# Boost-invariant Boltzmann-Vlasov equations for relativistic quark-antiquark plasma

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Boost-invariant solutions of Boltzmann-Vlasov equations for quark-antiquark plasma are studied. A simple example is presented showing an apparently general feature of the plasma: oscillations of quarks and antiquarks in the mean chromoelectric field. The energy loss of the plasma by electromagnetic radiation is estimated.

## I. INTRODUCTION

When two large nuclei collide at ultrahigh energies there might appear many exotic phenomena,[1] including formation of a transient quark-gluon plasma in space-time volumes considerably exceeding in scale the volumes active in nucleon-nucleon collisions. Such phenomena involve a multitude of degrees of freedom; thus, it is natural to attempt their description in terms of some theories or models of multicomponent systems.

One of the attractive possibilities is to employ relativistic hydrodynamics and, indeed, many attempts in this direction have been made, starting with the Landau classic papers from the 1950's.[2–4] However, it is not at all clear whether the conditions to justify its application are satisfied, for instance, whether the local thermal and chemical equilibriums are established fast enough and last long enough to leave any room for the relativistic hydrodynamics to work. In fact, one may suspect that the system of quarks and gluons which is formed in a central nucleus-nucleus collision in which local energy densities exceed 2—3 GeV/fm$^3$ spends all its lifetime approaching an equilibrium.

In this situation it is important to analyze the evolution in time of a bubble of a newly born quark-gluon plasma as a dynamical nonequilibrium process employing an approach which is based on the Boltzmann kinetic theory. The fundamentals of the relativistic transport equations for low-density weakly interacting systems are thoroughly discussed in Ref. 5. However, it is still not clear whether one can adapt and modify the results of Ref. 5 to the specific situations occurring in ultrarelativistic heavy-ion collisions.[6] Also, the transport equations for a quark-gluon plasma should be derived from the non-Abelian QCD and, so far, only the first steps in this direction have been taken.[7]

Altogether, the task of constructing a viable relativistic transport theory for quarks and gluons still looks formidable. Thus, since a frontal attack seems to be so difficult one may, in order to get some feeling for the physics of quark-gluon plasma, analyze some less forbidding yet interesting problems directly related to the main theme. The present paper presents such an attempt. It addresses itself to the problem of the Boltzmann-Vlasov equations in which the gauge field is treated classically and the distribution functions of quarks and antiquarks are the ensemble expectation values of the Wigner operators. In fact, such Boltzmann-Vlasov equations were explicitly written by Heinz[7] and here we discuss some of their solutions of interest (we hope) for understanding the behavior of the quark-gluon plasma. In this first step (and in line with the spirit of the Boltzmann-Vlasov approach) we neglect all collision terms.

The Boltzmann-Vlasov equations as they appear in Ref. 7 are still quite complicated and we impose on them the restrictions of invariance against boosts in one direction which we identify with the collision axis of the incident nuclei. This is the same boost invariance which was introduced and discussed by Bjorken:[3] Its physical significance is contained in the statement that the boost-invariant space-time evolution characterizes this part of the particle production process which populates the central region of rapidities and produces plateaus. In this sense our discussion of the solutions of the Boltzmann-Vlasov equations is limited to plasma created in the central region in between two nuclei just after their collision. Even in such a simplified situation, the solution we obtained reveals an interesting and apparently general phenomenon: oscillations of the quark-antiquark plasma in the self-consistently generated chromoelectric field. The frequency of the oscillations increases with increasing density of the plasma. Observation of such oscillations (e.g., by electromagnetic radiation which must accompany them) would be certainly of great interest. However, more work is needed to determine if this possibility has any chance to be realized in nature.

In Sec. II we write the Boltzmann-Vlasov equations for the SU(3) color group (spin of the quarks is neglected). Section III casts these equations into a boost-invariant form. Section IV deals with the general boost-invariant solutions and Sec. V with some very specific simple solutions resulting from static initial conditions which lead to oscillations of the plasma. The physical significance of these results is discussed in this section and Sec. VI, which also contains conclusions.

## II. BOLTZMANN-VLASOV EQUATIONS FOR RELATIVISTIC QUARK-GLUON PLASMA

The equations for quark (antiquark) distribution functions were given by Heinz.[7] They read[8,9]

$$p^\mu \partial_\mu g \pm \lambda p^\nu F^a_{\mu\nu} \partial^\mu_p g_a = 0 \ , \tag{2.1}$$

$$p^\mu \partial_\mu g_a - \lambda f_{abc} p^\mu A^b_\mu g_c \pm \frac{\lambda}{2} p^\nu F^b_{\mu\nu} \partial^\mu_p \mathrm{Tr}(\{Q^a, Q^b\} G) = 0 \ , \tag{2.2}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \lambda f_{abc} A^b_\mu A^c_\nu \tag{2.3}$$

