# Anomalous spin-1-meson decays from the gauged Wess-Zumino term

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We apply a recently proposed chiral-Lagrangian model to discuss the anomalous (unnatural-parity) decays of the low-lying vector and axial-vector mesons. First, about twenty "SU(3) relatives" of the $\omega \rightarrow 3\pi$ and $\omega \rightarrow \pi^0\gamma$ processes previously treated are investigated. The results confirm our confidence in the reliability of the method employed. We then claim that the obscure reaction $D(1285) \rightarrow \rho\pi\pi$ is an important prototype for studying anomalous processes which do not go through a vector-vector-pseudoscalar vertex. The present model based on a gauging (with phenomenological fields) of the Wess-Zumino term gives a reasonable description of this decay. It also suggests the suppression of processes like axial-vector meson$\rightarrow$vector meson plus photon. In the course of this work we take a fresh look at the $A_1$-$\rho$-$\pi$ system in the chiral-Lagrangian framework. We note that it is possible to explain together the $A_1$ mass and the widths for $A_1 \rightarrow \rho\pi$, $\rho \rightarrow 2\pi$, and $A_1 \rightarrow \pi\gamma$.

## I. INTRODUCTION

It seems generally accepted that regardless of what approach turns out to be best suited for describing QCD at low energies, the results should be describable by a chiral effective Lagrangian. In the limit of vanishing energy the relevant fields are members of the pseudoscalar nonet. The resulting $\sigma$-type models have been widely discussed in the literature. Recently Witten[1] has revived interest in the Wess-Zumino term[2] of this Lagrangian which is related to the non-Abelian flavor anomaly and which gives pseudoscalar vertices proportional to the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$. The simplest purely hadronic reaction mediated by this term is of the rather complicated type $K\bar{K} \rightarrow 3\pi$. Thus, it is very interesting to extend the chiral Lagrangian by introducing additional fields and to study vertices proportional to $\epsilon_{\mu\nu\alpha\beta}$. This extension should also increase the range of validity of the chiral Lagrangian away from zero energy up to 1 GeV or so.

The "classical" approach to this problem involves maintaining the chiral symmetry in a straightforward way by introducing vector and axial-vector mesons. This type of model[3] was of great theoretical interest about fifteen years ago. Very recently, it was applied[4] to the case where the Wess-Zumino term was present. The resulting description of the decays of the vector meson $\omega$ is quite promising. In the present paper we would like to further investigate that model. Before starting, it may be helpful to make some brief remarks to avoid confusion relating to the history of this subject.

In the effective-Lagrangian approach a truncation of the spectrum is of course necessary. For simplicity the scalar mesons will be neglected. (They could easily be included by using the linear rather than the nonlinear spin-zero-meson realization.) The vector mesons could also be chosen to transform nonlinearly under chiral transformations (this approach will in fact be given elsewhere[5]), but in the present context the presence of axial-vector fields seems very natural. Assuming that one has agreed to construct a chiral Lagrangian out of the $0^-$, $1^-$, and $1^+$ nonets, how should the interactions of the spin-1 fields be introduced? It seems almost inescapable to introduce them in the most symmetrical way possible and then to add additional terms to reduce this symmetry, if required by phenomenology. The most symmetrical way possible to introduce spin-1 nonets seems to be as gauge particles of chiral $U(3)\times U(3)$. The minimal additional terms which break the gauge symmetry but preserve $U(3)\times U(3)$ are degenerate mass terms. There is *a priori* no need for the model to be renormalizable. It is certainly not a fundamental gauging, but, on the other hand, it does not seem to be in conflict with the underlying fundamental color gauge theory. It is natural to ask why, if there is nothing basically wrong with it, this approach to extending the chiral Lagrangian has fallen out of favor. There seem to be two reasons: (i) The most general gauge Lagrangian contains, especially if SU(3)-symmetry breaking is taken into account, a fairly large number of terms. The fitting of the arbitrary parameters to experiment is thus rather difficult. [On the experimental side, the parameters of the $A_1$ meson have been steadily changing over the last fifteen years. (In the Particle Data Group table[6] its mass is now given as 1275 MeV rather than 1090 MeV.)] (ii) A belief somehow developed that an approximate gauge symmetry at the effective-Lagrangian level was in conflict with the fundamental color gauging.

The point of view we shall take is that the approximately gauged chiral model is a useful way station on the road to finding the complete low-energy $\mathscr{L}_{\text{eff}}$. The terms proportional to $\epsilon_{\mu\nu\alpha\beta}$ seem to have a more unique structure in this approach than the others and deserve special study. Two aspects of this program will be studied here: In Ref. 4 it was argued that the terms in $\mathscr{L}_{\text{eff}}$ responsible for the decay $\omega \rightarrow \pi^+\pi^-\pi^0$ are the two anomalous ones

$$\mathscr{L}_{VV\phi} = -g_{VV\phi}\epsilon_{\mu\nu\alpha\beta}\text{Tr}(\partial^\mu V^\nu \partial^\alpha V^\beta \phi) ,$$
$$\mathscr{L}_{V\phi\phi\phi} = ih\epsilon_{\mu\nu\alpha\beta}\text{Tr}(V^\mu \partial^\nu\phi\partial^\alpha\phi\partial^\beta\phi) \quad (1.1)$$
$$(\epsilon_{0123} = +1)$$

as well as the ordinary one,

 

$$\mathscr{L}_{V\phi\phi}=\frac{ig}{2}\mathrm{Tr}(V_\mu\phi\overleftrightarrow{\partial}^\mu\phi)\ . \tag{1.2}$$

Here $V_\mu$ is the vector-meson-nonet matrix and $\phi$ the pseudoscalar-meson matrix (a tilde is being dropped for simplicity). $g$ is the gauge coupling constant[7] determined from the $\rho$-meson width. The gauging[8] of the Wess-Zumino term gave the interesting prediction for the $VV\phi$ coupling constant,

$$g_{VV\phi}=\frac{3g^2}{16\pi^2F_\pi}\ , \tag{1.3}$$

where $F_\pi\simeq135$ MeV is the pion decay constant. The coefficient $h$ of the $V\phi\phi\phi$ term was also predicted to be

$$h=\frac{-g}{2\pi^2F_\pi^{\ 3}}\left[1-\tfrac{3}{4}\left[\frac{g^2F_\pi^{\ 2}}{m_\rho^{\ 2}}\right]+\tfrac{3}{32}\left[\frac{g^2F_\pi^{\ 2}}{m_\rho^{\ 2}}\right]^2\right],$$

and turned out to give only a small contact contribution to $\omega\rightarrow3\pi$. Notice that the $V\phi\phi$ interaction in (1.2) is taken to be of minimal type. In the present framework this corresponds to a particular choice of parameters as we will discuss in Sec. III. We adopt this choice because it is the simplest possibility. (It is also the one which emerges when the vector mesons are taken to transform nonlinearly, axial-vector mesons not being present. This case will be treated elsewhere.[5]) In Sec. II, we will study the nonet partners of the prototype processes $\pi^0\rightarrow2\gamma$, $\omega\rightarrow\pi^0\gamma$, and $\omega\rightarrow3\pi$ treated in Ref. 4. This will be done using (1.1), (1.2) and (1.3), as well as the usual vector-meson—photon coupling prescription

$$\mathscr{L}_{\mathrm{EM}}=\frac{\sqrt{2}e}{g}A_\mu\left[m_\rho^{\ 2}\rho_\mu^0+\tfrac{1}{3}m_\omega^{\ 2}\omega_\mu-\frac{\sqrt{2}}{3}m_\phi^{\ 2}\phi_\mu\right]+\cdots\ . \tag{1.4}$$

As has been extensively discussed in the literature,[9] this simple model can explain a large amount of experimental data. From our point of view it tends to support the validity of the present approach. The new features include the use of (1.3), the computation of $K^*\rightarrow K\pi\pi$, and the use of a current-algebra-type symmetry-breaking ansatz which might help to explain $K^*\rightarrow K\gamma$. In Sec. II, we also compute $\eta$ and $\eta'\rightarrow\pi\pi\gamma$ both by standard current algebra (no spin-1 particles) and with the inclusion of poles. The results for $\eta'\rightarrow\pi\pi\gamma$ provide a dramatic demonstration of the need to take vector-meson poles into account when working at energies close to 1 GeV.

