Analysis of $\psi/J \to p\bar{p}\pi^0$, $p\bar{p}\eta$, and $p\bar{p}\eta'$ decays

Rahul Sinha and Susumu Okubo
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 27 June 1984)

We analyze available experimental data on three-body decays of $\psi/J \to \pi^0 p\bar{p}$, $\eta p\bar{p}$, and $\eta' p\bar{p}$, as well as $\omega p\bar{p}$. The ratio $\Gamma(\psi \to \eta' p\bar{p})/\Gamma(\psi \to \eta p\bar{p})$ can be understood model-independently and is consistent with the validity of the quark-line rule. With respect to $\psi/J \to \pi^0 p\bar{p}$ decay, the soft-pion theorem suggests the dominance of the proton-pole diagram near the low-pion energy region. The present experimental data are consistent with the soft-pion theorem. The same method may be applicable for $\psi/J \to \pi\Sigma\bar{\Sigma}$ and $\pi\Xi\bar{\Xi}$ decays to extract direct informations on coupling parameters $g_{\pi\Sigma\bar{\Sigma}}$ and $g_{\pi\Xi\bar{\Xi}}$. Although we can qualitatively explain the energy spectrum of $\psi/J \to \pi^0 p\bar{p}$ decay, we have some difficulty in explaining the observed energy spectrum of $\psi/J \to \eta p\bar{p}$ decay.

I. INTRODUCTION AND SUMMARY OF MAIN RESULTS

Since its discovery in 1974, the charmonium $\psi/J(3100$ MeV) (hereafter referred to as ψ for simplicity) is now known[1] to decay into more than 50 different channels. Its narrow width of 63 keV together with leptonic decays of $\psi \to e\bar{e}$ and $\mu\bar{\mu}$ can be well understood[2,3] in terms of quantum chromodynamics (QCD). However, the individual exclusive decay rates are more difficult, in general, to analyze quantitatively by QCD, although radiative decay $\psi \to \gamma\chi_c$ has been successfully explained[4] by a potential model inspired by QCD. Also, the radiative decays $\psi \to \gamma\pi^0$, $\gamma\eta$, and $\gamma\eta'$ have been studied by many authors,[5] with some reasonable agreements with experiments. With respect to purely hadronic decays, the situation becomes more difficult. So far, the only reasonable way to deal with the problem appears to be limited to uses of purely phenomenological methods. Even so, only two-body decay modes such as $\psi \to B_8\bar{B}_8$ or P_9V_9 have been studied with some success on the basis of effective two-point quark currents[6] or by the vector-dominance model,[7] or by some other method.[8–10] Here, B_8, P_9, and V_9 refer to the baryon octet, the pseudoscalar nonet, and the vector nonet, respectively, in terms of the conventional (flavor) SU(3) classification.

The purpose of this note is to analyze the three-body decay mode $\psi \to P_9B_8\bar{B}_8$. Apart from the obvious desirability of explaining available experimental data, this process turns out to be of some intrinsic theoretical interest, which will be shortly explained. First, let us briefly sketch the following experimental facts[11,12] for the three-body decays: (i) The energy spectrum of the pion in $\psi \to \pi^0 p\bar{p}$ is now known.

$$\text{(ii) } \Gamma(\psi \to \pi^0 p\bar{p}):\Gamma(\psi \to \eta p\bar{p}):\Gamma(\psi \to \eta' p\bar{p}) \simeq 1.05\pm0.14:2.32\pm0.38:0.63\pm0.39 \,, \quad (1.1)$$

$$\text{(iii) } \Gamma(\psi \to \pi^0 p\bar{p})/\Gamma(\psi \to p\bar{p}) = 0.50\pm0.06 \,, \quad (1.2)$$

$$\text{(iv) } \Gamma(\psi \to \pi^0 p\bar{p})/\Gamma(\psi \to \pi^+ n\bar{p}) = 0.52\pm0.07 \,. \quad (1.3)$$

We note that the ratios of relativistic phase volume Ω for the decays are calculated to be

$$\Omega(\psi \to \pi^0 p\bar{p}):\Omega(\psi \to \eta p\bar{p}):\Omega(\psi \to \eta' p\bar{p}) = 1.050:0.252:0.039 \,. \quad (1.4)$$

From Eqs. (1.1) and (1.4), we can estimate the corresponding ratio of averaged matrix element $M_{\rm av}$ to be

$$\left| \frac{M_{\rm av}(\psi \to \eta' p\bar{p})}{M_{\rm av}(\psi \to \eta p\bar{p})} \right| \simeq 1.32\pm0.56 \,, \quad (1.5)$$

$$\left| \frac{M_{\rm av}(\psi \to \eta' p\bar{p})}{M_{\rm av}(\psi \to \pi^0 p\bar{p})} \right| \simeq 4.02\pm1.27 \,. \quad (1.6)$$

We first study the theoretical implication of Eq. (1.5). To this end, we consider a general η-η' mixing theory where physical η and η' may be expressed as[13]

$$\eta = S_1(\cos\theta_1\eta_8 - \sin\theta_1\eta_0) + S_1'\xi \,, \qquad \eta' = S_2(\sin\theta_2\eta_8 + \cos\theta_2\eta_0) + S_2'\xi' \,. \quad (1.7)$$

Here, η_8 and η_0 are octet and singlet components of 0^- nonet P_9, respectively, while ξ and ξ' stand for any other field which could mix with η_8 and η_0. For example, ξ may be gluonium, or $c\bar{c}$ pair, or radially excited states of η and η', or their linear combinations. All other symbols S_1, S_1', S_2, S_2', θ_1, and θ_2 are some constants. The simplest standard mass-mixing theory predicts

$$S_1 = S_2 = 1, \quad S_1' = S_2' = 0, \quad \theta_1 = \theta_2 \equiv \theta \quad (1.8)$$

with $\theta \simeq -11°$ for quadratic mixing and $-24°$ for linear mixing. Some more complicated choices for these constants can be found in Refs. 13—15. It may be instructive to rewrite Eq. (1.7) in terms of quark components for η_8 and η_0 as

$$\eta = S_1 \left[\sin(\theta_0 - \theta_1) \frac{1}{\sqrt{2}} \, | u\bar{u} + d\bar{d} \rangle - \cos(\theta_0 - \theta_1) \, | s\bar{s} \rangle \right] + S'_1 \xi ,$$

$$\eta' = S_2 \left[\cos(\theta_0 - \theta_2) \frac{1}{\sqrt{2}} \, | u\bar{u} + d\bar{d} \rangle + \sin(\theta_0 - \theta_2) \, | s\bar{s} \rangle \right] + S'_2 \xi , \tag{1.9}$$

where θ_0 is the canonical mixing angle;

$$\theta_0 = \arctan \frac{1}{\sqrt{2}} \simeq 35^\circ . \tag{1.10}$$

Now, let us consider a reaction

$$A + B \rightarrow C_1 + C_2 + \cdots + C_n + \eta \text{ (or } \eta') \tag{1.11}$$

and define the ratio of their matrix elements by

$$R_0 = \frac{M(A+B \rightarrow C_1 + C_2 + \cdots + C_n + \eta')}{M(A+B \rightarrow C_1 + C_2 + \cdots + C_n + \eta)} . \tag{1.12}$$

