

Pseudoscalar-meson decays into lepton pairs

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Some model-insensitive relations concerning $P \rightarrow l^+ l^-$ decay amplitudes are obtained and discussed. They are useful to discriminate among different theoretical predictions. Their violation by the experimental data (as seems already to be the case) should reveal the existence of new, unconventional effects.

In a previous paper¹ the authors have discussed the branching ratio $B^\pi \equiv \Gamma(\pi^0 \rightarrow e^+ e^-) / \Gamma(\pi^0 \rightarrow \gamma\gamma)$, showing that it depends essentially on a characteristic cutoff Λ governing the pion form factor but not on the (model-dependent and largely unknown) details of this form factor. The present Rapid Communication is an attempt to extend our analysis to other pseudoscalar-meson ($P = \pi^0, \eta, K_L$) decays into lepton pairs $P \rightarrow l^+ l^-$ ($l = e, \mu$), showing that the model insensitivity of our results can be maintained when considering the difference between the (separately model-dependent) real parts of two $P \rightarrow l^+ l^-$ amplitudes. In this sense, our findings share some similarity with the well-known theorem by Marciano and Sirlin² establishing that the dominant contribution (at order α) to the radiative corrections for the ratio of the total decay probabilities for $\pi \rightarrow e\nu$ and $\pi \rightarrow \mu\nu$ amounts to the introduction of the factor $[1 - 3\alpha/\pi \ln(m_\mu/m_e)]$ which is fully structure (or strong-interactions) independent. Unfortunately, we are unable to present our results in such a compact and elegant form, but we feel that they have some relevance in order to clarify the rather confused situation (both experimentally and theoretically) concerning the $P \rightarrow l^+ l^-$ decays.

Assuming the dominance of the two-photon contribution and following the notation of Ref. 1, the branching ratio B^P is given by

$$B^P \rightarrow l^+ l^- \equiv \frac{\Gamma(P \rightarrow l^+ l^-)}{\Gamma(P \rightarrow \gamma\gamma)} = 2 \left[1 - \frac{4m_l^2}{m_P^2} \right]^{1/2} \left[\frac{\alpha}{\pi} \frac{m_l}{m_P} \right]^2 |R(m_P^2)|^2, \quad (1)$$

where $m_P^2 = q^2$ and $m_l^2 = p^2$ are the squared masses of the pseudoscalar meson and the lepton, and

$$R(q^2) = \frac{2i}{\pi^2 m_P^2} \times \int \frac{d^4 k [q^2 k^2 - (q \cdot k)^2] F_P(k^2, (q-k)^2)}{(k^2 + i\epsilon)[(q-k)^2 + i\epsilon][(p-k)^2 - m_l^2 + i\epsilon]} \quad (2)$$

depends on the form factor $F_P(k^2, (q-k)^2)$ which contains the strong-interaction effects governing the $P \rightarrow \gamma\gamma \rightarrow l^+ l^-$ amplitude. The imaginary part of Eq. (2) may be written as

$$\text{Im}R(q^2) = \text{Im}R^{\text{PL}}(q^2) [1 - f_P(q^2)\theta(q^2 - \Lambda^2)] \quad (3)$$

and contains a model-independent (pointlike) contribution $\text{Im}R^{\text{PL}}(q^2)$ and a second, model-dependent one. The first is originated by the real $\gamma\gamma$ intermediate state and for the

whole range $0 < q^2 < \infty$ is given by

$$\text{Im}R^{\text{PL}}(q^2) = -\frac{\pi}{2\beta} \ln \frac{1+\beta}{1-\beta}, \quad \beta^2 \equiv 1 - \frac{4m_l^2}{q^2}. \quad (4)$$

The second one is due to the real, hadronic intermediate states allowed by the different models and, therefore, starts at some hadronic threshold Λ^2 . For reasonable values of Λ^2 ($m_\pi^2, m_K^2, m_\eta^2 < \Lambda^2$) Eqs. (1)–(4) lead to the well-known and model-independent unitary limits

$$\begin{aligned} B^{\pi^- \rightarrow ee} &> B_{\text{unit}}^{\pi^- \rightarrow ee} = 4.75 \times 10^{-8}, \\ B^{\eta^- \rightarrow \mu\mu} &> B_{\text{unit}}^{\eta^- \rightarrow \mu\mu} = 1.11 \times 10^{-5}, \\ B^{K^- \rightarrow \mu\mu} &> B_{\text{unit}}^{K^- \rightarrow \mu\mu} = 1.20 \times 10^{-5}, \\ B^{\eta^- \rightarrow ee} &> B_{\text{unit}}^{\eta^- \rightarrow ee} = 4.50 \times 10^{-9}, \end{aligned} \quad (5)$$