One might argue that the work above mostly tests the nonet model in conjunction with the $VV\phi$ vertex. It is clearly desirable to study anomalous processes mediated by different pieces of the gauged Wess-Zumino term. It seems that the most accessible reaction is the decay of the $D(1285)$ (the axial-vector $\omega$) into a $\rho$ meson and two pions. This decay is the proper axial-vector analog of $\omega\rightarrow3\pi$. From Eqs. (6.5) and (4.18) of Ref. 4 we extract the axial-vector analog of the $VV\phi$ term,

$$\mathscr{L}_{AA\phi}=\frac{-g^2}{16\pi^2F_\pi}\left[\frac{1-\gamma}{1+\gamma}\right]\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\partial_\mu A_\nu\partial_\alpha A_\beta\phi)\ . \tag{1.5}$$

Here $A_\nu$ is the physical axial-vector-meson matrix. $\gamma$ is an additional small correction to be discussed in Sec. III. It is amusing that the coefficient in (1.5) is one third of (1.3), reflecting the chiral U(3)$\times$U(3) breaking in the gauged Wess-Zumino term which satisfies the Bardeen form of the anomaly. There is also an anomalous contact term. To do the calculation one finally needs the nonanomalous $AV\phi$ vertex. This requires a detailed analysis which is carried out in Sec. III. There we show that the general gauge invariant (up to mass terms) chiral Lagrangian contains a sufficient number of free parameters to fit both the $\rho$ and $A_1$ widths. Neither the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation,[10] the Weinberg relation,[11] nor any relation between the two follows from chiral symmetry alone. As a general check on vector dominance in the axial-vector-meson region we note that it gives a value for $(A_1^+\rightarrow\pi^+\gamma)$ in agreement with the recent measurement.[12] The calculation of $D\rightarrow\rho\pi\pi$ is carried out and discussed in Sec. IV. The agreement with experiment seems reasonable, considering the experimental uncertainties and theoretical simplifications. It is also pointed out that the model predicts a number of unnatural-parity radiative decays of the axial-vector mesons to be suppressed.

Finally, in Sec. V, we briefly discuss a speculative extension of the gauged Wess-Zumino term and also two recent related papers.[13,14]

Some aspects of the present model would seem to warrant additional discussion. The first is that we are treating all particle families to be nonets rather than octets. One might wonder whether the U(1) problem is being taken into account since the $\eta'$ meson is being treated in a similar way to the pseudoscalar Goldstone octet. However, for the nonanomalous part of $\mathscr{L}_{\mathrm{eff}}$ it can be shown (see the second footnote of Ref. 4, for example) that the present treatment respects the anomalous U(1) Ward identity and becomes exact in the large-$N_c$ limit. We would like to stress that this situation is unchanged in the presence of the gauged Wess-Zumino term (satisfying either the left-right-symmetric or the Bardeen form of the anomaly). This is because, as briefly remarked in Ref. 4, the gauged Wess-Zumino term constructed with nonet pseudoscalar, vector, and axial-vector fields is invariant under global U(1)$_A$ transformations. A second question concerns the relation of our results to the current-algebra theorems [i.e., the formula (4.15) with $r=0$ of Ref. 4]. These theorems should be exact in a world of zero quark masses (which is expected to imply zero pseudoscalar-octet masses). To what extent the zero-quark-mass world approximates the real world can be roughly determined by comparing the predictions of (4.15) with experiment. At characteristic energies around 100 MeV ($\pi^0\rightarrow\gamma\gamma$) the agreement is good. At characteristic energies around 1 GeV the agreement is reasonable for $\eta'\rightarrow2\gamma$ but terrible for $\eta'\rightarrow\pi^+\pi^-\gamma$. (In Sec. II the current-algebra rate is shown to be about $\frac{1}{30}$ the experimental rate for this process.) The moral seems to be that to describe processes with characteristic energies around 1 GeV it is necessary to go considerably beyond the "soft-pion" theorems. Our point of view is that a systematic way to accomplish this goal is to develop an effective Lagrangian for the particles involved which displays the underlying symmetry structure of QCD. Incidentally our formulas for processes involving pseudoscalars and photons reduce to the soft-pion theorems when the vector masses go to infinity. For processes involving spin-1 mesons there are not any rigorous theorems which are the analogs of the spin-0 ones.

## II. OTHER ANOMALOUS PROCESSES WITH VECTOR MESONS

It is interesting to explore the "SU(3) partners" of the reactions $\omega\to3\pi$, $\omega\to\pi^0\gamma$, and $\pi^0\to\gamma\gamma$ treated in Ref. 4. This actually is more of a test of the details of the way SU(3)-symmetry breaking is introduced into the effective Lagrangian than of the anomalous vertices predicted by the present model.

First consider the anomalous vector-meson decays of the type $K^*\to K\pi\pi$. The computation is essentially identical to that of $\omega\to3\pi$ in Ref. 4, but there are a number of different modes. Isospin invariance predicts

$$\begin{aligned}&\Gamma(K^{*0}\to K^+\pi^0\pi^-)=\Gamma(K^{*+}\to K^0\pi^+\pi^0)\ ,\\&\Gamma(K^{*0}\to K^0\pi^0\pi^0)=\Gamma(K^{*+}\to K^+\pi^0\pi^0)\ ,\\&\Gamma(K^{*0}\to K^0\pi^+\pi^-)=\Gamma(K^{*+}\to K^+\pi^+\pi^-)\ ,\end{aligned}\tag{2.1}$$

so we can confine our attention to the $K^{*+}$ decays. The amplitude for

$$K_\mu^{*+}(p)\to K^a(K)+\pi^b(q)+\pi^c(q')$$

is denoted $M_\mu^{abc}$ and we find[15]

$$M_\mu^{abc}=-i\epsilon_{\mu\nu\alpha\beta}K^\nu q^\alpha q'^\beta F^{abc}\ ,\tag{2.2a}$$

$$F^{+-+}=-h+\frac{3g^3}{16\pi^2F_\pi}\left[\frac{1}{(p-K)^2+m_\rho{}^2}+\frac{1}{(p-q')^2+m_{K^*}{}^2}\right],\tag{2.2b}$$

$$F^{+00}=-\frac{3g^3}{32\pi^2F_\pi}\left[\frac{1}{(p-q)^2+m_{K^*}{}^2}-\frac{1}{(p-q')^2+m_{K^*}{}^2}\right].\tag{2.2c}$$

The third amplitude is related to the other two by isospin invariance:

$$F^{0+0}=\sqrt{2}(F^{+-+}-F^{+00})\ .\tag{2.3}$$

For $K^{*+}\to K^+\pi^-\pi^+$, the predicted rate is given by the integral

$$\Gamma(K^{*+}\to K^+\pi^-\pi^+)=\frac{m_{K^*}}{192\pi^3}\int\int dE^+dE^-[(\vec{q}^{\,-})^2(\vec{q}^{\,+2})-(\vec{q}^{\,-}\cdot\vec{q}^{\,+})^2](F^{+-+})^2\ ,\tag{2.4}$$

where $E^+$ and $E^-$ are the $\pi^+$ and $\pi^-$ energies. Carrying out the numerical integrations with the same values of the parameters as in Ref. 4 gives the predictions

$$\begin{aligned}&\Gamma(K^{+*}\to K^+\pi^-\pi^+)=10.2\ (6.2)\ \text{keV}\ ,\\&\Gamma(K^{+*}\to K^+\pi^0\pi^0)=0.05\ (0.03)\ \text{keV}\ ,\\&\Gamma(K^{+*}\to K^0\pi^+\pi^0)=20.3\ (12.3)\ \text{keV}\ .\end{aligned}\tag{2.5}$$

The values in parentheses are obtained by dividing by $(F_K/F_\pi)^2\simeq1.64$ and are probably more realistic. This correction is to be expected because in the simple version of the nonlinear $\sigma$ model being employed we are assuming

$$F_\pi=F_K=F_\eta=F_{\eta'}\tag{2.6}$$

which, at least for the $F_K$, does not quite agree with experiment. In a general current-algebra treatment (which can be accommodated in the effective-Lagrangian framework by adding derivative-type symmetry-breaking terms which would renormalize the pseudoscalar fields), the emission of a $K$ meson is always accompanied in the amplitude by a factor of $F_K{}^{-1}$. In (2.2) this corresponds to changing $F_\pi{}^{-1}$ to $F_K{}^{-1}$ in the coefficient of the pole term which very much dominates the contact term $h$. The 1984 Particle Data Group table[6] gives the bound on the sum of the three modes in (2.5),

$$\Gamma(K^*\to K\pi\pi)<26\ \text{keV}\ ,\tag{2.7}$$

so our results are consistent. It is interesting that the rate is predicted to be around the present bound.

