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Dynamical calculation of quark, lepton, and gauge-boson masses

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A model is proposed in which fermion and gauge-boson masses are calculable. Fermions get their masses from their own condensates and the low-lying gauge bosons such as Z or W get their masses from the condensate of fermions of higher generations. The short-distance contribution to chiral-symmetry breaking is important, in contrast to the technicolor theory. We have no difficulty concerning the flavor-changing neutral current.

I. INTRODUCTION

Spontaneously broken chiral symmetry in various theories has been studied extensively in the past partly in order to understand the origin of fermion masses. Nambu and Jona-Lasinio studied in their classical paper[1] the mechanism of dynamical mass generation in the theory with four-fermion interaction. Johnson, Baker, and Willey initiated the study of chiral-symmetry breaking in QED.[2] Lack of asymptotic freedom makes the introduction of some kind of cutoff unavoidable in these theories.

After the discovery of asymptotic freedom[3] people started working on chiral-symmetry breaking in non-Abelian gauge theories[4] making use of the Schwinger-Dyson equations. The existence of "regular" and "irregular" solutions has been shown in these theories. The regular solution corresponds to the case of spontaneous chiral-symmetry breaking and the irregular one to the case of explicit bare mass term in Lagrangian.

On the other hand, the investigation of chiral-symmetry breaking in lattice gauge theory has also been started.[5] It is rather difficult, however, to understand chiral-symmetry breaking without knowing the short-distance behavior of a given theory because the chiral symmetry concerns the existence or absence of bare mass term in the Lagrangian.

If we turn our attention to the phenomenological side, we notice that we do not have a satisfactory understanding of quark and lepton masses yet. Explicit introduction of Higgs bosons avoiding the dynamical approach leaves many arbitrary parameters unexplained.

Technicolor theory[6] has been proposed to remedy this situation but not without much difficulty of its own. The appearance of a relatively large flavor-changing neutral current is rather hard to overcome.

I would like to describe a model in this paper which is similar to the technicolor theory at least in its spirit. The essential difference is the importance of short-distance contributions to the chiral breaking in our model, in contrast to the technicolor case.

Let us discuss some general aspect of lepton and quark masses in our dynamical approach. Our model, which is a gauge theory without elementary Higgs field has also the chiral symmetry by nature or by an assumption. What causes the mass difference between electron and muon, for example, in such a theory? We know experimentally that there is no interaction in the relatively-low-energy region that is strong enough to cause the e-μ mass difference. We must, therefore, have some gauge interaction which is important only in the high-energy region yet to be reached, and is sufficiently strong to cause the mass difference. We note that the Schwinger-Dyson equation with constant running coupling gives a logarithmically divergent self-mass. Asymptotically free theory gives finite answer to the self-energy but the fact that the coupling decreases as slow as $(\log Q^2)^{-1}$ shows that the high-energy contribution to self-mass is still very important.

In the case of quarks the long-distance QCD contribution to the masses is separated out as the constituent quark mass. The current-algebra mass is defined to be the contribution from the short-distance gauge interactions including QCD. As a possible source of the gauge interaction the "generation gauge interaction" is examined in this paper together with the usual QCD and quantum-flavor-dynamics (QFD) interactions. We know that the generation gauge interaction must be completely broken at the energy scale yet to be reached. For the purpose of breaking the generation group a technicolorlike interaction is introduced. We also need an additional interaction to break the grand unified symmetry at approximately 10^{15} GeV.

In contrast to the technicolor theory, however, the Z and W masses originate from the condensates of higher generation of quarks which have masses in the TeV region. QCD coupling starts to increase as a function of energy in the TeV region until the energy reaches the grand unification scale because of these higher generations of quarks.

In Sec. II some basic properties of the Schwinger-Dyson equation are discussed. We propose our model in Sec. III. We also show that the model satisfies such basic conditions as the asymptotic freedom or the absence of triangle anomaly. In Sec. IV we discuss the possible pattern of symmetry breaking. Section V gives an approximate solution to the Schwinger-Dyson equation derived in Sec. IV. We can understand the existence of hierarchy by just looking at the solution. The result is also compared with experiment. The property of the solution in general is satisfactory except for a few discrepancies with experiment. I do not know at this moment whether this is due to the approximation adopted or to the inadequacy of the model. We discuss briefly the problem of Z and W masses in Sec. VI. Some other models we considered are presented in Sec. VII. Finally, general discussions and conclusions are given in Sec. VIII. Numerical analysis of the Schwinger-Dyson equation is presented in the Appendix.

II. SCHWINGER-DYSON EQUATION

Our tool for attacking the problem of chiral-symmetry breaking is the Schwinger-Dyson equation[7] (hereafter referred to as the SD equation). The SD equation in non-Abelian gauge theories has been extensively studied in the past[4] and its basic properties are well understood. Let us summarize these properties below.

ψ_L^α and ψ_R^i denote the left-handed and the right-handed massless fermion fields, respectively. Indices α and i refer to the gauge group G. We write the renormalized propagator Δ for this fermion system as

$$\Delta^{-1}=\begin{bmatrix} \frac{1-\gamma_5}{2}(1+A)_\alpha^\beta \not{p} & \frac{1-\gamma_5}{2}B_\alpha^j \\ \frac{1+\gamma_5}{2}B_i^\beta & \frac{1+\gamma_5}{2}(1+A)_i^j \not{p} \end{bmatrix} \tag{1}$$

$$=\begin{bmatrix} \frac{1-\gamma_5}{2}\delta_\alpha^\beta \not{p} & 0 \\ 0 & \frac{1+\gamma_5}{2}\delta_i^j \not{p} \end{bmatrix}+\Sigma\ . \tag{2}$$

B is nonvanishing only when the chiral symmetry is broken. The SD equation in the ladder approximation (Fig. 1) reads

$$\Sigma=\begin{bmatrix} & \bar{B} \\ B & \end{bmatrix}=\frac{3}{2(2\pi)^4 i}\int g^2(k^2)F_m\Sigma[(p-k)^2-\Sigma^2]^{-1} \times F_m\frac{d^4k}{k^2}\ , \tag{3}$$

where we have taken the Landau gauge ($A_\beta^\alpha=A_i^j\equiv 0$) for convenience. $g^2(k^2)$ is the gauge coupling and F_m is the representation matrix

$$F_m=\begin{bmatrix} (F_m^{(-)})_\beta^\alpha & 0 \\ 0 & (F_m^{(+)})_j^i \end{bmatrix}\ . \tag{4}$$

In order to understand the nature of this equation we write its cutoff version with the explicit bare mass

$$\Sigma=m_0(\Lambda)+\frac{3}{2(2\pi)^4 i}\int^\Lambda g^2F_m\Sigma[(p-k)^2-\Sigma^2]^{-1}F_m\frac{d^4k}{k^2}\ . \tag{5}$$

If we perform the perturbation expansion, there appear divergences from the integral and these are canceled by $m_0(\Lambda)$ in front. The chiral symmetry is implied by $m_0(\Lambda)\equiv 0$ for all values of the cutoff parameter Λ. A peculiarity of non-Abelian asymptotically free gauge theory is

$$\lim_{\Lambda\to\infty} m_0(\Lambda)=\lim_{\Lambda\to\infty}(\log\Lambda)^{-a}=0\ , \tag{6}$$

due to the renormalization effect. This means that whether we start from a chiral-symmetric theory or from a theory with explicit bare mass term we end up with the same SD equation (3) in the infinite-cutoff limit. This is why we expect two types of solutions for the equation (3); one which corresponds to the case of explicit bare mass (irregular solution) and the other corresponding to the broken chiral symmetry (regular solution). These solutions behave differently in the high-energy limit:

$$\Sigma_{\rm ir}(p)=O((\log p)^{-c})\ , \tag{7}$$

and

$$\Sigma_{\rm r}(p)=O(p^{-2}(\log p)^c)\ , \tag{8}$$

when $g^2(k^2)=O((\log k^2)^{-1})$. c is a number which depends only on the structure of the gauge group.