which have to be confronted with the experimental data

$$\begin{aligned} B_{\text{Ref. 3(a)}}^{\pi^- \rightarrow ee} &= (1.7 \pm 0.7) \times 10^{-7} \\ &= (3.6 \pm 1.5) B_{\text{unit}}^{\pi^- \rightarrow ee}, \end{aligned} \quad (6a)$$

$$\begin{aligned} B_{\text{Ref. 3(b)}}^{\pi^- \rightarrow ee} &= (1.82 \pm 0.61) \times 10^{-7} \\ &= (3.8 \pm 1.3) B_{\text{unit}}^{\pi^- \rightarrow ee}, \end{aligned}$$

$$\begin{aligned} B_{\text{Ref. 4}}^{\pi^- \rightarrow ee} &= (2.26 \pm_{1.11}^{2.43}) \times 10^{-7} \\ &= (4.8 \pm_{2.4}^{2.4}) B_{\text{unit}}^{\pi^- \rightarrow ee}, \end{aligned} \quad (6b)$$

$$\begin{aligned} B_{\text{Ref. 5}}^{\eta^- \rightarrow \mu\mu} &= (1.66 \pm 0.54) \times 10^{-5} \\ &= (1.5 \pm 0.49) B_{\text{unit}}^{\eta^- \rightarrow \mu\mu}, \end{aligned} \quad (7a)$$

$$\begin{aligned} B_{\text{Ref. 6}}^{\eta^- \rightarrow \mu\mu} &= (5.62 \pm 2.04) \times 10^{-5} \\ &= (5.06 \pm 1.84) B_{\text{unit}}^{\eta^- \rightarrow \mu\mu}, \end{aligned} \quad (7b)$$

$$\begin{aligned} B_{\text{Ref. 7}}^{K_L^- \rightarrow \mu\mu} &= (1.86 \pm 0.42) \times 10^{-5} \\ &= (1.56 \pm 0.35) B_{\text{unit}}^{K_L^- \rightarrow \mu\mu}, \end{aligned} \quad (8)$$

$$B_{\text{Ref. 8}}^{\pi^- \rightarrow ee} < 7.7 \times 10^{-4}. \quad (9)$$

Comparing Eqs. (5) with Eqs. (6)–(9) leads to the conclusion that a substantial contribution coming from the real part of the amplitude is required at least in some cases. In the remainder of this paper we proceed to a discussion of these real parts and some interesting relationships among them.

For convenience and also because some vector-meson-dominance-inspired amplitudes do not verify an unsubtracted dispersion relation, we introduce the following once-

subtracted one for $\text{Re}R(m_P^2)$:

$$\text{Re}R(m_P^2) = \text{Re}R(0) + \frac{m_P^2}{\pi} \int dq^2 \frac{\text{Im}R(q^2)}{(q^2 - m_P^2)q^2}. \quad (10)$$

We refer to the first and second terms of the right-hand side (RHS) of Eq. (10) as the soft-limit and dispersive contributions, respectively. The soft-limit contribution $\text{Re}R(0)$ is a function of the ratio m_l^2/Λ^2 of the only two surviving masses in that term: the lepton mass m_l and the cutoff Λ provided by the form factor in order to render convergent the otherwise logarithmically divergent $\text{Re}R(0)$. A simple calculation (along the lines of that by Marciano and Sirlin²) allows us to write

$$\text{Re}R(0) = 3 \ln \frac{m_l}{\Lambda} + K_{\text{MD}} + \dots, \quad (11)$$

where the dots refer to terms of order $(m_l/\Lambda)^2 \ln(m_l/\Lambda)$ or smaller terms, and K_{MD} is a model-dependent, but Λ -independent, constant. Numerical values of K_{MD} for the $\pi^0 \rightarrow e^+e^-$ decay are explicitly given in Ref. 1 and, for both vector-meson-dominance-inspired and constituent-quark models, they turn out to be of order unity. Thus, these model-dependent contributions can safely be neglected in the $\pi^0 \rightarrow e^+e^-$ decay where $m_l = m_e$ is certainly much smaller than any conceivable hadronic mass or cutoff Λ .