Now, suppose that the quark-line rule[16] (or Okubo-Zweig-Iizuka rule) is exact, and that effects of the unknown field ξ as well as mass-difference between η and η' are negligible for matrix elements concerned. If all quark constituents of $A, B, C_1, C_2, \ldots, C_n$ in the reaction Eq. (1.11) do not contain any strange quark s and $\bar{s}$, then only $u\bar{u} + d\bar{d}$ quark content in the right side of Eq. (1.9) can contribute to the reaction. Therefore, we must have[13]

$$R_0 = \frac{S_2 \cos(\theta_0 - \theta_2)}{S_1 \sin(\theta_0 - \theta_1)} , \tag{1.13}$$

which is independent of any reaction mode as well as of energy. Similarly, we find

$$R_0 = \frac{M(A+B \rightarrow C_1 + C_2 + \cdots + C_n + \eta + \eta')}{M(A+B \rightarrow C_1 + C_2 + \cdots + C_n + \eta + \eta)} , \tag{1.14a}$$

$$(R_0)^2 = \frac{M(A+B \rightarrow C_1 + C_2 + \cdots + C_n + \eta' + \eta')}{M(A+B \rightarrow C_1 + C_2 + \cdots + C_n + \eta + \eta)} , \tag{1.14b}$$

under the same conditions with the same universal constant R_0 given by Eq. (1.13).

In deriving these formulas, we have neglected possible contributions from $u\bar{u} + d\bar{d}$ components contained in radially excited states. Here, we consider the effect of only the first radially excited state $\tilde{\eta}_8$ and $\tilde{\eta}_0$, although our analysis can be easily generalized for inclusion of higher radially excited states. We rewrite $S'_1 \xi$ and $S'_2 \xi'$ in Eq. (1.7) now as

$$S'_1 \xi = \tilde{S}_1 (\cos\tilde{\theta}_1 \tilde{\eta}_8 - \sin\tilde{\theta}_1 \tilde{\eta}_0) + \cdots ,$$

$$S'_2 \xi' = \tilde{S}_2 (\sin\tilde{\theta}_2 \tilde{\eta}_8 + \cos\tilde{\theta}_2 \tilde{\eta}_0) + \cdots$$

for some constants $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{\theta}_1$ and $\tilde{\theta}_2$, where terms omitted in right sides refer to all other possible excited states as well as gluonium and so on. Since both $\tilde{\eta}_8$ and $\tilde{\eta}_0$ contain $u\bar{u} + d\bar{d}$ components, these can contribute to the ratio R_0 defined by Eq. (1.12). However, contributions from $\tilde{\eta}_8$ and $\tilde{\eta}_0$ for the matrix element of the reaction $A + B \rightarrow C_1 + \cdots + C_n + \eta$ (or η') will give, in general, different forms of dependence upon both energy and channels in comparison to those from η_8 and η_0. Therefore, the universal relation Eq. (1.13) will not be in general valid, once we take into account contributions from radially excited states. Since we expect to have $|\tilde{S}_1| \ll |S_1|$ and $|\tilde{S}_2| \ll |S_2|$ as in the calculation by Frank and O'Donnell,[14] this may account for the experimentally observed small fluctuation of $|R_0|$ from a constant, dependent upon different energy and channel. However, when we have a universal mixing relation

$$\frac{\tilde{S}_2 \cos(\theta_0 - \tilde{\theta}_2)}{\tilde{S}_1 \sin(\theta_0 - \tilde{\theta}_1)} = \frac{S_2 \cos(\theta_0 - \theta_2)}{S_1 \sin(\theta_0 - \theta_1)} , \tag{1.15}$$

then Eq. (1.13) still remains valid. It is interesting to note that such a relation is approximately valid up to the second radially excited states in a model investigated by Frank and O'Donnell.[14] If coefficients involving the gluonium component contained in η and η' satisfied a similar relation to Eq. (1.15), then the relation Eq. (1.13) would be again valid even when we consider the contribution from gluonium. Alternatively, let N and $\tilde{N}$ be $u\bar{u} + d\bar{d}$ components contained in η_8 (or η_0) and $\tilde{\eta}_8$ (or $\tilde{\eta}_0$), respectively. If the ratio λ defined by

$$\lambda = \frac{M(A+B \rightarrow C_1 + \cdots + C_n + \tilde{N})}{M(A+B \rightarrow C_1 + \cdots + C_n + N)} \tag{1.16}$$

is a constant independent of energy and channel, then Eq. (1.13) is now replaced by

$$R_0 = \frac{S_2 \cos(\theta_0 - \theta_2) + \lambda \tilde{S}_2 \cos(\theta_0 - \tilde{\theta}_2)}{S_1 \sin(\theta_0 - \theta_1) + \lambda \tilde{S}_1 \sin(\theta_0 - \tilde{\theta}_1)} . \tag{1.17}$$

Note that R_0 is still a constant independent of specific energy and channels. Moreover, if Eq. (1.15) is valid, then this reduces to Eq. (1.13). The constancy of λ defined by Eq. (1.16) is plausible in the quark-model, since we can roughly expect λ to be equal to the ratio of wave functions evaluated at origin for radially excited state and the ground state. Because of these considerations, the validity of Eq. (1.13) or (1.17) appears to be reasonable. Hereafter, we assume this to be so. Systematic study of the validity of Eq. (1.13) has been undertaken in Ref. 17 and summarized in Ref. 13 for various experimental results before 1976. From several experimental data on $\pi^{\pm} p \rightarrow N\eta$ (or η'), and $\Delta\eta$ (or η') as well as other reactions such as $p\bar{p} \rightarrow \pi^+ \pi^- \eta$ (or η') in various energy ranges of 1—200 GeV, it has been found in Ref. 13 that the absolute values $|R_0|$ defined by Eq. (1.12) are indeed concentrated in a relatively narrow range of

$$|R_0| = 0.5 \text{ to } 0.9 \tag{1.18}$$

for all channels and for all energies studied so far. The rather large variations of $|R_0|$ quoted above reflect partly large experimental errors. Some deviation of $|R_0|$ from the constancy may be attributable to contributions from radially excited states $\tilde{\eta}_8$ and $\tilde{\eta}_0$ as we have already discussed. Or it could be due to genuine violation of the quark-line rule especially for low-energy reactions where the mass difference between η and η' for matrix elements may not be negligible. Note that the quark-line rule ignores contributions from $s\bar{s}$ as well as gluonium components contained in physical η and η'. We cannot settle the question at the present since it requires a large collection of accurate experimental data for various reactions.