The other anomalous decay of the type (low-mass) vector meson $\to$ three pseudoscalar mesons is the Okubo-Zweig-Iizuka-rule-violating process $\phi(1020)\to3\pi$. We regard this decay as being due to a small $\omega\phi$ mixing:

$$\begin{aligned}&\omega_\mu=\omega_{\mu p}-\epsilon\phi_{\mu p}\ ,\\&\phi_\mu=\epsilon\omega_{\mu p}+\phi_{\mu p}\ .\end{aligned}\tag{2.8}$$

The subscript $p$ denotes the physical field. In our model the intermediate $\rho$ in the dominant pole terms can be on mass shell so it is a good approximation to simply calculate $\phi\to\rho\pi$. (We have verified this explicitly by computing $\phi\to3\pi$ using a $\rho$ propagator in which $m_\rho$ is replaced[15] by $m_\rho+i\Gamma_\rho/2$.) From the anomalous $VV\phi$ interaction term we find the simple formula

$$\Gamma(\phi\to\rho\pi)=\frac{9\epsilon^2g^4\,|\,\vec{q}_\pi\,|^3}{512\pi^5F_\pi{}^2}\ ,\tag{2.9}$$

where $\vec{q}_\pi$ is the pion momentum in the $\phi$ rest frame. Using $g\approx8.66$ as in Ref. 4 and the experimental value $\Gamma_{\text{expt}}(\phi\to\rho\pi)\simeq0.62$ MeV yields $|\epsilon|=0.076$. This is in fairly reasonable agreement with the value obtained[6] from the canonical-mass-mixing model, $\epsilon=-0.058$. The domination of $\phi\to3\pi$ by $\phi\to\rho\pi$ also seems to be in accord with experiment.[6]

Next let us turn to the radiative decays involving vector mesons as either external or internal lines. We have found that the effective action (6.5) of Ref. 4 gives the standard current-algebra formulas for $\pi^0\to2\gamma$, $\eta\to2\gamma$, and $\eta'\to2\gamma$ when treated with the vector-meson-dominance prescription (1.4). To compare with experiment requires a model for mixing in the complicated system $\eta$, $\eta'$, $\iota(1440)$,. . . . For simplicity we shall here just consider $\eta$ and $\eta'$ to mix:

$$\eta=\eta_P\cos\theta+\eta'_P\sin\theta\ ,$$
$$\eta'=-\eta_P\sin\theta+\eta'_P\cos\theta \tag{2.10}$$

with an angle $\theta\simeq-18°$. To take some account of SU(3)-symmetry breaking we shall introduce $F_\eta$ and $F_{\eta'}$ as discussed above. Then one has

$$\frac{\Gamma(\eta\to2\gamma)}{\Gamma(\pi^0\to2\gamma)}=2\left[\frac{\cos\theta}{\sqrt{6}}-\frac{2\sin\theta}{\sqrt{3}}\right]^2\left[\frac{F_\pi}{F_\eta}\right]^2\left[\frac{m_\eta}{m_\pi}\right]^3 ,$$
$$\frac{\Gamma(\eta'\to2\gamma)}{\Gamma(\pi^0\to2\gamma)}=2\left[\frac{\sin\theta}{\sqrt{6}}+\frac{2\cos\theta}{\sqrt{3}}\right]^2\left[\frac{F_\pi}{F_{\eta'}}\right]^2\left[\frac{m_{\eta'}}{m_\pi}\right]^3 . \tag{2.11}$$

Comparison with the 1984 Particle Data Group tables then implies $F_\eta/F_\pi=1.28\pm0.10$ and $F_{\eta'}/F_\pi\approx1$.

There are a dozen low-lying processes of the type $V\to\phi\gamma$ or $\phi\to V\gamma$. Their widths are all related in the present model by phase space and Clebsch-Gordan factors to that for $\omega\to\pi^0\gamma$ found in Ref. 4:

$$\Gamma(\omega\to\pi^0\gamma)=\frac{3\alpha g^2|\vec{q}_\pi|^3}{64\pi^4F_\pi^{\,2}},\quad \alpha\simeq\tfrac{1}{137}\ . \tag{2.12}$$

Equation (2.12) is derived from the anomalous term (1.1) and (1.3) together with the assumption of vector-meson dominance of the electromagnetic interaction (1.4). The result of this calculation for all possible decays of this type may be conveniently summarized by the following effective term ($A_\mu$ is the photon field):

$$\epsilon_{\mu\nu\alpha\beta}A_\mu\mathrm{Tr}[Q(\partial_\nu V_\alpha\partial_\beta\phi+\partial_\beta\phi\partial_\nu V_\alpha)]\ ,$$
$$Q=\mathrm{diag}(\tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3})\ . \tag{2.13}$$

[The overall normalization is given in (2.12).] The relative squared matrix elements, numerical predictions, and experimental values are given in Table I. Symmetry-breaking corrections $F_K/F_\pi\approx F_\eta/F_\pi\approx1.28$ are incorporated and the value $|\epsilon|=0.076$ obtained from $\phi\to\rho\pi$ is taken. It is seen that the overall pattern of agreement between experiment[16] and theory is quite satisfying. Note that the correction factor $(F_\pi/F_K)^2$ reduces the $K^{*0}\to K^0\gamma$ rate to a more acceptable value.

Finally consider the decays $\eta$, $\eta'\to\pi^+\pi^-\gamma$. $\eta'\to\pi^+\pi^-\gamma$ is observed only as $\eta'\to\rho^0\gamma$ which is well accounted for above. As in the case of $\pi^0\to2\gamma$ we can compute these processes either directly from the current-algebra formula [Eq. (4.15) of Ref. 4 with $r=0$ and $A\to eA$] or by using vector-meson dominance together with the anomalous $VV\phi$ and $V\phi^3$ vertices. We find for the widths

$$\Gamma\left[\begin{bmatrix}\eta\\ \eta'\end{bmatrix}\to\pi^+\pi^-\gamma\right]=\begin{bmatrix} m_\eta\left[\dfrac{\cos\theta}{\sqrt{6}}-\dfrac{\sin\theta}{\sqrt{3}}\right]^2\\ m_{\eta'}\left[\dfrac{\sin\theta}{\sqrt{6}}+\dfrac{\cos\theta}{\sqrt{3}}\right]^2\end{bmatrix}\times\int\int dE^+dE^-[(\vec{q}^{\,-})^2(\vec{q}^{\,+})^2-(\vec{q}^{\,-}\cdot\vec{q}^{\,+})^2]\mathscr{F}\ . \tag{2.14}$$

Calculating by the current-algebra (CA) formula and including the factor $(F_\pi/F_\eta)^2$ needed to get agreement for $\eta\to2\gamma$ yields

$$\mathscr{F}=\frac{1}{4\pi^4F_\pi^{\,4}}\begin{bmatrix}1/F_\eta^{\,2}\\ 1/F_{\eta'}^{\,2}\end{bmatrix}, \tag{2.15}$$

while using vector-meson dominance (VMD) yields

$$\mathscr{F}=\begin{bmatrix}(F_\pi/F_\eta)^2\\ (F_\pi/F_{\eta'})^2\end{bmatrix}\left[\frac{h}{g}-\frac{2g_{VV\phi}}{(q^++q^-)^2+m_\rho^{\,2}}\right]^2 . \tag{2.16}$$

Numerical integration over the appropriate phase-space volumes gives the predictions

TABLE I. Radiative vector-meson decays. Here $C=\cos\theta$ and $S=\sin\theta$ are defined in Eq. (2.10). The experimental data are taken from Ref. 6. Each mode has an additional phase-space factor $|\vec{q}|^3$. The overall normalizations follow from Eq. (2.12).