After Wick rotation, Eq. (3) reads

$$\Sigma(p)=\frac{3}{2(2\pi)^4}\int g^2(p-p')^2F_m\Sigma(p')[p'^2+\Sigma^2(p')]^{-1}F_m\frac{d^4p'}{(p-p')^2}\ , \tag{9}$$

where the integration variable was changed from k to $p'=p-k$. Subtracting $\Sigma(p_0)$ from both sides, we obtain

$$\Sigma(p)=\Sigma(p_0)+\frac{3}{2(2\pi)^4}\int F_m\Sigma(p')[p'^2+\Sigma^2(p')]^{-1}F_m\left[\frac{g^2[(p-p')^2]}{(p-p')^2}-\frac{g^2[(p_0-p')^2]}{(p_0-p')^2}\right]d^4p'\ . \tag{10}$$

This equation has a unique iterative solution for an arbitrary value of $\Sigma(p_0)$ with the asymptotic behavior $(\log p)^{-c}$ (Fig. 2). The solution behaves like the curve (a) of Fig. 2 when $\Sigma(p_0)$ is sufficiently large. It will, however, behave like the curve (b) when $\Sigma(p_0)$ is sufficiently decreased. We have thus the unique value of $\Sigma(p_0)$ which separates the case (b) from the case (a). This value of $\Sigma(p_0)$ gives the regular solution [curve (c)] with the asymptotic behavior $(p^2)^{-1}(\log p)^c$. This situation was checked numerically (Appendix) in the case of QCD although I have no rigorous proof to justify it.

FIG. 1. Ladder approximation to the self-energy part.

Now let us proceed to the case of massive vector bosons:

$$\Sigma(p)=\frac{3}{2(2\pi)^4}\int g^2[(p-p')^2]F_m\Sigma(p')[p'^2+\Sigma(p')^2]^{-1} \times F_m\frac{d^4p'}{(p-p')^2+m^2}\ . \tag{11}$$

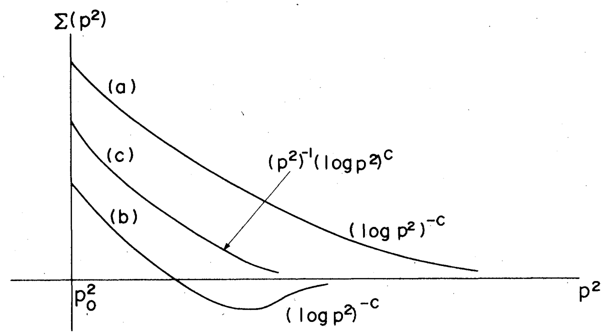

FIG. 2. Behavior of the solution to Schwinger-Dyson equation.

The question to be answered is if this equation always has a nontrivial solution. To see this let us examine the following equation which is closely related to Eq. (9):

$$\Sigma=\frac{3g^2}{2(2\pi)^4}\int^{|p^2|<\Lambda^2}(p'+\Sigma^2)^{-1}\frac{d^4p'}{p'^2+m^2} \,. \tag{12}$$

After integration we obtain

$$\frac{1}{k}=\frac{1}{1-\xi}\left[\ln(1+\lambda)-\xi\ln\frac{\lambda+\xi}{\xi}\right] , \tag{13}$$

where $k=(3/16\pi^2)g^2$, $\xi=\Sigma^2/m^2$, and $\lambda=\Lambda^2/m^2$. The right-hand side of this equation is a decreasing function of ξ for $\xi\geq 0$. It, therefore, has a solution only when $1/k\leq\ln(1+\lambda)$. A more rigorous argument[8] shows that Eq. (11) also has a solution when the coupling is larger than a certain critical value.

Equation (12) reads

$$1=\frac{3}{2(2\pi)^4}\int^{\Lambda^2}\frac{g^2}{p^2+\Sigma^2}\frac{d^4p}{p^2} , \tag{14}$$

when $m=0$. In this case the integral can be made arbitrarily large by making Σ small. It, therefore, has a solution for any value of g^2. Physically, infrared instability in this case is sufficient to make the normal vacuum unstable. Let us pay attention to the upper end of the integral. The integral can be made as large as we want by making Λ sufficiently large. This is true whether the vector boson is massive or not. In the asymptotic free theory the cutoff is naturally provided by the renormalization effect: $g^2\propto(\log p^2)^{-1}$. Suppose that the coupling behaves as shown in Fig. 3. The effective cutoff energy is approximately Λ in this case which could be as large as the grand unification mass. The contribution from short distances will be very important in such a situation. In the next section we will construct a model where this situation is realized. Finally, we observe that the solution to Eq. (14)

$$\Sigma=\Lambda\exp(-16\pi^2/3g^2) , \tag{15}$$

shows that we can have a very small mass although Λ may be as large as 10^{15} GeV, thus providing a possible solution to the hierarchy problem.

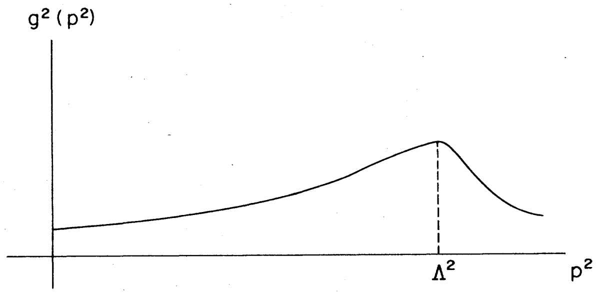

FIG. 3. Behavior of a running coupling which is asymptotically free and also infrared finite.

III. CONSTRUCTION OF THE MODEL

Our model consists of gauge bosons and fermions which belong to a certain representation of a gauge group G. G is a product of semisimple groups including grand-unified group U and also the "generation interaction" group G_1. We assume that the group U is broken down to SU(3)×SU(2)×U(1) at $\simeq 10^{15}$ GeV and then down to SU(3)×U(1) at $\simeq 10^2$ GeV. The group G_1 should be completely broken at $\sim 10\sim 10^3$ TeV or even higher. The only possible origin of the mass of all the fermions and gauge bosons is the fermion-pair condensation since we have no elementary Higgs scalars.

Our basic assumption is that all the "low-lying fermions" get their masses from the condensation of themselves. The attraction which binds these fermions and antifermions into pairs comes from the short-distance of both the unification and the generation gauge interactions. Specifically, the long-distant part of QCD will be neglected or only partly included. Thus, the mass of fermions which have QCD interaction will be close to the so-called "current-algebra mass."

Besides these two gauge interactions U and G_1 we need at least one more very strong interaction which gives rise to a condensate comparable in magnitude to the unification scale. This condensate breaks U down to SU(3)×SU(2)×U(1) and also G_1 down to some smaller group G_2. We call this group (T_u) the "unification technicolor group." T_u plays the role of the conventional technicolor group except that it violates $U\times G_1$ at the scale of 10^{15} GeV rather than SU(2)×U(1) at the scale of 10^2 GeV. If the group G_2 is an identity, this will be the whole story. But it turns out that G_2 is nontrivial in our model. We, therefore, need one more gauge group T_g which violates G_2 completely. We call this group (T_g) the "generation technicolor group."

Our gauge group, therefore, is $U\otimes G_1\otimes T_U\otimes T_g$, i.e., we assume the existence of four fundamental interactions:

(1) Unified gauge interaction of strong, weak, and electromagnetic forces.

