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Strange matter

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We explore the properties of quark matter in equilibrium with the weak interactions, containing comparable numbers of up, down, and strange quarks. Witten has recently conjectured that this "strange matter" may be absolutely stable. Using a Fermi-gas model including $O(\alpha_c)$ corrections we establish the conditions under which strange matter in bulk is stable and describe its characteristics. Augmenting our model with surface-tension and Coulomb effects we study strange matter with intermediate baryon number, $10^2 \lesssim A \lesssim 10^7$. For low baryon numbers $A \lesssim 10^2$, we replace the Fermi gas by the bag model and study shell effects and the approach to the bulk limit. Finally, we discuss the phenomenology of strange matter in all its forms.

I. INTRODUCTION

Witten has recently pointed out that quark matter with strangeness per baryon of order unity, "strange matter," may be stable and , if so, might have important cosmological consequences.[1] The proposal that quark matter rather than nuclear matter might be the ground state of QCD at finite baryon number is sufficiently radical to merit study in its own right, independent of any cosmological or astrophysical implications. In this paper we investigate three-flavor (u,d,s) quark matter in equilibrium with respect to the weak interactions at zero temperature and external pressure. Making reasonable assumptions we find that for reasonable values of certain QCD-related parameters (B, the energy difference between the "perturbative vacuum" and the true vacuum, essentially the "bag constant;" α_c, the QCD coupling; and m, the strange-quark mass), strange matter is absolutely stable. For parameter values where strange matter is stable we investigate its properties in bulk, for large baryon number $(10^2 \lesssim A \lesssim 10^7)$ and for low baryon number $(A \lesssim 10^2)$. Some of the properties we look at are its binding energy, electric charge, strangeness, stability against fission, and interactions with electrons. Finally, we briefly consider how strange matter in various forms would interact with ordinary matter.

By "quark matter" we mean a Fermi gas of $3A$ quarks which together constitute a single color-singlet baryon with baryon number A. We assume that the dynamics of confinement are well approximated by separating the quarks from the vacuum by a phase boundary and endowing the region in which the quarks live with a constant universal energy density B. B behaves dynamically like a pressure and maintains the quark gas at finite density and chemical potential. It has been shown[2] that a chemical potential μ provides a cutoff on the infrared divergences of QCD. Perturbative calculations of the properties of quark matter at large μ^2 are therefore reliable in precisely the same sense that QCD calculations of deep-inelastic phenomena are reliable at large Q^2. We will distinguish between two forms of quark matter: "strange matter" in which flavor equilibrium has been established by the weak interactions; and "nonstrange quark matter" consisting only of u and d quarks.

Witten's conjecture that strange matter may be stable is based on elementary symmetry considerations. The success of the nuclear shell model provides ample evidence that quarks do not roam freely inside of nuclei but are instead primarily confined to protons and neutrons. Thus, we know that the energy per baryon of nonstrange quark matter is higher than the energy per baryon of nuclei. (We ignore the possibility that nonstrange matter might be stable but not observed because the transition from nuclei to this state is highly suppressed due to grossly dissimilar wave functions.) Suppose we now introduce the strange quark which, for the moment, we take to be massless. Because of the exclusion principle it is energetically favorable to convert u and d quarks into s quarks via the weak interactions. This will continue until the Fermi energies of all flavors are the same and the energy per baryon has dropped by a factor of $(\frac{2}{3})^{1/4} \simeq 0.904$. For massive strange quarks this factor is closer to one, and it reaches one when the strange-quark mass is as large as the Fermi energy of the original nonstrange Fermi gas. At nuclear-matter densities this is about 300 MeV. Note that other quark flavors $(c,b,t$, etc.) do not appear because their masses are large compared to 300 MeV. Now suppose that the energy per baryon of nonstrange quark matter is just a few tens of MeV above the mass of the nucleon, 939 MeV. Then it is possible that strange matter, even with a nonzero strange-quark mass, has an energy per nucleon less than 939 MeV. The fact that symmetry-energy considerations favor the appearance of s quarks in quark matter at and above nuclear-matter densities was certainly well known (cf. Ref. 3). However, Witten appears to have been the first to entertain the possibility that strange matter is in fact bound.

In Sec. II, we investigate the properties of stable strange matter in bulk. Our study rests on several plausible assumptions. The first, as we have already mentioned, is that the system is well approximated by a Fermi gas separated from the vacuum by a phase boundary. We further assume that the effects of dynamical chiral-symmetry breakdown (e.g., dynamical quark masses, Goldstone pions) can be ignored in the quark gas so quarks are characterized by their current-algebra masses. Finally, we assume that the properties of the quark Fermi gas can be computed using renormalization-group-improved QCD perturbation theory. Unfortunately, at the momentum scale typical of the problem at hand (roughly $M_N/3$) α_c is not small. Other methods (e.g., lattice Monte Carlo simulations of QCD) may eventually yield information about quark matter; at present, perturbative QCD is the only tool available. Our study of strange matter in bulk is parametrized by B, and the strange-quark mass $m(\rho)$ and the QCD coupling $\alpha_c(\rho)$ renormalized at a mass scale ρ. Although we cannot resolve the question of stability decisively, we find that there are substantial "windows" in B, m, and α_c in which strange matter is stable and nonstrange quark matter is not. Within these windows we explore the characteristic binding energy, charge, strangeness, and density of strange matter. We also discuss the relation between the parameter values required for bulk strange-matter stability and the values inferred from ordinary hadronic physics.

Strange matter, stable in bulk, may also come in small chunks for which surface effects are important and which are too small to support an electron gas within. We call these chunks of strange matter "strangelets." These systems with baryon numbers lower than (very roughly) 10^7 are studied in Sec. III. However, we leave $A \lesssim 10^2$ to Sec. IV. We discuss the origin and effect of a surface tension and find that strange matter stable in bulk may be unbound for small enough baryon number. We also discuss the charge that develops on strangelets and finally how these charged systems neutralize themselves, like ordinary nuclei, by accumulating electron (or perhaps positron) clouds.

One of the striking features of strange matter is that for generic values of the parameters we have considered, it does not develop an instability toward fission as A increases. The physical reason for this is simple: Coulomb energy drives fission, but strange matter in the $m=0$ limit has equal numbers of u, d, and s quarks and is electrically neutral. Even when the strange-quark mass is not zero, the Coulomb energy is not sufficient to overcome the stabilizing effects of a small surface tension. This has important consequences for the phenomenology of strange matter which are discussed in Sec. V.

In Sec. IV we look in detail at low A, less than roughly 300. For small A the Fermi gas approximation breaks down and we calculate by explicitly filling orbitals in a hadronic bag model. For $A \leq 6$ order-α_c corrections are included but for $A > 6$ they are not. For $A > 6$ we study the approach to the bulk-matter limit and discover that finite-size effects in the bag model do, on the average, increase the energy per baryon with decreasing A. So strange matter may be unbound for small A even when parameters are chosen so that it is bound in bulk by many tens of MeV. We also use these calculations to estimate surface effects used in the previous discussions. Including order α_c corrections for $A \leq 6$, we conclude that it is unlikely that any strange hadrons with $A \leq 6$ are stable. A possible exception is the "dihyperon," which has been found to be nearly stable in quark models[4] and more recently in a Skyrme model.[5] In Sec. V we begin a discussion of the phenomenology of strange matter. We argue that because of its stability against fission strange matter could be found in lumps of any size. We discuss how these lumps would interact with nucleons and atoms emphasizing the dramatic differences between strange matter with positive and negative hadronic electric charge. Finally, we investigate strangelets with very low baryon number and discuss how these unstable systems would decay by alpha particle and nucleon emission.

If strange matter is stable, we can look forward to a wealth of new phenomena comparable to the richness of nuclear physics. The most pressing questions regarding stable strange matter are undoubtedly: Can it be found? and Can it be made? In this paper we attempt to provide enough information about the properties of strange matter to make it possible to address these questions. We have only begun a systematic study of the properties of strange matter. Our results are based on physically simple models which are fairly insensitive to detailed dynamical assumptions. We are aware, however, that at almost every turn, our calculations could be refined and improved upon. Ultimately, we must admit that the only definitive demonstration of the existence and properties of strange matter will come from experiment.

II. BULK PROPERTIES

By "bulk" quark matter we mean quark matter in aggregates large enough that surface effects can be ignored and that electrons (or positrons) bound to it by Coulomb forces are, in fact, inside the chunk and numerous enough to be treated as a degenerate Fermi gas. Our first object is to determine the conditions under which strange matter in bulk, at zero temperature and pressure, is the true ground state of the strong interactions. This requires the energy per baryon E/A of strange matter to be less than that of the nucleon, $M_N=939$ MeV. Actually for E/A between 930 and 939 MeV strange matter could decay by emission of nuclei accompanied by weak interactions to maintain flavor equilibrium. If E/A is less than 930 MeV an ordinary nucleus could, in principle, lower its energy by converting roughly one third of its quarks into strange quarks. However, this would require a very high-order weak interaction, while the rate for just fourth order is, for all practical purposes, zero. It is for this reason that nuclei may have been mistakenly taken to be the ground state of hadronic matter.