and $G$ is the distribution function whose explicit form in the color space is

$$G = \tfrac{1}{3} g + 2 g_a Q_a \ , \tag{2.4}$$

and $Q^a = \hat{\lambda}^a/2$ are the generators of the SU(3) group. $\lambda$ is the color coupling constant. The trace in Eq. (2.2) can be evaluated using Eq. (2.4) and the identity

$$\{Q_a, Q_b\} = \tfrac{1}{3}\delta_{ab} + d_{abc} Q_c \ , \tag{2.5}$$

and we obtain finally

$$p^\mu \partial_\mu g \pm \lambda p^\nu F^a_{\mu\nu} \partial^\mu_p g_a = 0 \ , \tag{2.6}$$

$$p^\mu \partial_\mu g^a - \lambda f_{abc} p^\mu A^b_\mu g_c \pm \frac{\lambda}{2} p^\nu F^b_{\mu\nu} \partial^\mu_p (\tfrac{1}{3} g \delta_{ab} + d_{abc} g^c) = 0 \ . \tag{2.7}$$

The system of Eqs. (2.6) and (2.7) represents nine equations for nine unknowns $g$, $g_a$ $(a = 1, \ldots, 8)$. They describe the behavior of quarks (antiquarks) in the external color field $A^a_\mu$.

In the Boltzmann-Vlasov formulation the gauge field is itself generated by quark and antiquark currents. One can thus write equations for $F^a_{\mu\nu}$. They read[7]

$$(D_\mu F^{\mu\nu})_a = j^\nu_a \ , \tag{2.8}$$

where the current $j^\nu_a$ is given by

$$j^\nu_a = \lambda \int dP\, p^\nu (g_a - \bar{g}_a) \tag{2.9}$$

with $\bar{g}_a$ being the distribution function of antiquarks.

To explain the physical meaning of the quark (antiquark) distribution function $g$ we observe that the integral of $g$ over an arbitrary volume of phase space gives the number of quarks (antiquarks) in this volume:

$$\int_{\Delta V \Delta P} d\sigma_\mu dP\, p^\mu g(x,p) = \Delta N_q \ , \tag{2.10}$$

where $d\sigma_\mu$ is the volume element in $x$ space, $d\sigma_0 = dV$ in the rest frame of the volume. Similarly, such an integral of $G_{AB}$ is proportional to the density matrix of quarks (antiquarks) in the color space:

$$\int_{\Delta V \Delta P} d\sigma_\mu dP\, p^\mu G_{AB}(x,p) = \Delta N_q \rho_{AB} \ . \tag{2.11}$$

Finally, we note that the energy-momentum tensor of the plasma is

$$T^{\mu\nu}_{\mathrm{mat}} = \int p^\mu p^\nu (g + \bar{g}) dP \ . \tag{2.12}$$

It satisfies the conservation law

$$\partial_\mu T^{\mu\nu}_{\mathrm{mat}} + \partial_\mu T^{\mu\nu}_{\mathrm{YM}} = 0 \ ,$$

where $T^{\mu\nu}_{\mathrm{YM}}$ is the energy-momentum tensor of the color field, provided the functions $g$ and $\bar{g}$ vanish sufficiently rapidly for $p^\mu \to \infty$.

## III. IMPLEMENTATION OF BOOST INVARIANCE

As explained in the Introduction, our aim is to discuss the boost-invariant solutions of the Boltzmann-Vlasov equations (2.6) to (2.8) which may, perhaps, be applicable to plasma created in very-high-energy nuclear collisions. Boost invariance implies quite severe restrictions on the gauge fields $A^a_\mu$ and on the quark (antiquark) distribution functions. In this section we shall explore these conditions in order to simplify the general system of Eqs. (2.6) to (2.8).

Let us first consider the gauge fields:[10] For transverse components we have

$$A^a_{x,y} \equiv \vec{A}^a(\vec{r}, t) = \vec{A}^a(\vec{s}, u) \ , \tag{3.1}$$

where $\vec{s} = (x,y)$ is the transverse position vector and $u$ is the boost-invariant variable

$$u = t^2 - z^2 \ . \tag{3.2}$$

For longitudinal and time components we have[10]

$$A^a_z(\vec{r}, t) = A^a_0(\vec{r}, t) = \psi^a(\vec{s}, u)/(t - z) \ . \tag{3.3}$$