| Process | Relative squared matrix element | Width (keV) Prediction | Width (keV) Experiment |
|---|---|---|---|
| $\rho^+\to\pi^+\gamma$ | 1 | 80 | $63\pm4$ |
| $\rho^0\to\pi^0\gamma$ | 1 | 80 | $63\pm4$ |
| $K^{*+}\to K^+\gamma$ | $(F_\pi/F_K)^2$ | 29 | $51\pm5$ |
| $K^{*0}\to K^0\gamma$ | $4(F_\pi/F_K)^2$ | 117 | $75\pm35$ |
| $\omega\to\pi^0\gamma$ | 9 | 800 | $789\pm92$ |
| $\rho^0\to\eta\gamma$ | $(F_\pi/F_\eta)^2(\sqrt{3}C-\sqrt{6}S)^2$ | 38 | $72.5\pm14$ |
| $\eta'\to\rho^0\gamma$ | $3(F_K/F_{\eta'})^2(\sqrt{3}S+\sqrt{6}C)^2$ | 77 | $93.1\pm25$ |
| $\omega\to\eta\gamma$ | $(F_\pi/F_\eta)^2(C/\sqrt{3}-S\sqrt{2/3})^2$ | 5 | $3.2\pm2.6$ |
| $\eta'\to\omega\gamma$ | $3(F_\pi/F_{\eta'})^2(S/\sqrt{3}+C\sqrt{2/3})^2$ | 7 | $8.4\pm2.7$ |
| $\phi\to\eta\gamma$ | $(F_\pi/F_\eta)^2(4C/\sqrt{6}+2S/\sqrt{3})^2$ | 68 | $67.7\pm9$ |
| $\phi\to\eta'\gamma$ | $(F_\pi/F_{\eta'})^2(4S/\sqrt{6}-2C/\sqrt{3})^2$ | 1 | |
| $\phi\to\pi^0\gamma$ | $9\epsilon^2$ | 5 | $6.5\pm1.9$ |

$$\Gamma_{\rm CA}\left[\begin{bmatrix}\eta\\ \eta'\end{bmatrix}\to\pi^+\pi^-\gamma\right]=\begin{bmatrix}21.8\text{ eV}\\ 3.0\text{ keV}\end{bmatrix},$$
$$\Gamma_{\rm VMD}\left[\begin{bmatrix}\eta\\ \eta'\end{bmatrix}\to\pi^+\pi^-\gamma\right]=\begin{bmatrix}105\text{ eV}\\ 80.4\text{ keV}\end{bmatrix}, \tag{2.17}$$

which should be compared to

$$\Gamma_{\rm expt}\left[\begin{bmatrix}\eta\\ \eta'\end{bmatrix}\to\pi^+\pi^-\gamma\right]=\begin{bmatrix}40.8\pm6.9\text{ eV}\\ 93.1\pm25\text{ keV}\end{bmatrix}. \tag{2.18}$$

It is clear that the current-algebra approach, which neglects the $\rho$ pole, is very badly wrong for the $\eta'$ mode while perhaps slightly better for the $\eta$ mode. Note that the vector-meson calculation for $\eta'$ gives an essentially identical result to the previous $\eta'\to\rho^0\gamma$ calculation, indicating that the contact term is not numerically important. It is interesting to remark that the current-algebra threshold theorems in (2.15) can be recovered from (2.16) by taking the $\rho$ mass to infinity. In this limit we see that $h=-g/2\pi^2F_\pi^{\,3}$.

## III. NONANOMALOUS PART OF $\mathcal{L}_{\rm eff}$

In this section we will discuss the $A_1$-$\rho$-$\pi$ system in the chiral Lagrangian of $0^-$, $1^-$, and $1^+$ nonet particles without the Wess-Zumino-type terms. A knowledge of this aspect of the problem is required to discuss the anomalous decay modes. We should remind the reader that above 1 GeV the experimental resonance spectrum is very complicated and for practical reasons has to be truncated. Since chiral Lagrangians are extrapolations from zero momentum transfer, which, however, work well below 1 GeV, we are, of course, pushing the present model to the limit of its applicability. Considering the great interest in this region, which appears to contain glueballs, etc., it seems very important to do so.

The experimental situation concerning the axial-vector mesons—most notably the $A_1$—has an interesting history. Nowadays it seems that the existence of the $A_1$ is well established, but its exact mass and width are still uncertain. Back in the heyday of chiral Lagrangians it was generally considered that the mass of the $A_1$ was given by the Weinberg relation[11] $m_{A_1}=\sqrt{2}m_\rho=1090$ MeV. However, it was recognized that this relation involved an additional assumption. The general chiral-invariant Lagrangian, as we shall see, contains the $A_1$ mass as an arbitrary parameter. Now the Particle Data Group table lists the $A_1$ mass as 1275 MeV and its width as $315\pm45$ MeV. The new higher $A_1$ is almost degenerate with the $D(1285)$ (the axial-vector $\omega$) which is more easily understandable on the basis of "ideal mixing" in the quark model. Nevertheless, some experiments ($\tau$ decay and $K^-\rho$ production) determine a lower mass $A_1$ of different width while others which have better statistics but involve more analysis ($\pi^-p$ production) favor the higher mass.[17]

Following the notation of Ref. 4, the chiral U(3)×U(3)-invariant part of our gauged chiral Lagrangian is

$$\mathcal{L}=-\frac{F_\pi^{\,2}}{8}\mathrm{Tr}(D_\mu U D_\mu U^\dagger)-\tfrac{1}{2}\mathrm{Tr}(F^L_{\mu\nu}F^L_{\mu\nu}+F^R_{\mu\nu}F^R_{\mu\nu})$$
$$-m_0^{\,2}\mathrm{Tr}(A_{L\mu}A_{L\mu}+A_{R\mu}A_{R\mu})+\mathcal{L}_\zeta+\mathcal{L}_\gamma+\cdots . \tag{3.1}$$

The first three terms are conventional; note that $D_\mu$ is the chiral covariant derivative and that $m_0$, the degenerate spin-1 mass, breaks gauge invariance but not chiral invariance. Two more gauge-invariant terms are needed to provide an adequate description of the $\rho$-$A_1$ system:

$$\mathcal{L}_\zeta=-i\zeta\,\mathrm{Tr}(D_\mu U D_\nu U^\dagger F^L_{\mu\nu}+D_\mu U^\dagger D_\nu U F^R_{\mu\nu}) , \tag{3.2a}$$

$$\mathcal{L}_\gamma=\gamma\,\mathrm{Tr}(F^L_{\mu\nu}UF^R_{\mu\nu}U^\dagger) . \tag{3.2b}$$

The quadratic part of (3.1) works out to be

$$-\tfrac{1}{4}(1-\gamma)\mathrm{Tr}[(\partial_\mu V_\nu-\partial_\nu V_\mu)^2]$$
$$-\tfrac{1}{4}(1+\gamma)\mathrm{Tr}[(\partial_\mu A_\nu-\partial_\nu A_\mu)^2] , \tag{3.3}$$
$$-\tfrac{1}{2}\mathrm{Tr}[\partial_\mu\phi\partial_\mu\phi-gF_\pi\partial_\mu\phi A_\mu+(m_0^{\,2}+g^2F_\pi^{\,2}/4)A_\mu A_\mu] ,$$

where $V_\mu=A_{L\mu}+A_{R\mu}$ and $A_\mu=A_{L\mu}-A_{R\mu}$. This is brought to standard diagonal form with the redefinitions:

$$V_\mu=(1-\gamma)^{-1/2}\tilde V_\mu ,$$
$$A_\mu=(1+\gamma)^{-1/2}\tilde A_\mu+\frac{g\tilde F_\pi}{2m_0^{\,2}}\partial_\mu\tilde\phi ,$$
$$\phi=Z^{-1}\tilde\phi , \tag{3.4}$$
$$Z=\left[1+\frac{g^2F_\pi^{\,2}}{4m_0^{\,2}}\right]^{-1/2} ,$$
$$F_\pi=Z^{-1}\tilde F_\pi ,$$

where the tilde quantities are the "physical" ones.[18] The vector and axial-vector masses are identified as

$$m_V^{\,2}=(1-\gamma)^{-1}m_0^{\,2} ,$$
$$m_A^{\,2}=(1+\gamma)^{-1}\left[m_0^{\,2}+\frac{g^2\tilde F_\pi^{\,2}}{4}\right] . \tag{3.5}$$