(2) Gauge interaction which exists among different generations.

(3) Technicolorlike interaction which breaks both $U\otimes G_1$ to much smaller groups at $\simeq 10^{15}$ GeV.

(4) Technicolorlike interaction which breaks the generation group completely.

We do not consider the unification of all these interactions in this paper. In our model, therefore, all the physical quantities are calculable in terms of four parameters, in principle.

The next question to be answered is obvious: What are the U, G_1, T_1, and T_g? Popular candidates for U are SU(5) (Ref. 9) or SO(10) (Ref. 10). Phenomenologically we prefer SU(5) because of the following reason: In the conventional SO(10) model the entire one generation of low-lying fermions belong to a single representation of $\underline{16}$. This means that the mass splittings inside the generation should come from U only. This does not seem to be plausible since, for example, the fact that m_u is less than m_d, whereas m_c is larger than m_s, will be unexplainable. We, therefore, adopt SU(5) as the unification group U where some of the up quarks, down quarks, and leptons belong to different representations.

A restriction on G_1 comes from the behavior of the coupling strength as shown in Fig. 3. We assume this behavior for all the gauge couplings of $SU(3)_c \times SU(2)\times U(1)$ and also of G_2. We, therefore, must have more than eight generations to make $SU(3)_c$ nonasymptotically free. Couplings for U and G_1 must satisfy the asymptotic freedom. The last condition to be satisfied is the absence of anomaly.

After so many trials in search for the candidate for G_1 which satisfies all the above conditions we find that the SU(5) group is a possible candidate for G_1 as well as for U. The groups T_u and T_g which are needed to violate U and G do not play an important role in determining the mass of low-lying fermions and gauge bosons. They, nevertheless, are restricted severely by the condition of asymptotic freedom and of absence of anomaly. We find that $T_g = SU(5)$ with $T_u = SU(2)$ is a possible solution. We, therefore, consider $SU(5)_u \times SU(5)_g \times SU(5)_{tg} \times SU(2)_{tu}$ as our fundamental gauge group. Under this group usual low-lying fermions transform like $(10,10,1,1,)\oplus(\bar{5},5,1,1)\oplus(5,\bar{5},1,1)$; there are ten generations of quarks and leptons.

Fermions which belong to $(1,\bar{5},5,1)\oplus(1,\bar{5},\bar{5},1)$ have $SU(5)_{tg}$ interactions and can produce a large condensate which violates G_2 completely. Note also that the G_1 anomaly produced by low-lying fermions is completely canceled by this representation.

Finally, we have superheavy fermions which belong to the representations $[(1,5,1,2,)\oplus(1,\bar{5},1,2)]\oplus[(5,1,1,2)\oplus(\bar{5},1,1,2)]$. These are responsible to violate U and G_1 down to $SU(3)\times SU(2)\times U(1)$. Note that G_2 in our case is also $SU(3)\times SU(2)\times U(1)$. We can calculate[11] the β function for each group U, G_1, T_g, and T_u. We find

$$
\begin{aligned}
f_u{}^2(M)&=\frac{24\pi^2}{11\ln M/\Lambda_u}\,,\\
f_g{}^2(M)&=\frac{24\pi^2}{\ln M/\Lambda_g}\,,\\
f_{tg}{}^2(M)&=\frac{4\pi^2}{15\ln M/\Lambda_{tg}}\,,\\
f_{tu}{}^2(M)&=\frac{12\pi^2}{\ln M/\Lambda_{tu}}\,.
\end{aligned}
\tag{16}
$$

All four interactions, therefore, are asymptotically free. Below the condensates of $(1,5,1,2)\oplus(1,\bar{5},1,2)$ and $(5,1,1,2)\oplus(\bar{5},1,1,2)$, both U and G_1 break down to $SU(3)\times(SU(2)\times U(1)$. Coupling strengths f_3, f_2, and f_1 for U and g_3, g_2, and g_1 for G_1 have the following form:

$$
f_3{}^2=-\frac{24\pi^2}{7\ln M/\Lambda_3}\,,\quad f_2{}^2=\frac{-24\pi^2}{18\ln M/\Lambda_2}\,,\quad
f_1{}^2=-\frac{24\pi^2}{40\ln M/\Lambda_1}\,,
\tag{17}
$$

and

$$
g_3{}^2=-\frac{24\pi^2}{17\ln M/\Xi_3}\,,\quad g_2{}^2=-\frac{24\pi^2}{28\ln M/\Xi_2}\,,\quad
g_1{}^2=-\frac{24\pi^2}{50\ln M/\Xi_1}\,.
\tag{18}
$$

At much lower energy the generation group breaks down completely. This situation is illustrated in Fig. 4, Λ_1, Λ_2, Λ_3, Ξ_1, Ξ_2, and Ξ_3 are related to Λ_u and Λ_g by

$$
\Lambda_1=\left[\frac{M_0}{\Lambda_u}\right]^{11/40}M_0,\quad \Lambda_2=\left[\frac{M_0}{\Lambda_u}\right]^{11/18}M_0\,,\quad
\Lambda_3=\left[\frac{M_0}{\Lambda_u}\right]^{11/7}M_0\,,
\tag{19}
$$

and

$$
\Xi_1=\left[\frac{M_0}{\Lambda_g}\right]^{1/50}M_0,\quad \Xi_2=\left[\frac{M_0}{\Lambda_g}\right]^{1/28}M_0\,,\quad
\Xi_3=\left[\frac{M_0}{\Lambda_g}\right]^{1/17}M_0\,,
\tag{20}
$$

where M_0 is the grand-unification scale which can be determined by calculating the condensate

$$\langle(1,5,1,2),(1,\bar{5},1,2)\rangle \text{ or } \langle(5,1,1,2),(\bar{5},1,1,2)\rangle\,.$$

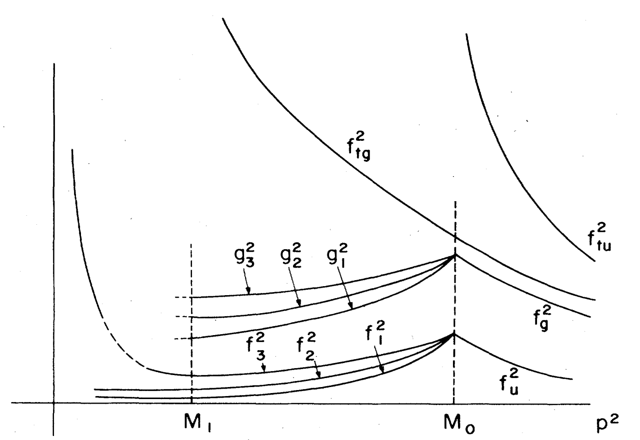

FIG. 4. Various couplings of our model.

IV. PATTERN OF SYMMETRY BREAKING

We now apply a general method described in Sec. II to the specific model of Sec. III. No rigorous argument will be given regarding the stability of the solution we present. This problem is worth looking into only when our model is phenomenologically adequate. We, therefore, concentrate on the phenomenological side of the model in this paper.

