$-\bar{A}_1 A_2 \bar{B}_2 B_2 C_1 \bar{C}_2$	$-A_1 A_2 \bar{B}_2 B_2 \bar{C}_1 \bar{C}_2$
ϕ_1^3	$\bar{A}_1 A_2 \bar{B}_2 B_1 C_1 \bar{C}_1$	$A_1 A_2 \bar{B}_2 \bar{B}_1 \bar{C}_1 C_1$
ϕ_1^2	$-\bar{A}_2 A_2 \bar{B}_2 B_1 C_2 \bar{C}_1$	$-\bar{A}_2 A_2 \bar{B}_2 \bar{B}_1 C_2 C_1$
$-\frac{3}{2}\phi_3^3$	$-\frac{3}{2}\bar{A}_1 A_1 \bar{B}_2 B_2 C_1 \bar{C}_1$	$-\frac{3}{2} A_1 \bar{A}_1 \bar{B}_2 B_2 \bar{C}_1 C_1$
$\frac{1}{\sqrt{2}}(\phi_1^1 - \phi_2^2)$	$\frac{1}{\sqrt{2}}(\bar{A}_2 A_2 \bar{B}_1 B_1 C_1 \bar{C}_1 - \bar{A}_2 A_2 \bar{B}_2 B_2 C_2 \bar{C}_2)$	$\frac{1}{\sqrt{2}}(\bar{A}_2 A_2 B_1 \bar{B}_1 \bar{C}_1 C_1 - A_2 \bar{A}_2 B_2 \bar{B}_2 \bar{C}_2 C_2)$

From above one sees that the internal structure of the gluons obtained by the three different methods is identical. We may consider the structure of each gluon separately. From Eqs. (4) and (7)–(11), all ϕ_3^2's have the same charges A_1, $\bar{A}_2$, $\bar{C}_1$, and C_2 but have two different B charges, i.e., $(B_1,\bar{B}_1)$ and $(B_2,\bar{B}_2)$. The interaction vertex of ϕ_3^2 with a quark is shown in Fig. 2(a), where B denotes B_1 or $\bar{B}_2$, (A',A'') denotes $(A_1,\bar{A}_2)$ or $(\bar{A}_2,A_1)$, and (C',C'') denotes $(\bar{C}_1,C_2)$ or $(C_2,\bar{C}_1)$.

We note that Fig. 2(b) has the same physical meaning as Fig. 2(a). However, Fig. 2(a) has one more annihilation and one more creation process than Fig. 2(b) and, therefore, has a much smaller probability to appear. Moreover, the pair of B charges could annihilate after it appears. Hence, it seems very natural to identify the real gluon ϕ_3^2 as a boson consisting of four charges A_1, $\bar{A}_2$, $\bar{C}_1$, and C_2. Similarly the charges of the gluons ϕ_3^1, ϕ_2^1, ϕ_2^3, ϕ_1^3, and ϕ_1^2 may be, respectively, $(A_1,\bar{A}_2,\bar{B}_1,B_2)$, $(\bar{B}_1,B_2,C_1,\bar{C}_2)$, $(\bar{A}_1,A_2,C_1,\bar{C}_2)$, $(\bar{A}_1,A_2,\bar{B}_2,B_1)$, and $(\bar{B}_2,B_1,C_2,\bar{C}_1)$. From this point of view, there should be more than one pair of charges annihilated in ϕ_3^3 and $(\phi_1^1-\phi_2^2)$. Actually this can be proved directly using Eqs. (16), (25), (26), and the relations[1]

$$2U_3=\tfrac{1}{2}(\tau_{A3}-\tau_{B3}) \ , \tag{27}$$

$$2(T_3-V_3)=\tfrac{1}{2}(-\tau_{C3}-\tau_{A3}+\tau_{B3}) \ . \tag{28}$$

One sees that each of the gluons ϕ_3^3 and $\phi_1^1-\phi_2^2$ consists of one pair of charges, and its interaction vertex with a quark is similar to that of Fig. 4.

