Repulsive and attractive planar walls in general relativity

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The method of a previous paper is generalized to yield all solutions to Einstein's equations for planar-symmetric walls composed of surface energy density σ and tension τ, with constant ratios in the physical range $\tau/\sigma \leq 1$. Special cases include domain walls, walls of cosmic strings, dust walls, and (when $\tau < 0$) pressure walls. Attention is focused on the sense in which a wall is gravitationally repulsive when the tension is strong, the sense in which repulsion gives way to attraction as τ/σ is reduced below the value $\frac{1}{2}$, and the way in which these features are reflected in the motion of the wall. Also studied is the way in which the unique static planar-symmetric solution fits within the classes of solutions.

I. INTRODUCTION

In recent papers[1,2] solutions to Einstein's equations were obtained for certain solitonlike structures associated with phase transitions that, particle-physics theories suggest, might occur in the early universe. Attention was focused on domain walls, three-dimensional timelike hypersurfaces whose stress energy consists of surface energy density and isotropic tension in two spatial dimensions, with the magnitude of tension equal to the surface energy density. In Ref. 1 all solutions to Einstein's equations for domain walls exhibiting spherical or planar symmetry were found. Of particular interest was the sense in which the gravitational field of a domain wall is repulsive. It was found that observers who wish to remain next to a domain wall must accelerate toward it, and that this feature is connected with the strong tension in the wall. The implications for the possible roles of domain walls in the early universe were discussed briefly.

The purpose of the present paper is to extend the methods of Ref. 1 to yield all solutions to Einstein's equations for planar walls with a constant ratio of tension τ to surface energy density σ satisfying $\tau/\sigma \leq 1$. (Outside this range, some observers measure negative energy densities.) This extension includes, in addition to domain walls ($\tau/\sigma = 1$), walls composed of isotropically distributed cosmic strings ($\tau/\sigma = \frac{1}{2}$), dust walls ($\tau/\sigma = 0$), and pressure walls ($\tau/\sigma < 0$); and it permits exploration of the precise way in which repulsion yields to attraction as the tension is steadily decreased. Throughout this paper our notation and units will conform to those of Ref. 1. Hence $\hbar = c = 1$, and G_N denotes the gravitation constant.

II. THE BASIC EQUATIONS GOVERNING PLANAR WALLS

As in Ref. 1, we seek solutions to Einstein's equations for spacetimes whose sources are confined to infinitesimally thin, three-dimensional timelike hypersurfaces. Denote such a hypersurface by S, and let ξ^a be its unit spacelike normal. The intrinsic metric on S is

$$h_{ab} = g_{ab} - \xi_a \xi_b \,, \tag{2.1}$$

where g_{ab} is the four-metric of spacetime. Let ∇_a denote the covariant derivative of spacetime, and let

$$D_a \equiv h_a{}^b \nabla_b, \quad \pi_{ab} \equiv D_a \xi_b = \pi_{ba} \,, \tag{2.2}$$

where π_{ab} is the extrinsic curvature of S. By familiar methods,[1,3] the Gauss-Codazzi formalism leads to the equations

$$\begin{aligned} S_{ab} &= \int dl\, T_{ab} \\ &= -\frac{1}{8\pi G_N}(\gamma_{ab} - h_{ab}\gamma_c{}^c), \quad h_{ac} D_b S^{cb} = 0 \,, \\ & h_{ac} D_b \tilde{\pi}^{cb} - D_a \tilde{\pi}_b{}^b = 0, \quad \tilde{\pi}_{ab} S^{ab} = 0 \,, \\ & {}^3R + [\tilde{\pi}_{ab}\tilde{\pi}^{ab} - (\pi_a{}^a)^2] = -16\pi^2 G_N{}^2 [S_{ab}S^{ab} - \tfrac{1}{2}(S_a{}^a)^2] \,, \end{aligned} \tag{2.3}$$

where

$$\gamma_{ab} \equiv \pi_{+ab} - \pi_{-ab}, \quad \pi_{ab} \equiv \tfrac{1}{2}(\pi_{+ab} + \pi_{-ab}) \,. \tag{2.4}$$

Here S_{ab}, the surface energy tensor, is the integral of the stress energy tensor T_{ab} through S, the subscripts $\pm$ refer to values on either side of S, and ${}^3R$ is the three-dimensional Ricci scalar of S.

