Nonlinear effective Lagrangian for pseudoscalar mesons in broken $SU(3)\times SU(3)$

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A symmetry-breaking term with derivative coupling $\mathrm{Tr}[(a'+b'\lambda_8+c'\lambda_3)M\partial_\mu M\partial_\mu M^\dagger+\mathrm{H.c.}]$ originating from the $(3,\bar{3})+(\bar{3},3)$ quark mass term is added to the minimal nonlinear meson Lagrangian for a unified description of $SU(3)\times SU(3)$ breaking. These effects arise from a rescaling of the meson fields which gives automatically the K_{l3} Callan-Treiman relation. SU(3) breaking for f_K/f_π and f_{η_8}/f_π is understood to come from the current quark mass m_s. Using this result, we find $\Gamma(\eta\rightarrow 2\gamma)=289\pm 20$ eV for $\theta_P=10.5°$, in good agreement with the measured value 324 ± 50 eV. It is also shown that the $K\rightarrow 3\pi$ slope parameters and relative decay rates are unaffected by this symmetry breaking.

The standard minimal effective Lagrangian given by Cronin and by Coleman, Wess, and Zumino in a nonlinear realization of chiral symmetry[1,2] is known to reproduce all the soft-pion amplitudes obtained by the current-algebra (CA) technique in the simple case[3] where $SU(3)\times SU(3)$ is broken by the meson mass term (but not the full quark mass term in the QCD Lagrangian). In particular, the successful calculation of the slope parameters and the decay amplitudes for $K\rightarrow 3\pi$ in terms of the $K\rightarrow 2\pi$ decay rates is a confirmation of the validity of the nonlinear-effective-Lagrangian technique, which, as stressed[4] by Weinberg, is more powerful than the standard CA technique, since the calculation can be performed unambiguously by summing all tree graphs generated by this effective Lagrangian. More important is the fact that terms quadratic in meson momenta can be kept, in contrast with the CA derivation where some of the external momenta must vanish. The minimal effective Lagrangian is, however, unable to account for $SU(3)\times SU(3)$-breaking effects other than those given by the meson mass term and must be modified to reproduce the Callan-Treiman relation between K_{l2} and K_{l3} form factors[5] in the $SU(2)\times SU(2)$ limit, i.e.,

$$f_+(0)+f_-(0)=\frac{f_K}{\sqrt{2}f_\pi}\ , \tag{1}$$

which expresses the deviation from exact SU(3) symmetry in terms of f_K/f_π. The minimal effective Lagrangian gives instead of (1)

$$f_-(0)=0,\quad f_K=f_\pi\ .$$

This holds because, as a result of the invariance under $SU(3)\times SU(3)$ of the kinetic term, no renormalization of the meson fields is needed, and therefore symmetry-breaking terms in the vector and axial-vector currents are absent. It is thus clear that nonminimal terms with derivative coupling must be present to account for $f_-(0)$ and f_K/f_π. Such a term must, together with the nonderivative-coupling meson mass term, originate from the quark mass term

$$\sum_q m_q(\bar{q}_R q_L+\bar{q}_L q_R)$$

in the QCD Lagrangian[6,7] and therefore transforms as the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)_L\times SU(3)_R$. With derivative coupling, this term will produce a symmetry-breaking term for the vector and axial-vector currents as well as a renormalization of the meson fields. In this paper we shall first show that the modified effective Lagrangian with a derivative coupling for the symmetry-breaking term added will lead automatically to the K_{l3} Callan-Treiman relation [Eq. (1)]. We then use this modified Lagrangian to analyze the $SU(3)\times SU(3)$-symmetry-breaking effects in $\eta\rightarrow 2\gamma$ and nonleptonic K decays.