We also know that ordinary nuclei are made of nucleons and not of a two-flavor (up and down) quark phase. Therefore, the energy per baryon of nonstrange quark matter must exceed the lowest energy per baryon found in nuclei, which is about 930 MeV for iron. However, applying this requirement to nonstrange quark matter *in bulk* is not restrictive enough. At large but finite baryon number A, we believe the energy per baryon of nonstrange quark matter to be lower than it is when A goes to infinity (the bulk case). Strange matter does the opposite: its energy per baryon decreases with A. The origin of this behavior is explained in Sec. IV. We define Δ to be the difference between the energy per baryon of nonstrange quark matter in bulk and for $A \approx 250$. Therefore, we require the energy per baryon of nonstrange quark matter in bulk to exceed $930+\Delta$ MeV. In Sec. IV we estimate Δ to be 4 MeV.

Our model of strange matter is a degenerate Fermi gas of u, d, and s quarks and electrons (or positrons) with "chemical" equilibrium maintained by the weak-interaction processes $d \leftrightarrow u+e+\bar{\nu}_e$, $s \leftrightarrow u+e+\bar{\nu}_e$, and $s+u \leftrightarrow u+d$. At equilibrium the chemical potentials obey

$$\begin{aligned} \mu_d &= \mu_s \equiv \mu , \\ \mu_u + \mu_e &\equiv \mu , \end{aligned} \tag{2.1}$$

so only two are independent. The neutrinos play no role and are ignored. Massive neutrinos could be bound to strange matter since it has a net weak charge of order, its strangeness. This provides a source for the neutral weak boson Z^0 which, in turn, generates a potential with a depth of approximately $G_F n_S/2\sqrt{2}$ to which massive neutrinos bind. Typically this is 5 eV, so neutrinos with masses which are not negligible compared to 5 eV will be found within bulk strange matter.[6] The neutrino gas is so dilute that it has no effect on the dynamics.

For pedagogical purposes we first ignore one gluon exchange inside the Fermi gas and set $\alpha_c=0$. In this case the thermodynamic potential is a sum of contributions from each species:

$$\begin{aligned} \Omega_u &= -\frac{\mu_u{}^4}{4\pi^2} , \\ \Omega_d &= -\frac{\mu_d{}^4}{4\pi^2} , \\ \Omega_e &= -\frac{\mu_e{}^4}{12\pi^2} , \\ \Omega_s &= -\frac{1}{4\pi^2}\left[\mu_s(\mu_s{}^2-m^2)^{1/2}(\mu_s{}^2-\tfrac{5}{2}m^2) \right. \\ &\qquad \left. + \tfrac{3}{2}m^4 \ln\frac{\mu_s+(\mu_s{}^2-m^2)^{1/2}}{m} \right] . \end{aligned} \tag{2.2}$$

We assume that only the strange quark has a mass m. Corrections for small u- and d-quark masses are small. The number density of each species n_a, is given by $n_a = -\partial\Omega_a/\partial\mu_a$ with $a=u,d,s,e$. Any bulk system must have a zero electric charge so we require

$$\tfrac{2}{3}n_u - \tfrac{1}{3}n_d - \tfrac{1}{3}n_s - n_e = 0 . \tag{2.3}$$

Together, Eqs. (2.1) and (2.3) leave only one independent chemical potential, say μ.

The energy density carried by the fermions is $\sum_a(\Omega_a + n_a\mu_a)$. In addition, the vacuum associated with this phase is assumed to carry a positive energy per unit volume B, so the total energy density is

$$\epsilon = \sum_a (\Omega_a + \mu_a n_a) + B . \tag{2.4}$$

The baryon-number density is

$$n_A = \tfrac{1}{3}(n_u + n_d + n_s) . \tag{2.5}$$

The equilibrium configuration of the system which balances Fermi pressure against vacuum pressure is determined by

$$\frac{\partial}{\partial\mu}(\epsilon/n_A) = 0 , \tag{2.6}$$

or equivalently,

$$\sum_a \Omega_a = -B . \tag{2.7}$$

Given values of B and m we can determine the μ that satisfies Eq. (2.6). This in turn gives us the equilibrium number densities and energy density, or equivalently, the energy per baryon. Our results are given in Fig. 1(a) for $\alpha_c=0$. The contours give m versus $B^{1/4}$ for fixed values of E/A. To the right of the contour with $E/A=939$ MeV strange matter is unstable against emission of nucleons. The vertical line gives the value of $B^{1/4}$ for which nonstrange quark matter has an energy per baryon of $930+\Delta$ MeV. The value of Δ in this case is determined in Sec. IV. To the left of this line, nuclei with high atomic numbers would be unstable against decay into nonstrange quark matter. The allowed range of values of $B^{1/4}$ narrows as m grows.

Note in Fig. 1 that our "allowed" values of m and $B^{1/4}$ include regions where m is arbitrarily large. For very large values of m strange matter has no strange quarks so there is, in fact, no distinction between strange and nonstrange quark matter. However, for these values of m (and $B^{1/4}$), E/A is larger than 930 MeV so this quark matter is, in fact, unstable against α-particle emission, although it will not emit nucleons.

In Fig. 2(a) we have given examples of how the strangeness per baryon varies with the strange-quark mass. (We define S to be the number of strange quarks, not its negative as is conventional.) The line marked 0 corresponds to $\alpha_c=0$, at a fixed E/A of 899 MeV. The strangeness per baryon decreases with m and goes to zero when the strange-quark mass is roughly one third of 899 MeV. Figure 2(b) displays the density of the same system as a function of the strange-quark mass.

We have also calculated the hadronic electric charge per baryon which equals the number of electrons per baryon. For $m=0$, the equilibrium configuration has equal numbers of u, d, and s quarks and is electrically neutral. As m grows, strange quarks are depleted and the system develops a positive charge. When m is so large that there are no strange quarks in the system the hadronic electric charge per baryon is 0.0056. In Fig. 1(a) the nearly horizontal lines are lines of constant hadronic electric charge per baryon with the indicated values. Apparently the number of electrons is always very small and therefore their contribution to the energy is also small.

The vacuum pressure B which we have assumed holds quark matter together is, in reality, a simple model for the long-range, confining interactions in QCD. In addition, we must include QCD modifications of the quark Fermi gas at shorter distance scales. We avoid double counting, at least in principle, because the pressure B maintains the system at finite values of the chemical potentials μ_a.