The largest condensates we expect are

$$\langle (1,5,1,2),(1,\bar{5},1,2)\rangle \;, \tag{21}$$

and

$$\langle (5,1,1,2),(\bar{5},1,1,2)\rangle \;. \tag{22}$$

T_{tu} interaction is assumed to be so large that these condensates violate U and G_1 down to $SU(3)\times SU(2)_w\times U(1)$ and $SU(3)_g\times SU(2)_g\times U(1)_g$, respectively. If we write down the SD equation for $\langle (5,1,1,2),(\bar{5},1,1,2)\rangle$, for example, we immediately see that the solution can have one of the following three forms with respect to the $SU(5)_u$ representations:

$$\begin{bmatrix} b &&&& \\ & b &&& \\ && b && \\ &&& b & \\ &&&& b \end{bmatrix},\quad \begin{bmatrix} b &&&& \\ & b &&& \\ && b && \\ &&& b & \\ &&&& -b \end{bmatrix},$$

or

$$\begin{bmatrix} b &&&& \\ & b &&& \\ && b && \\ &&& -b & \\ &&&& -b \end{bmatrix}. \tag{23}$$

The first choice does not violate $SU(5)_u$ at all, the second choice violates it to $SU(4)\times U(1)$ and the last choice down to $SU(3)\times SU(2)\times U(1)$. If the relatively small force due to $SU(5)_u$ gauge-boson exchange is important besides the strong $SU(2)_{tu}$, we would have the first choice. We simply assume in this paper that the third choice is most stable, or, if it is not, then the time required to decay into the stable state is long enough.

The generation group $SU(3)\times SU(2)\times U(1)$ should be broken completely above the TeV region. This will be done by the condensate of

$$\langle (1,\bar{5},5,1),(1,\bar{5},\bar{5},1)\rangle \;, \tag{24}$$

which is reduced to a matrix

$$\begin{bmatrix} \langle (\bar{3},1)(2),(\bar{3},1)(2)\rangle & \langle (\bar{3},1)(2),(1,2)(-3)\rangle \\ \langle (1,2)(-3),(\bar{3},1)(+2)\rangle & \langle (1,2)(-3),(1,2)(-3)\rangle \end{bmatrix}, \tag{25}$$

with respect to the $SU(3)_g\times SU(2)_g\times U(1)_g$ representation. Each element of the matrix behaves as a singlet in $SU(5)_{tg}$ representation. We see that the off-diagonal element should be nonvanishing in order that no residual symmetry exists. There exists such a solution except that here again we do not know its stability against other solutions. We assume the existence of a mass scale M_1 where the generation group $SU(3)\times SU(2)_g\times U(1)_g$ breaks down completely.

A scenario for the fate of 10 generations of low-lying fermions will now be given. First of all we have

$$(10,10)=[(3,2)(1)+(3^*,1)(-4)+(1,1)(6)] \otimes[(3,2)(1)+(3^*,1)(-4)+(1,1)(6)]\;, \tag{26}$$

$$(\bar{5},\bar{5})=[(3^*,1)(2)+(1,2)(-3)]\otimes[(3^*,2)(2)+(1,2)(-3)]\;, \tag{27}$$

and

$$(\bar{5},5)=[(3^*,1)(2)+(1,2)(-3)]\otimes[(3,1)(-2)+(1,2)(3)]\;, \tag{28}$$

where the first bracket refers to the color-electroweak group and the second bracket refers to the generation group. Let us first discuss the quarks of charge $\frac{2}{3}$. The strongest attractive force will be present in the following channels:

$$\langle [(3,2)(1),(3,2)(1)]\otimes[(3^*,1)(-4),(3^*,1)(-4)]\rangle \;, \tag{29}$$

and

$$\langle [(3,2)(1),(3^*,1)(-4)]\otimes[(3^*,1)(-4),(3,2)(1)]\rangle \;. \tag{30}$$

Either in Eq. (29) or in (30) the first entry in the bracket refers to the color-electroweak group and the second one refers to the generation group. Only one component of the SU(2) doublet will have this type of condensation. The other component can be transformed away. Thus, six out of ten generations of $\frac{2}{3}$ quarks become massive through this condensation. The next higher condensation will be

$$\langle [(3,2)(1),(3,2)(1)]\otimes[(3^*,1)(-4),(3,2)(1)]\rangle \;. \tag{31}$$

There is another solution where the role of generation group and the color-electroweak group is interchanged. We suggest that solution (31) is more stable due to the fact that the generation group is already completely broken while $SU(3)_c$ gives a strong attraction in the long distance. This condensate exhausts three out of remaining four generations. The smallest condensate is given by

$$\langle [(3,2)(1),(1,1)(6)]\otimes[(3^*,1)(-4),(1,1)(6)]\rangle \;. \tag{32}$$

We see that the generation interaction gives a repulsive force in this channel in contrast to all the other nine channels. This is the reason why the u quark is so much lighter than the other quarks.

For the charge $-\frac{1}{3}$ quarks we will first have the following condensates

$$\langle [(3,2)(1),(3,2)(1)]\otimes[(3^*,1)(2),(3^*,1)(2)]\rangle \;, \tag{33}$$

$$\langle [(3,2)(1),(3^*,1)(-4)]\otimes[(3^*,1)(2),(3,1)(-2)]\rangle \;. \tag{34}$$

These six generations of quarks will be as heavy as six heavy generations of charge $\frac{2}{3}$ quarks. The remaining four generations of quarks have the origin of masses from the condensates

$$\langle [(3,2)(1),(3,2)(1)+(1,1)(6)] \otimes [(3^*,1)(2),(1,2)(-3)+(1,2)(3)] \rangle \,. \tag{35}$$

Ten generations of leptons have the same group structure as charge $-\frac{1}{3}$ quarks for the condensates except that the color-electroweak part transforms like (1,1)(6) or (1,2)(−3) instead of (3,2)(1) or (3*,1)(2).

Let us now write down the SD equation for the condensates mentioned above. Indices $a,b,c,\ldots$ hereafter denote color SU(3), $m,n,\ldots$ denote electroweak SU(2), $\alpha,\beta,\gamma,\ldots$ refer to the generation SU(3), and $\mu,\nu,\ldots$ refer to the generation SU(2). Indices for each wave function in $B=\langle\bar\psi\psi\rangle$ will be separated by a semicolon as in $B^{\alpha}{}_{;\beta}$. Corresponding to the condensates in Eq. (29) we have

$$\begin{aligned} B^{am\alpha\mu}{}_{;b\beta} = {} & \frac{3}{2(2\pi)^4}\int f_3{}^2 F^a_{ia'} B^{a'm\alpha\mu}{}_{;b'\beta}[(p-k)^2+\bar B^{b'\beta';}{}_{a''m'\alpha'\mu'}B^{a''m'\alpha'\mu'}{}_{;b''\beta}]^{-1}F^{b''}_{ib}\frac{d^4k}{k^2} \\ & +\frac{3}{2(2\pi)^4}\int g_3{}^2 F^\alpha_{i\alpha'} B^{am\alpha'\mu}{}_{;b'\beta}[(p-k)^2+\bar B^{b'\beta';}{}_{a'm'\alpha''\mu'}B^{a'm'\alpha''\mu'}{}_{;b\beta''}]^{-1}F^{\beta''}_{i\beta}\frac{d^4k}{k^2+\mu_i{}^2} \\ & +\frac{3}{2(2\pi)^4}\int \frac{f_1{}^2}{30} B^{am\alpha\mu}{}_{;b'\beta}[(p-k)^2+\bar B^{b'\beta';}{}_{a'm'\alpha'\mu'}B^{a'm'\alpha'\mu'}{}_{;b\beta}]^{-1}\frac{d^4k}{k^2} \\ & +\frac{3}{2(2\pi)^4}\int \frac{g_1{}^2}{30} B^{am\alpha\mu}{}_{;b'\beta}[(p-k)^2+\bar B^{b'\beta';}{}_{a'm'\alpha'\mu'}B^{a'm'\alpha'\mu'}{}_{;b\beta}]^{-1}\frac{d^4k}{k^2+\mu_0{}^2}\,, \end{aligned} \tag{36}$$