The internal structure of the horizontal bosons can be obtained exactly parallel to the case of the gluons with the color triplet of quarks replaced by the generation triplet of quarks, the $\vec{\tau}_A$'s replaced by the $\vec{\tau}'_A$'s, and the two spinors (A_2A_1) and $(-\bar{A}_1\bar{A}_2)$ replaced by $(A_2-\bar{A}_1)$ and $(A_1\bar{A}_2)$. The normalized wave functions ϕ_i^j relating to the (u_R,t_R,c_R)-generation triplet are tabulated in Table I. The internal charges of the real horizontal bosons ϕ_3^2, ϕ_3^1, ϕ_2^1, ϕ_2^3, ϕ_1^3, and ϕ_1^2 may be, respectively, $(\bar{A}_2,\bar{A}_1,C_2,C_1)$, $(\bar{A}_2,\bar{A}_1,B_1,B_2)$, $(B_1,B_2,\bar{C}_1,\bar{C}_2)$, $(A_1,A_2,\bar{C}_1,\bar{C}_2)$, $(A_1,A_2,\bar{B}_2,\bar{B}_1)$, and $(\bar{B}_2,\bar{B}_1,C_2,C_1)$. We note that Eqs. (25) and (26) are also true by replacing τ_{A3} by τ'_{A3}, but Eqs. (21)–(24) change slightly.

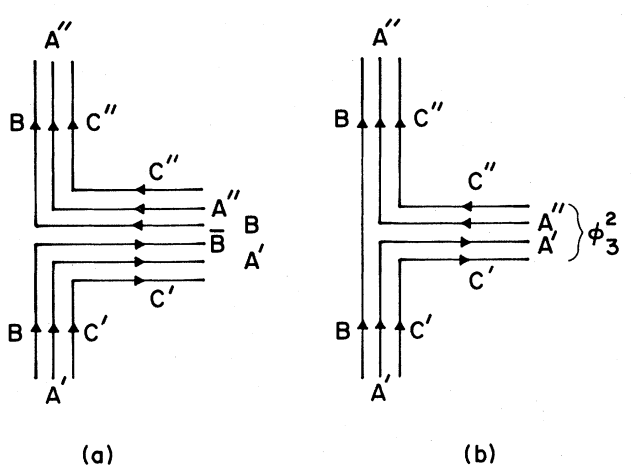

FIG. 2. The interaction vertex of a gluon ϕ_3^2 with a quark (A',B,C') where (A',A'') denotes $(A_1,\bar{A}_2)$ or $(\bar{A}_2,A_1)$, B denotes B_1 or $\bar{B}_2$, and (C',C'') denotes $(\bar{C}_1,C_2)$ or $(C_2,\bar{C}_1)$.

The structure of weak bosons can be conveniently obtained by means of the third method. By rewriting Eq. (4) of Ref. 1 as

$$\begin{aligned}2(-I_{W3})&=[I_W^-,I_W^+]\\&=-\tfrac{1}{4}(\tau'_{A3}+\tau'_{B3}+\tau'_{C3}-3\tau'_{A3}\tau'_{B3}\tau'_{C3}) \ ,\end{aligned} \tag{29}$$

one sees that I_W^-, I_W^+, and $(-I_{W3})$ may correspond to the three generators I^+, I^-, and I_3 of the $SU(2)'_W$. The wave functions of the weak bosons can be obtained as above by putting

$$W^+\left(\sim\frac{A_u^1-iA_u^2}{\sqrt{2}}\right)=I_W^- , \quad W^-\left(\sim\frac{A_u^1+iA_u^2}{\sqrt{2}}\right)=I_W^+ \ ,$$

and

$$A_u^3\sim -I_{W3} \ .$$

Thus, we have as a typical example

$$\begin{aligned}W^+&=\tau'_{A-}\tau'_{B-}\tau'_{C+}+\tau'_{A-}\tau'_{B+}\tau'_{C-}+\tau'_{A+}\tau'_{B-}\tau'_{C-}\\&=\bar{A}_1\bar{A}_2\bar{B}_1\bar{B}_2C_1C_2+\bar{A}_1\bar{A}_2B_1B_2\bar{C}_1\bar{C}_2+A_1A_2\bar{B}_1\bar{B}_2\bar{C}_1\bar{C}_2 \ ,\end{aligned} \tag{30}$$

$$\begin{aligned}W^-=&-A_1A_2B_1B_2\bar{C}_1\bar{C}_2-A_1A_2\bar{B}_1\bar{B}_2C_1C_2\\&-\bar{A}_1\bar{A}_2B_1B_2C_1C_2 \ ,\end{aligned} \tag{31}$$

$$A_u^3\sim\tfrac{1}{8}(A_1\bar{A}_1-\bar{B}_2B_2-C_2\bar{C}_2-3A_1\bar{A}_1\bar{B}_2B_2C_2\bar{C}_2) \ . \tag{32}$$