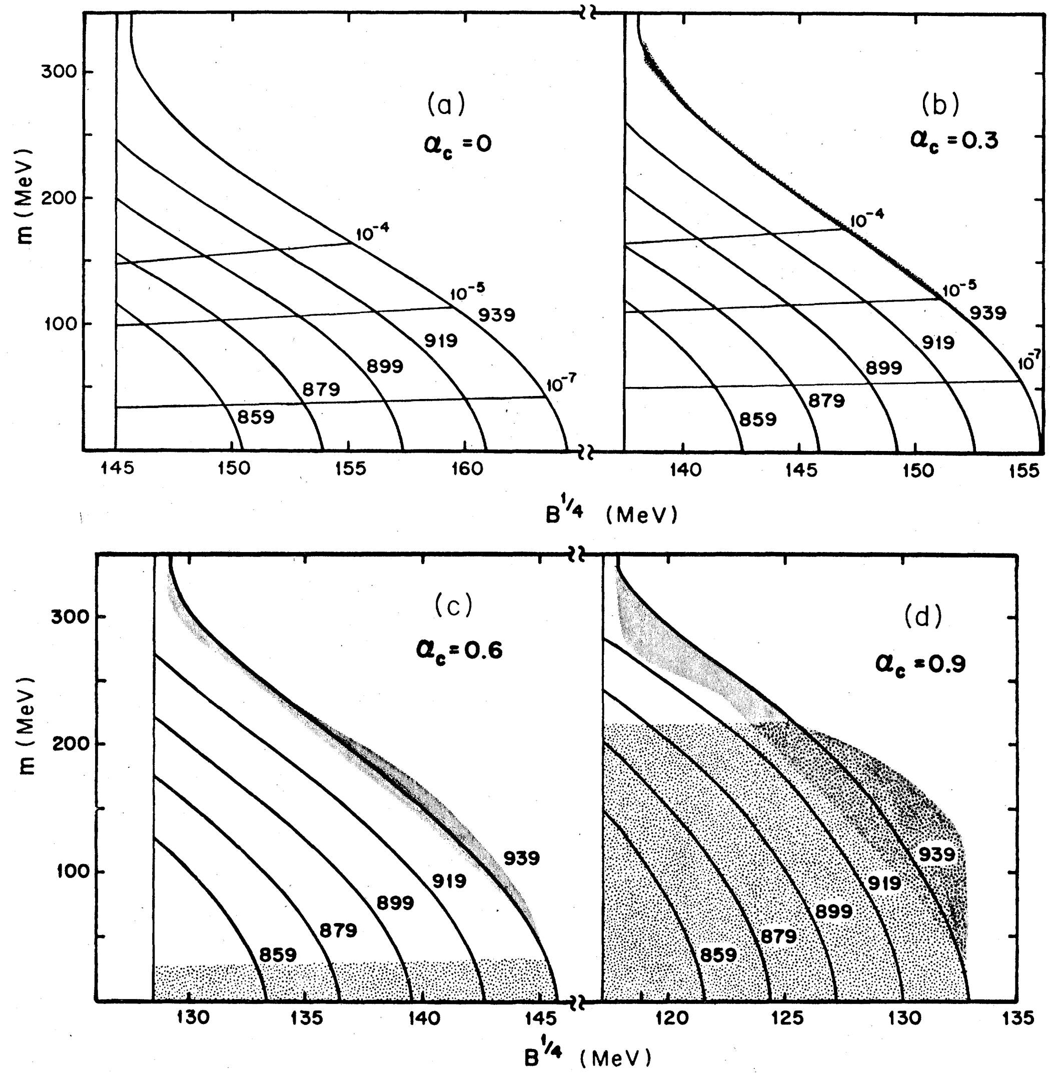

FIG. 1. Contours of fixed E/A in the $B^{1/4}$-m plane for $\alpha_c=0$, 0.3, 0.6, and 0.9. The vertical line at the left of each figure is the minimum $B^{1/4}$ for which nonstrange quark matter is unbound (see text). In (a) and (b) the nearly horizontal lines are contours of fixed hadronic electric charge per baryon as marked. In (c) and (d) the dotted regions are regions of negative hadronic electric charge. The grey shading around the 939 contour represents the same contour calculated using different renormalization schemes (see text).

These in turn provide an infrared cutoff for the QCD perturbation expansion inside quark matter, allowing one to develop a renormalization-group-improved perturbation expansion for quark matter. We use these methods to calculate the $O(\alpha_c)$ corrections to the properties of bulk strange matter.

In all such schemes it is necessary to choose a renormalization point ρ, at which $m=m(\rho)$ and $\alpha_c=\alpha_c(\rho)$ are defined. In principle, observables are independent of ρ. In practice, when α_c is not small and only first-order corrections are included, the choice of ρ matters. We believe ρ should be identified with a mass scale typical of the problem at hand (μ) in order to eliminate large logarithms ($\ln\rho/\mu$) in higher orders. The same arguments are used to motivate renormalizing at Q^2 in deep-inelastic processes. Earlier workers have chosen to renormalize the quark mass "on shell," i.e., $\rho=m$. Since the quark mass enters into Ω at zeroth order in α_c, a change in its renor-

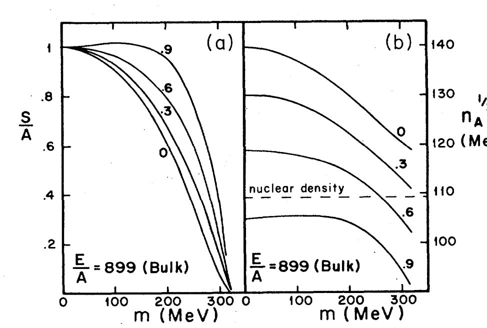

FIG. 2. (a) The bulk strangeness per baryon as a function of the strange-quark mass for $\alpha_c=0$, 0.3, 0.6, and 0.9, all at $E/A=899$ MeV. (b) The bulk baryon-number density as a function of the strange-quark mass for $\alpha_c=0$, 0.3, 0.6, and 0.9, all at $E/A=899$ MeV.

malization enters at $O(\alpha_c)$. [Changes in the definition of α_c enter only at $O(\alpha_c^2)$ and can be ignored here.] To order α_c, renormalizing on shell does not produce singularities as m goes to zero. However, in the calculation of certain physical observables like the sign of the electric charge density, $\alpha_c \ln m/\mu$ terms can compete with order-one terms when m is small and the calculation becomes unreliable because in higher orders one expects higher powers of the logarithm. (All of these terms are multiplied by m so there is no singularity in the $m \to 0$ limit.) We avoid this difficulty by renormalizing at a scale typical of the chemical potentials in the problem at hand: Specifically, we choose $\rho = M_N/3 = 313$ MeV.

The thermodynamic potentials to $O(\alpha_c)$ renormalized at ρ can be obtained from the results of Ref. 2:

$$\Omega_u = -\frac{\mu_u^4}{4\pi^2}\left[1-\frac{2\alpha_c}{\pi}\right], \quad \Omega_d = -\frac{\mu_d^4}{4\pi^2}\left[1-\frac{2\alpha_c}{\pi}\right],$$
$$\begin{aligned}\Omega_s = -\frac{1}{4\pi^2}\Bigg\{&\mu_s(\mu_s^2-m^2)^{1/2}(\mu_s^2-\tfrac{5}{2}m^2)+\tfrac{3}{2}m^4\ln\frac{\mu_s+(\mu_s^2-m^2)^{1/2}}{m}\\ &-\frac{2\alpha_c}{\pi}\left[3\left\{\mu_s(\mu_s^2-m^2)^{1/2}-m^2\ln\frac{\mu_s+(\mu_s^2-m^2)^{1/2}}{\mu_s}\right\}^2-2(\mu_s^2-m^2)^2+3m^4\ln^2\frac{m}{\mu_s}\right.\\ &\left.+6\ln\frac{\rho}{\mu_s}\left\{\mu_s m^2(\mu_s^2-m^2)^{1/2}-m^4\ln\frac{\mu_s+(\mu_s^2-m^2)^{1/2}}{m}\right\}\right]\Bigg\},\end{aligned} \tag{2.8}$$

where $\alpha_c = \alpha_c(\rho)$ and $m = m(\rho)$. Ω_s can be verified to be independent of ρ to order α_c,

$$\frac{d\Omega}{d\rho} = \frac{\partial\Omega}{\partial\rho} + \frac{\partial m}{\partial\rho}\frac{\partial\Omega}{\partial m} + \frac{\partial\alpha_c}{\partial\rho}\frac{\partial\Omega}{\partial\alpha_c} = 0, \tag{2.9}$$

provided $\alpha_c(\rho)$ and $m(\rho)$ obey the usual renormalization-group equations

$$\frac{\partial m}{\partial\rho} = -\frac{2\alpha_c}{\pi}\frac{m}{\rho}, \tag{2.10}$$

$$\frac{\partial\alpha_c}{\partial\rho} = O(\alpha_c^2). \tag{2.11}$$

To obtain the previous prescription set $\rho = m$.

In Figs. 1(b)–1(d) we show the results of our calculations for $\alpha_c = 0.3$, 0.6, and 0.9 all with $\rho = 313$ MeV. The significance of the contours is the same as for Fig. 1(a). Note that the allowed values of $B^{1/4}$ decrease as α_c increases. This is because one-gluon-exchange effects inside of a hadron are repulsive. The left-most vertical line in each case corresponds to nonstrange matter having an energy of $930+\Delta$ where we use the same Δ for α_c not zero as we did for α_c equal to zero.

We have also displayed a measure of the uncertainty which comes from changing the renormalization point and working only to order α_c. In principle, a change in renormalization point ρ can be compensated by an appropriate change in m and α_c as indicated by Eqs. (2.9)–(2.11). Changes in α_c are $O(\alpha_c^2)$, so at $O(\alpha_c)$ a change in ρ should be compensated by a rescaling of m according to Eq. (2.10). However, order-α_c^2 corrections make this compensation less than perfect. We have recalculated the energy contours at 939 MeV with ρ changed by $\pm 25\%$ and then rescaled m according to Eq. (2.10). The grey shaded regions around the 939 MeV contours shown in Figs. 1(b)–1(d) are bordered by the rescaled curves renormalized at $\rho = 313$ MeV $\pm 25\%$. This gives a rough guide to the uncertainties inherent in our calculations. Clearly, they grow with α_c.

In quark matter one-gluon exchange is repulsive if the quarks are massless and relativistic and attractive if the quarks are massive and nonrelativistic. One-gluon exchange therefore shifts the chemical equilibrium in the direction of more strange quarks. Of course, in the absence of interactions strange quarks are depleted simply because they have mass. For a small value of the strange-quark mass and a large value of α_c it can happen that strange quarks are more abundant in quark matter than the massless up or down quarks. When this occurs strange matter has a negative hadronic electric charge and carries a sea of positrons to guarantee its neutrality.