where $F^a_{ia'}$ stands for SU(3) matrices of fundamental representation, μ_i for the $SU(3)_g$ gauge-boson masses close to M_1, and μ_0 for $U(1)_g$ gauge-boson mass, which is also close to M_1. Z and W masses will be neglected. Equation (30) leads to a SD equation similar to Eq. (29) with the replacement of indices:

$$B^{am\alpha\mu}{}_{;b\beta} \to B^{am}{}_{\beta;b}{}^{\alpha\mu}\,. \tag{37}$$

Equation (31) gives a SD equation with the force in the 3×3 channel of generation group:

$$\begin{aligned} B^{am\alpha\mu}{}_{;b}{}^{\beta\nu} = {} & \frac{3}{2(2\pi)^4}\int f_3{}^2 F^a_{ia'} B^{a'm\alpha\mu}{}_{;b'}{}^{\beta'\nu'}[(p-k)^2+\bar B^{b'}{}_{\beta'\nu';a''m'\alpha'\mu'}B^{a''m'\alpha\mu'}{}_{;b''}{}^{\beta\nu}]^{-1}F^{b''}_{ib}\frac{d^4k}{k^2} \\ & +\frac{3}{2(2\pi)^4}\int g_3{}^2 F^\alpha_{ia'} B^{am\alpha'\mu}{}_{;b'}{}^{\beta'\nu'}[(p-k)^2+\bar B^{b'}{}_{\beta'\nu';a''m'\alpha''\mu'}B^{a''m'\alpha''\mu'}{}_{;b}{}^{\beta'\nu}]^{-1}(-F^{\beta}_{i\beta'})\frac{d^4k}{k^2+\mu_i{}^2} \\ & +\frac{3}{2(2\pi)^4}\int g_2{}^2 F^\mu_{i\mu'} B^{am\alpha\mu'}{}_{;b'}{}^{\beta'\nu'}[(p-k)^2+\bar B^{b'}{}_{\beta'\nu';a''m'\alpha'\mu''}B^{a''m'\alpha'\mu''}{}_{;b}{}^{\beta\nu'}]^{-1}(-F^{\nu'}_{i\nu})\frac{d^4k}{k^2+\mu_i{}^2} \\ & +\frac{3}{2(2\pi)^4}\int \frac{f_1{}^2}{15} B^{am\alpha\mu}{}_{;b'}{}^{\beta'\nu'}[(p-k)^2+\bar B^{b'}{}_{\beta'\nu';a'm'\alpha'\mu'}B^{a'm'\alpha'\mu'}{}_{;b}{}^{\beta\nu}]^{-1}\frac{d^4k}{k^2} \\ & -\frac{3}{2(2\pi)^4}\int \frac{g_1{}^2}{60} B^{am\alpha\mu}{}_{;b'}{}^{\beta'\nu'}[(p-k)^2+\bar B^{b'}{}_{\beta'\nu';a'm'\alpha'\mu'}B^{a'm'\alpha'\mu'}{}_{;b}{}^{\beta\nu}]^{-1}\frac{d^4k}{k^2+\mu_0{}^2}\,. \end{aligned} \tag{38}$$

The lightest charge $\frac{2}{3}$ quark (u quark) is described by the SD equation:

$$\begin{aligned} B^{am}{}_{;b} = {} & \frac{3}{2(2\pi)^4}\int f_3{}^2 F^a_{ia'} B^{a'm}{}_{;b'}[(p-k)^2+\bar B^{b'}{}_{;a''m'}B^{a''m'}{}_{;b''}]^{-1}F^{b''}_{ib}\frac{d^4k}{k^2} \\ & +\frac{3}{2(2\pi)^4}\int \frac{f_1{}^2}{15} B^{am}{}_{;b'}[(p-k)^2+\bar B^{b'}{}_{;a'm'}B^{a'm'}{}_{;b}]^{-1}\frac{d^4k}{k^2} \\ & -\frac{3}{2(2\pi)^4}\int \tfrac{3}{5}g_1{}^2 B^{am}{}_{;b'}[(p-k)^2+\bar B^{b'}{}_{;a'm'}B^{a'm'}{}_{;b}]^{-1}\frac{d^4k}{k^2+\mu_0{}^2}\,. \end{aligned} \tag{39}$$

We also have SD equations for charge $-\frac{1}{3}$ quarks and leptons. We do not bother to write down all these equations here but make a few remarks in passing. The two-body force in the $-\frac{1}{3}$ quark-antiquark channel has a contribution from the QCD interaction, whereas the lepton channel has only $U(1)_Y$ interaction besides, of course, the force due to the generation interaction. The generation interaction is common to the quark channel and to the lepton channel. This does not necessarily mean that we have to take the same type of solution in each case. As will be discussed in the next section we have to choose different type of solution in each case to explain the experimental fact.

V. APPROXIMATE SOLUTION TO SD EQUATION AND PHENOMENOLOGY

Equation (36) has the solution of the form

$$B^{am\alpha\mu}{}_{;b\beta} = \delta^a_b \delta^\alpha_\beta \delta_1^{2m}\delta_1^{2\mu} B\,. \tag{40}$$

Substituting this form in Eq. (36) we obtain

$$B=\frac{3}{2(2\pi)^4}\int f_3{}^2\tfrac{4}{3}B[(p-k)^2+B^2]^{-1}\frac{d^4k}{k^2}+\frac{3}{2(2\pi)^4}\int g_3{}^2\tfrac{4}{3}B[(p-k)^2+B^2]^{-1}\frac{d^4k}{k^2+M_1{}^2}$$
$$+\frac{3}{2(2\pi)^4}\int f_1{}^2\tfrac{1}{15}B[(p-k)^2+B^2]^{-1}\frac{d^4k}{k^2}+\frac{3}{2(2\pi)^4}\int g_1{}^2\tfrac{1}{15}B[(p-k)^2+B^2]^{-1}\frac{d^4k}{k^2+M_1{}^2}\,, \tag{41}$$

where $\mu_i{}^2$ and $\mu_0{}^2$ are replaced by the approximate value $M_1{}^2$. This equation has a unique "regular solution" which corresponds to the case of broken chiral symmetry. One can solve this equation numerically without much difficulty. The method is to first subtract at some point $k_0{}^2$, obtaining an equation similar to Eq. (10) of Sec. II. Then one solves this equation iteratively changing the value of $B(k_0{}^2)$ until one gets the solution of type (c) as is illustrated in Fig. 2.

Since our object is to see if our model is phenomenologically sound, we are satisfied with the following crude approximation: Instead of using the coupling of Eqs. (16) and (17) we use $f_3{}^2(\Lambda_s{}^2)$ or $g_3{}^2(\Lambda_s{}^2)$ where Λ_s is presumably close to M_0 and we cut off the integral at Λ_s. Then Eq. (41) reduces to

$$1=\frac{1}{4\pi^2}f_3{}^2(\Lambda_s)\ln\frac{\Lambda_s}{B}+\frac{1}{4\pi^2}g_3{}^2(\Lambda_s)\ln\frac{\Lambda_s}{M_1}\,, \tag{42}$$

where $f_1{}^2$ and $g_1{}^2$ contributions are neglected for the moment. This has the solution

$$B=\Lambda_s\exp\left[\frac{-4\pi^2}{f_3{}^2(\Lambda_s)}\right]\exp\left[\frac{g_3{}^2(\Lambda_s)}{f_3{}^2(\Lambda_s)}\ln\frac{\Lambda_s}{M_1}\right]. \tag{43}$$