As previously stated, the dispersive integral in Eq. (10) receives a model-independent or pointlike contribution coming from the two-real-photons intermediate state which turns out to be

$$\begin{aligned} \text{Re}R^{\text{PL}}(m_P^2) &= \frac{m_P^2}{\pi} \int_0^\infty dq^2 \frac{\text{Im}R^{\text{PL}}(q^2)}{(q^2 - m_P^2)q^2} \\ &= \frac{1}{4\beta_P} \ln^2 \frac{1 - \beta_P}{1 + \beta_P} + \frac{\pi^2}{12\beta_P} - \frac{1}{\beta_P} \text{Li}_2 \left(\frac{\beta_P - 1}{\beta_P + 1} \right), \end{aligned} \quad (12)$$

where use has been made of Eq. (4),

$$\beta_P^2 \equiv 1 - 4m_l^2/m_P^2,$$

and

$$\text{Li}_2(x) \equiv - \int_0^x dt \ln(1-t)/t$$

is the dilogarithm function. A series expansion for $\text{Re}R^{\text{PL}}(m_P^2)$ leads immediately to

$$\text{Re}R^{\text{PL}}(m_P^2) = \ln^2 \frac{m_l}{m_P} + \frac{\pi^2}{12} + \dots, \quad (13)$$

where one has neglected terms of the type $(m_l/m_P)^2 \times \ln(m_l/m_P)$ and smaller. However, the dispersive integral in Eq. (10) receives a second type of contribution generated by real intermediate states other than the two-photon one just considered (containing at least one hadron) with threshold at $q^2 = \Lambda^2$. Such a model-dependent (dispersive) contribution to the real part of the $\pi^0 \rightarrow e^+e^-$ amplitude has already been considered for several models in Ref. 1. Here, we will discuss the general case considering alternatively two extreme situations. First, let us assume that the hadronic channels open so slowly [or, equivalently, that the function $f_P(q^2)$ in Eq. (3) is so close to zero] that its contribution can be neglected. In this case, the real part of the on-shell amplitude $\text{Re}R(m_P^2)$ is simply given by the sum of Eqs.

(11) and (12). Alternatively, one may assume that the opening of the hadronic channels is so drastic that $f_P(q^2) = 1$ for $q^2 > \Lambda^2$. In this case, one should add to Eqs. (11) and (12) the new contribution

$$\begin{aligned} \text{Re}R^\Lambda(m_P^2) &= - \frac{m_P^2}{\pi} \int_{\Lambda^2}^\infty dq^2 \frac{\text{Im}R^{\text{PL}}(q^2)}{(q^2 - m_P^2)q^2} \\ &\simeq \ln \frac{m_l}{\Lambda} \ln \left[1 - \frac{m_P^2}{\Lambda^2} \right] + \frac{1}{2} \text{Li}_2 \left[\frac{m_P^2}{\Lambda^2} \right]. \end{aligned} \quad (14)$$

The different models for the form factor F_P discussed in the literature correspond to intermediate situations between these two extremely opposite ones. This is illustrated in Fig. 1, where we have plotted the model-dependent part of the integrand in Eq. (10), $\text{Im}R(q^2)/q^2$, for different choices. Therefore, one can conclude that for any reasonable model the real part of the $P \rightarrow l^+l^-$ amplitude, $\text{Re}R(m_P^2)$, is given by the sum of Eqs. (11), (12), and a (model-dependent and unknown) fraction r of Eq. (14), i.e.,

$$\text{Re}R(m_P^2) = \text{Re}R(0) + \text{Re}R^{\text{PL}}(m_P^2) + r \text{Re}R^\Lambda(m_P^2). \quad (15)$$

with $0 < r < 1$.

Starting from Eq. (15) one can now easily discuss the situation concerning the $\pi^0 \rightarrow e^+e^-$ decay for which one obviously has $m_e \ll m_\pi \ll \Lambda$. The contribution of the first two terms of the RHS of Eq. (15) is dominated by just the first term of Eqs. (11) and (13). Moreover, the third, r -dependent term in Eq. (15) is negligible and, therefore, one has

$$\text{Re}R(m_\pi^2) \simeq 3 \ln \frac{m_e}{\Lambda} + \ln^2 \frac{m_e}{m_\pi}. \quad (16)$$

More precisely, for the reasonable values of Λ , $\Lambda = m_\rho \simeq m_\omega$, or $\Lambda = 2m_q$ (where m_q is the constituent mass of u or d quarks), Eq. (16) gives¹

$$\text{Re}R(m_\pi^2) \simeq -22 + 31 \simeq +9. \quad (17)$$

Unfortunately, there is an important cancellation between the two terms of Eq. (16) but, in spite of this, the error affecting Eq. (17) has been estimated to be around some 20%, i.e., $\text{Re}R(m_\pi^2) = +9 \pm 2$, and comes essentially from the constant term K_{MD} neglected in Eq. (11).