Hereafter we assume the exact validity of Eq. (1.13) or (1.17). Then, the best value for $|R_0|$ can be perhaps inferred from more recent high-energy precision experimental studies of the reaction $\pi^- p \to \eta n$ and $\eta' n$. Apel *et al.*[18] find

$$|R_0| \simeq 0.80 \pm 0.05 \tag{1.19a}$$

from data on 15- and 40-GeV/c pion momenta, while Stanton *et al.*[19] give essentially the same value

$$|R_0| \simeq 0.82 \pm 0.02 \tag{1.19b}$$

from 8.45-GeV/c data. Also, these values are consistent with earlier values[13] determined from $p\bar{p} \to \pi^+\pi^-\eta$ (or η') at rest. Similarly, we note the validity[13] of

$$\frac{\Gamma(\eta' \to \rho^0\gamma)}{\Gamma(\rho \to \eta\gamma)} = 3\,|R_0|^2 \left[\frac{k'}{k}\right]^3 , \qquad \frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \eta\gamma)} = 3\,|R_0|^2 \left[\frac{k'}{k}\right]^3 \tag{1.20}$$

under the same condition as before, where k and k' are magnitudes of the final photon momentum in the decay modes concerned, and where we have assumed the exact ideal mixing for the ω-ϕ complex. If we use new values for two-photon decay rates[20] of η and η' given by $\Gamma(\eta \to \gamma\gamma) = 0.344 \pm 0.044$ keV and $\Gamma(\eta' \to \gamma\gamma) = 5.3 \pm 0.6$ keV, then Eq. (1.20) is now consistent with values of $|R_0|$ given by Eq. (1.19), rather than the previous smaller values[13] for $|R_0|$. Various theoretical estimates of R_0 have been given in Ref. 13. Here, we simply note that the recent model of Ref. 14 will give a small value of $R_0 \simeq 0.43$, neglecting contributions from radially excited states. Also, the model of Rosenzweig, Salamone, and Schechter[15] leads to $R_0 = 0.66$, while that by Gault and Rimmer[15] gives $R_0 = 1.56$.

We cannot experimentally determine values of S_1, S_2, θ_1, θ_2, etc., separately from $|R_0|$. However, there are reactions from which we can obtain information on other combinations of these constants. Here, we only mention one of them,[13]

$$\frac{M(\psi \to \eta'\phi)}{M(\psi \to \eta\phi)} = -\frac{S_2 \cos(\theta_0 - \theta_2)}{S_1 \sin(\theta_0 - \theta_1)} ,$$

where we have assumed exact ideal mixing for the ω-ϕ complex and neglected contributions from possible radially excited states. If we assume the simple mass-mixing scheme Eq. (1.8), then we can give more definite statements. First, Eq. (1.13) reduces to

$$R_0 = \cot(\theta_0 - \theta) . \tag{1.13'}$$

The value of $|R_0|$ given by Eq. (1.19a) implies then

$$\theta \simeq -16° \pm 2° \tag{1.21}$$

which is intermediate between linear-mass-mixing and quadratic-mass-mixing values. Note that another solution $\theta = 86°$ is excluded from other considerations (see Ref. 13). Genz[21] analyzes experimental data for $p\bar{p} \to \pi^0\pi^0$, $\pi^0\eta$, $\pi^0\eta'$, $\eta\eta$, etc., at rest with the quark-line rule in a more generalized sense, and finds

$$\theta \simeq -(13° \text{ to } 16°) \pm 6° .$$

Also, the decay rates for new values of $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$ are consistent[20,22] with

$$\theta \simeq -18° \pm 5° .$$

On the theoretical side, a QCD-inspired calculation[23] indicates

$$\theta \simeq -(17° \text{ to } 20°)$$

while a study based upon the two-gluon-exchange diagram gives[22]

$$\theta \simeq -13° .$$

Summarizing, we suggest that the η-η' mixing theory together with the quark-line rule is experimentally well satisfied. Returning to our original relation Eq. (1.13) or (1.17), it must also be valid for decays $\psi \to p\bar{p}\eta$ and $p\bar{p}\eta'$. It is gratifying to see that the experimental ratio of Eq. (1.5) is consistent with $|R_0| \simeq 0.80 \pm 0.05$ given by Eq. (1.19a). The same conclusion based upon the more restricted mass-mixing scheme Eq. (1.8) has been noted also in Ref. 21. In this connection, the branching ratio of η_c(3000 MeV) decaying into $\eta\pi^+\pi^-$ channel has been measured[24] to be

$$B_r(\eta_c \to \eta\pi^+\pi^-) \equiv \frac{\Gamma(\eta_c \to \eta\pi^+\pi^-)}{\Gamma(\eta_c \to \text{all})} \simeq (3.0 \pm 1.7) \times 10^{-2} .$$

Then, using the value of $|R_0| \simeq 0.8$, we predict

$$B_r(\eta_c \to \eta'\pi^+\pi^-) \equiv \frac{\Gamma(\eta_c \to \eta'\pi^+\pi^-)}{\Gamma(\eta_c \to \text{all})} \simeq (2.17 \pm 1.23) \times 10^{-2} .$$

Next, we consider the pion energy (or essentially equivalent to the square of invariant $p\bar{p}$ mass) spectrum in $\psi \to p\bar{p}\pi^0$ decay. In view of the soft-pion theorem,[25] the

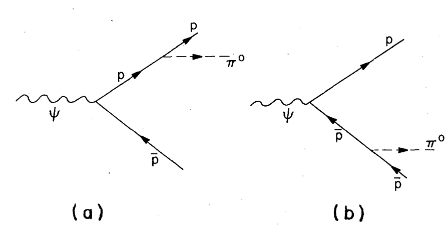

FIG. 1. Proton-pole Feynman diagrams for $\psi \to p\bar{p}\pi^0$ decay.