It proves convenient to introduce a renormalized-coupling constant $\tilde g$:

$$\tilde g=(1-\gamma)^{-1/2}g , \tag{3.6}$$

so that $gV_\mu=\tilde g\tilde V_\mu$. Notice that we may rewrite $Z^2$ in terms of physical quantities as

$$Z^2=1-\frac{(\tilde{g}\tilde{F}_\pi)^2}{4m_V^2}=\left[\frac{1-\gamma}{1+\gamma}\right]\left[\frac{m_V}{m_A}\right]^2 . \quad (3.7)$$

The first equality in (3.7) corresponds to the KSRF relation[10] for the special choice $Z^2=\frac{1}{2}$, while the second equality would be Weinberg's formula[11] for $Z^2=\frac{1}{2}$ and $\gamma=0$. We see that in general these two relations do not imply each other. The arbitrary choices $Z^2=\frac{1}{2}$ and $\gamma=0$ do not fit the experimental data, as we shall see. Collecting terms gives the $V\phi\phi$ interaction (with a nonminimal piece)

$$\mathscr{L}_{V\phi\phi}=+\frac{i\tilde{g}}{2}\mathrm{Tr}(\tilde{V}_\mu\tilde{\phi}\overleftrightarrow{\partial}_\mu\tilde{\phi}) +\frac{i\tilde{g}\delta}{2m_V^2}\mathrm{Tr}[(\partial_\mu\tilde{V}_\nu-\partial_\nu\tilde{V}_\mu)\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}] ,$$

$$\delta=1-Z^2-2\left[\frac{Z^4}{1-Z^2}\right]\frac{\zeta\tilde{g}}{(1-\gamma)^{1/2}} . \quad (3.8)$$

Now let us agree to drop all tildes so that all quantities are the physical ones. For many purposes the following $AV\phi$ vertex which follows from (3.1) is very important:

$$\mathscr{L}_{AV\phi}=\frac{ig^2F_\pi}{4Z^2}\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\mathrm{Tr}(V_\mu[A_\mu,\phi])+\frac{ig^2F_\pi}{4M_V^2Z^2}(1-\delta)\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\mathrm{Tr}\{(\partial_\mu V_\nu-\partial_\nu V_\mu)[A_\mu,\partial_\nu\phi]\}$$
$$+\frac{ig^2F_\pi}{4m_V^2}\left[\frac{1+\gamma}{1-\gamma}\right]^{1/2}\mathrm{Tr}\{(\partial_\mu A_\nu-\partial_\nu A_\mu)[V_\mu,\partial_\nu\phi]\}-\frac{i}{F_\pi}\frac{\gamma}{\sqrt{1-\gamma^2}}\mathrm{Tr}[(\partial_\mu A_\nu-\partial_\nu A_\mu)(\partial_\mu V_\nu-\partial_\nu V_\mu)\phi] . \quad (3.9)$$

It should be remarked that the addition of SU(3)-symmetry-breaking terms with well defined chiral transformation properties will modify the formulas above. For the present we will assume that the physical masses of the various particles adequately describe SU(3)-symmetry breaking.

It is convenient to write the momentum-space amplitude for, say, $A^+_{1\mu}(p)\to\rho^+_\nu(k)+\pi^0(q)$ in the form

$$T_{\mu\nu}(p,q)=\sqrt{2}[T_S(p,q)\delta_{\mu\nu}+T_D(p,q)q_\mu q_\nu+\cdots] , \quad (3.10)$$

where the expressions for $T_S$ and $T_D$ are obtained from (3.9) and are given in the Appendix. For the physical $A_1\to\rho\pi$ decay $T_S$ is the $S$-wave and $T_D$ the $D$-wave amplitude. Other Lorentz tensor structures exist in (3.10) but they vanish both for the physical decay as well as for the application with off-shell $A_1$ to be discussed in Sec. IV. We now have the ingredients to discuss the traditional difficulty of this model—simultaneously fitting the $A_1$ mass as well as $\rho$ and $A_1$ widths. In older treatments[10] the gauge-invariant Lagrangian was most commonly described by only two parameters $g$ and $\zeta$. Now that we have included the gauge-invariant term (3.2b) it is a simple matter in principle to trade the parameters $g$, $\gamma$, and $\zeta$ for the experimental quantities $\Gamma(A_1\to\rho\pi)$, $\Gamma(\rho\to2\pi)$, and $m(A_1)$. The formulas for $\Gamma(A_1\to\rho\pi)$ and $\Gamma(\rho\to2\pi)$ are given in the Appendix. If one chooses $m(A_1)=1275$ and $\Gamma(A_1)=315$ MeV, as suggested by the Particle Data Group, one finds two possible solutions

(i) $\gamma=0.25,\ \ \zeta=0.34\ ,\ \ g=10.1\ ,$

(ii) $\gamma=-0.26,\ \ \zeta=0.065,\ \ g=7.0\ .$

Considering that both the mass and width of the $A_1$ are not yet conclusively established, it seems safer to study the system for a range of values rather than just accepting the above choices. Furthermore, the predicted physical quantities turn out to have a sensitive dependence on $g$. To understand the nature of this system it proves convenient to fix the $\rho\pi\pi$ interaction in (3.8) to be of standard type (minimal momentum dependence) by setting

$$\delta=0 . \quad (3.11)$$

This was the procedure followed in Ref. 4. With (3.11). $\Gamma(\rho\to2\pi)$ depends only on $g$. This is graphed in Fig. 1 which shows that $\Gamma(\rho)$ changes only moderately as $g$ changes [$g\approx8.66$ corresponds to $\Gamma_{\rm exp}=154$ MeV]. Figure 1 also shows the dependence of $\Gamma(A_1\to\rho\pi)$ on $g$ for various values of $m_{A_1}$ [which can, by (3.7), be regarded as a free parameter]. Note that the $A_1$ width depends strongly on $g$. For $m(A_1)=1275$ MeV, $\Gamma(A_1)=315$ MeV is predicted for the points $g\approx9.6$ and $g\approx7.6$. It is clear that these correspond, respectively, to cases (i) and (ii) above. This indicates that setting $\delta=0$ does not change the essential features of the model. We shall adopt this choice in the following. Note also that $\Gamma(A_1\to\rho\pi)$ rather than $\Gamma(A_1\to3\pi)$ is being plotted in Fig. 1. Thus, the (uncertain) experimental $A_1$ width must be interpreted as an upper bound on $\Gamma(A_1\to\rho\pi)$. In Fig. 2 we have

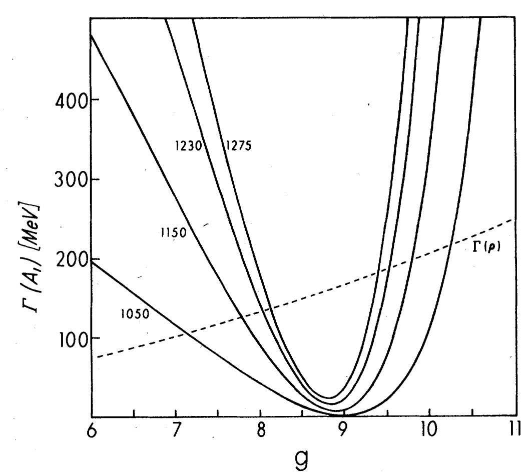


FIG. 1. $A_1\to\rho\pi$ width for four values of $m(A_1)$ plotted against the (renormalized) coupling constant $g$.