The available experimental data, Eqs. (6), require $|\text{Re}R(m_\pi^2)| \simeq 35$ and, therefore, they are clearly incompatible with our prediction, Eq. (17). Furthermore, they are also incompatible with many other theoretical predictions⁹⁻¹³ based on the same widely accepted context of two-photon dominance, which essentially agrees with Eq. (17). An exception is the pioneering work by Drell¹⁴ where the first terms of the RHS of Eqs. (16) and (17) are not obtained. [This is a consequence of using a rather unphysical form factor and postulating an unsubtracted dispersion relation, whose validity cannot be proved, for $R(q^2)$.] Therefore, the theoretical analysis (based on two-photon dominance) suggests that the available data could have somehow overestimated the $\pi^0 \rightarrow e^+e^-$ decay rate and that future experiments could reduce the corresponding branching ratio somewhat above the unitary bound quoted in Eq. (5). New data on this decay are highly desirable since they will fix, through Eq. (16) and in a practically model-independent way, the hadronic mass scale Λ . Moreover, if they confirm the available experimental results (which imply¹ $\Lambda \simeq 4 \times 10^5$

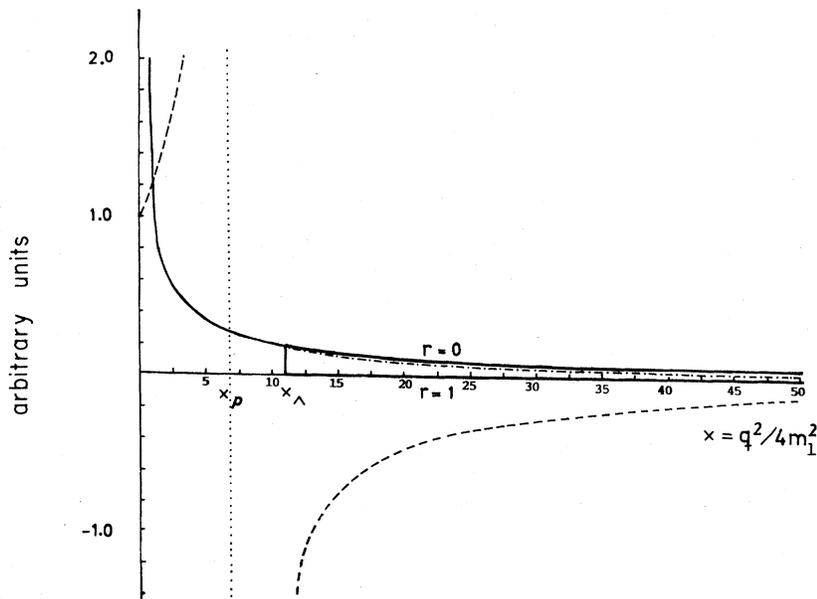


FIG. 1. $\text{Im}R(q^2)/q^2$ as a function of $x \equiv q^2/4m_l^2$ for the two extreme cases discussed in the text (solid curves) and for a vector-meson-dominance model (dot-dashed curve). The dashed curve is the pole term, $(1 - q^2/m_p^2)^{-1}$, modulating the previous curves in Eq. (10). The plot represents the most unfavorable case, $\eta \rightarrow \mu^+ \mu^-$, where $x_\eta = 7 = x_\Lambda/2$. The model insensitivity is much more apparent for the $\pi^0 \rightarrow e^+ e^-$ amplitude, where $x_\pi \approx 1.7 \times 10^4 \ll x_\Lambda \approx 5.7 \times 10^5$.

GeV), one should believe that there are new and unexpected effects (i.e., non- $\gamma\gamma$ -mediated) behind the $\pi^0 \rightarrow e^+ e^-$ transition (see below).