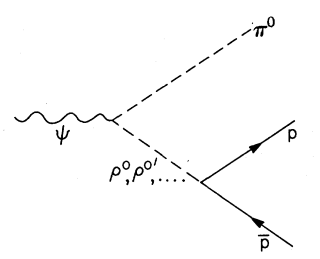

FIG. 2. ρ-meson-pole Feynman diagrams for $\psi \to p\bar{p}\pi^0$ decay.

matrix element for $\psi \to p\bar{p}\pi^0$ will vanish, in general, in the soft-pion limit $E_\pi \to 0$. The exception for this statement is for proton-pole diagrams of Fig. 1, where the proton propagator also vanishes in the soft-pion limit so as to give a finite answer in the limit.

We expect that for small values of the pion energy E_π or equivalently for large values of invariant $p\bar{p}$ mass, the dominant contribution would come from these proton-pole diagrams. In this connection, we note that ρ and ρ' pole diagrams depicted in Fig. 2 will give zero contribution for the zero-pion-momentum limit but also for nonzero pion mass, because of the selection rule due to conservation of parity and angular momentum for $\psi \to \pi^0\rho^0$ decay. In any case, the pion energy spectrum should be essentially model-independent for small values of the pion energy E_π, since decay matrix elements for $\psi \to p\bar{p}$ as well as the pion-nucleus coupling constant $g_{\pi N\bar{N}}$ are experimentally reasonably well established. Indeed, the present experimental value is consistent with this observation as we will see from Fig. 3, and we conclude that the soft-pion theorem is applicable for $\psi \to \pi^0 p\bar{p}$ decay. We remark that the same method may also be applicable to $\psi \to \pi\Sigma\bar{\Sigma}$ and $\pi\Xi\bar{\Xi}$ decays for us to extract values of $g_{\Xi\bar{\Xi}\pi}$ and $g_{\Sigma\bar{\Sigma}\pi}$, directly from the pion-energy spectrum of these decays. Finally, the presence of the Adler's zero has been previously established[26] for the decay $\psi' \to \pi\pi\psi$.

So far, our analysis has been largely model-independent. However, the rest of our discussion is very model-dependent with many uncertainties. First, let us consider the pion energy spectrum for large values of E_π. As we see from Fig. 3, the proton-pole diagrams of Fig. 1 give

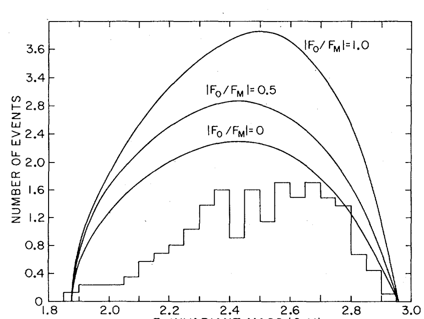

FIG. 3. Plot of $[d\Gamma(\psi \to p\bar{p}\pi^0)]/dM_{p\bar{p}}/\Gamma(\psi \to p\bar{p})$, where $M_{p\bar{p}}$ stands for the invariant mass of the $p\bar{p}$ system. The calculated value takes into account only the proton-pole diagram of Fig. 1. The experimental values are taken from Ref. 11. The soft-pion limit corresponds to large $M_{p\bar{p}}$.

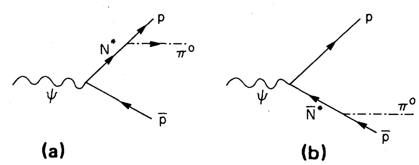

FIG. 4. N^*-pole diagrams for $\psi \to p\bar{p}\pi^0$ decay, where N^* refers to any nucleon isobar.

nearly twice as large a value in comparison to experimental data for large-E_π regions. This implies that other types of Feynman diagrams must contribute for the mode with negative interference. Since the production of nucleon isobars N^* in the mass range 1400–1600 MeV appears to be present in the experimental data, we have to take into account the N^*-pole diagrams of Fig. 4. In the next section, we will demonstrate that the inclusion of Fig. 4 together with the proton-pole diagram can explain the experimental energy spectrum, provided that both $N^*(1440$ MeV) with $J^P = \frac{1}{2}^+$ and $N^+(1535$ MeV) with $J^P = \frac{1}{2}^-$ are both present with comparable rates in $\psi \to N^*\bar{p}$ mode.

In the above discussion, we have neglected the contribution from ρ and ρ' pole diagrams of Fig. 2. First, the contribution from the ρ pole can be shown to be small with negligible interference terms with baryon pole diagrams. The same remark applies also for that from the ρ' pole, provided that ρ' couples with the nucleon with the same coupling constant as the ρ-nucleon interaction. Moreover, the inclusion of ρ and ρ' pole terms may give double counting of the same quark graphs. The reason is as follows. The proton-pole diagram of Fig. 1(a) is realized as quark graphs of Fig. 5. Note that in Fig. 5(c), three gluons interact with three different quark lines inside the nucleon. On the other hand, the ρ-pole diagram may be represented as the quark diagrams of Fig. 6. We see that Figs. 6(a) and 6(b) are topologically equivalent to Figs. 5(a) and 5(b), respectively. However, Fig. 6 does not contain an analog of Fig. 5(c). This fact may suggest that the baryon-pole diagrams are perhaps more representative of underlying quark diagrams. Note that the duality principle[27] indicates the importance of quark graphs rather than ordinary Feynman diagrams. Nevertheless, the calculation for ρ-ρ' pole diagrams will be discussed in Sec. III.

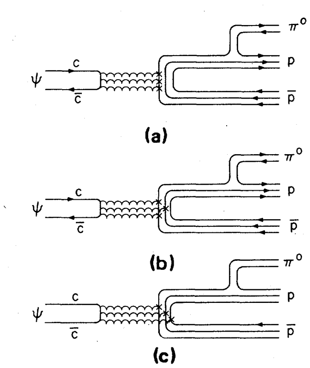

FIG. 5. Quark diagrams corresponding to proton-pole graph of Fig. 1. Three curly lines refer to three gluons.

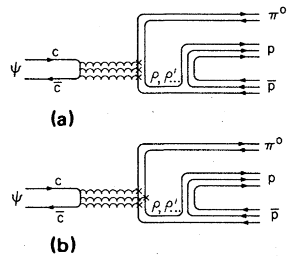

FIG. 6. Quark diagrams corresponding to ρ-meson-pole graph of Fig. 2.