We assume that the surface energy tensor

$$S^{ab} = \sigma u^a u^b - \tau(h^{ab} + u^a u^b) \,, \tag{2.5}$$

where u^a is the four-velocity of an observer in S who sees no energy flux and who measures surface energy density σ and tension τ. Further, we assume that

$$\tau = \Gamma \sigma \,, \tag{2.6}$$

where Γ is a constant ≤ 1. We exclude $\Gamma > 1$ because in such a case, as one shows by using Eq. (2.5), observers exist who measure negative energy densities. (If $\Gamma < 0$, we are dealing with pressure rather than tension.) Finally, we assume that S is a "planar wall," i.e., that it is homogeneous and isotropic in its two space dimensions and that the geometry is reflection symmetric. It follows that $\tilde{\pi}_{ab} = 0$ and that the geometry of spacetime is of the form

$$ds^2=e^{2\nu(t,|z|)}(-dt^2+dz^2)+B(t,|z|)(dx^2+dy^2)\ , \qquad (2.7)$$

with the location of the wall given by $z=0$.

The solutions to the vacuum Einstein equations off the wall separate into the two classes studied in Ref. 1. For class I the generic vacuum solution is

$$B(t,|z|)=F(t-|z|)\ ,$$
$$e^{2\nu}=\frac{F'(t-|z|)K(t+|z|)}{F^{1/2}(t-|z|)}\ , \qquad (2.8)$$

where F and K are specifiable functions. Throughout this paper a prime denotes the derivative of a function with respect to its argument. For class II, the generic vacuum solution is

$$B(t,|z|)=F(t-|z|)+G(t+|z|)\ ,$$
$$e^{2\nu}=C_0\frac{F'(t-|z|)G'(t+|z|)}{[F(t-|z|)G(t+|z|)]^{1/2}}\ , \qquad (2.9)$$

where F and G are specifiable and C_0 is a constant.

A complete set of equations consists of Eqs. (2.3)—(2.6), and either (2.8) or (2.9).

III. CLASS-I PLANAR WALLS

A. The solutions

A procedure parallel to that of Sec. IV C 1 of Ref. 1 enables one to reduce the problem of finding class-I solutions to that of solving the equations (assuming $z>0$ for definiteness)

$$e^{2\nu(t,z)}=F'(t-z)K(t+z)/F^{1/2}(t-z)\ , \qquad (3.1a)$$

$$\nu_{,z}|_{+}=2\pi G_N(1-2\Gamma)\sigma e^{\nu(t,0)}\ , \qquad (3.1b)$$

$$e^{\nu(t,0)}=F'(t)/4\pi G_N\sigma F(t)\ , \qquad (3.1c)$$

and

$$\sigma=\sigma(t)=C_1F^{-(1-\Gamma)}(t)\ , \qquad (3.1d)$$

with the conditions $F(t-z)$ and $F'(t)>0$. Here C_1 is a constant. The last of Eqs. (2.3) is not needed, because it is implied by Eqs. (3.1). Also, note that Eq. (3.1d) expresses the local conservation law that work done against tension (by pressure) during expansion is stored in (removed from) surface energy.

Equations (3.1a)—(3.1c) imply that

$$K(t)=C_0F^{1/2-2\Gamma}(t)\ , \qquad (3.2)$$

which conveniently replaces Eq. (3.1b). Equations (3.1) and (3.2) then yield

$$C_1=1/4\pi G_NC_0^{1/2}\ . \qquad (3.3)$$

It follows that the generic class-I solution is of the form

$$e^{2\nu(t,z)}=C_0\frac{F'(t-|z|)F'(t+|z|)}{F^{1/2}(t-|z|)F^{2\Gamma-1/2}(t+|z|)}\ ,$$
$$B(t,z)=F(t-|z|)\ ,$$
$$\sigma=1/4\pi G_NC_0^{1/2}F^{1-\Gamma}(t)\ ,$$
$$F(t-|z|)>0,\quad F'(t)>0\ . \qquad (3.4)$$