For quark (antiquark) distributions one has

$$f(\vec{r}, t; \vec{p}, E) = f(\vec{s}, u; \vec{p}_\perp, m^2, w) \ , \tag{3.4}$$

where $f$ stands for any of the $g$, $\bar{g}$, $g_a$, and $\bar{g}_a$ functions, $m$ is the quark mass, and $w$ is another boost-invariant variable:

$$w = p_z t - E z \ . \tag{3.5}$$

We shall now express the operators entering Eqs. (2.6) and (2.7), using formulas (3.1) to (3.5). Using the definition (2.3) and Eqs. (3.1) to (3.5) we obtain

$$p^\gamma F^a_{\mu\nu} \partial^\mu_p f = -2v \dot{\psi}^a f' - (\vec{p}_\perp \cdot \vec{\nabla} \psi^a - \lambda f_{abc} \psi^b \vec{A}^c \cdot \vec{p}_\perp) f' - 2v \dot{\vec{A}}^a \cdot \vec{\nabla}_p f - \frac{E - p_z}{t - z} (\vec{\nabla} \psi^a + \lambda f_{abc} \vec{A}^b \psi^c) \cdot \vec{\nabla}_p f + F^a_{12} \vec{n} \cdot (\vec{p}_\perp \times \vec{\nabla}_p f) \ , \tag{3.6}$$

where

$$v = Et - p_z z = [w^2 + (m^2 + p_\perp{}^2) u]^{1/2} \ , \tag{3.7}$$

and $\vec{n}$ is the unit vector in $z$ direction. The dot denotes the partial derivative with respect to $u$ and the prime denotes that with respect to $w$. Operators $\vec{\nabla}$ and $\vec{\nabla}_p$ are defined as

$$\vec{\nabla} = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right], \quad \vec{\nabla}_p = \left[ \frac{\partial}{\partial p_x}, \frac{\partial}{\partial p_y} \right]. \tag{3.8}$$

It is also not difficult to see that

$$p^\mu \partial_\mu f = 2v\dot{f} + \vec{p}_\perp \cdot \vec{\nabla} f , \tag{3.9}$$

$$p^\mu A_\mu^a = \frac{E-p_z}{t-z} \psi^a - \vec{p}_\perp \cdot \vec{A}^a . \tag{3.10}$$

Using Eq. (3.6), and Eqs. (3.9) and (3.10), one can implement boost invariance in the basic equations (2.6) and (2.7) for $g$ and $g_a$ ($a=1,\ldots,8$). We shall, however, not write explicitly the resulting equations here since they are rather lengthy. In the present exploratory paper we shall discuss in more detail only a much simpler problem corresponding to pure longitudinal motion of the plasma, with both color field and quark-antiquark distribution functions independent of $x$ and $y$. In this case the equations for $g$ and $g_a$ simplify considerably. One has

$$2v\dot{g} \mp 2\lambda W_a g_a = 0 , \tag{3.11}$$

$$2v\dot{g}_a \mp \tfrac{1}{3}\lambda W_a g \mp \lambda d_{abc} W_b g_c - \lambda \frac{E-p_z}{t-z} f_{abc} \psi_b g_c = 0 , \tag{3.12}$$

where $W_a$ is the operator defined as

$$W_a = v\dot{\psi}^a \frac{\partial}{\partial w} + \left[ v\dot{\vec{A}}^a + \frac{\lambda}{2} \frac{E-p_z}{t-z} f_{abc} \vec{A}^b \psi^c \right] \cdot \vec{\nabla}_p . \tag{3.13}$$

Since there is no transverse motion the dependence of $f$ on $\vec{p}_\perp$ must be $\sim\delta(\vec{p}_\perp)$ and thus the expression $\vec{\nabla}_p f$ is singular at $\vec{p}_\perp = 0$. To remove such singular terms from Eqs. (3.11) and (3.12) the transverse components of the gauge fields must vanish:

$$\vec{A}^a = \dot{\vec{A}}^a = 0 , \tag{3.14}$$

and thus finally the equations for quark and antiquark distribution functions with no transverse motion read

$$\dot{g} \mp \lambda \dot{\psi}^a g_a' = 0 , \tag{3.15}$$

$$2\dot{g}_a \mp \tfrac{1}{3}\lambda \dot{\psi}^a g' \mp \lambda d_{abc} \dot{\psi}^b g_c' - \lambda f_{abc} \frac{E-p_z}{t-z_-} \psi_b g_c / v = 0 . \tag{3.16}$$

Let us now turn to Eq. (2.8) for the gluon field $A_\mu^a$. Using the boost-invariance restrictions, we obtain for the transverse components

$$4(u\dot{\vec{A}}^a)^{\cdot} + 2\vec{\nabla}\dot{\psi}^a - 2\lambda f_{abc} \dot{\psi}^b \vec{A}^c - 4\lambda f_{abc} \psi^b \dot{\vec{A}}^c + (\vec{n} \times \vec{\nabla}) F_{12}^a + \lambda f_{abc} (\vec{n} \times \vec{A}^b) F_{12}^c = \vec{j}^a , \tag{3.17}$$

where $\vec{j}_a$ is the transverse component of the current

$$\vec{j}_a = \lambda \int dP\, \vec{p}_\perp (g_a - \bar{g}_a) . \tag{3.18}$$

Adding and subtracting longitudinal and time components, one obtains two additional equations