This effect occurs when first order in α_c contributions to Ω, dominate over the zeroth order, making us suspicious that it may not be stable with respect to including higher orders in α_c. Indeed, we find that the regions of parameter space in which the hadronic electric charge is negative are regions in which our calculations are rather sensitive to the choice of renormalization point. We should note, however, that had we renormalized on shell the hadronic electric charge of bulk strange matter would have been negative throughout most of the region of its stability for $\alpha_c \geq 0.3$. This reflects large logarithms for small m and signals the breakdown of perturbation theory at rather small α_c in that renormalization scheme.

The strangeness excess can be seen in Fig. 2(a), where the strangeness per baryon is larger than one for small m at $\alpha_c = 0.9$. In Figs. 1(c) and 1(d) the dotted regions give the values of the parameters for which the baryon charge could be negative. The region is bounded by the charge-zero contour renormalized at $\rho = 313 + 25\%$ and then rescaled back in m by Eq. (2.10). Thus, the dotted region has our renormalization uncertainties already expressed. We conclude that negative hadronic electric charge on strange matter appears to be allowed by the parameters of our theory but only for small values of m_s and large α_c. The (rather catastrophic) phenomenological consequences

TABLE I. Parameters obtained from bag-model fits to light-hadron spectra.

Reference	$B^{1/4}$ (MeV)	m (MeV)	α_c
7	145	280	2.2
8	149	283	2.0
9	120	340	2.8
10	$\sim$200–220	288	$\lesssim 1$

of this possibility are discussed in Sec. V.

Looking at Fig. 1, it is natural to ask whether or not previous determinations of the parameters $B^{1/4}$, m, and α_c lie within one of the windows of stability. Some examples, obtained from fits of light-hadron spectra in the hadronic bag model, are given in Table I. The values of α_c are very large; too large, in fact, for first-order perturbation theory to be reliable in the calculation of Ω. Note Ω in Eq. (2.8) changes sign when α_c exceeds $\pi/2$.

Even if it were possible to perform reliable calculations with such large values of α_c, there are several reasons why the bulk parameters should not be compared directly to the parameters determined in bag-model fits to light-hadron spectra. First, no complete calculation of $O(\alpha_c)$ radiative corrections have been made in the context of the bag model. Therefore, we cannot assign a renormalization point ρ_{bag} to the parameters m and α_c listed in Table I and cannot compare them with our parameters renormalized at $\rho = M_{N/3}$. Both α_c and m decrease with increasing ρ, so the smaller value of m required for stability of strange matter could merely reflect the fact that $\rho_{\text{bag}} \lesssim \rho$. Second, bag fits to light-hadron spectra invariably introduce other, less fundamental parameters. For example, all fits in Table I include a "zero-point" energy Z_0/R; in addition, Refs. 9 and 10 include parameters associated with gluon and/or quark self-energies. Fitting to hadron spectra invariably couples the output values of m, α_c, and $B^{1/4}$ to those of the more phenomenological additional parameters. Finally, the parameters extracted from bag-model fits depend sensitively on details of the lowest-bag-quark wave function, the $1s_{1/2}$ orbital. The values of m in the table are larger than estimates in other models. This can be attributed entirely to the small matrix element of the operator $\bar{\psi}\psi(\approx \frac{1}{2})$ in the lowest bag orbital. The value of α_c in bag fits is determined largely by baryon hyperfine splittings which are (roughly speaking) proportional to α_c times a product of quark (color) magnetic moments. The same model predicts baryon (electro) magnetic moments which are too small by a factor of $\approx \frac{2}{3}$. A change in the quark wave function which brings baryon magnetic moments into agreement with experiment would yield a value of α_c smaller by a factor of $\approx \frac{4}{9}$. In light of these difficulties we cannot hope to decide conclusively on the basis of bag-model fits whether or not strange matter is stable. We can say, however, that the windows or stability are large, the values of $B^{1/4}$ and m which lead to stability are not unusual, and the windows do not diminish with increasing α_c.

III. LARGE BARYON NUMBER

In this section we analyze chunks of strange matter containing enough quarks so the Fermi gas approximation is valid, but small enough that the effects of surface tension cannot be neglected—strangelets. For the largest surface tensions we consider, the energy per baryon due to surface effects will be negligible (less than 1 MeV) for baryon numbers larger than roughly 10^6 or 10^7. In Sec. IV we estimate that the Fermi gas approximation including surface tension is reasonably accurate for systems of baryon numbers larger than about 50. So in this section we limit our attention to $10^2 \lesssim A \lesssim 10^7$. Spherical strangelets with baryon numbers between 10^2 and 10^7 will typically have radii of between 5 and 200 fm, which is less than the electron's Compton wavelength. Thus, we begin by assuming that there are no electrons (or positrons) inside strangelets, so they are charged and Coulomb effects must be considered. The consistency of this assumption is explored at the end of this section.

We treat a finite chunk of strange matter as a bit of bulk strange matter to which we add Coulomb and surface energies. The quark Fermi gas is characterized by the chemical potentials μ_u, μ_d, and μ_s. Weak interactions still maintain the relation $\mu_d = \mu_s$ $(\equiv \mu)$, but the abundance of up quarks relative to down and strange quarks is affected by the Coulomb energy, so Eq. (2.3) is no longer valid, and there are two independent chemical potentials, μ and μ_u. (Here we ignore possible flavor dependence in the surface energy which would spoil the relation $\mu_d = \mu_s$. Later we will find that the surface energy is likely to be flavor dependent, but the effects on this calculation are small.) We assume Ω_a and $n_a = -\partial\Omega_a/\partial\mu_a$ are still given by Eq. (2.8). In this section we take the renormalization point to be fixed at $\rho = 313$ MeV and do not consider the effects of changing it. In reality both Ω_a and n_a are certainly modified by surface effects which are both quark-mass and α_c dependent. It is always possible to lump all those modifications into a "surface tension" σ, which we treat as a parameter in this section. Later we explore the range of values of σ and its dependence on m and α_c.

We find the flavor composition and size of a strangelet by minimizing the energy,

$$E = \sum_a (\mu_a n_a + \Omega_a) V + 4\pi\sigma \left[\frac{3V}{4\pi}\right]^{2/3} + BV + \frac{16\pi^2\alpha}{15}\left[\frac{3V}{4\pi}\right]^{5/3} n_Z{}^2 \quad (3.1)$$

with respect to μ and μ_u, at fixed baryon number, $A = \frac{1}{3}\sum_a n_a V$. Here $n_Z = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$ and we have assumed the charge is uniformly distributed throughout a spherical chunk.

To isolate the Coulomb effects we first ignore surface effects and set $\sigma = 0$. In Fig. 3 the charge on a strangelet with large A is displayed as a function of A for various choices of parameters for which bulk strange matter is bound. The charge Z grows as $A^{1/3}$ for A large with the constant of proportionality depending on the bulk properties of strange matter:

$$Z=\frac{15}{32\pi^2\alpha}\left[\frac{\mu-\mu_u}{n_A^{1/3}}\right]_{\text{bulk}}A^{1/3}+O(A^0)\ . \qquad (3.2)$$

The charge-to-baryon-number ratio for chunks of strange matter is much lower than for nuclei. In nuclei the symmetry-energy minimum occurs when $Z=A/2$, whereas the Coulomb energy vanishes when $Z=0$. Since the Coulomb energy grows like $Z^2/A^{1/3}$ (at constant density), nuclei are eventually destabilized by Coulomb effects. They fission and the periodic table ends roughly at $A=250$. Strange matter owes its low charge to the happy coincidence that the Coulomb energy vanishes at the symmetry-energy minimum for massless quarks, $n_u=n_d=n_s$. Only the strange-quark mass upsets this equilibrium. We see in Fig. 3 that the charge increases as m increases, but for each m it decreases with increasing α_c. This effect has the same explanation as in the bulk case. For $\alpha_c=0.9$, $m=150$ MeV we have an example where the charge is negative. We cannot overemphasize the importance of the low charge-to-baryon-number ratio of strange matter. Because of it the physics of strange matter differs radically from nuclear physics.