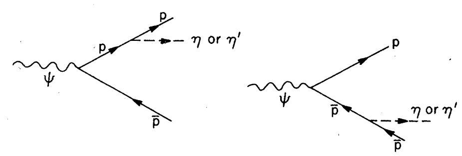

FIG. 7. Proton-pole Feynman diagrams for $\psi \to p\bar{p}\eta$ and $p\bar{p}\eta'$ decays.

Finally, we will turn our attention to decays $\psi \to p\bar{p}\eta$ and $p\bar{p}\eta'$. We have already noted that their decay ratio of Eq. (1.5) agrees well with the general principle based upon the quark-line rule without recourse to dynamical detail. However, the large ratio given in Eq. (1.6) is more difficult to explain. We have already remarked that the proton-pole diagram of Fig. 1 gives the dominant contribution for $\psi \to p\bar{p}\pi^0$ decay. But the same mechanism cannot be correct for $\psi \to p\bar{p}\eta$ and $p\bar{p}\eta'$ decays because of the following reason. Consider the proton-pole diagram of Fig. 7. Assuming the general mixing model, Eq. (1.7), together with the exact validity of the quark-line rule as well as the flavor SU(3) group, the coupling constants of η and η' with the nucleon are expressed as

$$g_{\eta N\bar{N}} = S_1 \sin(\theta_0 - \theta_1)(3f-d) g_{\pi N\bar{N}} \,,$$
$$g_{\eta' N\bar{N}} = S_2 \cos(\theta_0 - \theta_2)(3f-d) g_{\pi N\bar{N}} \,. \quad (1.22)$$

Here, f and d are the standard f and d coefficients with normalization $f+d=1$. Note that the ratio $R_0 = g_{\eta' N\bar{N}}/g_{\eta N\bar{N}}$ satisfies the general universal equations (1.13). From the known value of the coupling constants $g_{\pi N\bar{N}}$ and $g_{k\Lambda\bar{N}}$, it is, in general, believed[28] that the d/f ratio is approximately 2.7. Assuming the mass-mixing result Eq. (1.8) with $\theta_1 = \theta_2 \equiv \theta \simeq -16°$ in conformity to Eq. (1.19), we then calculate

$$g_{\eta' N\bar{N}}/g_{\pi N\bar{N}} = \begin{cases} 0.05 & \text{if } d/f = 2.7 \\ 0.21 & \text{if } d/f = 2.0 \,, \end{cases} \quad (1.23)$$

which are too small in comparison to the value given in Eq. (1.6). Therefore, we have to consider N^*-pole diagrams which are analogs of Fig. 4. Since the experimental decay rates of $\Gamma(N^* \to \eta N)$ are rather large for both N^*(1440 MeV) and N^*(1535 MeV), these are indeed now the dominant diagrams. Although we can roughly account for total experimental decay rates $\Gamma(\psi \to p\bar{p}\eta)$ and $\Gamma(\psi \to p\bar{p}\eta')$ in terms of these diagrams, it gives unfortunately an incorrect spectrum for the decay especially for ηp mass plot in comparison to the experimental data. However, in view of many uncertainties both experimentally and theoretically, this fact may not be serious. In this connection, we have also computed the contribution from the proton-pole diagram for $\psi \to p\bar{p}\omega$. The total decay rate so computed amounts only to one-tenth of the experimental value, and we conclude that the baryon-isobar pole diagrams must be also important for $\psi \to p\bar{p}\omega$ decay.

Last, we would like to comment on the ratio of Eq. (1.3). If the charge independence based upon SU(2) symmetry is exact, then the ratio should be precisely $\frac{1}{2}$ which is consistent with Eq. (1.3). Moreover, the SU(2) requires the identical energy spectrum for π^0 and π^+ mesons in decays $\psi \to \pi^0 p\bar{p}$ and $\pi^+ n\bar{p}$, respectively. The present experiment appears to be consistent with it. This fact may imply that the SU(2)-violating one-photon intermediate diagram should give negligible contribution for $\psi \to \pi N\bar{N}$ decays. An example for such a SU(2)-violating process is two-steps mode $\psi \to \pi^+\pi^-$ followed by virtual decay $\pi^- \to n\bar{p}$. Note that the corresponding mode $\psi \to \pi^0\pi^0$ followed by $\pi^0 \to p\bar{p}$ is forbidden by charge-conjugation invariance. The validity of the SU(2) invariance is also consistent with near experimental equality of $\Gamma(\psi \to p\bar{p})$ and $\Gamma(\psi \to n\bar{n})$. However, the one-photon-exchange diagram may give[6,7] a sizeable contribution for $\psi \to V_9 P_9$ decays through its interference with the normal SU(2)-preserving three-gluon-exchange diagram.

II. CONTRIBUTION FROM BARYON-POLE DIAGRAMS

Before we analyze $\psi \to \pi^0 p\bar{p}$, we will first discuss the $\psi \to p\bar{p}$ mode since it is relevant for our evaluation of the proton-pole diagram Fig. 1. The S-matrix element is written as

$$S(\psi \to p\bar{p}) = -(2\pi)^4 i \delta^{(4)}(q-p+p') \left[\frac{m^2}{2q_0 p_0 \,|\, p_0' \,|\, V^3} \right]^{1/2} \bar{u}(p) \left[F_M \gamma_\mu \epsilon^\mu(q) + \frac{1}{2m} F_0 (p+p')_\mu \epsilon^\mu(q) \right] v(p') \,, \quad (2.1)$$

with $p_0 > 0$ but $p_0' < 0$, where m is the mass of the proton and $\epsilon^\mu(q)$ is the polarization vector of ψ with four momentum q. The dimensionless real decay constants F_M and F_0 are defined by

$$\langle p\bar{p} \,|\, (\Box + M^2)\psi_\mu(x) \,|\, 0 \rangle = \left[\frac{m^2}{p_0 \,|\, p_0' \,|\, V^2} \right]^{1/2} \bar{u}(p) \left[F_M \gamma_\mu + \frac{1}{2m} F_0 (p+p')_\mu \right] v(p') e^{i(p-p')x} \,, \quad (2.2)$$

in terms of the interpolating field operator $\psi_\mu(x)$ for ψ/J with mass M. It is sometimes more convenient to use the electric and magnetic coupling parameters, F_E and F_M in whose terms F_0 is expressed as

$$F_0=\frac{4m^2}{M^2-4m^2}(F_M-F_E)\ . \tag{2.3}$$

The total decay rate for $\psi\to p\bar{p}$ is now calculated to be

$$\Gamma(\psi\to p\bar{p})=\frac{1}{12\pi}\left[|F_M|^2+\frac{2m^2}{M^2}|F_E|^2\right](M^2-4m^2)^{1/2}\ . \tag{2.4}$$