Taub's[4] analyses imply that these solutions are flat in the vacuum off the wall and hence can be transformed to Minkowski form there. To see this explicitly, one performs the coordinate transformation

$$t^*-z^*\equiv 2C_0^{1/2}F^{1/2}(t-z)\ ,$$
$$t^*+z^*\equiv[C_0^{1/2}/(\tfrac{3}{2}-2\Gamma)]F^{3/2-2\Gamma}(t+z)$$
$$+(1/2C_0^{1/2})F^{1/2}(t-z)(x^2+y^2)\ , \qquad (3.5)$$
$$x^*\equiv F^{1/2}(t-z)x,\quad \text{and } y^*\equiv F^{1/2}(t-z)y\ ,$$

for $z\geq 0$, and similarly for $z\leq 0$. (The special case $\Gamma=\frac{3}{4}$ requires a slightly special transformation.) In terms of the new coordinates, the metric takes the Minkowski form

$$ds^2=-dt^{*2}+dz^{*2}+dx^{*2}+dy^{*2}\ . \qquad (3.6)$$

The surface energy density on the wall is now given by

$$\sigma=[4^{\Gamma}\pi G_NC_0^{\Gamma-1/2}(t^*-z^*)^{2(1-\Gamma)}]^{-1}\ . \qquad (3.7)$$

B. The motion of the wall

The problem of interpreting the solutions boils down to that of determining the motion of the wall in the Minkowski coordinate system. Setting $z=0$ in Eqs. (3.5) yields

$$x^{*2}+y^{*2}+z^{*2}=t^{*2}+\frac{(4C_0)^{2\Gamma-1}}{(4\Gamma-3)}(t^*-z^*)^{4(1-\Gamma)} \qquad (3.8)$$

for the location of the wall. Note that $\partial t^*/\partial t>0$ on the wall, since F and $F'>0$.

In Ref. 1 the implications of Eq. (3.8) for domain walls, i.e., for $\Gamma=1$, were discussed. It was noted that, in the Minkowski coordinates, the domain wall is really not planar at all; but rather that, upon extension of the solution into the region $t^*-z^*<0$, the domain wall is bent into a sphere that completely encloses the original $z>0$ side of the wall. The sphere comes in from infinity, slows down, turns around, and heads back out to infinity, all the while maintaining constant outward acceleration $2\pi G_N\sigma$. Because of reflection symmetry, a similar statement is valid for the $z<0$ side of the wall. It was pointed out that this behavior is permitted on both sides because of the lack of demand for asymptotic flatness.

In this paper our interest lies in the implications of Eq. (3.8) for $\Gamma<1$. For such cases, it is easily seen from Eq. (3.8), upon extension of the solutions, that the intersection of the wall with a surface of constant t^* is again a closed surface completely enveloping the original $z>0$ side of the wall. (All statements have analogs for $z<0$.) The maximum and minimum values $z^*_{\max}(t^*)$ and $z^*_{\min}(t^*)$ of z^* on the wall are obtained by setting $x^*=y^*=0$ in Eq. (3.8), which yields

$$t^*-z^*_{\max}=0$$

and

$$t^*+z^*_{\min}=\frac{(4C_0)^{2\Gamma-1}}{3-4\Gamma}(t^*-z^*_{\min})^{3-4\Gamma}\ . \qquad (3.9)$$

For the extended solutions, the surface energy density

(3.7) now blows up at $z^* = z^*_{max}$. Another useful equation is an expression for the derivative of z^*_{min},

$$\frac{dz^*_{min}}{dt^*} = \frac{(4C_0)^{2\Gamma-1}(t^* - z^*_{min})^{2-4\Gamma} - 1}{(4C_0)^{2\Gamma-1}(t^* - z^*_{min})^{2-4\Gamma} + 1}, \tag{3.10}$$

obtained by differentiating Eq. (3.9).

Equations (3.8)—(3.10) reveal the existence of several interesting subclasses marked by distinctive wall motion.

1. $\frac{3}{4} < \Gamma < 1$

Equations (3.9) and (3.10) imply that $(t^* + z^*_{min}) \to -\infty$ and $dz^*_{min}/dt^* \to +1$ as $(t^* - z^*_{min}) \to 0_+$; and that $(t^* + z^*_{min}) \to 0_-$ and $dz^*_{min}/dt^* \to -1$ as $(t^* - z^*_{min}) \to +\infty$. Thus the world line $z^* = z^*_{min}(t^*)$ comes in from $z^* = -\infty$, all the while accelerating toward negative z^*. It turns around when the numerator in Eq. (3.9) vanishes and then heads back out to $z^* = -\infty$. An observer following this world line experiences acceleration

$$a^{z^*} = z^{*\prime\prime}_{min}/[1-(z^{*\prime}_{min})^2]^{3/2} = 2\pi G_N(1-2\Gamma)\sigma \tag{3.11}$$

toward the wall, according to Eqs. (3.7)—(3.10), in agreement with the general result (2.15) of Ref. 1. Also Eq. (3.8) implies that $(x^{*2} + y^{*2})$ at fixed z^* increases as t^* increases, asymptotically approaching t^* as $t^* \to \infty$.