Before writing down the effective Lagrangian, let us use the standard CA technique to obtain the derivative coupling term from the symmetry-breaking quark mass term $\mathscr{L}_{\mathrm{SB}}$:

$$\mathscr{L}_{\mathrm{SB}}=\sum_{q=u,d,s} m_q(\bar{q}_L q_R+\bar{q}_R\bar{q}_L)\ . \tag{2}$$

In the soft-pion limit, the off-shell matrix element

$$\mathscr{M}(p_1,p_2)=\langle\pi^0(p_1)|\mathscr{L}_{\mathrm{SB}}(0)|\pi^0(p_2)\rangle,\quad q=p_2-p_1 \tag{3}$$

is given by[8]

$$\mathscr{M}(p_1,p_2)\underset{p_1,p_2\rightarrow 0}{\sim}-\left(\frac{2}{f_\pi^2}\right)[p_{1\mu}p_{2\nu}\mathscr{M}_{\mu\nu}+(p_{1\mu}+p_{2\nu})\mathscr{M}_\mu] + \text{commutator piece}\ , \tag{4}$$

where

$$\mathscr{M}_{\mu\nu}=i^2\int d^4x\, d^4y\, \exp(ip_1\cdot x-ip_2\cdot y)\times\langle 0|T[A_{3\mu}(x)A_{3\nu}(y)\mathscr{L}_{\mathrm{SB}}(0)]|0\rangle\ . \tag{4'}$$

The commutator terms in (13) correspond to a nonderivative-coupling term (p_1,p_2 independent) given as

$$\mathrm{Tr}[(a+b\lambda_8)(M+M^\dagger)]\ ,$$

and nonleading terms which do not concern us here. The T-product part $\mathscr{M}_{\mu\nu}$ and other terms of the form $p_1\cdot p_2$ to leading order in pion momenta give rise to the derivative coupling $\partial_\mu\Phi_3\partial_\mu\Phi_3$ term we are looking for. Since $\mathscr{L}_{\mathrm{SB}}$ is a $(3,\bar{3})+(\bar{3},3)$ piece, the derivative-coupling part must be a $(3,\bar{3})+(\bar{3},3)$ term. The simplest choice is a linear combination of two $(3,\bar{3})+(\bar{3},3)$ terms[9] constructed from the Cronin meson-coupling matrix M which transforms linearly as a $(3,\bar{3})$ under $SU(3)_L\times SU(3)_R$:

$$\mathrm{Tr}(\alpha M\partial_\mu M\partial_\mu M^\dagger+\beta\partial_\mu M^\dagger\partial_\mu MM+\mathrm{H.c.})\ . \tag{5}$$

Our modified effective Lagrangian is then

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_m+\mathscr{L}'_m \ , \tag{6}$$

where

$$\mathscr{L}_m=\frac{1}{8f^2}\mathrm{Tr}[(a+b\lambda_8+c\lambda_3)(M+M^\dagger)] \ ,$$
$$\mathscr{L}'_m=-\tfrac{1}{2}\mathrm{Tr}[(a'+b'\lambda_8+c'\lambda_3)\partial_\mu\Phi\partial_\mu\Phi+\text{other terms}] \ . \tag{7}$$

$\mathscr{L}'_m$ is the new derivative-coupling $(3,\bar{3})$ term obtained from (5). From the quark mass matrix m_q we have

$$(a+b\lambda_8+c\lambda_3)=\lambda m_q, \quad (a'+b'\lambda_8+c'\lambda_3)=\lambda' m_q \ .$$

The total Lagrangian expanded as powers of Φ is

$$\mathscr{L}=\tfrac{1}{2}\sum_{i=1,8}(1+\alpha_i)\partial_\mu\Phi_i\partial_\mu\Phi_i-\tfrac{1}{2}\mu_i^2\Phi_i^2+\text{other terms}$$
$$\left(\Phi=\sum_{i=1,8}\Phi_i\frac{\lambda_i}{\sqrt{2}}\right) \ . \tag{8}$$

It should be stressed that the presence of a $(3,\bar{3})$ derivative-coupling term is a direct consequence of the Goldstone-boson character of the pseudoscalar mesons [see Eq. (4)]. In the past, numerous workers[10] have invoked the existence of the scalar-meson fields as well as an SU(3)-asymmetric vacuum to account for f_K/f_π. Such assumptions are not needed in our analysis, however. [In our effective Lagrangian the vacuum is SU(3)-symmetric and $\langle 0|\bar{q}q|0\rangle$ has a common value.]