According to Eq. (3.2), for large A the Coulomb energy drives Z/A to zero. This is not the preferred charge-to-baryon ratio for strange matter in bulk (unless $m=0$), so the $A\rightarrow\infty$ limit of our finite A calculations does not

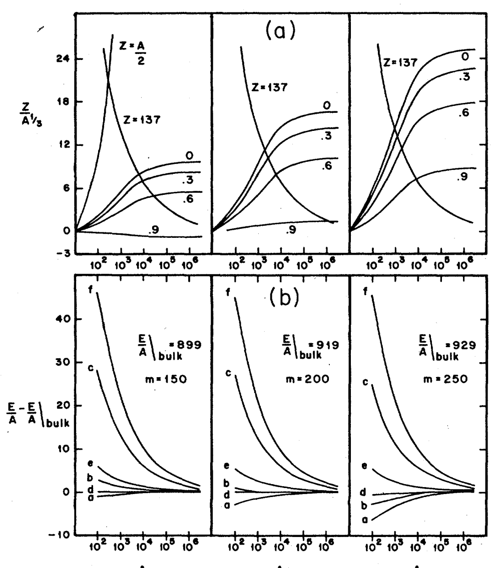

FIG. 3. (a) $Z/A^{1/3}$ versus A for parameters such that $E/A=899$, 919, and 929 MeV in bulk. For each E/A, curves are shown at $\alpha_c=0$, 0.3, 0.6, and 0.9. The curve $Z=A/2$ represents the approximate nuclear charge versus baryon number. (b) E/A versus A for parameters such that $E/A=899$, 919, and 929 MeV in bulk. The small letters refer to the surface tension $\sigma^{1/3}$ and α_c as follows:

	$\sigma^{1/3}$ (MeV)	α_c
a	0	0
b	40	0
c	80	0
d	0	0.9
e	40	0.9
f	80	0.9

coincide with the bulk limit. It would if electrons had been included. The Coulomb energy per baryon is positive and, using Eq. (3.2), is proportional to $A^{-2/3}$. Nevertheless, a glance at Fig. 3 indicates that as $A\rightarrow\infty$, E/A approaches its limit *from below*. The reason is the increase with A of the symmetry energy driven by the constraint $Z/A\rightarrow 0$. In the absence of surface effects, therefore, the energy per baryon grows with A, so chunks with large A should fission into smaller pieces. However, the preferred charge-to-baryon ratio in bulk is rather small, so the destabilizing effects of the Coulomb interaction are weak and can easily be swamped by surface effects.

Surface effects may be either "intrinsic" or "dynamical." An intrinsic surface tension σ_I would be a characteristic of the phase boundary separating the true vacuum of QCD from the perturbative vacuum inside hadrons. (We assume σ_I is positive. If it were not, the vacuum would be unstable toward filling up with a foam of phase boundaries.) This type of surface tension would contribute to the masses of ordinary mesons and baryons. An intrinsic surface tension elevates the coordinates describing the phase boundary to be independent dynamical variables and would give rise to true surface excitations somewhere in the baryon and meson spectrum. The spacing of surface excitations would be proportional to $1/\sigma_I$. The absence of any candidates for surface states in the baryon spectrum suggests that σ_I is very small. Since the spacing of quark excitations is proportional to $B^{1/4}$ we expect $\sigma_I^{1/3}\ll B^{1/4}$. (Here we identify B with the bag constant of the bag model.)

A dynamical surface tension σ_D arises from calculable corrections to the Fermi gas approximation. In general, σ_D depends on α_c, quark masses, and whatever parameters characterize the boundary. The dynamical surface tension expected in the bag model is discussed further in Sec. IV. Generally, a strange-quark mass, a scale-dependent renormalization scheme, or a phase boundary which is not perfectly sharp will all give rise to a nonvanishing dynamical surface tension σ_D. In Sec. IV, we estimate σ_D in the bag model for $\alpha_c=0$ and find $\sigma_D\lesssim(70\text{ MeV})^3$.

In Fig. 3 we also display the energy per baryon as a function of A for $\sigma=0$, $(40\text{ MeV})^3$ and $(80\text{ MeV})^3$. It should be noted that the charge-to-baryon-number ratio is almost totally insensitive to σ. The $\sigma=0$ curves increase with A due to Coulomb effects. For $\sigma=(40\text{ MeV})^3$, E/A falls with A for most values of parameters and for $\sigma=(80\text{ MeV})^3$ the curves fall sharply as A increases. The value of $\sigma^{1/3}$ is to be compared to $B^{1/4}$, so we see that a reasonable value of $\sigma^{1/3}$ completely overwhelms the Coulomb effects in the energy. Thus, it seems quite unlikely that strangelets would fission.

Surface effects can increase the energy per baryon substantially for small A leading to instability and a lower bound on the baryon number for which strange matter is bound. For small values of σ and ϵ (the energy per baryon of bulk matter) the minimum A is too small to analyze with the methods of this section so we postpone the discussion until Sec. IV. For large values of σ and/or weakly bound bulk strange matter, the minimum A is large enough so only surface and Coulomb corrections need be included.

A strangelet is unstable, in principle, if E/A exceeds the mass of a nuclear system of the same baryon number. However, even if E/A exceeds M_N a strangelet would not convert (in any finite time) en masse into nucleons because the process would require a very-high-order weak interaction. The practical measure of stability is the "separation energy" dE/dA required to remove a baryon from a strangelet. If dE/dA exceeds M_N, neutrons evaporate from the surface. If dE/dA is less than M_N but greater than $M_\alpha/4$ (M_α is the mass of the α particle), then α particles are emitted, though this process is inhibited by a Coulomb barrier.

With these considerations in mind, let us study finite A effects for a "typical" choice of parameters. Since Coulomb effects are small we ignore them leaving

$$E=\epsilon A+(36\pi)^{1/3}\sigma A^{2/3}/n_A{}^{2/3}+O(A^{1/3}) , \qquad (3.3)$$

where ϵ and n_A are the bulk energy and baryon densities. Suppose, as an example, $\epsilon=919$ MeV, $n_a=(110\text{ MeV})^3$, and $\sigma=(80\text{ MeV})^3$. Then dE/dA exceeds $M_\alpha/4$ for $A \lesssim 1900$. Below this limit strange matter would decay by α-particle emission (the rate depends on Z/A), occasionally punctuated by a weak interaction necessary to maintain a favorable flavor composition. For $A \lesssim 1100$, E/A exceeds M_N, but the strangelet does not emit nucleons until $A \lesssim 320$, at which point dE/dA exceeds M_N. Below this limit it decays more rapidly by evaporating neutrons (again accompanied occasionally by a weak interaction). Thus, strangelets with low A might be quasistable, and decay radioactively by chains of α, β (and γ) emission.

The charge-to-baryon ratio in strangelets is small. In Fig.3 we have drawn the $Z=137$ contour to show how large the baryon number must be to attain this charge for various choices of typical parameters. Strangelets with positive charges of less than 100 will look like ordinary, but unusually heavy, atoms. Electrons will surround the core in atomic orbitals and the Bohr radius of the innermost shell, $1/mZ\alpha$, will be much larger than the size of the strangelet.

It is known from quantum electrodynamics that point charges of arbitrarily large charge cannot exist. For Z greater than 137 it is energetically favorable to produce electron-positron pairs; the positrons escape to infinity leaving the electrons, tightly bound, to screen the charge. However, if the charge is distributed over a small volume, larger values of Z can be tolerated. It has been shown[11] that a Z of 169 can exist for a sphere whose charge density is equal to the observed nuclear charge density. For Z greater than 169 two positrons can be emitted making two units of charge available for screening. At $Z=182$ another two units are available. It is only for Z's greater than around 300 that substantial screening occurs.[12]

Strangelets have a much lower charge density than ordinary nuclei. Thus, it seems likely that strangelets could support much larger charges than matter with the charge density of ordinary nuclei. A charge as large as 1000 seems reasonable. A strangelet of charge 1000 would typically have a baryon number of roughly 10^6. For a density of $(110\text{ MeV})^3$, a system with $A=10^6$ has a radius of 112 fm. A core of charge 1000 is surrounded by an atomic cloud of (very close to) 1000 electrons. It would be useful to know the percentage of these electrons that are actually spending most of their time inside the core. One approach is to solve the Dirac equation directly for the system at hand. The potential is $\alpha Z/r$ outside the core, but is quadratic in r inside. Finding all the energy levels required for 1000 electrons is clearly a very involved task. Another approach is to use a relativistic Thomas-Fermi model for the electron cloud and solve for the self-consistent potential. This is more promising but beyond the scope of this paper. The most we can say is that the naive "Bohr radius" of the innermost electron $1/mZ\alpha$ is 53 fm, which is close to the size of the core, so at $Z=1000$ we are near the transition to bulk matter. The transition from large A with electrons outside of the strangelet to bulk with electrons within should be totally smooth although somewhat difficult to calculate.