2. $\frac{1}{2} < \Gamma < \frac{3}{4}$

Equations (3.9) and (3.10) imply that $(t^* + z^*_{min}) \to 0_+$ and $dz^*_{min}/dt^* \to +1$ as $(t^* - z^*_{min}) \to 0_+$; and that $(t^* + z^*_{min}) \to \infty$ and $dz^*_{min}/dt^* \to -1$ as $(t^* - z^*_{min}) \to \infty$. Thus the world line $z^* = z^*_{min}(t^*)$ emerges from the origin in the positive z^* direction, all the while accelerating toward negative z^*. It turns around when the numerator in Eq. (3.9) vanishes and then heads out to $z^* = -\infty$. An observer following this world line accelerates towards the wall in accord with Eq. (3.11). Also, Eq. (3.8) implies that $(x^{*2} + y^{*2})$ at fixed z^* increases as t^* increases, asymptotically approaching t^{*2} as $t^* \to \infty$.

3. $\Gamma = \frac{1}{2}$

This special case, it turns out, marks the dividing line between "repulsive" and "attractive" solutions. It merits special attention not only for this reason, but also because it corresponds to a wall of isotropically and uniformly distributed cosmic strings, topological structures of one spatial dimension with tension equal to the linear mass-energy density. Equation (3.9) implies that $z^*_{min} = 0$ for all t^*. In fact, according to Eq. (3.8), one obtains the solution at time t^*_1 from that at time t^*_0 by multiplying each spatial coordinate by (t^*_1/t^*_0). Thus the wall expands away from the origin linearly in time t^*, and an observer moving with it does not accelerate.

4. $\Gamma < \frac{1}{2}$

This case includes dust walls ($\Gamma = 0$) and walls with pressure ($\Gamma < 0$). Equations (3.9) and (3.10) imply that $(t^* + z^*_{min}) \to 0_+$ and $dz^*_{min}/dt^* \to -1$ as $(t^* - z^*_{min}) \to 0_+$; and that $(t^* + z^*) \to \infty$ and $dz^*/dt^* \to +1$ as $(t^* - z^*) \to \infty$. Thus the world line $z^* = z^*_{min}(t^*)$ emerges from the origin in the negative z^* direction, all the while accelerating toward positive z^*. It turns around when the numerator in Eq. (3.9) vanishes and then heads out to $z^* = +\infty$. An observer following this world line accelerates away from the wall in agreement with Eq. (3.11).

C. Summary

The properties of class I walls can be summarized as follows.

In each case the wall is bent into a closed surface enveloping the original $z > 0$ side of the wall. The geometry is flat off the wall. At time t^* the maximum and minimum values of z^* on the wall satisfy Eqs. (3.9). For ratios $\tau/\sigma \equiv \Gamma$ satisfying $\frac{3}{4} < \Gamma < 1$, the wall comes in from $z^* = -\infty$. For $\Gamma < \frac{3}{4}$ the wall emerges from the origin of the Minkowski coordinate system. For $\Gamma > \frac{1}{2}$ the gravitational field of the wall is uniform and repulsive. An observer at fixed x and y in the original coordinates must accelerate toward the wall with acceleration (3.11) in order to comove with it. (Vilenkin,[5] working in the linearized approximation, was the first to show that walls with $\Gamma > \frac{1}{2}$ are repulsive.) For $\Gamma = \frac{1}{2}$ the wall is neither repulsive nor attractive, and a comoving observer must not accelerate. For $\Gamma < \frac{1}{2}$ the gravitational field of the wall is uniform and attractive, and a comoving observer must accelerate toward the wall with acceleration (3.11). A similar statement is valid for the $z < 0$ side of the wall.

IV. CLASS II PLANAR WALLS

A. The solutions

A procedure parallel to that of Sec. IV C 2 of Ref. 1 enables one to reduce the problem of finding class-II solutions to that of solving the equations (assuming $z > 0$ for definiteness)

$$e^{2\nu(t,z)} = C_0 F'(t-z)G'(t+z)/[F(t-z)+G(t+z)]^{1/2}, \tag{4.1a}$$

$$\nu_{,z}|_+ = 2\pi G_N(1-2\Gamma)\sigma e^{\nu(t,0)}, \tag{4.1b}$$

$$e^{\nu(t,0)} = [F'(t) - G'(t)]/4\pi G_N \sigma[F(t)+G(t)], \tag{4.1c}$$

and

$$\sigma = \sigma(t) = C_1[F(t)+G(t)]^{-(1-\Gamma)}, \tag{4.1d}$$

along with $[F(t-z)+G(t+z)]$ and $[F'(t)-G'(t)] > 0$. Again Eq. (2.3) is redundant.