For the Lagrangian (8) to describe the physics of pseudoscalar mesons, the quadratic terms with derivative coupling must be identified with the kinetic term. Hence Φ_i must be expressed in terms of the physical field operators φ_i as

$$\Phi_i=Z_i\varphi_i, \quad Z_i=(1+\alpha_i)^{-1/2} \ . \tag{9}$$

The vector and the axial-vector currents are then $(i,j,k=1,\ldots,8)$

$$V_{\mu i}=\sum_{j,k} if_{ijk}(1+\alpha_j)\Phi_j\partial_\mu\Phi_k+\cdots \ ,$$
$$A_{\mu i}=(1+\alpha_i)\partial_\mu\Phi_i+\cdots \ . \tag{10}$$

In terms of the physical meson fields, the Lagrangian (8) becomes

$$\mathscr{L}=\tfrac{1}{2}\sum_i\left(\partial_\mu\varphi_i\partial_\mu\varphi_i-\frac{\mu_i^2}{(1+\alpha_i)}\varphi_i^2\right)+\cdots \ . \tag{8'}$$

The vector and axial-vector currents are given by

$$V_{\mu i}=\sum_{j,k} if_{ijk}\left(\frac{1+\alpha_j}{1+\alpha_k}\right)^{1/2}\varphi_j\partial_\mu\varphi_k+\cdots \ ,$$
$$A_{\mu i}=\sqrt{1+\alpha_i}\frac{\partial_\mu\varphi_i}{f}+\cdots \ . \tag{10'}$$

The masses and the decay constants of pseudoscalar mesons can be read off immediately from (8′) and (10′). We have (neglecting isospin-breaking effects)

$$m_\pi^2=\frac{\lambda m_1}{1+\lambda' m_1}, \quad f_\pi=f^{-1}(1-km_\pi^2)^{-1/2} \ ,$$
$$m_K^2=\frac{\lambda m_4}{1+\lambda' m_4}, \quad f_K=f^{-1}(1-km_K^2)^{-1/2} \ , \tag{11}$$
$$\bar{m}_8^2=\frac{\lambda m_8}{1+\lambda' m_8}, \quad f_{\eta_8}=f^{-1}(1-k\bar{m}_8^2)^{-1/2} \ ,$$

where

$$m_1=\tfrac{1}{2}(m_u+m_d), \quad m_4=\tfrac{1}{2}(m_1+m_s), \quad m_8=\tfrac{1}{3}(m_1+2m_s) \ .$$

The ratio f_K/f_π is

$$\frac{f_K}{f_\pi}=\left(\frac{1-km_\pi^2}{1-km_K^2}\right)^{1/2}\simeq 1+\tfrac{1}{2}k(m_K^2-m_\pi^2) \ , \tag{12}$$

which reproduces the measured value for $k\simeq 2m_p^{-2}$. In terms of f_K/f_π, we get from (11)

$$\frac{m_\pi^2}{m_K^2}=\left(\frac{f_K^2}{f_\pi^2}\right)\frac{m_1}{m_4}, \quad \frac{m_\pi^2}{\bar{m}_8^2}=\left(\frac{f_{\eta_8}^2}{f_\pi^2}\right)\frac{m_1}{m_8} \ , \tag{13}$$

which are the expressions for the pseudoscalar-meson masses in terms of the current quark masses obtained by the CA technique. From the measured π,K masses we deduce

$$\frac{m_1}{m_4}=0.047 \ll 1 \ ,$$

which tells us that $SU(2)\times SU(2)$ is a much better symmetry[6] than $SU(3)\times SU(3)$, and for all practical purposes we can set $m_1=0$. In this limit (11) then determines f as the inverse of the pion decay constant f_π. The deviation from unity for f_K/f_π given in (12) is then due entirely to the large value of m_s.