IV. STRANGELETS WITH SMALL BARYON NUMBER

Strange matter with low A, less than several hundred, is not adequately described by a simple Fermi-gas model. This is especially true for very low A. In this section we replace the Fermi gas by the hadronic bag model and estimate the properties of strangelets with low A by explicitly populating the quark orbitals in a spherical bag. We treat the bag model[7] with caution and explore the sensitivity of our results to the most model-dependent ingredients. The bag description goes over to the Fermi gas for large A so we can use these explicit calculations to estimate the surface corrections to the Fermi gas. We do this for nonstrange quark matter as well as for strange matter and we find the power-law correction in A in agreement with what we expect from general considerations of surface corrections to the Fermi gas. For very low A, ≤ 6, we include order-α_c corrections, neglected for $A>6$, and we find that this model, fortunately, does not predict the existence of stable strange hadrons with $A \leq 6$. A possible exception occurs at $A=S=2$, the previously predicted dihyperon. This state, while not exactly strange matter, is interesting in its own right and should be further sought experimentally.[13–15]

We begin for $A>6$ by considering the bag model for three flavors of quarks, giving only the strange quark a mass m. The bag model is well described elsewhere[7] so we will be brief. The Hamiltonian includes quark kinetic energies, bag energy (BV), and a phenomenological zero-point energy ($-Z_0/R$). Beyond the first shell it is prohibitively difficult to include gluon-exchange effects so we take $\alpha_c=0$. For each flavor each energy eigenmode can hold $3(2j+1)$ quarks. (j is the angular momentum of the mode.) At a fixed A we determine N_u, N_d, N_s to minimize the total energy subject to the constraint $3A=N_u+N_d+N_s$. Of course, at each stage the radius of the system is also varied to minimize the energy.

In Fig. 4(a) we show E/A versus A for the choice $m=150$ MeV, $B^{1/4}=150$ MeV, and $Z_0=2.0$. The quark orbitals included in this calculation are $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $1g_{9/2}$, $2p_{3/2}$, and $2p_{1/2}$. For this choice of parameters E/A in bulk is 903

MeV. (These calculations ignore Coulomb effects which are small in strangelets.) For this choice of parameters strange matter is unbound for A less than ~ 70. This demonstrates a crucial characteristic of strange matter: it is stable in bulk but unbound for sufficiently low A.

In Fig. 4(a), the deviations from a smooth approach to the bulk limit as $A \to \infty$ are due to shell effects and are most pronounced at low A. In Fig. 4(b) the strangeness is displayed as a function of A and the bulk value of S/A, 0.81, is shown as the dashed line. Again shell effects are obvious. Fitting E/A to a constant plus $A^{-1/3}$ corrections we obtain an estimate for $\sigma_D{}^{1/3}$ of approximately 70 MeV. Similar estimates of $\sigma_D{}^{1/3}$ for $m=100$ and 200 MeV in both cases give approximately 60 MeV. This is the basis of our choice of a range of σ_D used in Sec. III.

We study nonstrange quark matter at low A because we wish to establish the minimum value of $B^{1/4}$ for which nuclei are effectively stable. Coulomb effects are included because nonstrange matter with low A, like nuclei, is not charge neutral. Figure 5 displays the results of filling energy levels up to $A=250$. In addition to the orbitals listed in the strangelet case, we include here the $1g_{7/2}$, $1h_{11/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1i_{13/2}$, $1h_{9/2}$, $2f_{7/2}$, $1j_{15/2}$, $2f_{5/2}$, and $1i_{11/2}$ levels. $B^{1/4}$ has been chosen to be 145.0 MeV, so the minimum of E/A, which occurs at $A \approx 250$, is 930 MeV. With this value of $B^{1/4}$ bulk nonstrange quark matter has an energy per baryon of 934 MeV. This fixes the parameter Δ, required in Sec. II, to be $934-930=4$ MeV.

Comparing Fig. 5 with Fig. 4, it appears that strange and nonstrange quark matter approach their bulk limits quite differently. For strange-quark matter, Coulomb effects are negligible and the approach to bulk is dominated by surface tension ($\sigma A^{-1/3}$). For nonstrange quark matter the dynamical surface tension vanishes (see below). Because α is small, Coulomb effects are not important for $A \lesssim 150$, so E/A falls toward its bulk value (with $N_u = N_d$) like $A^{-2/3}$. For large values of A, Coulomb effects increase and force nonstrange matter toward charge neutrality, $N_d = 2N_u$, and toward a higher bulk value of E/A. Thus, for values of A beyond those shown in Fig. 5, E/A will rise to 934 MeV.

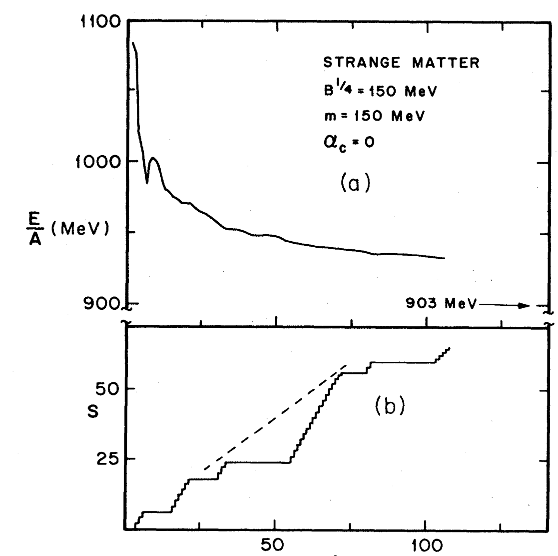

FIG. 4. (a) E/A versus A in the hadronic bag model for u, d, and s quarks. Parameters are such that E/A in bulk is 903 MeV. (b) S versus A in the hadronic bag model for u, d, and s quarks. Parameters are such that E/A in bulk is 903 MeV.

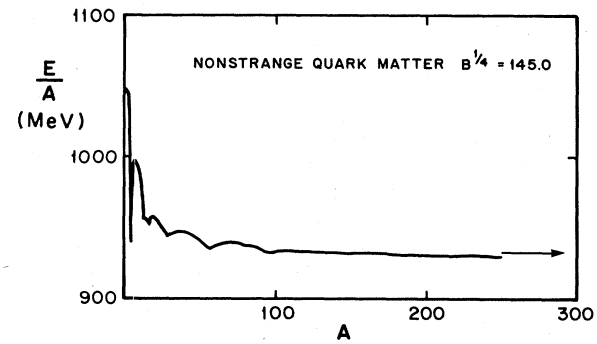

FIG. 5. E/A versus A in the hadronic bag model for u and d quarks, including Coulomb energy. Parameters are such that E/A in bulk is 934 MeV.

These calculations have been done with $\alpha_c=0$ because of the prohibitive difficulties encountered in including perturbative corrections beyond the $1s_{1/2}$ level. There is another method for studying the approach to the Fermi-gas limit which is more amenable to QCD corrections, but does not allow us to follow the detailed fluctuations due to shell closures, namely, the multiple-reflection-expansion method pioneered by Balian and Bloch.[16] To our knowledge their treatment of finite-size corrections has only been worked out completely for $\alpha_c=0$ and for massless or nonrelativistic particles. Their analysis can be extended to $\alpha_c \neq 0$ and perhaps to $m \sim \mu$. Even with the present limitations we shall be able to draw some insights into the low-A behavior of strange matter. For illustrative purposes we consider the massless case. The (suitably smoothed) density of states $[dN/dk \equiv \rho(k)]$ in a cavity may be expanded asymptotically in powers of k ($k=E$ for $m=0$) (Ref 16):

$$\rho(k) = g\left[\frac{Vk^2}{2\pi^2} + C_s kS + \frac{C_R}{8\pi}\int d^2s\left[\frac{1}{R_1}+\frac{1}{R_2}\right] + \cdots\right], \tag{4.1}$$

where V, S, R_1, and R_2 are the volume, surface area, and local radii of curvature. g is the degeneracy factor. The first term is universal, the others depend on the spin of the confined particles and the form of the confining boundary condition. A similar expansion holds in the nonrelativistic limit when $k \ll m$ and $k^2=2mE$. Successive terms in Eq. (4.1) can be calculated from terms in a reflection expansion for the confined propagator as described in Ref. 16. For a confined gas of massless, noninteracting quarks, $C_s=0$ (Ref. 17) and $C_R=-\frac{1}{6}\pi$ (Ref. 18). It can be shown that the absence of a surface area correction to $\rho(k)$ implies that the energy per baryon E/A approaches its bulk limit like $A^{-2/3}$ rather than $A^{-1/3}$ as might be naively expected. Order-α_c effects do not change this conclusion. We have verified this behavior explicitly for nonstrange quark matter without a Coulomb interaction and have obtained a good fit to the $A^{-2/3}$ coefficient of E/A using the values of C_S and C_R just quoted. The somewhat more complicated case of nonstrange quark matter with a Coulomb interaction has already been discussed in conjunction with Fig. 5.

In the nonrelativistic limit the bag-boundary condition goes over to the Dirichlet case studied by Balian and Bloch.[16] This leads to a positive $A^{-1/3}$ correction to the energy per baryon corresponding to a positive dynamical surface tension, σ_D.[19] This limit is not of direct physical interest since strange quarks are relativistic in the systems we consider. Nevertheless, to the extent strange quarks are present we are led to expect positive $A^{-1/3}$ corrections and, effectively, a positive dynamical surface tension.