We now turn to the discussion of the differences between the three purely electromagnetic (EM) and physically relevant amplitudes $\pi^0 \rightarrow e^+ e^-$, $\eta \rightarrow e^+ e^-$, and

$\eta \rightarrow \mu^+ \mu^-$. Only two of such differences are independent in our context and for them one immediately gets

$$\begin{aligned} & \text{Re}R_{\eta \rightarrow \mu\mu}(m_\eta^2) - \text{Re}R_{\eta \rightarrow ee}(m_\eta^2) \\ &= \ln \frac{m_\mu}{m_e} \left\{ 3 + \ln \frac{m_\mu m_e}{m_\eta^2} + r \ln \left[1 - \frac{m_\eta^2}{\Lambda^2} \right] + \dots \right\}, \quad (18) \end{aligned}$$

$$\text{Re}R_{\pi \rightarrow ee}(m_\pi^2) - \text{Re}R_{\eta \rightarrow \mu\mu}(m_\eta^2) = 3 \ln \frac{\Lambda_\eta}{\Lambda_\pi} - 3 \ln \frac{m_\mu}{m_e} + \ln \frac{m_e m_\mu}{m_\pi m_\eta} \ln \frac{m_e m_\eta}{m_\mu m_\pi} + r \ln \frac{m_\eta}{\Lambda_\eta} \ln \left[1 - \frac{m_\eta^2}{\Lambda_\eta^2} \right]. \quad (19)$$

In Eq. (18) the constant terms K_{MD} and $\pi^2/12$ appearing in Eqs. (11) and (13) have exactly cancelled and the dots refer to terms such as $(m_l/\Lambda)^2 \ln(m_l/\Lambda)$ or even smaller. Similarly, the third, model-dependent term represents for any reasonable choice of Λ , $\Lambda^2 > m_\eta^2$, only a negligible correction to the model-independent contribution coming from the first two terms. Thus, Eq. (18) leads to

$$\text{Re}R_{\eta \rightarrow \mu\mu}(m_\eta^2) - \text{Re}R_{\eta \rightarrow ee}(m_\eta^2) \approx -30 \pm 2. \quad (20)$$

The meaning and the validity of Eq. (18) is quite similar to that of the Marciano-Sirlin theorem. The main difference is due to the fact that, being the model- and Λ -dependent corrections not always negligible, one has to consider differences between two amplitudes rather than their ratio. Equation (20) can be fruitfully exploited to obtain one of the $\eta \rightarrow \mu^+ \mu^-$ or $\eta \rightarrow e^+ e^-$ branching ratios from the knowledge of the other one. Indeed, if one accepts the values quoted in Eq. (7a) or (7b) for $B^{\eta \rightarrow \mu\mu}$, one immediately obtains $B^{\eta \rightarrow ee} = 2.4 B_{\text{unit}}^{\eta \rightarrow ee}$ or $1.7 B_{\text{unit}}^{\eta \rightarrow ee}$, respectively.

Similar considerations apply to the difference between the real part of the $\pi^0 \rightarrow e^+ e^-$ and $\eta \rightarrow \mu^+ \mu^-$ amplitudes, Eq. (19). In this case, however, one has to invoke approximate SU(3) symmetry $\Lambda_\eta \approx \Lambda_\pi$ for the different cutoffs Λ , and

one simply obtains

$$\text{Re}R_{\pi \rightarrow ee}(m_\pi^2) - \text{Re}R_{\eta \rightarrow \mu\mu}(m_\eta^2) \approx +12 \pm 2. \quad (21)$$

The inconsistency of this relation with the available data has already been discussed in Ref. 1. Indeed, the data imply $|\text{Re}R_{\pi \rightarrow ee}(m_\pi^2)| \approx 35$ [from Eqs. (6a) and (6b)] and $|\text{Re}R_{\eta \rightarrow \mu\mu}(m_\eta^2)| \approx 4$ [from Eq. (7a)] or ≈ 11 [from Eq. (7b)]. Notice, however, that Eq. (21) is fully consistent with the experimental value deduced from Eq. (7a) and our theoretical prediction Eq. (17). Equation (21) allows also for a critical review of the existing theoretical predictions. It is fully respected in the models by Quigg and Jackson,⁹ by Bergström,¹⁰ and by Babu and Ma¹¹ in spite of the use of very different form factors. The same applies to the work by Pratap and Smith¹⁵ once some computational errors have been corrected.¹⁶