Before getting to the Callan-Treiman relation for K_{l3} form factors, let us discuss the higher-order corrections to the meson masses. To first order in SU(3) breaking, the mass term transforms like an SU(3) octet which gives the usual Gell-Mann–Okubo (GMO) mass formula for pseudoscalar mesons. Neglecting higher-order terms in (11), we have

$$m_\pi^2=\lambda m_1, \quad m_K^2=\lambda m_4, \quad m_{\eta_8}^2=\bar{m}_8^2=\lambda m_8 \ ,$$

which clearly obeys the GMO relation satisfied by m_1, m_4, and m_8. When higher-order effects are included, the π and K masses are still given by (11) but $\bar{m}_8^2$ is not the whole η_8 mass since second order in SU(3) breaking can also contribute to η_8 mass as can be easily seen from the e term in the effective Lagrangian[1] of Cronin. The octet mass is then

$$m_{\eta_8}^2=\bar{m}_8^2+\delta\bar{m}_8^2 \ ,$$

where δm_8^2 is second order in SU(3) breaking. A possible dynamical origin for this term may be found in the quark-model $q\bar{q}$ annihilation terms via gluon intermediate states.[11] Note that to all orders in the current quark masses we find two GMO-type formulas:

$$4f_K^2=3f_{\eta_8}^2+f_\pi^2 \ , \tag{14}$$

$$4m_K^2f_K^2=3\bar{m}_8^2f_{\eta_8}^2+m_\pi^2f_\pi^2 \ . \tag{15}$$

Using the above relations, we obtain the following expression for the η-η' mixing angle θ_P:

$$\sin^2\theta_P = \frac{\sin^2\theta_0}{(1+\delta)} - \frac{(m_K{}^2 - m_\pi{}^2)}{(m_{\eta'}{}^2 - m_\eta{}^2)}\frac{\delta}{(1+\delta)} + \frac{\delta m_8{}^2}{m_{\eta'}{}^2 - m_\eta{}^2} \geq 0$$

$$\left[\delta = \frac{1}{3}\left(1 - \frac{f_\pi{}^2}{f_K{}^2}\right)\right] , \quad (16)$$

and θ_0 is the value of the mixing angle ($\theta_0 = 10.5°$) obtained from the usual GMO mass formula for $m_{\eta_8}{}^2$. It is clear from (16) that the second term is as large as the first term and cannot be neglected. This term and the first term make a *negative contribution* to $\sin^2\theta_P$, showing that $\delta m_8{}^2$ must be large enough to satisfy the positivity condition of $\sin^2\theta_P$ since $m_{\eta'}{}^2 > m_\eta{}^2$. The precise value of θ_P is thus sensitive to second-order SU(3)-breaking effects[12] other than those given by the GMO mass formula.

For the vector currents in K_{l3} decay $K^+ \rightarrow \pi^0 + l^+ + \nu_e$, we have from (10′)

$$V_\mu^{K^+} = \frac{1}{\sqrt{2}}\left[\frac{1}{2}\left(\frac{f_K}{f_\pi} + \frac{f_\pi}{k_K}\right)K^+\overleftrightarrow{\partial}_\mu\pi^0 + \frac{1}{2}\left(\frac{f_K}{f_\pi} - \frac{f_\pi}{f_K}\right)\partial_\mu(K^+\pi^0)\right] , \quad (17)$$

from which we read off immediately

$$f_+ = \frac{1}{\sqrt{2}}\frac{1}{2}\left(\frac{f_K}{f_\pi} + \frac{f_\pi}{f_K}\right), \quad f_- = \frac{1}{\sqrt{2}}\frac{1}{2}\left(\frac{f_K}{f_\pi} - \frac{f_\pi}{f_K}\right) , \quad (18)$$

which satisfies the Callan-Treiman relation [Eq. (1)]. (No variation of f_+, f_- with the momentum transfer q^2 is considered here.)