Equation (31) has a solution with the following tensor structure:

$$B^{am\alpha\mu}{}_{;b}{}^{\beta\nu}=\delta^{\mu\nu}\delta^a_b(\epsilon^{\alpha\beta\gamma}iB_\gamma+\delta^\alpha_3\delta^\beta_3B_0)\,, \tag{44}$$

where we can set $B_1=B_2=0$. The first term corresponds to $\underline{3}^*\in\underline{3}\times\underline{3}$ and the second term to $\underline{6}\in\underline{3}\times\underline{3}$. Representation $\underline{6}$ has the repulsion from the generation interaction. The approximate solution reads

$$B_3=\Lambda_s\exp[-4\pi^2/f_3{}^2(\Lambda_s)]\times\exp\left[\frac{g_3{}^2(\Lambda_s)}{2f_3{}^2(\Lambda_s)}\ln\Lambda_s/M_1+\frac{9g_2{}^2(\Lambda_s)}{16f_3{}^2(\Lambda_s)}\ln\Lambda_s/M_1\right], \tag{45}$$

and

$$B_0=\Lambda_s\exp[-4\pi^2/f_3{}^2(\Lambda_s)]\times\exp\left[-\frac{g_3{}^2(\Lambda_s)}{4f_3{}^2(\Lambda_s)}\ln\frac{\Lambda_s}{M_1}+\frac{9g_2{}^2(\Lambda_s)}{16f_3{}^2(\Lambda_s)}\ln\frac{\Lambda_s}{M_1}\right]. \tag{46}$$

All the other SD equations can be approximated in a similar manner. We summarize the result for the solution of the case of charge $\frac{2}{3}$ quarks in Table I. In Table I the group representation is with respect to the generation group. We also have

$$A=\overline{\Lambda}_s\exp\{[g_3{}^2(\Lambda_s)/f_3{}^2(\Lambda_s)]\ln(\Lambda_s/M_1)\}\,, \tag{47}$$

$$B=\overline{\Lambda}_s\exp\{[\tfrac{1}{2}g_3{}^2(\Lambda_s)+\tfrac{9}{16}g_2{}^2(\Lambda_s)]\times\ln(\Lambda_s/M_1)/f_3{}^2(\Lambda_s)\}\,, \tag{48}$$

$$C=\overline{\Lambda}_s\exp\{[\tfrac{-1}{4}g_3{}^2(\Lambda_s)+\tfrac{9}{16}g_2{}^2(\Lambda_s)+\tfrac{3}{16}g_1{}^2(\Lambda_s)]\times\ln(\Lambda_s/M_1)/f_3{}^2(\Lambda_s)\}\,, \tag{49}$$

$$D=\overline{\Lambda}_s\exp\{-[3g_1{}^2(\Lambda_s)/f_3{}^2(\Lambda_s)]\ln(\Lambda_s/M_1)\}\,, \tag{50}$$

and

$$E=\overline{\Lambda}_s\exp\left[\frac{-9}{20}\frac{g_1{}^2(\Lambda_s)}{f_3{}^2(\Lambda_s)}\ln\Lambda_s/M_1\right], \tag{51}$$

where

$$\overline{\Lambda}_s=\Lambda_s[\exp-4\pi^2/f_3{}^2(\Lambda_s)]\,. \tag{52}$$

We have, therefore, six generations of charge $\frac{2}{3}$ quarks with the mass $\simeq A$. Third (top quark) and the fourth generation of quarks lie very close to each other.

$$m_t=B\,,$$

and

TABLE I. Solution in case of charge $\frac{2}{3}$ quarks.

	$(3^*,1)$			$(3,2)$						$(1,1)$
				A						
$(3^*,1)$					A					
						A				
	A			B						
		A			B					
$(3,2)$			A			C				
							B			D
								B		D
									C	D
$(1,1)$							D	D	D	E

TABLE II. Solution in case of quarks of charge $-\frac{1}{3}$.

	(3,1)(-2)			(3*,1)(2)			(1,2)(3)		(1,2)(−3)	
	A									
(3*,1)(−4)		A								
			A							
				$\bar{A}$			F^-			
					$\bar{A}$			F^+		
(3,2)(1)						$\bar{A}$			C	
								F^-		
										F^+
									$-C$	
(1,1)(6)							G	G		

$$m_{t'}=B+2D^2/B \ . \tag{53}$$

Charm and up quark masses are $m_c=C$ and $m_u=D^2/C$, respectively.

For the quark of charge $-\frac{1}{3}$ we choose a solution shown in Table II. In Table II we have

$$\bar{A}=\bar{\Lambda}_s \exp\left[\frac{1}{f_3^{\,2}(\Lambda_s)}[g_3^{\,2}(\Lambda_s)-g_1^{\,2}(\Lambda_s)/10] \times \ln(\Lambda_s/M_1)\right], \tag{54}$$

$$F^{\pm}=\bar{\Lambda}_s \exp\left[\frac{1}{f_3^{\,2}(\Lambda_s)}[\tfrac{9}{16}g_2^{\,2}(\Lambda_s)\pm\tfrac{3}{80}g_1^{\,2}(\Lambda_s)] \times \ln(\Lambda_s/M_1)\right], \tag{55}$$

and

$$G=\bar{\Lambda}_s \exp\left[\frac{-9}{40}\frac{g_1^{\,2}}{f_3^{\,2}}\ln(\Lambda_s/M_1)\right], \tag{56}$$

while for the leptons we have Table III. In Table III we have

$$\bar{\Lambda}_l=\Lambda_s \exp\frac{-4\pi^2}{f_0^{\,2}(\Lambda_s)}\ , \tag{57}$$

and

$$C^-=\bar{\Lambda}_s \exp\left[\frac{-1}{4f_0^{\,2}(\Lambda_s)}[\tfrac{3}{4}g_2^{\,2}(\Lambda_s)+\tfrac{3}{10}g_1^{\,2}(\Lambda_s)]\ln\frac{\Lambda_s}{M_1}\right]. \tag{58}$$

The important difference between the charge $-\frac{1}{3}$ quark and the lepton is the interchange of (1,2)(3) and (1,2)(−3). This choice is made purely for the phenomenological reason. Leptonlike choice for the quark would give m_s which is less than m_d or equivalently an unacceptably large value for the Cabibbo angle. On the other hand, e/μ ratio would become too large if we use quarklike solution for the leptons. Stability of the solution will not be discussed here as before.

For the low-lying four generations of charge $\frac{2}{3}$ quarks we get the following expressions:

$$m_{t'}\simeq m_t=\bar{\Lambda}_s Z^{1/2}Y^{9/16}X^{3/16}\ , \tag{59}$$

$$m_c=\bar{\Lambda}_s Z^{-1/4}Y^{9/16}X^{3/16}\ , \tag{60}$$

and

$$m_u=\bar{\Lambda}_s Z^{1/4}Y^{-9/16}X^{-27/80}\ . \tag{61}$$

Here

$$X=\exp\left[\frac{g_1^{\,2}(\Lambda_s)}{f_3^{\,2}(\Lambda_s)}\ln\frac{\Lambda_s}{M_1}\right], \tag{62}$$

TABLE III. Solution for leptons.