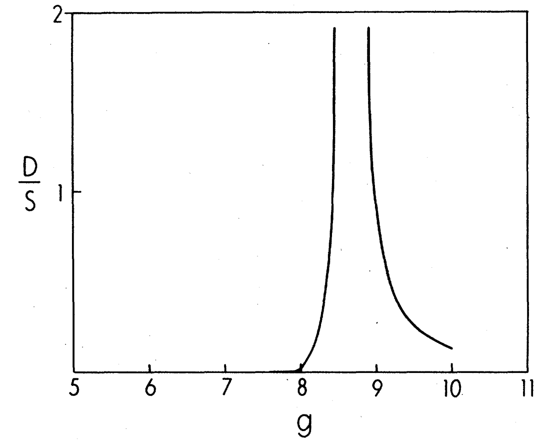


FIG. 2. The ratio of $D$-wave to $S$-wave contribution $(D/S)$ to the $A_1 \to \rho\pi$ width, plotted against $g$. Here $m(A_1)=1275$ MeV, but the curve is almost the same for $m(A_1)=1200$ MeV. Note that $D/S$ becomes negligible for $g \lesssim 8$.

displayed the ratio of the $D$-wave to $S$-wave contributions to the $A_1 \to \rho\pi$ width. Since the $\tau$ decay data[17] indicate that the $S$ wave is strongly dominant, the lower value of $g$ is favored. A simultaneous fit of $\Gamma(A_1 \to \rho\pi)$ and $\Gamma(\rho \to 2\pi)$ is seen to suggest the more recent higher values of $m(A_1)$ when we take $\delta=0$.

As a check on the assumptions of vector-meson dominance and our choice of parameters we can estimate the width for $A_1^+ \to \pi^+\gamma$, which is a *natural-parity* process. Thus, we multiply the $A_1^+ \to \pi^+\rho^0$ rate by $(\sqrt{2}\,e/g)^2$ [see Eq. (1.4)] and take the different $S$-wave phase-space factors into account. This yields

$$\Gamma(A_1^+ \to \pi^+\gamma) = \left[\frac{\sqrt{2}\,e}{g}\right]^2 \frac{|\vec{q}(A_1 \to \pi\gamma)|}{|\vec{q}(A_1 \to \pi\rho)|}\Gamma(A_1^+ \to \pi^+\rho^0)$$

$$\approx 730 \text{ keV} \tag{3.12}$$

for $\Gamma(A_1)=300$ MeV and $g=7.6$. This is consistent with the recent measurement[12] $\Gamma_{\text{expt}}(A_1^+ \to \pi^+\gamma)=640 \pm 246$ keV.

## IV. ANOMALOUS DECAYS OF AXIAL-VECTOR MESONS

An interesting feature of the present model is that it enables one to calculate unnatural-parity processes involving axial-vector mesons. To our knowledge pure hadronic processes of this type have not been previously discussed in the literature. These involve anomalous interactions different from the $VV\phi$ vertex of (1.3) which has, in one form or another, been widely treated. Since our gauging of the Wess-Zumino term involves both vector and axial-vector mesons, the same effective action which predicts the strength of the $VV\phi$ vertex also gives the anomalous axial vertices. The prototype anomalous interaction involving axials is the $AA\phi$ vertex in (1.5).

To begin we would like to remark that the structure of the effective anomalous interaction $[\Gamma'_{\text{WZ}}=\Gamma_{\text{WZ}}(U,A_L,A_R)-\Gamma_{\text{WZ}}(1,A_L,A_R)$; see (4.18) of Ref. 4] when combined with the idea of vector-meson dominance immediately gives a number of interesting predictions. These follow from the observation that $\Gamma'_{\text{WZ}}$ vanishes when $U$ is set equal to one. Expanding $U=1+2i\phi/F_\pi+\cdots$ shows that every term must contain at least one pseudoscalar field. Terms involving just spin-1 mesons will vanish. The simplest example is the anomalous $AVV$ vertex, for which we thus expect the following *virtual* processes to vanish:

$$\begin{aligned} &D \to \rho\rho,\ D \to \omega\omega,\ A_1 \to \rho\omega\ , \\ &Q \to K^*\rho,\ Q \to K^*\omega,\ Q \to K^*\phi\ . \end{aligned} \tag{4.1}$$

[Here $D$ is the $D(1285)$ and $Q$ is the appropriate strange axial-vector meson.] Complete vector-meson dominance of the electromagnetic interaction then implies

$$\begin{aligned} \Gamma(D \to \rho^0\gamma) &= \Gamma(D \to \omega\gamma) = \Gamma(A_1{}^0 \to \rho^0\gamma) = \Gamma(A_1{}^0 \to \omega\gamma) \\ &= \Gamma(Q \to K^*\gamma) = 0\ , \end{aligned} \tag{4.2}$$

as well as the vanishing of

$$\begin{aligned} &D \to \gamma + (\text{virtual } \gamma)\ , \\ &D \to \gamma + \mu^+\mu^-\ , \\ &A_1{}^0 \to \gamma + (\text{virtual } \gamma)\ , \end{aligned} \tag{4.3}$$

etc. Equation (4.3) shows that "Primakoff effect" production of $D$ and $A_1{}^0$ by an incoming photon is completely suppressed in the present approximation. The vanishing of a process like $D \to$ two real photons is, of course, guaranteed by Yang's theorem[20] (which holds for either axial or vector-meson decays). Equations (4.2) and (4.3) amount to a kind of off-shell Yang's theorem. Notice that the vertices in (4.1) would be allowed if we had used $\Gamma_{\text{WZ}}(U,A_L,A_R)$ rather than $\Gamma'_{\text{WZ}}(U,A_L,A_R)$ as our basic anomalous term. The use of $\Gamma'_{\text{WZ}}$ reproduces the $\pi^0 \to 2\gamma$ theorem with vector-meson dominance and correctly normalizes the rates for anomalous *vector-meson* decays (see Ref. 4 and Sec. II of this paper).

Now let us consider the pure hadronic unnatural-parity (anomalous) decays of the axial-vector mesons; these are the analogs of $\omega$, $\phi \to 3\pi$, and $K^* \to K\pi\pi$. One's first thought is to look for a reaction like axial-vector meson$\to$two pseudoscalar mesons. However, a parity-conserving vertex of this type is seen to vanish by partial integration. The possibilities with three-body final states are

$$\begin{aligned} &D(1285) \to \rho\pi\pi\ , \\ &E(1420) \to \rho\pi\pi\ , \\ &A_1 \to \omega\pi\pi\ , \\ &Q \to K^*\pi\pi\ . \end{aligned}$$

At present only the first of these has been measured with a width[6]

$$\Gamma(D \to \rho\pi\pi) < \Gamma(D \to 4\pi) = 10.4 \pm 3.9 \text{ MeV}\ . \tag{4.4}$$

(The experimental analysis is based on the assumption that the $4\pi$ state appears as $\rho\pi\pi$.) The branching ratio into this mode accounts for 40% of the total width. We shall study this particular reaction here. Note that isospin invariance implies equal rates for the three final states $\rho^0\pi^+\pi^-$, $\rho^+\pi^0\pi^-$, and $\rho^-\pi^0\pi^+$. For simplicity we take $D(1285)$ to be the ideally mixed combination, corresponding to its status as an axial $\omega$.

With momenta and polarizations labeled from

$$D_\alpha(p) \to \rho_\beta^+(k) + \pi^-(q^-) + \pi^0(q^0) , \quad (4.5)$$

the general amplitude $\epsilon_\alpha(D)\epsilon_\beta(\rho)A^{\alpha\beta}$ may be parametrized as

$$\begin{aligned} A^{\alpha\beta} = {} & i\epsilon^{\alpha\beta\mu\nu}(p_\mu q_\nu^- f_1 + p_\mu q_\nu^0 f_2 + q_\mu^0 q_\nu^- f_3) \\ & + i\epsilon^{\alpha\sigma\mu\nu} p_\sigma q_\mu^- q_\nu^0 [(q^-)^\beta f_4 - (q^0)^\beta f_5] \\ & + i\epsilon^{\beta\sigma\mu\nu} p_\sigma q_\mu^- q_\nu^0 [(q^-)^\alpha f_6 + (q^0)^\alpha f_7] . \end{aligned} \quad (4.6)$$