Note that f_+ is clearly of second order in SU(3) breaking in agreement with the Ademollo-Gatto theorem. Although the above expressions for K_{l3} form factors[13] are just the old CA result, the effective-Lagrangian treatment leads us beyond the physics of CA, which is rather limited in being unable to say much about the origin of SU(3)×SU(3)-symmetry breaking. The modified nonlinear effective Lagrangian gives us a unified description of all symmetry-breaking effects arising from the current quark masses. *In particular, the deviation from unity for f_K/f_π is now understood in terms of the large value of m_s.* Having established that the modified Lagrangian with derivative coupling for the symmetry-breaking term $(3,\bar{3})+(\bar{3},3)$ given by (7) is the pseudoscalar-meson effective Lagrangian in broken SU(3)×SU(3), we can now use this to analyze symmetry-breaking effects for more complicated processes involving pseudoscalar mesons usually not covered by the CA technique. As examples, we shall now discuss the electromagnetic $\eta \rightarrow 2\gamma$ decay and the nonleptonic $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ decay.

(a) Two-photon decay mode of η. In the soft-pion limit, the two-photon decay mode of π^0, η, η' proceeds via the electromagnetic Adler-Bell-Jackiw anomaly.[14] In a theory with quarks, the coefficient of the anomaly is independent of quark masses and hence flavor independent. The effective Lagrangian for π^0, η, $\eta' \rightarrow 2\gamma$ is then given by[15]

$$\mathscr{L}_{\rm EM} = -fG\,{\rm Tr}\{Q^2[\Phi + O(f^2\Phi^3)]\} ,$$

$$G = \frac{e^2N}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} , \quad (19)$$

and Q is the charge operator acting in flavor-SU(3) space. To obtain the $\eta' \rightarrow 2\gamma$ decay, we have to use nonet symmetry and include the η' field Φ_0 in Φ. Apart from the Lorentz-invariance kinematic factor $(\epsilon_{\mu\nu\rho\sigma}\epsilon_\mu\epsilon'_\nu k_\rho k'_\sigma)$ we have

$$A(\pi^0 \rightarrow 2\gamma) = -\frac{\alpha}{\pi}\left(\frac{2S\sqrt{2}}{f_\pi}\right) , \quad (20a)$$

$$A(\eta \rightarrow 2\gamma) = -\frac{\alpha}{\pi}\left(\frac{2S\sqrt{2}}{f_\pi}\right)\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_{\eta_8}}\cos\theta_P + 2\sqrt{2}\sin\theta_P\right) \quad (20b)$$

($S = \frac{1}{2}$ for quarks with 3 colors as extra degree of freedom).

Since Φ_8 is the unrenormalized field operator, the physical decay amplitude $\eta_8 \rightarrow 2\gamma$ is obtained by using the physical field φ_8 defined by Eq. (9). This explains the SU(3)-violation factor f_π/f_{η_8} in (20b) for the $\eta \rightarrow 2\gamma$ amplitude. This factor is also of first order in SU(3) breaking as is the $\sin\theta_P$ term, and cannot be neglected. Note that our expression for the $\eta \rightarrow 2\gamma$ amplitude is only correct to first order in SU(3) breaking. Using the value of f_π/f_{η_8} from (14) and taking $\theta_P = 10.5°$, we have

$$R_{\rm theor} = \frac{A(\eta \rightarrow 2\gamma)}{A(\pi^0 \rightarrow 2\gamma)} = \frac{1.27}{\sqrt{3}} = 0.73 ,$$

$$\Gamma_{\rm theor}(\eta \rightarrow 2\gamma) = 289 \pm 20 \text{ eV} , \quad (21)$$

in good agreement with the measured values[16]

$$R_{\rm expt} = 0.78 \pm 0.06, \quad \Gamma_{\rm expt}(\eta \rightarrow 2\gamma) = 324 \pm 50 \text{ eV} .$$

Note that the calculated decay rate without SU(3) violation for f_{η_8} ($f_{\eta_8} = f_\pi$) is larger than data by 20% ($\Gamma_{\rm calc} = 409 \pm 28$ eV). The success of this calculation indicates that θ_P should be around $\theta_0 = 10.5°$, the naive GMO-mass-formula value. (A value for θ_P much smaller than θ_0 would lead to a decay rate far below the data.)