We finally turn our attention to the interesting $K_L \rightarrow \mu^+ \mu^-$ decay and discuss only the part of the amplitude corresponding to the $\gamma\gamma$ intermediate state, R^{EM} . This part has a piece \bar{R}^{EM} coming from the $K_L \rightarrow \pi^0, \eta, \eta' \rightarrow \mu^+ \mu^-$ channel and, therefore, having a threshold Λ_{K_L} which is approximately the same as in the above-discussed $\pi^0, \eta \rightarrow \mu^+ \mu^-$ decays. In consequence, we

have

$$\begin{aligned} & \text{Re}R_{\eta \rightarrow \mu\bar{\mu}}(m_\eta^2) - \text{Re}\bar{R}_{K_L^{\text{EM}} \rightarrow \mu\bar{\mu}}(m_{K_L}^2) \\ & \simeq \ln \frac{m_\mu^2}{m_\mu m_K} \ln \frac{m_K}{m_\eta} + r \ln \frac{m_\mu}{\Lambda} \ln \left[\frac{\Lambda^2 - m_\eta^2}{\Lambda^2 - m_K^2} \right], \quad (22) \end{aligned}$$

which, numerically, is small when compared to $\text{Im}R^{\text{EM}}$. A similar numerical result can be obtained by assuming the dominance of the η pole in the amplitude \bar{R}^{EM} . This has been used by several authors¹⁷ as an estimate of R^{EM} when analyzing the nonphotonic contributions to the $K_L \rightarrow \mu^+\mu^-$ decay. However, we stress that there could be additional photonic contributions to R^{EM} due to the presence of the combined weak and electromagnetic contributions. The prototype of these new terms has been studied in Ref. 18 and applied to the $K_L \rightarrow \mu^+\mu^-\gamma$ decay. It turns out that its corresponding Λ_{K_L} might be much different than Λ . Since these types of contributions vanish when the intermediate photons are on their mass shell, we are not able to incorporate them in Eq. (22). It follows that it is quite difficult at present to know the room left to nonphotonic contributions in the $K_L \rightarrow \mu^+\mu^-$ amplitude.¹⁹

Before concluding let us briefly discuss several models including nonphotonic contributions even in the previously considered π^0 and $\eta \rightarrow l^+l^-$ decays. Most of these models^{10, 20-22} conclude that the contributions of Z and Higgs bosons (as well as other electroweak effects) are really negligible when compared with the photon-photon contribution which we have assumed to be the dominant one. Apparently one significant exception is the work by Tupper and Samuel.²³ These authors postulate a once-subtracted dispersion relation analogous to Eq. (10) with a subtraction constant $\text{Re}R(0)$ receiving two types (photonic and weak) of contributions. Their sum is fixed from low-energy phenomenology and current-algebra arguments and is found to be much smaller than the values implied by Eq. (11). Therefore, our (two-photon-dominated) amplitudes $R(q^2)$ and the corresponding ones (containing an additional, weak contribution) of Ref. 23 differ essentially by the constant term given in Eq. (11). This term cancels when writ-

ing the relation

$$\text{Re}R_{\pi \rightarrow ee}(m_\pi^2) - \text{Re}R_{\eta \rightarrow ee}(m_\eta^2) = -18 \pm 2$$

[obtained when adding Eqs. (20) and (21)], which should be satisfied by the results of Ref. 23. This turns out to be the case. Moreover, the predictions of Ref. 23 for $B^{\pi \rightarrow ee}$ and $B^{\eta \rightarrow \mu\mu}$ are both in good agreement with the data. In spite of all that, the model of Ref. 23 is rather controversial²⁴ since it is hard to understand how its additional, weak contribution to $\text{Re}R(0)$ (which is expected to be small^{10, 20-22}) can compensate the photonic one [which is known to be large; see Eq. (11) and Refs. 1 and 16].

We have therefore reached the following main conclusions: (i) Assuming the dominance of the two-photon contribution one can obtain simple expressions for the differences between the model-dependent real parts of the $P \rightarrow l^+l^-$ amplitudes which turn out to be essentially model independent (a situation which is similar to that concerning the theorem by Marciano and Sirlin on the $\pi \rightarrow e\nu\gamma$ and $\pi \rightarrow \mu\nu\gamma$ decays); in other words, from the knowledge of the real part of a single $P \rightarrow l^+l^-$ amplitude we obtain safe predictions for the analogous part of the other ones. (ii) Our results allow for a simple discrimination among theoretical models based on two photon dominance; in particular, one observes that only those compatible with a $\pi^0 \rightarrow e^+e^-$ branching ratio not far from the unitary limit are consistent with our analysis. (iii) Some of our relations are in sharp disagreement with the available experimental data; the failure could be attributed to the difficulties of the experiment, but if it is not so and the new data persist in violating our relations, one should believe that new effects are present behind the $P \rightarrow l^+l^-$ decays (see Refs. 10 and 23).

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