To summarize first, nonstrange quark matter approaches its bulk limit, like $A^{-2/3}$, even including $O(\alpha_c)$ corrections; second, strange matter should exhibit $A^{-1/3}$ corrections which vanish either when $m \to 0$ (all quark species massless) or when $m \gg \mu$ (only massless u and d quarks present). This behavior has been observed in our hadronic bag-model calculations, although we worked only at $\alpha_c = 0$. For strange matter we found that the $A^{-1/3}$ coefficient peaked at m near 150 MeV.

If strange matter with small enough A were stable, light nuclei would decay into it and heavier nuclei might emit it in processes analogous to α decay. Since $S \sim A$ for strange matter and each unit of strangeness costs a factor of $G_F{}^2$ in the decay rate, such instability would only be of practical importance if it occurs for very small A, say, $A \lesssim 10$. It has long been known that QCD spin- and color-dependent forces favor the appearance of strange quarks in multiquark systems.[20] For $A < 6$ we can include order-α_c effects in the bag model so it is both important to ask and possible to study in the context of these models, whether light, strange multiquark hadrons might be absolutely stable. The calculations are done in the same way as for $A > 6$ except that order-α_c effects are included. We choose to work with bag parameters fit to the baryon spectrum ($B_{\text{bag}}{}^{1/4} = 145$ MeV, $m_{\text{bag}} = 280$ MeV, $\alpha_{\text{bag}} = 2.2$, $Z_0 = -1.84$). As was explained in Sec. II, the relationship between the parameters used in our bulk calculations and those fit to the hadron spectrum is rather uncertain. For very low A we are investigating the possible existence of a stable ($E/A < 939$ MeV) strange baryon so it makes most sense to use parameter values which correctly fit the known baryon masses.

Figure 6 shows the results of this calculation. For each $A \leq 4$ we plot the mass per baryon of the lightest nonstrange multiquark hadron and for each $A \lesssim 6$ the lightest strange multiquark hadron. No multiquark system is stable. The nearest approach for $S \neq 0$ is at $A = 2$, $S = 2$. This doubly strange dihyperon has been studied extensively.[4,5,14,15] More careful theoretical estimates of its mass range from 2000 MeV (Ref. 21) to 2240 MeV (Ref. 22). It is not inconceivable that it is stable. Remarkably, the possibility of an absolutely stable H (as it is known) is not ruled out by experiment. As long as M_H were not too much less than $2M_N$ nuclear decays like ${}_ZA \to {}_Z(A-2) + H$, which are second-order weak, would be very rare.

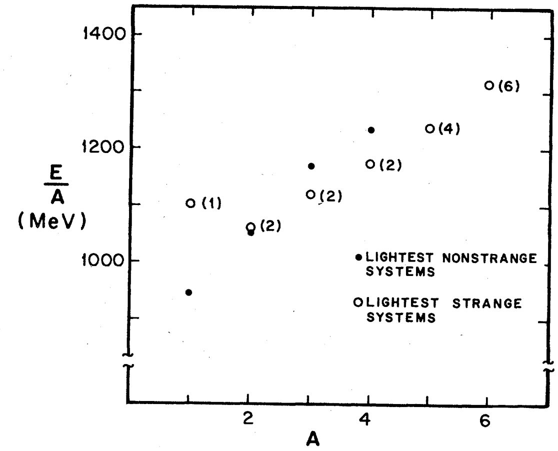

FIG. 6. E/A for $A \leq 6$. Solid dots are lightest nonstrange system. Open dots are lightest system with strange quarks. Figure in parenthesis is the number of s quarks in the lightest strange system.

Though not exactly strange matter as originally conceived of (stabilized by symmetry energy), nevertheless, the discovery of a stable or nearly stable dihyperon would be of great interest.

Of course, the energies shown in Fig. 6 change if the model parameters are changed. One might worry that a different choice of parameters, less desirable for $A = 1$, would make some multiquark bag with $A < 6$ stable. We have looked at a wide range of parameters and believe this does not happen (except perhaps for $A = S = 2$ as already noted). The quark kinetic energies and the one-gluon-exchange energy dominate the S and A dependence of bag-model masses. For fixed A the kinetic energy rises roughly linearly with the number of strange quarks. On the other hand, as A increases so does R, the bag radius, consequently, quark kinetic energies decrease. For a wide range of parameters multiquark bags with $A \leq 6$ would be stable were it not for the effects of one-gluon exchange, but the most stable state would have $S = 0$, then $S = 1$, etc. A stable multiquark bag with $S = 0$ or 1 could not have been missed experimentally, so one-gluon exchange would have to reverse the order of states, destabilizing states with $S = 0$ and 1, but leaving a state with $S \geq 2$ stable. Color magnetic interactions do, in fact, favor the inclusion of strange quarks[20] (as evidenced in Fig. 6), but they are not sufficient to reverse the ordering of states except, perhaps, when $A = 2$. We conclude that it is unlikely that strange matter with $A \leq 6$ is stable, with the possible exception of the dihyperon.

V. PHENOMENOLOGY OF STRANGE MATTER

If strange matter is stable in bulk, it is quite likely to be stable over a wide range of baryon numbers down to some minimum, $A_{\min}$, which depending on parameters lies roughly between 10 and 1000. Here we sketch some of the characteristics of strange matter, in its variety of forms, which will influence attempts to search for it in various settings. In his paper, Witten[1] has already discussed some properties of bulk quark matter, notably in the neutron-rich environment within a neutron star. We refrain from a discussion of that subject and refer the interested reader to his paper. It is convenient to divide our considerations into two classes: those applicable to bulk matter, and those dealing with strangelets with $A \lesssim 10^7$.

A. Bulk matter

If strange matter is stable in bulk it can absorb ordinary nucleons in exothermic reactions accompanied by occasional weak interactions necessary to maintain flavor equilibrium. In addition to the usual semileptonic processes, e.g., $u \rightarrow s + e + \bar{\nu}_e$, the nonleptonic process $u + d \rightarrow s + u$ is likely to be important. Except when m is small and α_c is large, strange matter has positive hadronic electric charge. As Witten has noted, a Coulomb barrier prevents this system from absorbing the nuclei of ordinary atoms and renders it inert in contact with ordinary matter. The height of the barrier is just the electron chemical potential. For "generic" parameter choices we find a fairly equal distribution of flavors, for example, $B^{1/4} = 133.49$ MeV, $\alpha_s = 0.6$, and $m = 150$ MeV corresponding to E/A of 899 MeV, we find $n_u:n_d:n_s = 1:1.09:0.906$. The hadronic electric charge per baryon is small so there are relatively few electrons and $\mu_e = 9$ MeV. This electrostatic potential at the surface of bulk matter is sufficient to prevent it from absorbing ordinary matter at low energy. There is no barrier to absorbing neutrons, so strange matter would grow without limit in a neutron-rich environment such as the interior of a neutron star[1] or a conventional nuclear reactor here on earth.

If bulk strange matter has negative hadron electric charge the situation is radically different. This alternative occurs for large α_c and small m where QCD effects overcome the suppression of strange quarks. It is sensitive to higher-order effects in α_c, but it is not clear whether higher-order effects will make it more or less likely. Strange matter with negative hadron electric charge would be neutralized by positrons. Ordinary atoms would be attracted to it and absorbed. In contact with a supply of ordinary matter it would grow without limit, renewing its appetite by weak interactions and its positron cloud by weak interactions as well as e^+e^- pair creation at its surface. Clearly, negatively charged strange matter would have disastrous consequences for any ordinary matter it touches. It probably could not be tolerated at any level on earth or in ordinary stellar environments.

Two features of the gravitational interactions of bulk strange matter deserve mention. First, since it is very dense even small chunks cannot be supported by material forces at the earth's surface. For typical values of the energy per baryon and the density we find that the gravitational force at the earth's surface exceeds 1 eV /Å when the radius of the chunk of strange matter exceeds 5 Å. This places an approximate upper limit of about 10^{17} on the baryon number and 1.6×10^{-7} gm on the mass of any lump of quark matter which one might hope to find at the surface of the earth. Second, in large enough amounts quark matter would be unstable against gravitational collapse. Fechner and Joss[23] have studied the dynamics of quark stars and conclude that the upper limit on their mass coming from gravitational instability is 2 solar masses.