(b) Nonleptonic kaon decays in broken SU(3)×SU(3). In the past ten years, since the advent of quantum chromodynamics (QCD) as a possible theory of strong interactions, the nature of nonleptonic weak interactions has been greatly clarified.[17,18] Although the origin of the $\Delta I = \frac{1}{2}$ rule remains to be explained, the form of the effective nonleptonic-decay Lagrangian is probably still the one given by Cronin many years ago. In the standard Weinberg-Salam model with left-handed currents,[18] it consists of two operators transforming as (8,1) and (27,1) representations under $SU(3)_L \times SU(3)_R$. We shall neglect the (27,1) piece and consider only the octet part which gives rise to the $\Delta I = \frac{1}{2}$ rule. The effective Lagrangian for the octet piece is given in terms of $M(f\Phi)$ as[1]

$$\mathscr{L}_{\rm NL}(\text{octet}) = -c\frac{G_F}{\sqrt{2}f^2}{\rm Tr}(\lambda_6(\partial_\mu\Phi\partial_\mu\Phi + if\{\partial_\mu\Phi, [\Phi, \partial_\mu\Phi]\})) + \mathscr{L}_{\rm NL}^{(4)} + \cdots . \quad (22)$$

By expressing Φ in terms of the physical φ we see that the symmetry-breaking effects for K-π transition, $K \rightarrow 2\pi$ decay, and the direct interactions for $K \rightarrow 3\pi$ decays from the rescaling factor $(1+\alpha_K)^{1/2}$ due to the renormalization of

Φ_K. Since SU(2)×SU(2) is almost an exact symmetry, $\alpha_\pi \approx 0$ and symmetry-breaking effects are due entirely to the rescaling of the kaon fields. We see immediately that the form of the matrix elements for K-π transition, $K_S \to \pi\pi$, and the direct $K \to 3\pi$ amplitudes are the expressions obtained by Cronin with c replaced by

$$c_{\rm eff} = c\left(\frac{f_\pi}{f_K}\right) \ . \tag{23}$$

The measured $K_S \to 2\pi$ decay rate gives

$$c_{\rm eff} = 1.1 \pm 0.1, \quad c = 1.40 \pm 0.13 \ . \tag{24}$$

With $c_{\rm eff}$ determined, the $K \to 3\pi$ amplitude can now be calculated by summing all tree graphs obtained from the total Lagrangian, the sum of $\mathscr{L}$ and $\mathscr{L}_{\rm NL}$. As explained above, SU(3)-breaking effects for the direct term (obtained from $L_{\rm NL}^{(4)}$) can be absorbed into $c_{\rm eff}$. The other contributions to $K \to 3\pi$ amplitude come from two terms. The first term is given by a K-π mixing followed by the strong process $\pi \to 3\pi$, which has no SU(3)×SU(3) breaking and can be given in the SU(2)×SU(2) limit. Hence a SU(3) violation for this term is due entirely to K-π transition and is contained in $c_{\rm eff}$. The second term comes from the $K \to K\pi\pi$ transition followed by a K-π transition. This second term, however, can be neglected in the physical decay amplitude (i.e., on-mass-shell pions) since the K-π matrix elements are proportional to $m_\pi{}^2$. Thus the $K \to 3\pi$ decay amplitude can be obtained directly from Cronin's expressions with a slight modification in the replacement of c by $c_{\rm eff}$. The $K \to 3\pi$ slope parameters and the relative amplitude in terms of the $K \to 2\pi$ rates are not affected by this replacement. The good agreement with experiment for the slope parameters and the decay amplitude must be considered as evidences for the validity of the nonlinear-effective-Lagrangian approach to nonleptonic K decays. We can also deduce from this success that $\mathscr{L}_{\rm NL}$ must have a dominant component transforming as (8,1) under $\text{SU}(3)_L \times \text{SU}(3)_R$.

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