In reality, $\psi\to p\bar{p}$ is experimentally observed in the reaction $e+\bar{e}\to\psi\to p\bar{p}$. If θ is the angle between the incoming electron and the final proton, then the angular dependence of the production cross section is

$$1+\alpha\cos^2\theta\ ,$$

where α is given by

$$\alpha=\frac{|F_M|^2-(4m^2/M^2)|F_E|^2}{|F_M|^2+(4m^2/M^2)|F_E|^2}\ . \tag{2.5}$$

The numerical value of α has been determined to be

$$\alpha=\begin{cases}1.45\pm0.56 \ \ (\text{Ref. }11), & (2.6\text{a})\\ 0.61\pm0.23\ \ (\text{Ref. }12)\ . & (2.6\text{b})\end{cases}$$

Since Eq. (2.5) implies $|\alpha|<1$, Eq. (2.6a) can be consistent only if $F_E\sim0$. More precisely, we find

$$|F_E|^2\le0.14|F_M|^2\ , \tag{2.7a}$$

$$|F_E|^2=(0.45\pm0.27)|F_M|^2\ , \tag{2.7b}$$

respectively, for Eqs. (2.6a) and (2.6b). Together with the total rate given by Eq. (2.4), then we estimate $|F_E|$ and $|F_M|$. However, for evaluation of $\Gamma(\psi\to\pi^0p\bar{p})$, we have to know the values of $|F_0|$ and $|F_M|$ instead, as we will see shortly. This will introduce another unknown phase factor δ defined by

$$F_0/F_M=|F_0|/|F_M|e^{i\delta}\ . \tag{2.8}$$

Together with Eq. (2.3), Eqs. (2.6) and (2.8) then require

$$|\delta|\le22^\circ,\quad |F_0|/|F_M|\simeq(0.34\text{ to }0.82) \tag{2.9a}$$

$$|\delta|\le58^\circ,\quad |F_0|/|F_M|\simeq(0.09\text{ to }1.07) \tag{2.9b}$$

for two values of α given by Eqs. (2.6a) and (2.6b), respectively.

We will now turn to $\psi\to\pi^0p\bar{p}$ decay. First, we discuss the proton-pole diagram of Fig. 1. The pion-nucleon interaction can be chosen either as pseudoscalar-pseudoscalar (PS-PS) form:

$$H_1=ig_{N\bar{N}\pi}\bar{N}\gamma_5\vec{\tau}N\cdot\vec{\pi} \tag{2.10a}$$

or pseudoscalar-pseudovector (PS-PV) form:

$$H_1'=\frac{1}{2m}g_{N\bar{N}\pi}\bar{N}\gamma_5\gamma_\mu\vec{\tau}N\cdot\partial^\mu\vec{\pi}\ , \tag{2.10b}$$

where $g_{N\bar{N}\pi}$ is the standard pion-nucleon coupling constant with

$$(g_{N\bar{N}\pi})^2/4\pi\simeq14.8\ . \tag{2.10c}$$

In accordance with the equivalence theorem[29] on PS-PS and PS-PV couplings in the lowest order, both interactions H_1 and H_1' give exactly the same final result for the present problem, if we add both diagrams Figs. 1(a) and 1(b). Especially, the soft-pion theorem is manifest for PS-PV form Eq. (2.10b). The S-matrix element for the proton-pole diagram of Fig. 1 is now calculated to be

$$S_1(\psi\to\pi^0p\bar{p})=(2\pi)^4\delta^{(4)}(q-p+p'-k)\times\left[\frac{m^2}{4q_0k_0p_0|p_0'|V^4}\right]^{1/2}M_1\ , \tag{2.11}$$

$$M_1=g_{N\bar{N}\pi}\bar{u}(p)\gamma_5\left[\left[\frac{(k\cdot\gamma)(\epsilon\cdot\gamma)}{2p\cdot k+k^2}+\frac{(\epsilon\cdot\gamma)(k\cdot\gamma)}{2p'k-k^2}\right]F_M+\left[\frac{p'\cdot\epsilon}{2p\cdot k+k^2}-\frac{p\cdot\epsilon}{2p'\cdot k-k^2}\right]\times(k\cdot\gamma)\frac{F_0}{m}\right]v(p')\ , \tag{2.12}$$

where k_μ is the four-momentum of the outgoing pion with $k^2=m_\pi{}^2$. Note that for the soft-pion limit $k_\mu\to0$, M_1 remains finite because of the vanishing denominator. Strictly speaking, F_M and F_0 here depend upon values of pk and $p'k$ since the intermediate proton state is virtual. However, for the soft-pion limit, the off-shell effect is expected to be very small, although we must be careful about its effect for large-pion-energy regions.

As we explained in the previous section, the soft-pion theorem implies that M_1 will give the dominant contribution for small values of E_π. Therefore, we suppose for a while that it will give the whole contribution without any correction. Then, as we will show in the Appendix, the interference term proportional to $\text{Re}(F_0F_M^*)$ contains a factor $k^2=m_\pi{}^2$ which is very small and can be negligible. This implies that the differential decay rate contains essentially only two terms proportional to $|F_M|^2$ and $|F_0|^2$, but not to interference effect $\text{Re}(F_0F_M^*)$. In Fig. 3, we have plotted our calculation of the $p\bar{p}$-mass distribution $R(M_{p\bar{p}})$ defined by

$$R(M_{p\bar{p}})=\frac{1}{\Gamma(\psi\to p\bar{p})}\frac{d}{dM_{p\bar{p}}}\Gamma(\psi\to p\bar{p}\pi^0) \tag{2.13}$$

for three choices of $|F_0|/|F_M|=0$, 0.5, and 1.0, and compared them to the experimental data of Ref. 11. Here, $M_{p\bar{p}}$ refers to the invariant mass of the $p\bar{p}$ system. Note that the high (or low) values of $M_{p\bar{p}}$ correspond to low (or high) values of the pion energy E_π since

$$M_{p\bar{p}}{}^2=(M_\psi{}^2+M_\pi{}^2)-2M_\psi E_\pi\ . \tag{2.14}$$

We see that our calculations reproduce fairly well the high-mass distribution of the $p\bar{p}$ system in both shape and magnitude especially for $F_0=0$. However, the values of $|F_0|/|F_M|=0.5$ and 1.0 are still consistent with data, especially when we compare them with the experimental data of Ref. 12. This confirms the validity of the soft-pion theorem. However, for smaller values of $M_{p\bar{p}}$, our calculation tends to give almost twice as large values in comparison to the data. Indeed, the total decay rates $\Gamma(\psi\to p\bar{p}\pi^0)$ are calculated to be