$$4u\ddot{\psi}^a - 2\nabla^2\psi^a - 2u\,\vec{\nabla}\dot{\vec{A}}^a - 4\lambda f_{abc}\psi^b\ddot{\psi}^c - 2\lambda f_{abc}\,\vec{\nabla}(\vec{A}^b\psi^c) - 2\lambda f_{abc}(\vec{A}^b\vec{\nabla}\psi^c) - 2\lambda u f_{abc}\vec{A}^b\dot{\vec{A}}^c - 2\lambda^2 f_{abc} f_{cde}\vec{A}^b\vec{A}^d\psi^e = \lambda \int dP(w+v)(g_a - \bar{g}_a) \tag{3.19}$$

and

$$4u\ddot{\psi} + 2u\,\vec{\nabla}\dot{\vec{A}}^a + 2u\lambda f_{abc}\vec{A}^b\dot{\vec{A}}^c = \lambda \int dP(w-v)(g_a - \bar{g}_a) . \tag{3.20}$$

These equations determine the behavior of the self-consistent gauge fields $\vec{A}^a$ and $\psi^a$. For purely longitudinal motion with vanishing transverse components of the fields they simplify greatly. Equation (3.17) for transverse components vanishes identically, and Eqs. (3.19) and (3.20) turn into

$$4u\ddot{\psi}^a - 4\lambda f_{abc}\psi^b\ddot{\psi}^c = \lambda \int dP(w+v)(g_a - \bar{g}_a) , \tag{3.21}$$

$$4u\ddot{\psi}^a = \lambda \int dP(w-v)(g_a - \bar{g}_a) . \tag{3.22}$$

In the next section we shall discuss some simple solutions of Eqs. (3.15) and (3.16), and (3.21) and (3.22).

## IV. A SIMPLE SOLUTION OF BOLTZMANN-VLASOV EQUATIONS FOR BOOST-INVARIANT PLASMA

Equations (3.15) and (3.16), and (3.21) and (3.22), describe the behavior of boost-invariant plasma having no transverse velocity and extending infinitely in the $x$-$y$ plane. To have some idea of this behavior, we shall now present a class of solutions of these equations and discuss its physical significance. To keep the discussion as elementary as possible, we shall restrict ourselves to distribution functions $G$ [as defined by Eq. (2.4)] in the simplest possible form

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G \end{bmatrix} . \tag{4.1}$$

This assumption implies that there are only quarks (antiquarks) of one color. It is certainly a simplification which misses some potentially interesting physics related to the non-Abelian character of our problem. However, it allows for an explicit (and nontrivial) solution of Boltzmann-Vlasov equations which, as shown below, reveals some interesting phenomena. Discussion of more complicated situations including quarks of different colors and transitions between them would, of course, also be very interesting. We hope to pursue this problem in further publications.

The form (4.1) and Eq. (2.4) imply that only the functions $g$ and $g_8$ are nonvanishing. It is then natural to assume [as seen from Eqs. (3.21) and (3.22)] that only $\psi_8$ is nonvanishing. Using these conditions, Eqs. (3.15) and (3.16) simplify to

$$2\dot{G}\mp\lambda\dot{\Psi}G'=0\ , \tag{4.2}$$

where

$$\Psi=-2\psi_8/\sqrt{3}\ . \tag{4.3}$$

Furthermore, it is readily seen from Eqs. (3.21), (3.22), and (2.4) that the field $\Psi$ satisfies the equation

$$\begin{aligned}4u\ddot{\Psi}&=\tfrac{2}{3}\lambda\int dP(G-\bar{G})(w+v)\\&=\tfrac{2}{3}\lambda\int dP(G-\bar{G})(w-v)\ .\end{aligned} \tag{4.4}$$

To solve Eq. (4.2) we observe that the characteristic lines $w=w(u)$ obey the equation

$$\frac{dw}{du}=\mp\frac{\lambda}{2}\dot{\Psi}\ , \tag{4.5}$$

and thus

$$w-w_0=\mp\frac{\lambda}{2}[\Psi(u)-\Psi(u_0)]\equiv\mp\lambda H(u,u_0)\ , \tag{4.6}$$

where $w_0=w(u_0)$.

On the other hand, it is seen from Eq. (4.2) that $G(u,w)$ is constant along the characteristic lines given by Eq. (4.6). Thus we obtain the solution in the form

$$G(u,w)=G(u_0,w\pm\lambda H(u,u_0))\ . \tag{4.7}$$

We see that the solution of Eq. (4.2) is easily obtained from initial conditions at $u=u_0$, provided that the gauge field $\Psi(u)$ is known. This gauge field must, in turn, be determined from Eq. (4.4). We shall now turn to this problem.