Equations (4.1a)—(4.1c) imply that at $z = 0$

$$\frac{F''}{F'} - \frac{G''}{G'} + (\tfrac{1}{2} - 2\Gamma)\frac{F'-G'}{F+G} = 0 \tag{4.2}$$

and

$$C_0 = \frac{1}{(4\pi G_N \sigma)^2} \frac{F'-G'}{F'G'(F+G)^{3/2}}, \tag{4.3}$$

which replace Eqs. (4.1b) and (4.1c). Differentiation of Eq. (4.3) and substitution from Eq. (4.1d) yield

$$C_0\frac{F'+G'}{F'-G'}\left[\frac{F''}{F'}-\frac{G''}{G'}+(\tfrac{1}{2}-2\Gamma)\frac{F'-G'}{F+G}\right]=0 . \quad (4.4)$$

If $F'+G'\neq 0$, this equation implies Eq. (4.2). If $F'+G'=0$, Eq. (4.2) itself implies that $\Gamma=\frac{1}{4}$, since $F'-G'>0$. It follows that one obtains the class-II solutions by solving the first-order Eqs. (4.3) and (4.1d), with the proviso that $\Gamma=\frac{1}{4}$ when $F'+G'=0$.

In all cases one brings a class-II solution to the canonical form of Refs. 1 and 3 by performing the coordinate transformation

$$F(t-z)=\tfrac{1}{2}(X+Y),\quad G(t+z)=\tfrac{1}{2}(X-Y) . \quad (4.5)$$

In terms of the new coordinates,

$$ds^2=\frac{C_0}{4}\frac{(-dX^2+dY^2)}{X^{1/2}}+X(dx^2+dy^2) . \quad (4.6)$$

The surface energy density of the wall is now given by

$$\sigma=C_1X^{-(1-\Gamma)} , \quad (4.7)$$

and the problem of completing the solution process again boils down to that of determining the motion of the wall in the new coordinates. Note, in this connection, that

$$\frac{\partial Y}{\partial t}=-\frac{\partial X}{\partial z}=F'-G'>0 ,$$
$$\frac{\partial Y}{\partial z}=-\frac{\partial X}{\partial t}=-(F'+G') . \quad (4.8)$$

Hence Y increases with time t on the wall. Further, the unit normal to the wall has components

$$\xi^X=-(F'-G')e^{-\nu}<0,\quad \xi^Y=-(F'+G')e^{-\nu} . \quad (4.9)$$

These expressions enable one to determine which side of the wall in the $X-Y$ coordinates corresponds to $z>0$.

B. The motion of the wall

1. $F'+G'\neq 0$

Combining Eqs. (4.1d) and (4.3) yields

$$C_0=\frac{1}{(4\pi G_NC_1)^2}\frac{(F'-G')^2}{F'G'(F+G)^{2\Gamma-1/2}} . \quad (4.10)$$

Eliminating F and G via Eqs. (4.5) then yields

$$(2\pi G_NC_1)^2C_0[(dX/dY)^2-1]X^{2\Gamma-1/2}=1 \quad (4.11)$$

on the wall. Note that $dX/dY=\text{const}$ if $\Gamma=\frac{1}{4}$. All solutions with $\Gamma=\frac{1}{4}$, whether or not $F'+G'\neq 0$, are discussed separately in Sec. IV B 2.

If $C_0<0$, X is a spatial coordinate, Y is a time coordinate, and the motion of the wall is determined by

$$\left[\frac{dX}{dY}\right]^2=1-\frac{1}{(2\pi G_NC_1)^2|C_0|X^{2\Gamma-1/2}} . \quad (4.12)$$

In the $X-Y$ coordinates, the $z>0$ side of the wall is that containing $X=0$, where a curvature singularity exists.