	(3,1)(−2)			(3*,1(2)			(1,2)(3)		(1,2)(−3)		
	A										
(3*,1)(−4)		A									
			A								
				$\bar{A}$					F^+		
(3,2)(1)					$\bar{A}$		F^-				
						$\bar{A}$		C^-			$\times(\bar{\Lambda}_l/\bar{\Lambda}_s)$
										F^+	
(1,1)(6)								F^-			
							$-C^-$				
									G^{-1}	G^{-1}	

$$Y=\exp\left[\frac{g_2{}^2(\Lambda_s)}{f_3{}^2(\Lambda_s)}\ln\frac{\Lambda_s}{M_1}\right], \tag{63}$$

and

$$Z=\exp\left[\frac{g_3{}^2(\Lambda_s)}{f_3{}^2}\ln\frac{\Lambda_s}{M_1}\right]. \tag{64}$$

Similarly for the charge $-\frac{1}{3}$ quarks we have

$$m_{b'}=\overline{\Lambda}_s X^{3/80} Y^{9/16}, \tag{65}$$

$$m_b=\overline{\Lambda}_s X^{-3/80} Y^{9/16}, \tag{66}$$

$$m_s=\overline{\Lambda}_s Y^{-3/16} X^{3/40} \tag{67}$$

and

$$m_d=\overline{\Lambda}_s X^{-9/40}. \tag{68}$$

We also have the following expression for the Cabibbo angle:

$$\tan\theta_C=(m_u/m_c)^{1/2}. \tag{69}$$

We also have

$$\tan\theta'=m_d/m_b \tag{70}$$

for the mixing between d and b. b' has the zero mixing in this approximation. If we choose

$$\overline{\Lambda}_s=0.5\text{ GeV},\ X=Y=100,\text{ and } Z=10^4, \tag{71}$$

we get

$$m_t\simeq 1580\text{ GeV},\quad m_c=1.58\text{ GeV},\qquad m_u=0.08\text{ GeV}. \tag{72}$$

We also have

$$m_{b'}=7\text{ GeV},\quad m_b=5\text{ GeV},\quad m_s=0.48\text{ GeV} \tag{73}$$

and

$$m_d=0.12\text{ GeV}.$$

From these values for the masses we get

$$\tan\theta,=(\tfrac{8}{158})^{1/2}, \tag{74}$$

and also

$$\tan\theta'=0.06. \tag{75}$$

Let us now turn to discussions of lepton masses:

$$m_e=\overline{\Lambda}_l\exp\left[\frac{-1}{4f_0{}^2}(\tfrac{3}{4}g_2{}^2+\tfrac{3}{10}g_1{}^2)\ln\frac{\Lambda_s}{M_1}\right], \tag{76}$$

$$m_\mu=\overline{\Lambda}_l\exp\left[\frac{1}{4f_0{}^2}\frac{9}{10}g_1{}^2\ln\frac{\Lambda_s}{M_1}\right], \tag{77}$$

$$m_\tau=\overline{\Lambda}_l\exp\left[\frac{9}{16f_0{}^2}(g_2{}^2-\tfrac{1}{15}g_1{}^2)\ln\frac{\Lambda_s}{M_1}\right], \tag{78}$$

and

$$m_{\tau'}=\overline{\Lambda}_l\exp\left[\frac{9}{16f_0{}^2}(g_2{}^2+\tfrac{1}{15}g_1{}^2)\ln\frac{\Lambda_s}{M_1}\right], \tag{79}$$

where

$$f_0{}^2=\tfrac{3}{10}f_1{}^2=\tfrac{3}{10}\alpha f_3{}^2 \tag{80}$$

where α being a number which should not be much different from 1. We get the following values for the choice $=1.5$:

$$m_e/m_\mu=\tfrac{1}{146},\quad m_\tau/m_\mu=21.5,\text{ and } m_{\tau'}/m_\tau=2.15. \tag{81}$$

We have, therefore, $m_{\tau'}\cong 4$ GeV.

Let us now compare these results with experiment. The values of m_u, m_d, and m_s seem to suggest that we are talking about something between current-algebra mass and the constituent quark mass. Although the calculation we performed above should be taken only as an order-of-magnitude estimation, the agreement in general is excellent. There are some problems, however. First is the value of very small $m_{b'}$ and $m_{\tau'}$. The heavy lepton of mass $\simeq 4$ GeV seems to be excluded by DESY PETRA as well as SLAC PEP experiments.[12] It seems very hard in our model to push τ' much higher without drastically altering the value of other masses because $m_{b'}$ and $m_{\tau'}$ are the least sensitive to the choice of parameters as one can see from Eqs. (65) and (79). The experimental situation[13] for the fourth generation b' is less clear although there is no positive evidence for its existence around 7 GeV.[14] Another problem is in the Kobayashi-Maskawa (KM) matrix.[15] Our KM matrix reads

$$V_{\mathrm{KM}}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta' & 0 & \sin\theta'\\ 0 & 1 & 0\\ -\sin\theta' & 0 & \cos\theta'\end{bmatrix}=\begin{bmatrix}\cos\theta\cos\theta' & \sin\theta & \cos\theta\sin\theta'\\ -\sin\theta\cos\theta' & \cos\theta & -\sin\theta\sin\theta'\\ -\sin\theta' & 0 & \cos\theta'\end{bmatrix}, \tag{82}$$

where θ and θ' are given in Eqs. (74) and (75), respectively. Our values for V_{ub} and V_{cb} are

$$V_{ub}=\cos\theta\sin\theta'=0.06, \tag{83}$$

and

$$V_{cb}=-\sin\theta\sin\theta'=-3\times10^{-3}. \tag{84}$$

Experiment[16] shows that the role of c and u must be interchanged in our V_{ub} and V_{cb}. Whether this is due to the wrong choice of the solution or due to the inadequacy of our model remains to be seen.

VI. GAUGE-BOSON MASS

We can calculate the gauge-boson mass using the so-called Schwinger mechanism.[17] In our ladder approximation (Fig. 5) the gauge-boson self-energy part can be written as

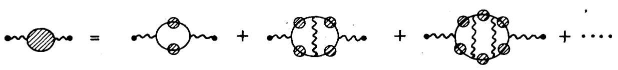

FIG. 5. Ladder approximation to the boson self-energy part.

$$(k^2g^{\mu\nu}-k^\mu k^\nu)\pi_{ij}(k^2)$$
$$=\frac{-i}{(2\pi)^4}\int \mathrm{Tr} g_i g_j T_i^\mu \Delta(p) T_j^\nu \Delta(p-k) d^4p \ . \quad (85)$$

The gauge-boson masses are calculated to be

$$\mu^2{}_{ij}=-\lim_{k^2\to 0} k^2\pi_{ij}(k^2) \ . \quad (86)$$

Here

$$\Gamma_i^\mu=\gamma^\mu \begin{bmatrix} p_L & \\ & p_R \end{bmatrix} \begin{bmatrix} F_i^{(-)} & \\ & F_i^{(+)} \end{bmatrix}$$

in the lowest order with p_L and p_R being projection operators to the left-handed and to the right-handed fermions, respectively. Higher-order effects to Γ_i^μ are taken into account by utilizing the following Ward-Takahashi identity:

$$\Gamma^\mu=(\not p-\Sigma)F-F(\not p-\not k-\Sigma) \ . \quad (87)$$

We put

$$\Gamma^\mu=\begin{bmatrix} a_-\gamma^\mu & \bar b k^\mu+\bar c p^\mu \\ bk^\mu+cp^\mu & a_+\gamma^\mu \end{bmatrix} . \quad (88)$$

Then Eq. (87) gives

$$bk^2+ck\cdot p=F^{(+)}B\,|_{(p-k)^2}-BF^{(-)}\,|_{p^2} \ , \quad (89)$$

and

$$\bar b k^2+\bar c\, k\cdot p=F^{(-)}\bar B\,|_{(p-k)^2}-BF^{(+)}\,|_{p^2} \ , \quad (90)$$

where $F^{(\pm)}$ and B are defined in Sec. II. From these equations we calculate the residue of the pole at k^2 in $\pi_{ij}(k^2)$. After some approximation we obtain

$$\begin{aligned}\mu^2{}_{ij}=\frac{2}{(2\pi)^4}\int d^4p\, g_i g_j \mathrm{Tr}[&F_i^{(-)}(p^2+\bar BB)^{-1}(\bar BBF_j^{(-)}-\bar BF_j^{(+)}B)(p^2+\bar BB)^{-1}\\ &+F_i^{(+)}(p^2+B\bar B)^{-1}(B\bar BF_j^{(+)}-BF_j^{(-)}\bar B)(p^2+B\bar B)^{-1}\\ &+\tfrac{1}{2}F_i^{(-)}(p_+{}^2\bar BB)^{-1}(F_j^{(-)}\bar BB-\bar BBF_j^{(-)})(p^2+\bar BB)^{-1}\\ &+\tfrac{1}{2}F_i^{(+)}(p^2+B\bar B)^{-1}(F_j^{(+)}B\bar B-B\bar BF_j^{(+)})(p^2+B\bar B)^{-1}] \ , \end{aligned} \quad (91)$$

where the Wick rotation has already been performed. The approximation adopted does not necessarily respect the QED gauge invariance. We, therefore, can use the above formula only for the purpose of order-of-magnitude estimation at best.