We observe that the second equality in Eq. (4.4) implies

$$\int dP(G-\bar{G})v=0\ . \tag{4.8}$$

Taking into account that

$$\begin{aligned}dP&=dp_z dE\ \delta(E^2-p_z{}^2-m^2)\\&=dv\,dw\ \delta(v^2-w^2-m^2u)=\frac{dw}{2v}\ ,\end{aligned} \tag{4.9}$$

we see that the natural way to satisfy the condition (4.8) is to require that the difference $G-\bar{G}$ be an odd function of $w$. Since both $G$ and $\bar{G}$ are positive, we conclude that

$$G(u,w)=\bar{G}(u,-w)\ . \tag{4.10}$$

This condition is consistent with the solution (4.7). Indeed, if Eq. (4.10) is satisfied at $u=u_0$, Eq. (4.7) guarantees that it is satisfied at all $u$.

Using Eqs. (4.6), (4.9), and (4.10), Eq. (4.4) can be written as

$$4u\ddot{H}=\tfrac{1}{3}\lambda\int\frac{dw\,w}{v}G(u_0,w+\lambda H(u,u_0))\ . \tag{4.11}$$

In such a way the gauge field $\Psi$ is determined by the quark distribution function at $u=u_0$ and the values of $\Psi(u_0)$ and $\dot{\Psi}(u_0)$.

To continue our discussion, it is necessary to assume some initial conditions for quark distribution functions. In the present paper we restricted ourselves to the discussion of the simplest case of the static initial conditions. This will be described in the next section.

## V. STATIC INITIAL CONDITIONS

We shall assume that at a certain initial time $t_0$ the quark and antiquark distributions are static at $z=0$, i.e.,

$$G(u_0,w)=d\delta(w)\ . \tag{5.1}$$

It seems to us a natural starting point and in the present preliminary analysis we restrict ourselves to this case. We would like to stress, however, that other initial conditions can also be treated using the general method outlined in Sec. IV.

Equations (5.1) and (4.12) imply

$$G(u,w)=d\delta(w-m\sqrt{u}\,\Pi)\ , \tag{5.2}$$

where

$$\Pi=-\frac{\lambda H}{m\sqrt{u}}\ . \tag{5.3}$$

From Eqs. (4.12) and (5.2) we obtain the following equation for $H$

$$\ddot{H}=\frac{\lambda d}{12u}\frac{\Pi}{(1+\Pi^2)^{1/2}}\ . \tag{5.4}$$

To obtain an insight into the character of the solutions of Eq. (5.4) it is instructive to consider the asymptotic solution at $u=\tau^2\to\infty$. As is shown in Appendix A the solution is oscillating

$$H\propto\tau^{3/4}\sin(\Omega\tau^{1/2}+\text{const})\ , \tag{5.5}$$

with $\Omega^2=\frac{4}{3}\lambda^2 d/m$.

The field strength $F^{03}\sim\dot{H}$ behaves as

$$F^{03}\propto\tau^{-3/4}\cos(\Omega\tau^{1/2}+\text{const})\ , \tag{5.6}$$

and thus is also oscillating and eventually tends to 0. Finally, the behavior of $m\Pi$ which, as seen from Eqs. (3.5) and (5.2), determines longitudinal momentum of the quarks at $z=0$ is

$$m\Pi\propto\tau^{-1/4}\sin(\Omega\tau^{1/2}+\text{const})\ . \tag{5.7}$$

We have again oscillations with very slowly decreasing amplitude.

It is interesting to speculate what is a realistic range of parameters, i.e., what values of $d$, $m$, and $\tau_0$ are relevant for high-energy collisions of heavy ions. Following Bjorken[3] we take $\tau_0=1$ fm and we find for energy density at $\tau=\tau_0$

$$dE/dV=n_{\text{eff}}\frac{dE}{dy}\frac{1}{2\tau_0}\ , \tag{5.8}$$

where $n_{\text{eff}}$ is the effective number of "wounded" nucleons per unit of the transverse surface and $dE/dy$ is the energy density (per unit of rapidity) in nucleon-nucleon collisions. This quantity should be equal to the sum of the energy densities of quarks and antiquarks. We thus obtain

$$\frac{md}{\tau_0}=n_{\text{eff}}\frac{dE}{dy}\frac{1}{2\tau_0} . \tag{5.9}$$

This gives

$$md \lesssim \tfrac{1}{2} n_{\text{eff}}\frac{dE}{dy} . \tag{5.10}$$

Using $dE/dy \simeq 2$ GeV, and[11] $n_{\text{eff}}=2\ \text{fm}^{-2}$ we obtain

$$md \lesssim 2\ \text{GeV}\,\text{fm}^{-2} . \tag{5.11}$$

On the other hand, a similar argument applied to particle multiplicities yields

$$d=n_{\text{eff}}\frac{dn}{dy} , \tag{5.12}$$

where $dn/dy$ is the density of produced particles (per unit rapidity) in nucleon-nucleon collisions. Taking $dn/dy \sim 4$ we have

$$d \simeq 8\ \text{fm}^{-2} , \tag{5.13}$$

and then from Eq. (5.11) we obtain[12]

$$m \simeq 250\ \text{MeV} . \tag{5.14}$$

It should be emphasized here that the values (5.13) and (5.14) refer to the average situation. Fluctuations with much larger densities are possible.[10,13]