For $\Gamma>\frac{1}{4}$ the wall comes in from $X=+\infty$, reaches a minimum X, and then heads back out to $X=+\infty$. With regard to the way observers must accelerate in order to keep up with the wall, this motion is somewhat misleading. In fact, for $\frac{1}{4}<\Gamma<\frac{1}{2}$ observers comoving with the wall must accelerate away from it, even though the wall appears to be accelerating, in the $X-Y$ coordinates, away from the $z>0$ side. The explanation lies in the fact that the geometry is not flat.

For $\Gamma<\frac{1}{4}$ the motion consists of two pieces. On one piece, the wall comes in from the limiting value of X at which the right-hand side of Eq. (4.12) vanishes and hits the singularity at $X=0$. On the other piece, the wall emerges from $X=0$ and asymptotically approaches the above limiting value of X.

If $C_0>0$, X is a time coordinate, Y is a spatial coordinate, and the motion of the wall is determined by

$$\left[\frac{dY}{dX}\right]^2=1-\frac{1}{1+(2\pi G_NC_1)^2C_0X^{2\Gamma-1/2}} . \quad (4.13)$$

For all Γ the wall emerges from the singularity at $X=0$, with Y increasing as time advances. The derivative dY/dX asymptotically approaches $+1$ if $\Gamma>\frac{1}{4}$ and 0 if $\Gamma<\frac{1}{4}$.

2. $\Gamma=\frac{1}{4}$

The value $\Gamma=\frac{1}{4}$ is special because, whether or not $F'+G'\neq 0$, Eq. (4.2) now implies

$$G(t)=C_2F(t)+C_3 , \quad (4.14)$$

where C_2 and C_3 are constants. It then follows from Eqs. (4.5) that

$$X=[(1+C_2)Y+2C_3]/(1-C_2) \quad (4.15)$$

on the wall. A condition relating the constants, namely,

$$C_0=(1-C_2)^2/(4\pi G_N)^2{C_1}^2C_2 , \quad (4.16)$$

follows from Eqs. (4.1d), (4.3), (4.5), and (4.15).

If $C_0<0$, $C_2<0$ and the wall emerges from the timelike singularity at $X=0$, unless $C_2=-1$. When $C_2=-1$ (i.e., $F'+G'=0$), the wall remains at the constant value $X=X_{\text{wall}}=C_3$, and the solution is static. It is clear that this yields the essentially unique static solution for planar walls. Indeed, the coordinate transformation

$$X=1-4\pi G_NC_1Z,\quad Y=4\pi G_NC_1T , \quad (4.17)$$

brings the static solution to the form

$$ds^2=\frac{-dT^2+dZ^2}{(1-4\pi G_N\sigma {X_{\text{wall}}}^{3/4}Z)^{1/2}}+(1-4\pi G_N\sigma {X_{\text{wall}}}^{3/4}Z)(dx^2+dy^2) , \quad (4.18)$$

with $\sigma=C_1{X_{\text{wall}}}^{-3/4}$. The choice $X_{\text{wall}}=1$ yields the form quoted in Ref. 1.

If $C_0>0$, $C_2>0$ and the wall emerges from the spacelike singularity at $X=0$ and follows the straight-line path of Eq. (4.15). Note that $dY/dX>0$ because $F'-G'>0$.

For all class-II solutions one easily shows that the acceleration vector of an observer following the wall has nonvanishing components

$$a^X=-(1-2\Gamma)2\pi G_N\sigma(F'-G')e^{-\nu}\,,$$
$$a^Y=-(1-2\Gamma)2\pi G_N\sigma(F'+G')e^{-\nu}\,, \quad (4.19)$$

and hence that

$$a^b\xi_b=(1-2\Gamma)2\pi G_N\sigma \quad (4.20)$$

as required.

V. CONCLUDING REMARKS

In this paper we have completed a program begun in Ref. 1, that of obtaining all solutions to Einstein's equations for planar walls with constant ratios $\tau/\sigma\leq 1$. We have seen how repulsion gives way to attraction as this ratio is reduced below $\frac{1}{2}$, and how the unique static solution finds its place within the classes of solutions.

From the standpoint of the cosmology/particle physics interface, perhaps the two most interesting solutions are those for domain walls ($\tau/\sigma=1$) and walls composed of isotropically distributed cosmic strings ($\tau/\sigma=\frac{1}{2}$). It is clear from the behavior of the corresponding class-I solutions, those free of curvature singularities, that the scale factor of a domain-wall-dominated universe, with walls stretched over the horizon (about one per horizon), will expand like t^2 (t=cosmic time);[1,6] while the scale factor of a string-wall-dominated universe will expand like t.[7]

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