$$\frac{\Gamma(\psi\to p\bar{p}\pi^0)}{\Gamma(\psi\to p\bar{p})}=\begin{cases}0.89 \text{ for } |F_0|/|F_M|=0,\\ 1.08 \text{ for } |F_0|/|F_M|=0.5,\\ 1.21 \text{ for } |F_0|/|F_M|=1.0,\end{cases} \quad (2.15)$$

which should be compared to 0.50 ± 0.06 of Eq. (1.2). This fact may be partly due to the off-shell effect of the virtual (instead of real) proton intermediate state as we remarked already. This implies that we have to replace F_M, for example, as

$$\frac{1}{\Delta^2}F_M\to\frac{1}{\Delta^2}F_M\phi(\Delta^2) \quad (2.16)$$

for $\Delta^2=2p\cdot k+k^2$ or $2p'\cdot k-k^2$, where the form factor $\phi(\Delta^2)$ is normalized to $\phi(0)=1$. Since we expect $|\phi(\Delta^2)|\le 1$, this will certainly lower the total decay rate as well as the partial rates for high pion energy E_π. However, since we have no reliable way of calculating $\phi(\Delta^2)$, we consider other Feynman diagrams which can destructively interfere with the proton-pole contribution and hence will lower total decay rate. We take into account the isospin-$\frac{1}{2}$ N^*-pole diagram of Fig. 4, since N^*-resonance production in the mass region 1400—1550 MeV has been experimentally observed[12] for $\psi\to N^*\bar{p}$. In passing, we note that there is no evidence for production of Δ(1232 MeV) with $I=\frac{3}{2}$ in $\psi\to\Delta\bar{p}$. This is consistent with our theoretical expectation, since the exact SU(2) invariance will forbid $\psi\to\Delta\bar{p}$ in absence of the SU(2)-violating one-photon-exchange process. For mass ranges 1400—1600 MeV, there are three known baryon isobars; 1440 MeV ($J^P=\frac{1}{2}^+$), 1520 MeV ($J^P=\frac{3}{2}^-$), and 1535 MeV ($J^P=\frac{1}{2}^-$). We neglect here the contribution from the $J^P=\frac{3}{2}^-$ isobar, partly for simplicity and partly from expectation that its effect will be small due to centrifugal barriers. We now define analogs of magnetic and electric form factors by

$$\langle N_\alpha\bar{p}|(\Box+M^2)\psi_\mu(x)|0\rangle=\left[\frac{mm_\alpha}{p_0'|p_0|V^2}\right]^{1/2}\bar{u}_\alpha(p')\left[F_M^{(\alpha)}\gamma_\mu+\frac{1}{2m}F_0^{(\alpha)}(p+p')_\mu+G^{(\alpha)}(p-p')_\mu\right]v(p)e^{i(p'-p)\cdot x}, \quad (2.17a)$$

$$\langle N_\beta\bar{p}|(\Box+M^2)\psi_\mu(x)|0\rangle=\left[\frac{mm_\beta}{p_0'|p_0|V^2}\right]^{1/2}\bar{u}_\beta(p')\gamma_5\left[F_M^{(\beta)}\gamma_\mu+\frac{1}{2m}F_0^{(\beta)}(p+p')_\mu+G^{(\beta)}(p-p')_\mu\right]v(p)e^{i(p'-p)\cdot x}, \quad (2.17b)$$

where N_α and N_β stand for N^*(1440 MeV) with $J^P=\frac{1}{2}^+$ and N^* (1535 MeV) with $J^P=\frac{1}{2}^-$, respectively. $G^{(\alpha)}$ (or $G^{(\beta)}$) in Eq. (2.17) can be expressed in terms of $F_M^{(\alpha)}$ (or $F_M^{(\beta)}$) and $F_0^{(\alpha)}$ (or $F_0^{(\beta)}$) by the constraint $\partial^\mu\psi_\mu(x)=0$.

The total decay rate for $\psi\to N^{(\pm)}\bar{p}$ with $N^{(+)}=N_\alpha$ and $N^{(-)}=N_\beta$ is calculated to be

$$\Gamma(\psi\to N^{(\pm)}\bar{p})=\frac{2k^3}{3\pi}\frac{1}{M^2-(m\pm m^*)^2}\times\left[|F_M^{(\pm)}|^2+\frac{2m^2}{M^2}|F_E^{(\pm)}|^2\right], \quad (2.18a)$$

$$F_0^{(\pm)}=\frac{4m^2}{M^2-(m\pm m^*)^2}\left[\frac{m\pm m^*}{2m}F_M^{(\pm)}-F_E^{(\pm)}\right], \quad (2.18b)$$

$$k^2=\frac{1}{4M^2}[M^2-(m^*-m)^2][M^2-(m^*+m)^2]. \quad (2.18c)$$

m^*, $F_M^{(\pm)}$, and $F_0^{(\pm)}$ stand for m_α, $F_M^{(\alpha)}$, and $F_0^{(\alpha)}$ or m_β, $F_M^{(\beta)}$, and $F_0^{(\beta)}$, depending upon identification $N^{(+)}=N_\alpha$ and $N^{(-)}=N_\beta$. Our formula reproduces, of course, Eq. (2.4) for the special case of $N_\alpha=p$ with $m^*=m$. Note also that Eqs. (2.18) are formally invariant under $m^*\to-m^*$ and $\alpha\leftrightarrow\beta$.

The experimental data are not sufficiently accurate enough at the present so as to distinguish N_α and N_β contributions, separately. Then, we estimate

$$|F_M^{(\alpha)}|^2+\frac{2m^2}{M^2}|F_E^{(\alpha)}|^2=(0.55\pm0.28)\left[|F_M|^2+\frac{2m^2}{M^2}|F_E|^2\right]\frac{1}{1+a^2}, \quad (2.19a)$$

$$|F_M^{(\beta)}|^2+\frac{2m^2}{M^2}|F_E^{(\beta)}|^2=(1.57\pm0.81)\left[|F_M|^2+\frac{2m^2}{M^2}|F_E|^2\right]\frac{a^2}{1+a^2}, \quad (2.19b)$$

where we have set

$$a=\left[\frac{\Gamma(\psi\to N_\beta\bar{p})}{\Gamma(\psi\to N_\alpha\bar{p})}\right]^{1/2}. \quad (2.20)$$