One remaining quantity, which we found difficult to estimate, is the intensity of the chromoelectric field $F_{03}$ at the initial time $t=t_0$. Unfortunately, the results are quite sensitive to this parameter. To fix it, we used its relation to the average transverse momentum of the quarks produced by tunneling in the uniform chromoelectric field (see Ref. 14 and references quoted there). We have

$$\langle p_\perp{}^2+m^2\rangle \simeq \lambda F_{03}/4\pi , \tag{5.15}$$

i.e.,

$$F_{03}=4\pi\langle p_\perp{}^2+m^2\rangle/\lambda . \tag{5.16}$$

Assuming $\langle p_\perp{}^2+m^2\rangle \simeq (200\ \text{MeV})^2$ [in line with our value of the effective transverse mass given by Eq. (5.14)] and $\lambda=1$, we thus obtain

$$F_{03}=F_{03}^{8} \simeq 10\ \text{fm}^{-2} . \tag{5.17}$$

In further calculations this value of $F_{03}^{8}=2\sqrt{3}\dot{H}$ was used. One should keep in mind, however, that it may well be different in real plasma.

In Fig. 1 the quantity $m\Pi$, i.e., the value of the quark momentum at $z=0$, is plotted versus $\sqrt{t}$ for $m=200$ MeV $=1$ fm, $\lambda=1$, $F_{03}^{8}=10\ \text{fm}^{-2}$, and three values of $d$. One sees a rather different behavior for the different $d$ values. For large $d$ oscillations set in already at $t \sim 2$ fm whereas for small $d$ the magnitude of the quark momentum increases up to $t \sim 25$ fm (which, most likely, is already a completely unphysical region). This behavior is easy to understand: at large densities the field generated by $q\bar{q}$ forces is strong and can quickly stop quarks and antiquarks moving under the influence of the initial field.

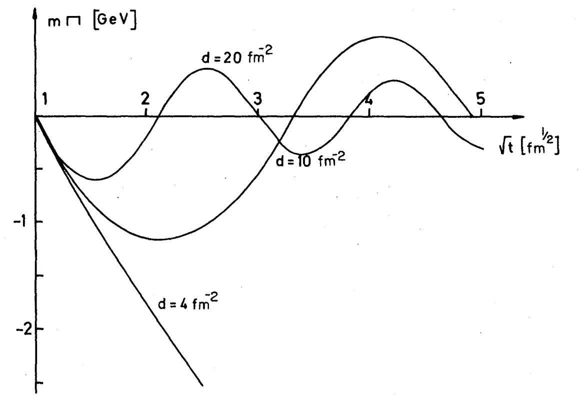


FIG. 1. Momentum of quarks at $z=0$ plotted versus $\sqrt{t}$.

At small densities the self $q\bar{q}$ field is weak and the process takes much longer time. In fact, as is easily seen from Eq. (5.4), in the limit $d \to 0$ there are no oscillations.

In Fig. 2 the field strength $F_{03}^{8}$ is plotted versus $\sqrt{t}$. The observed pattern is similar to that seen in Fig. 1.

The acceleration of quarks and antiquarks by a chromoelectric field, as exhibited in Figs. 1 and 2, implies that they should emit electromagnetic radiation. The emission rate can be estimated using the well-known formula[15] for energy loss of an accelerated charged particle. As shown in Appendix B, the resulting energy loss per unit time and unit rapidity is

$$\frac{d\epsilon}{dy\,dt}=\tfrac{2}{3}d\frac{q^2}{4\pi}S\left[\frac{d\Pi}{dt}\right]^2 . \tag{5.18}$$

Here $q$ is the electric charge of the quark and $S$ is the transverse size of the volume occupied by plasma.

In Fig. 3 $d\epsilon/dy\,dt$ is plotted versus $t$ for $S=100\ \text{fm}^2$ (which seems a realistic value for central collisions of two heavy nuclei[11]) and for

$$\frac{q^2}{4\pi}=\tfrac{1}{2}\left(\tfrac{1}{9}+\tfrac{4}{9}\right)\tfrac{1}{137} . \tag{5.19}$$

Other parameters are the same as in Figs. 1 and 2.

One sees from Fig. 3 that the loss of energy by electromagnetic radiation is rather substantial and thus may even be perhaps another observable feature of the plasma.[16] The estimate of the frequency spectrum would also be very interesting. It is much more sensitive, however, to

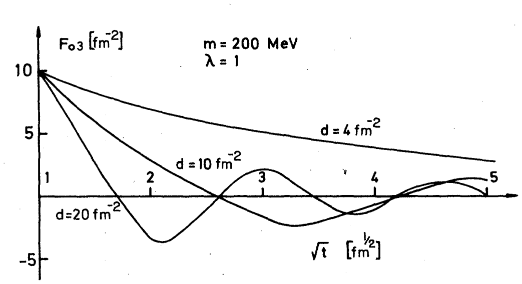


FIG. 2. Chromoelectric field intensity plotted versus $\sqrt{t}$.