The main contribution to the Z and W mass comes from the higher six generations of charge $-\frac{1}{3}$ and $\frac{2}{3}$ quarks all of which have the mass

$$m_h=\bar\Lambda_s\times A\simeq 5 \text{ TeV} \ . \quad (92)$$

Z or W mass is given by

$$\mu^2\simeq\frac{9}{(2\pi)^4}\int d^4p\, f_2{}^2\frac{2m_h{}^2}{p^2(p^2+m_h{}^2)} \ ,$$
$$\simeq f_3{}^2m_h{}^2 \ . \quad (93)$$

We can calculate the Weinberg angle in this approximation but the appearance of large photon mass makes it rather unlikely to be trustworthy.

VII. OTHER MODELS

Other than the model described in previous sections we have tried several other models all of which seem to have certain kind of unsatisfactory nature.

A. $G_u\times G_g$ models

G_u stands for the grand-unification group and G_g stands for the generation group. We have found no anomaly-free and asymptotically free model of this kind which explains the hierarchial pattern of symmetry breaking.

B. SO(10)×SO(10)×SO(10) (Ref. 18)

We consider the following representations: (16,10,1), (10,1,16), and (1,16,10). The first group is the unification group, the next one corresponds to the generation group and the last one is introduced to break the two previous symmetries at the level of grand-unification scale. This theory is asymptotically free and is, of course, anomaly free. Major trouble stems from the fact that already mentioned in Sec. II: SO(10) as a unification group forces us to explain the mass splitting inside a generation only in terms of color-electroweak interaction which is not at all an adequate thing to do.

C. SU(5)×SU(5)×SU(5)

The trouble here is that the single SU(5) interaction should be responsible for breaking the unification and also the generation SU(5) at the level of grand-unification scale. We find no reasonable assignment which breaks the generation group completely without leaving extra symmetry. Thus, we are more or less forced to the model considered in previous sections.

VIII. DISCUSSIONS AND CONCLUSIONS

Recent data from UA1 (Ref. 19) provides a restriction on the number of neutrinos. The measurement is done for

$$\frac{\sigma_Z B_{Z\to e^+e^-}}{\sigma_W B_{W\to e\nu}}=\frac{\sigma_Z}{\sigma_W}\frac{\Gamma_{Z\to e^+e^-}}{\Gamma_{W\to e\nu}}\frac{\Gamma_W^{\rm tot}}{\Gamma_Z^{\rm tot}}\ . \tag{94}$$

If we use theoretical values for the quantities on the right-hand side of Eq. (94) except for $\Gamma_Z^{\rm tot}$, we can estimate $\Gamma_Z^{\rm tot}$ which in turn is related to the number of neutrinos. $N_\nu<6$ at 90% confidence level is claimed. Since we have an extra channel $W\to\tau'\nu$ in our model, the $\Gamma_W^{\rm tot}$ is increased by approximately 10%. This has the tendency to increase the number of neutrinos. On the other hand, $Z\to b'\bar{b}'$ is open in our model which tends to decrease the number of allowed neutrinos. The next effect seems to give an even stricter restriction. If we increase the level of confidence to 95%, however, even 20 neutrinos are acceptable. We think our ten-generation model is far from excluded at this point. The next point I want to make is the existence of broken global symmetries in our model. Each representation gives rise to the chiral U(1)×U(1) symmetry one of which is the exact symmetry corresponding to the conservation of the fermion number in a given representation and the other U(1) is broken spontaneously. One gets several axionlike objects[20] which become massive only due to the instanton effect. We have not attempted in this paper to calculate these masses. The calculation of the composite-Higgs-boson masses is also left for the future publication.

In conclusion, I claim that in spite of some difficulties and ambiguities mentioned above the present model seems to be worth studying in more detail. I also believe that the general approach described in this paper is worth paying attention to although the validity of the concrete model chosen here remains to be seen.

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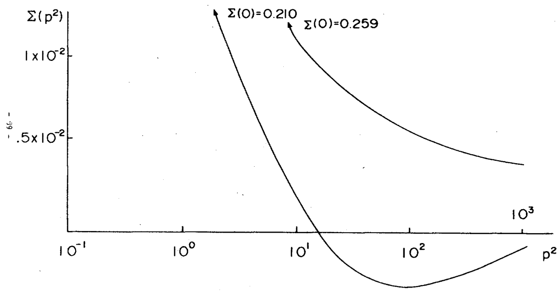

FIG. 6. Numerical solution to Eq. (A6).

APPENDIX

In this appendix we analyze numerically the equation

$$\Sigma(x)=\int_0^\infty dy\,\Sigma(y)[y+\Sigma(y)^2]^{-1}G(y)\frac{x+y+A}{2x}\times\{1-[1-4xy/(x+y+A)^2]^{1/2}\}\ , \tag{A1}$$

which we obtain from Eq. (11) by performing the angular integration. A stands for the vector-boson mass properly normalized. We assume that the coupling $G(y)$ has the following form:

$$G(y)=\begin{cases}G, & y<M\ ,\\ G\ \ln(M/M_c)/\ln(Y/M_C), & y\geq M\ ,\end{cases} \tag{A2}$$

since we have

$$\frac{x+y+A}{2x}\{1-[1-4xy/(x+y+A)^2]^{1/2}\}=\frac{2y}{x+y+A+[(x+y+A)^2-4xy]^{1/2}}\ , \tag{A3}$$

this expression reduces to

$$\frac{2y}{x+y+|x-y|} \tag{A4}$$

in the massless limit. Using the identity

$$\frac{d^2}{dx^2}x\left[\frac{2y}{x+y+|x-y|}\right]=-\delta(x-y)\ , \tag{A5}$$

we can reduce (A1) to differential equation in the massless limit. We solve Eq. (A1) directly here and show the existence of two types of solutions with different asymptotic behavior. For this purpose we use the subtracted form of Eq. (A1)

$$\Sigma(x)=\Sigma(0)+\int_0^\infty dy\,\Sigma(y)[y+\Sigma(y)^2]G(y)\left[\frac{2y}{x+y+A+[(x+y+A)^2]^{1/2}-4xy}-\frac{y}{y+A}\right]\ . \tag{A6}$$

No difficult problems arise when we try to obtain an iterative solution to this equation since the integrand is well behaved at both ends of the integral. For the purpose of illustration we set our parameters in the following way:

$$A=0.00 \tag{A7}$$

$$M=0.36\ , \tag{A8}$$

$$M_C=0.16\ , \tag{A9}$$

and

$$G = 0.52 . \tag{A10}$$

The result of numerical computation is shown in Fig. 6 confirming the claim we have made in Sec. II of the text.

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