Because $D \to \rho\pi\pi$ involves two spin-1 particles the general amplitude is seen to contain seven different terms, in contrast to $\omega \to 3\pi$ which has a unique kinematical structure. Actually $f_6$ and $f_7$ turn out to vanish in the present model. The formula for the width expressed in terms of $f_1 \to f_5$ is given in the Appendix. As in the case of $\omega \to 3\pi$ there are both pole and contact contributions to the amplitude (see Fig. 3). The pole pieces require the anomalous $DA_1\pi$ vertex from (1.5) as well as the ordinary $A_1\rho\pi$ vertex of (3.10) and (A2). The contact term is obtained by collecting the $D\rho\pi\pi$ pieces in $\Gamma'_{WZ}$ using the formula (4.18) of Ref. 4 as well as (3.4) of the present paper. The resulting expression is given in the Appendix. In Fig. 4 we plot [for $m(A_1)=1275$ MeV and $m(A_1)=1200$ MeV] the numerical evaluation of $\Gamma(D \to \rho\pi\pi)$ as a function of $g$. The previous discussion of the $A_1\rho\pi$ vertex showed that there were two possible regions of $g$ (around 7.6 and 9.6) which gave acceptable $A_1$ widths. The lower values predicted a negligible $D/S$ ratio while the higher values gave a somewhat higher $D/S$ ratio. We see that the $D \to \rho\pi\pi$ calculation favors the lower values of $g$. For $g$ around 7.6 we obtain $\Gamma(D \to \rho\pi\pi) \approx 3$ to 3.5. This is a little low compared to the experimental value in (4.4) but is definitely of the right approximate size. We remark that the contributions from the two pole diagrams in Fig. 3 tend to interfere with each other. Furthermore, the contact term, though not the major piece, interferes with the sum of the two pole diagrams. To test the stability of this calculation we have computed with $T_D$ set to zero and $T_S$ taken to be constant determined to fit $\Gamma(A_1 \to \rho\pi)$. This yielded results similar to those of Fig. 4. Notice that the predicted value of $\Gamma(D \to \rho\pi\pi)$ tends to increase (in the low-$g$ range)

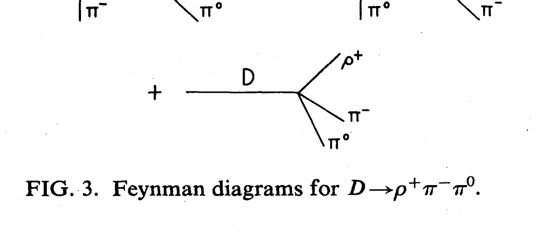


FIG. 3. Feynman diagrams for $D \to \rho^+\pi^-\pi^0$.

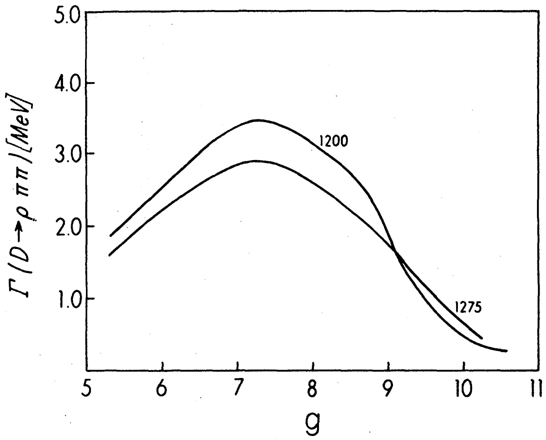


FIG. 4. $\Gamma(D \to \rho\pi\pi)$ plotted against $g$ for two values of $m(A_1)$.

with decreasing $A_1$ mass. This is due to the fact that the $A_1$ pole then gets closer to the physical region. One can imagine two ways to improve this calculation. First, one might compute in the present framework $D \to 4\pi$ rather that $D \to \rho\pi\pi$. This seems too complicated for an initial study of this process. Second, one might seek a systematic improvement of the present framework by including all sources of SU(3)-symmetry breaking as well as taking into account scalar mesons and excited pseudoscalars which exist in the same energy range as the axials. This is also extremely complicated to carry out reliably.

For completeness in case the mass of the $A_1$ turns out after all to be below 1145 MeV (the current evidence[17] favors 1275 MeV with the 1200–1275 range perhaps acceptable) we give the formula:

$$\Gamma(D \to A_1\pi) = \left[\frac{1-\gamma}{1+\gamma}\right]^2 \frac{g^4|\vec{q}\,|^3}{512\pi^5 F_\pi^{\,2}} . \quad (4.7)$$

This yields a width of about 6 MeV for $m(A_1)=1050$ MeV.

Finally we note that the value of $g$ which seems appropriate for axial-vector-meson decays seems to be roughly 10% lower than the best value for the vector-meson decays. If we use the lower value, decay amplitudes like vector $\to$ pseudoscalar + photon will be decreased by about 10% and the $\omega \to 3\pi$ amplitude by about 20%.

## V. FURTHER DISCUSSION

We have seen in Sec. II and in Ref. 4 that the present model gives quite a nice overall account of the unnatural-parity decays of the low-lying vector and pseudoscalar mesons. In order to discuss the unnatural-parity decays of the axial-vector mesons we first demonstrated (Sec. III) that the nonanomalous $\pi\rho A_1$ interaction could be satisfactorily treated in the chiral-Lagrangian framework. Then the anomalous process $D(1285) \to \rho\pi\pi$, which is apparently the only one already measured for axial-vector mesons, was found also (Sec. IV) to be reasonably well described. Experimental information on other decays of the type $A \to V\phi\phi$ and of the predicted suppression for electromagnetic transitions like $A \to V\gamma$ would be very helpful for further testing of this model.

An obvious theoretical improvement would be to add new physical particles as well as SU(3)-symmetry-breaking terms with definite chiral transformation properties. A possibly more subtle question involves the extension of the gauged Wess-Zumino term. In order to reproduce the $\pi^0 \to 2\gamma$ theorem in the present framework it was found necessary to use the Bardeen rather then the left-right-symmetric form of the anomaly,

$$\Gamma'_{WZ}(U,A_L,A_R)=\Gamma_{WZ}(U,A_L,A_R)-\Gamma_{WZ}(1,A_L,A_R) \quad (5.1)$$

with $U=\exp(2i\phi/F_\pi)$ and $\Gamma_{WZ}$ given in (4.18) of Ref. 4. It was noted in Ref. 4 that there are two theoretical problems associated with the use of (5.1) which, it was hoped, could be solved by introducing additional fields. The first (and this is also a problem for the left-right-symmetric form) is that the equations of motion imply extra constraints owing to the anomalies. The second is that (5.1) is not invariant under axial transformations, although it is invariant for vector $U(3)\times U(1)_A$. Now the structure of (5.1) strongly tempts us to make a speculative suggestion as to how both problems may be solved at once. If the theory contains another matrix $U'=\exp(2i\phi'/F'_\pi)$ which transforms like $U$ and which corresponds to fields $\phi'$ which are not excited at low energies we may simply replace (5.1) by

$$\Gamma_{WZ}(U,A_L,A_r)-\Gamma_{WZ}(U',A_L,A_R)\ . \quad (5.2)$$

Equation (5.2) is, by construction, fully $U(3)_L\times U(3)_R$ invariant and has no anomaly, so there are no constraints on the equations of motion. At low energies the predictions of the present model would be unaltered.

Finally, we remark on two related reports which appeared after Ref. 4 was written. Kramer, Palmer, and Pinsky[13] proposed to calculate anomalous vector-meson decays like $\omega\to 3\pi$ without developing an effective Lagrangian which includes spin-1 particles. Their method is based on an extrapolation of $\Gamma_{WZ}(U)$ using a form of vector-meson dominance. Their prediction for $\Gamma(\omega\to 3\pi)$ is about one third of the experimental value. However, their main goal seems to be the evaluation to the low-energy theorem for the anomalous part of the weak current. Somewhat related approaches[21] were proposed by Freund and Zee and by Rudaz. Adkins and Nappi[14] propose a simple Lagrangian which consists of the usual chiral SU(2) nonlinear $\sigma$ model to which the $\omega$ particle [treated as a chiral SU(2) singlet] is added. An anomalous $\omega\pi\pi\pi$ contact-interaction term with an arbitrary numerical coefficient [to be fit from $\Gamma(\omega\to 3\pi)$ is postulated. While such a Lagrangian seems reasonable for their purpose—setting a scale to stabilize the soliton without using a "Skyrme term"—it gives a very unrealistic description of low-energy meson dynamics. In the first place it does not include the $\rho$ mesons. Second, it is very unlikely that the $\omega\to 3\pi$ decay and the associated radiative processes discussed in Sec. II go mainly through a contact term rather than a $VV\phi$ vertex (the original Gell-Mann—Sharp—Wagner model[22]). For example, one sees experimentally[6] that $\eta'$ decays to $\rho^0\gamma$ rather than (nonresonant $\pi+\pi-\gamma$ and that $\phi\to 3\pi$ is dominated by $\phi\to\pi\rho$. Also, it was noted in Ref. 4 that in a theoretical framework which gives *both* the $VV\phi$ vertex and the contact $V\phi\phi\phi$ term [and also correctly predicts $(\omega\to 3\pi)$] the contact contribution is fairly negligible. Finally, the $VV\phi$ term is enhanced over the $V\phi\phi\phi$ term in the large-$N_C$ limit of QCD.