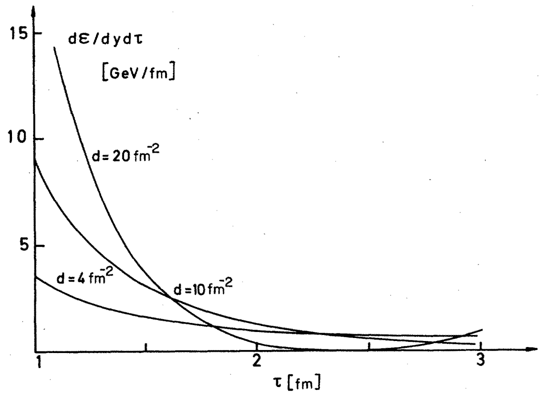


FIG. 3. Electromagnetic energy loss per unit rapidity and unit of time.

the details of the assumed initial conditions. We thus feel that this problem should wait till more realistic initial conditions are analyzed.

Nevertheless, one sees from Fig. 3 that quarks and antiquarks may be a rather efficient source of electromagnetic radiation. We feel it might be worthwhile to look for this effect experimentally.[16]

## VI. CONCLUSIONS AND OUTLOOK

In an attempt to generalize Bjorken's boost-invariant description[3] of quark-antiquark plasma possibly created in heavy-ion collisions, we have applied to this problem the Boltzmann-Vlasov equations formulated recently for color interactions by Heinz.[7] Following the Bjorken treatment, we have restricted ourselves to the purely longitudinal motion of the plasma, extending infinitely and uniformly in the direction transverse to the direction of motion. The Boltzmann-Vlasov equations for this one-dimensional problem were written down. Even in this simple case, however, they are coupled differential equations for 18 components of $q$ and $\bar{q}$ color densities and 8 components of the gauge fields. They seem to be rather difficult (but probably not impossible) to solve. To obtain some insight into the physics of the problem, we thus considered an even more restricted case, namely, when only quarks (antiquarks) of one color are present in the plasma. In this case all non-Abelian features of the equations disappear and they simplify considerably.

We have found an explicit solution for such a one-color system in terms of initial conditions at a given initial proper time $t_0$. A particular solution for static initial conditions (i.e., all quarks and antiquarks at rest at $t=t_0$ and $z=0$) was discussed in some detail. This solution describes a damped oscillatory motion: quarks and antiquarks vibrate in the self-consistently created mean chromoelectric field. The effect is simple to understand: the movement of quarks and antiquarks in opposite directions implies an attractive force between them and thus we obtain oscillations. The damping of oscillations is an effect of the longitudinal expansion (implied by the condition of boost invariance[3]).

One important consequence of these oscillations is that at a given point, for example, $z=0$, the longitudinal momentum of quarks and antiquarks fluctuates. These fluctuations can be translated into fluctuations in rapidity and thus perhaps be an observable feature of the plasma. Another potentially observable effect is the electromagnetic radiation. We estimated it and found that it may perhaps be important in the total-energy balance.

The frequency of the momentum oscillations depends primarily on the ratio $d/m$, where $d$ is the initial density of the plasma at $t=t_0$ and $m$ is the effective (transverse) mass of the quarks (antiquarks). For large $d/m$ the frequency is large. The magnitude of oscillations and the effective time after which they dominate the behavior of plasma depend on ratio $\mathscr{E}/d$ where $\mathscr{E}$ is the intensity of the chromoelectric field at $t=t_0$. For large $\mathscr{E}/d$ the amplitude of oscillations is large but they show up only at unrealistically large times.

We thus conclude that taking into account the self-interaction of the plasma leads to a rather dramatic qualitative change in its behavior,[17] which may perhaps be observable.[18]

Unfortunately, this conclusion refers, strictly speaking, only to the rather idealized case which we investigated, and it shall require further work to estimate to what extent the effects indicated in our analysis have a chance to survive in a more realistic situation. In particular, it would be interesting to know the consequences of (a) finite transverse size of the system, (b) the transverse motion of the plasma, (c) more realistic initial conditions (e.g., Maxwell distribution of $q$ and $\bar{q}$ velocities), and (perhaps most important) (d) the possibility of creation and annihilation of $q\bar{q}$ pairs. All these effects will tend to smear the clear-cut picture described above.

Finally, let us remark that, from the theoretical point of view, it is of obvious interest to investigate the non-Abelian terms of the equations, i.e., to allow for transition of color between quarks (this amounts to considering a nondiagonal color density matrix). We are presently studying this question.