Since, in the Weinberg-Salam model,

$$A_u^3=\frac{g'}{(g^2+g'^2)^{1/2}}A_u-\frac{g}{(g^2+g'^2)^{1/2}}Z_u \ ,$$

where A_u and Z_u are, respectively, the photon and the Z-boson fields, the corresponding internal wave functions of A_u and Z_u in the above example are

$$A\sim\frac{-(g^2+g'^2)^{1/2}}{8g'}(-A_1\bar{A}_1+\bar{B}_2B_2+C_2\bar{C}_2) \ , \tag{33}$$

$$Z\sim\frac{-3(g^2+g'^2)^{1/2}}{8g}A_1\bar{A}_1\bar{B}_2B_2C_2\bar{C}_2 \ . \tag{34}$$

The corresponding vertex W^+W^-Z is shown in Fig. 3. We note that the same weak-boson wave functions are obtained by the second method. However, there might be one or two pairs annihilated in the boson Z.

The photon wave function Eq. (33) can be obtained independently from the generator of the U(1) symmetry, namely,[1]

$$2I'_3=\tau'_{A3}+\tau'_{B3}+\tau'_{C3} \ .$$

The corresponding vertex S_R-photon-S_R is shown in Fig. 4.

The internal wave of the Higgs boson H^0 is obtained by means of the second method, and is very similar to the Z-boson wave function. In addition to the usual quantum-number difference, the difference between their internal

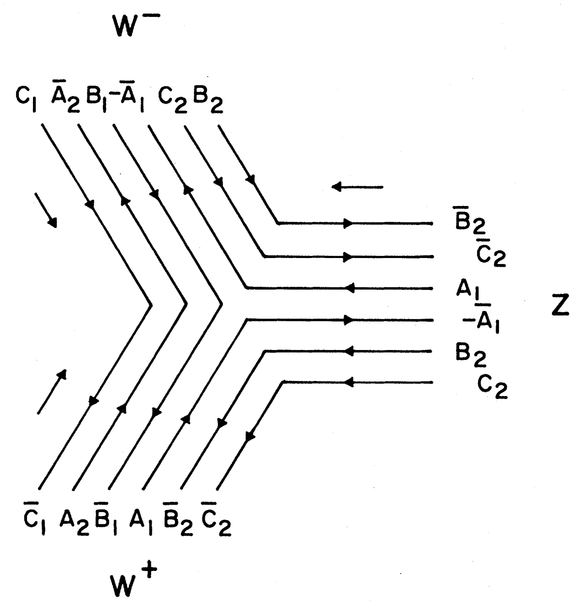

FIG. 3. Diagram of a vertex W^+W^-Z.

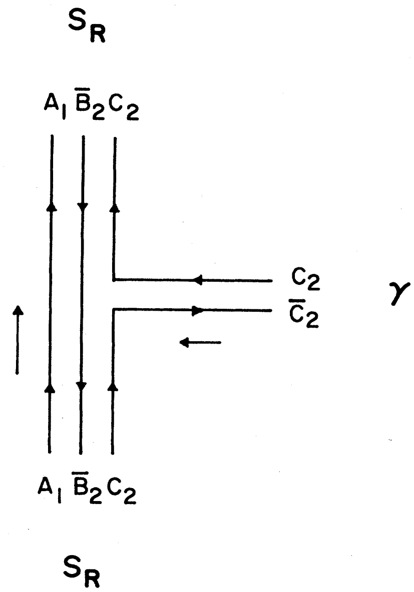

FIG. 4. The diagram of the vertex S_R-photon-S_R.

wave functions might be the number of pairs contained.

In summary, we find the internal wave functions of all known bosons in the framework of Ref. 1 by means of methods which can be applied on any composite symmetry. We have pointed out that the usual gauge-theory dynamics of the quarks, leptons, and bosons could remain true for this composite model. Several characteristic properties of the model have also been pointed out.

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*On leave of absence from the Department of Physics, Nankai University, Tianjin, China.

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[5]Our labeling of the wave function is slightly different from those of Ref. 1. See the explanation below Eq. (6).

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