B. $A \lesssim 10^7$

Strangelets with low baryon number display a wide variety of behavior depending on their stability and charge-to-mass ratio. Strangelets are unstable at low values of A. In principle, they are unstable if E/A exceeds M_N. This is irrelevant for practical purposes for the same reason the analogous instability is irrelevant for nuclei when their energy per baryon exceeds that of quark matter: the decay *en masse* is inhibited by a factor of G_F to a high power. A more serious instability occurs if it is energetically favorable to emit ordinary nuclei with low baryon number from the surface. The flavor composition of light nuclei, $N_u:N_d:N_s = 1:1:0$, differs drastically from strange matter. This inhibits the emission of all but the lightest nuclei for two reasons: first, the likelihood of finding a correlated collection of N nonstrange quarks within a strangelet falls quickly with N; and second, the Q value for the decay falls with N (and eventually goes negative) because the residual strangelet is far from flavor equilibrium. Therefore, the first instability of importance is α-particle emission. We define the practical lower limit of stability $A_{\min}$ by the condition $dE/dA > M_\alpha/4$ for $A < A_{\min}$.

For baryon numbers below $A_{\min}$ strangelets decay by a complicated chain of radioactive decays. α-particle emission eventually drives the strange matter out of flavor equilibrium. dE/dA decreases below $M_\alpha/4$ and α emission ceases until (primarily strangeness-changing) weak decays reestablish flavor equilibrium and allow renewed α emission. The process resembles the radioactive decay of a heavy nucleus like uranium. There are several important differences, however. First, the α process is much faster in strange matter because the Coulomb barrier is lower. For example, the electrostatic potential energy of an α particle at the surface of a strangelet with $A = 250$, and $Z = 13$ (typical of $\alpha_c = 0.6$, $E/A = 899$ MeV, and $M = 150$ MeV) is 6 MeV as compared to 36 MeV in uranium. Since the α decay rate varies exponentially with the barrier height we expect the process to be much faster in strange matter than in heavy nuclei. On the other hand, the β process is, if anything, slower because the $\Delta S = 1$ weak interactions are inhibited by a factor of $\sin^2\theta_c \cong 0.04$. Eventually, as a strangelet decays, dE/dA exceeds M_N and direct emission of protons and, in particular, neutrons becomes possible. As before, however, weak interactions, necessary to equilibrate flavors, slow the decay process down. Finally, for the lowest values of A, dE/dA may exceed M_Λ and hyperons may be emitted. Interestingly, the decay process could be initiated for an otherwise stable chunk of strange matter by a collision which strips off enough baryons to reduce A below $A_{\min}$. We are not at present able to estimate the lifetime of radioactive strange matter, however, it seems quite possible that the duration of some radioactive decay chain is comparable to the age of the Universe. So anomalous patterns of radioactive decay might be a signature for strange matter in terrestrial searches.

TABLE II. Some sample strangelets.

	m (MeV)	$\sigma^{1/3}$ (MeV)	A	Z	S	E/A (MeV)
A	150	80	316	12	283	925
B	200	40	1000	54	794	921
C	150	80	10^6	535	9.1×10^5	901
D	225	80	3.2×10^5	868	2.4×10^5	932

The most striking signature of strange matter is its very low charge-to-baryon-number (or mass) ratio. This is a generic characteristic, true for all of our parameter values, even when the strange-quark mass is large. To get some feel for the magnitude of the numbers we consider it useful to list in Table II a few "typical" examples (all with $\alpha=0.6$, $B^{1/4}=133.5$ MeV). Examples A and B could manifest themselves as anomalously superheavy isotopes of magnesium and xenon, respectively. C and D, on the other hand, would appear as new superheavy elements with Z/A unlike ordinary nuclei. D is radioactive. Even if the parameters entering our calculations were known precisely and the calculations themselves were highly reliable, it would be difficult to be specific about the prediction of novel, "strange isotopes" and "elements." The reason lies in the very large range of baryon numbers for which strange matter is stable. We expect new species for all A greater than $A_{\rm min}$ and for all Z greater than the associated $Z_{\rm min}$.

Since Z grows only slowly with A one would expect many stable strange isotopes for each value of Z. This can be illustrated for large A with the aid of Eq. (3.2). A change in Z of one unit requires a change in A, $\Delta A=3A/Z$ which can be very large. Clearly, looking for strange matter, or even using existing searches to place a meaningful limit on its abundance will be a subtle and difficult undertaking.

Finally, it is of practical importance to know if stable strange matter exists or can be made in quantity. Negatively charged strange matter, either as strangelets or in bulk, does not exist on earth. If it is stable and could be created it would react exothermically with ordinary matter, converting everything it touched into more of itself. Positively charged strange matter would have no such immediate apocalyptic consequences. Nevertheless, its propensity to absorb neutrons exothermically without limit has implications for energy production which could have great importance.

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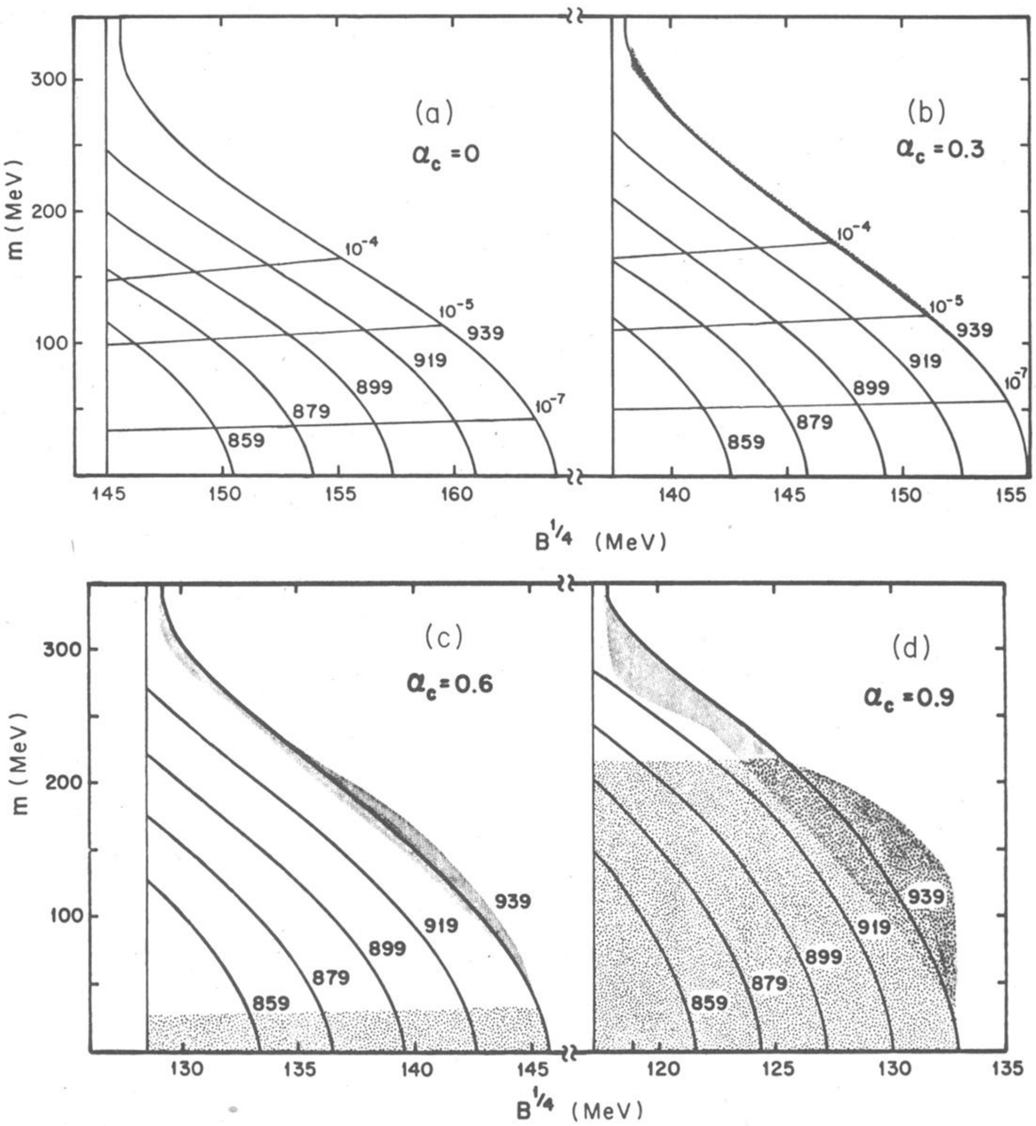

FIG. 1. Contours of fixed E/A in the $B^{1/4}$-m plane for $\alpha_c = 0$, 0.3, 0.6, and 0.9. The vertical line at the left of each figure is the minimum $B^{1/4}$ for which nonstrange quark matter is unbound (see text). In (a) and (b) the nearly horizontal lines are contours of fixed hadronic electric charge per baryon as marked. In (c) and (d) the dotted regions are regions of negative hadronic electric charge. The grey shading around the 939 contour represents the same contour calculated using different renormalization schemes (see text).