## ACKNOWLEDGMENTS

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## APPENDIX

Here we give formulas relating to the calculation of the nonanomalous $\rho$ and $A_1$ decays as well as the anomalous $D\to\rho\pi\pi$ decay mode.

The $\rho$ width into $2\pi$ derived from (3.8) is

$$\Gamma(\rho\to\pi\pi)=\frac{1}{12\pi}\frac{|\vec{q}\,|^3 g^2}{m_\rho{}^2}\left[1-\frac{\delta}{2}\right]^2 , \quad (\mathrm{A1})$$

where $\vec{q}$ is the momentum of one pion in the $\rho$ rest frame. Note that $g$ in (A1) is actually the renormalized quantity $\tilde{g}$. All of the following equations are written in terms of renormalized quantities.

Including off-shell pieces the $A^+_{1\mu}(p)\to\rho^+_\nu(k)+\pi^0(q)$ amplitude in (3.10) is

$$T_{\mu\nu}(p,q)=\sqrt{2}[\delta_{\mu\nu}T_S(p,q)+q_\mu q_\nu T_D(p,q)+p_\mu k_\nu R_1+p_\mu q_\nu R_2+q_\mu k_\nu R_3]\ ,$$

$$T_S(p,q)=-\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\left[\frac{1-Z^2}{Z^2F_\pi}\right][m_V{}^2+(1-\delta)k\cdot q]-\left[\frac{1+\gamma}{1-\gamma}\right]^{1/2}\left[\frac{1-Z^2}{F_\pi}\right]p\cdot q-\frac{2\gamma}{F_\pi\sqrt{1-\gamma^2}}p\cdot k\ ,$$

$$T_D(p,q)=\left[\frac{1+\gamma}{1-\gamma}\right]^{1/2}\left[\frac{1-Z^2}{F_\pi}\right]-\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\left[\frac{1-Z^2}{Z^2F_\pi}\right](1-\delta)-\frac{2\gamma}{F_\pi\sqrt{1-\gamma^2}}\ , \quad (\mathrm{A2})$$

$$R_1=\frac{2\gamma}{F_\pi\sqrt{1-\gamma^2}}\ ,$$

$$R_2=\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\left[\frac{1-Z^2}{Z^2 f_\pi}\right](1-\delta)+\frac{2\gamma}{F_\pi\sqrt{1-\gamma^2}}\ ,$$

$$R_3=\left[\frac{1+\gamma}{1-\gamma}\right]^{1/2}\left[\frac{1-Z^2}{F_\pi}\right]-\frac{2\gamma}{F_\pi\sqrt{1-\gamma^2}}\ .$$

Only $T_S$ and $T_D$ on mass shell contribute to the following formula for the width:

$$\Gamma(A_1\to\rho\pi)=\frac{|\vec{q}\,|}{2\pi m^2(A_1)}\left[\tfrac{2}{3}T_S{}^2+\tfrac{1}{3}\left[\frac{E_\rho}{m_\rho}T_S+\frac{m(A_1)}{m_\rho}|\vec{q}\,|^2 T_D\right]^2\right]. \tag{A3}$$

The nonzero amplitudes $f_1\ldots f_5$ for $D\to\rho\pi\pi$ defined in (4.6) are predicted to be

$$\begin{aligned}
f_1&=-\frac{ig^2}{16\pi^2}\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\left[\frac{2}{F_\pi{}^2}-\frac{g^2}{2m_\rho{}^2}\right]+\frac{ig^2}{8\pi^2F_\pi}\left[\frac{1-\gamma}{1+\gamma}\right]\frac{1}{(p-q^-)^2+(m_A+i\Gamma_A/2)^2}T_S(p-q^-,q^0)\ ,\\
f_2&=+\frac{ig^2}{16\pi^2}\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\left[\frac{2}{F_\pi{}^2}-\frac{g^2}{2m_\rho{}^2}\right]-\frac{ig^2}{8\pi^2F_\pi}\frac{1}{(p-q^0)^2+(m_A+i\Gamma_A/2)^2}T_S(p-q^0,q^-)\ ,\\
f_3&=\frac{ig^2}{4\pi^2}\left[\frac{1-\gamma}{1+\gamma}\right]^{1/2}\left[\frac{1}{F_\pi{}^2}-\frac{g^2}{2m_\rho{}^2}\right],\\
f_4&=\frac{ig^2}{8\pi^2F_\pi}\left[\frac{1-\gamma}{1+\gamma}\right]\frac{1}{(p-q^0)^2+(m_A+i\Gamma_A/2)^2}T_D(p-q^0,q^-)\ ,\\
f_5&=\frac{ig^2}{8\pi^2F_\pi}\left[\frac{1-\gamma}{1+\gamma}\right]\frac{1}{(p-q^-)^2+(m_A+i\Gamma_A/2)^2}T_D(p-q^-,q^0)\ .
\end{aligned} \tag{A4}$$

Finally, the formula for the $D\to\rho^+\pi^-\pi^0$ width expressed in terms of $f_1$ to $f_5$ is (in the $D$ rest frame):

$$\begin{aligned}
&\Gamma(D(p)\to\rho^+(q^+)+\pi^-(q^-)+\pi^0(q^0))\\
&\quad=\tfrac{1}{3}\Gamma(D\to\rho\pi\pi)\\
&\quad=\frac{1}{192\pi^3 m(D)}\int\int dE^-dE^0\left\{f_1{}^2\left[2(p\cdot q^-)^2-2m_D{}^2m_\pi{}^2\frac{\Delta}{m_\rho{}^2}\right]+f_2{}^2\left[2(p\cdot q^0)^2-2m_D{}^2m_\pi{}^2-\frac{\Delta}{m_\rho{}^2}\right]\right.\\
&\qquad+f_3{}^2\left[2(q^0\cdot q^-)^2-2m_\pi{}^4-\Delta\left[\frac{1}{m_D{}^2}+\frac{1}{m_\rho{}^2}\right]\right]+f_4{}^2\Delta[m_\pi{}^2-(q^-\cdot q^+)^2/m_\rho{}^2]\\
&\qquad+f_5{}^2\Delta[m_\pi{}^2-(q^0\cdot q^+)^2/m_\rho{}^2]+f_1f_2[4m_D{}^2q^0\cdot q^-+4(p\cdot q^0)(p\cdot q^-)+2\Delta/m_\rho{}^2]\\
&\qquad+f_1f_3[4m_\pi{}^2p\cdot q^0+4(p\cdot q^-)(q^0\cdot q^-)-2\Delta/m_\rho{}^2]\\
&\qquad+f_2f_3[-4m_\pi{}^2p\cdot q^--4(p\cdot q^0)(q^0\cdot q^-)+2\Delta/m_\rho{}^2]\\
&\qquad+\frac{2\Delta}{m_\rho{}^2}q^-\cdot q^+f_1f_4+f_1f_5[2\Delta(q^0\cdot q^+/m_\rho{}^2-1)]\\
&\qquad+f_2f_4[2\Delta(1-q^-\cdot q^+/m_\rho{}^2)]-\frac{2\Delta}{m_\rho{}^2}q^0\cdot q^+f_2f_5+\frac{2\Delta}{m_\rho{}^2}q^-\cdot q^+f_3f_4\\
&\qquad\left.+\frac{2\Delta}{m_\rho{}^2}q^0\cdot q^+f_3f_5+f_4f_5[-2\Delta q^0\cdot q^--2\Delta(q^-\cdot q^+)(q^0\cdot q^+)/m_\rho{}^2]\right\},
\end{aligned}$$

where $\Delta=-m_D{}^2(\vec{q}\,^-\times\vec{q}\,^+)^2$ and $f_if_j$ is an abbreviation for $\mathrm{Re}(f_if_j^*)$.

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