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## APPENDIX A: ASYMPTOTIC SOLUTION OF EQ. (5.4)

We seek a solution in the form

$$H(u)=u^{\gamma}\sin(\Omega u^{\beta}) , \tag{A1}$$

with $\gamma<\frac{1}{2}$ so that $\Pi\to 0$ at $u\to\infty$. With these conditions we have

$$\Pi/(1+\Pi^2)^{1/2}\to\Pi \quad \text{at } u\to\infty , \tag{A2}$$

and thus Eq. (5.4) can be written as

$$\ddot{H} = -\frac{\lambda^2 d}{12m} u^{-3/2} H \; . \tag{A3}$$

Substituting Eq. (A1) into Eq. (A3), we obtain

$$\gamma(\gamma-1)u^{\gamma-2}\sin(\Omega u^{\beta}) + (2\gamma+\beta-1)\beta\Omega u^{\gamma+\beta-2}\cos(\Omega u^{\beta}) - (\beta\Omega)^2 u^{\gamma+2\beta-2}\sin(\Omega u^{\beta}) = -\frac{\lambda^2 d}{12m} u^{\gamma-3/2}\sin(\Omega u^{\beta}) \; . \tag{A4}$$

We thus conclude that $2\gamma+\beta-1=0$ and $\gamma+2\beta-2=\gamma-\frac{3}{2}$. Consequently, we obtain

$$\beta = \tfrac{1}{4}, \quad \gamma = \tfrac{3}{8} \; . \tag{A5}$$

Furthermore, it follows from Eq. (A4) that $(\beta\Omega)^2 = \lambda^2 d/12m$, i.e.,

$$\Omega^2 = \frac{4}{3}\frac{\lambda^2 d}{m} \; . \tag{A6}$$

## APPENDIX B: ELECTROMAGNETIC ENERGY LOSS

The energy loss per unit of time of an accelerated particle with electric charge $q$ is given by

$$\frac{dE_{\text{elec}}}{dt} = \frac{2}{3}\frac{q^2}{4\pi} a^2 \; , \tag{B1}$$

where $a$ is the four-acceleration. Observing that the four-vector

$$(\vec{\Pi}, \Pi_0 = (1+\Pi^2)^{1/2})$$

is just the four-velocity of quarks, we have

$$a^2 = \left[\frac{d\Pi_0}{ds}\right]^2 - \left[\frac{d\Pi}{ds}\right]^2 = -\left[\frac{d\Pi}{ds}\right]^2 \Big/ \Pi_0^2 = -\left[\frac{d\Pi}{dt}\right]^2 , \tag{B2}$$

and thus

$$\frac{dE_{\text{elec}}}{dt} = -\frac{2}{3}\frac{q^2}{4\pi}\left[\frac{d\Pi}{dt}\right]^2 . \tag{B3}$$

Consequently, the electromagnetic energy emitted by quarks and antiquarks per unit time and per unit volume of the plasma is

$$\frac{d\epsilon}{dt} = \frac{d}{t}\frac{dE_{\text{elec}}}{dt} \; , \tag{B4}$$

and the emission per unit time and per unit rapidity interval is

$$\frac{d\epsilon}{dy\,dt} = Sd\frac{dE_{\text{elec}}}{dt} \; , \tag{B5}$$

where $S$ is the transverse cross section of the plasma and where we have used the relation[3] $\Delta z = t\,\Delta y$. In heavy nuclei[11] $S$ may be of the order of 100 $\text{fm}^2$.

$d\Pi/dt$ can be calculated if the solution of the Eq. (5.4) is known. Indeed, since

$$\Pi = -\frac{\lambda}{m}\frac{H}{t} \; ,$$

we have

$$\frac{d\Pi}{dt} = -\frac{\lambda}{m}\left[\frac{1}{t}\frac{dH}{dt} - \frac{H}{t^2}\right] = -\frac{\lambda}{m}\left[2\dot{H} - \frac{H}{u}\right] . \tag{B6}$$

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[1]See, e.g., (a) *Quark Matter Formation and Heavy Ion Collisions, 1982,* proceedings of the Bielefeld Workshop, edited by M. Jacob and H. Satz (World Scientific, Singapore, 1982), and (b) *Quark Matter '83,* proceedings of the Third International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Brookhaven National Laboratory, 1983, edited by T. W. Ludlam and H. E. Wegner [Nucl. Phys. **A418**, 1C (1984)].

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coupling to the color fields were discussed by G. Baym, Phys. Lett. **B138**, 18 (1984).

[18]At this point it is worth mentioning that the existence of oscillations in $q\bar{q}$ plasma was suggested by U. Heinz [Ref. 1(b), p. 603c]. He pointed out that such oscillations may be a consequence of two-stream instability. As seen from the formulas of Sec. V and of Appendix A, our solution is stable and thus represents a different type of plasma behavior than that conjectured by Heinz.