Next, we have to compute matrix elements of Fig. 4. To this end, we define $N^*N\pi$ interactions by

$$H_\alpha=\frac{1}{2m}g_\pi^{(\alpha)}\bar{N}_\alpha\gamma_5\gamma_\mu\vec{\tau}N\cdot\partial_\mu\vec{\pi}+\text{H.c.},$$
$$H_\beta=\frac{1}{2m}g_\pi^{(\beta)}\bar{N}_\beta\gamma_\mu\vec{\tau}N\cdot\partial_\mu\vec{\pi}+\text{H.c.}, \quad (2.21)$$

which are compatible with the soft-pion theorem. Then, the decay rate for $N^* \to N\pi$ with $N^* = N_\alpha$ or N_β is calculated as

$$\Gamma(N^* \to N\pi) = \frac{3}{8m^2(m^*)^2} \frac{(g_\pi^*)^2}{4\pi}(m^* \pm m)^2 \times [(m^* \mp m)^2 - m_\pi{}^2]k \;, \quad (2.22a)$$

$$k^2 = \frac{1}{4(m^*)^2}[(m^*-m)^2 - m_\pi{}^2][(m^*+m)^2 - m_\pi{}^2] \;, \quad (2.22b)$$

where m^* and g_π^* refer to m_α or m_β and $g_\pi^{(\alpha)}$ or $g_\pi^{(\beta)}$, respectively, and where the plus and minus signs correspond to two cases of $N^* = N_\alpha$ (plus) and $N^* = N_\beta$ (minus), respectively. From these formulas, we estimate

$$\frac{1}{4\pi}(g_\pi^{(\alpha)})^2 = 1.30 \pm 0.68 \;, \quad \frac{1}{4\pi}(g_\pi^{(\beta)})^2 = 0.40 \pm 0.19 \;. \quad (2.23)$$

Now, we are in a position to compute contributions from Figs. 1 and 4. However, since we do not know values of ratios F_0^*/F_M^* for $N^* = N_\alpha$ and N_β, we assume for simplicity $F_0 = F_0^{(\alpha)} = F_0^{(\beta)} = 0$ as well as real relative phases among F_M, $F_M^{(\alpha)}$ and $F_M^{(\beta)}$. In Table I, we give contributions for ratios

$$\Gamma(\psi \to p\bar{p}\pi^0)/\Gamma(\psi \to p\bar{p})$$

and

$$\Gamma(\psi \to p\bar{p}\eta)/\Gamma(\psi \to p\bar{p})$$

from various pole terms.

We should remark that absolute signs for contributions from interference terms between p, N_α, and N_β are undetermined in Table I. However, since only interference between p and N_α is numerically significant in view of the different parity of N_β, this affects mostly the former. If we choose now $a^2 = 0.3$ [see Eq. (2.20) for the definition of a] with destructive interference between p and N_α, then we find

TABLE I. The ratios of $\Gamma(\psi \to p\bar{p}\pi^0)$ and $\Gamma(\psi \to p\bar{p}\eta)$ to $\Gamma(\psi \to p\bar{p})$, computed on the basis of baryon-pole diagrams Figs. 1 and 4. See the text for details.

	$\frac{\Gamma(\psi \to p\bar{p}\pi^0)}{\Gamma(\psi \to p\bar{p})}$	$\frac{\Gamma(\psi \to p\bar{p}\eta)}{\Gamma(\psi \to p\bar{p})}$
p	0.89	0.0007
N_α	$\frac{0.29 \pm 0.21}{1+a^2}$	$\frac{0.40 \pm 0.32}{1+a^2}$
N_β	$(0.09 \pm 0.06)\frac{a^2}{1+a^2}$	$(0.44 \pm 0.32)\frac{a^2}{1+a^2}$
$N_\alpha p$	$(0.52 \pm 0.44)(1+a^2)^{-1/2}$	$(0.03 \pm 0.03)(1+a^2)^{-1/2}$
$N_\beta p$	$(0.0003 \pm 0.0002)\frac{a}{(1+a^2)^{1/2}}$	$-(0.015 \pm 0.012)\frac{a}{(1+a^2)^{1/2}}$
$N_\alpha N_\beta$	$-(0.07 \pm 0.07)\frac{a}{1+a^2}$	$-(0.41 \pm 0.48)\frac{a}{1+a^2}$

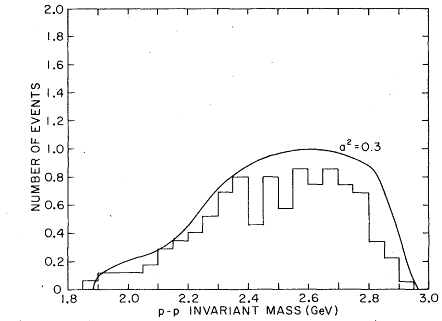

FIG. 8. Plot for $[d\Gamma(\psi \to p\bar{p}\pi^0)/dM_{p\bar{p}}]/\Gamma(\psi \to p\bar{p})$, where the calculated value takes account of proton as well as N^*-pole contributions. See the text for details.

$$\Gamma(\psi \to p\bar{p}\pi^0)/\Gamma(\psi \to p\bar{p}) \simeq 0.65 \pm 0.24 \quad (2.24)$$

in agreement with the present experimental value of 0.5 ± 0.064. Also, in Figs. 8 and 9, we have plotted mass spectra of the decay with respect to variable $M_{p\bar{p}}$ and $M_{p\pi^0}$. We see that the agreement of our calculations with experiments is satisfactory. Also the interference term between p and N_α does not seriously affect the low-pion-energy spectrum corresponding to the soft-pion theorem. In conclusion, we may say that we can explain the experimental data for $\psi \to \pi^0 p\bar{p}$ decay satisfactorily.

We next turn our attention to the decay $\psi \to \eta p\bar{p}$, since the discussion for $\psi \to \eta' p\bar{p}$ can be done exactly in the same way. As we have emphasized in the previous section, the proton-pole diagram now gives negligible contribution. Indeed, now contributions from N_α and N_β are dominant for the present case. We note that N_α(1440 MeV) can decay into $p\eta$ because of its large width of ~200 MeV. Therefore, in calculating the coupling parameter $g_\eta^{(\alpha)}$ and $g_\eta^{(\beta)}$ of η-N-N_α and η-N-N_β interactions, we must consider the large widths of these isobars. The procedure is rather model dependent. However, we estimate

$$\frac{1}{4\pi}(g_\eta^{(\alpha)})^2 \simeq 9.10 \pm 5.58 \;, \quad \frac{1}{4\pi}(g_\eta^{(\beta)})^2 \simeq 7.07 \pm 3.46 \quad (2.25)$$

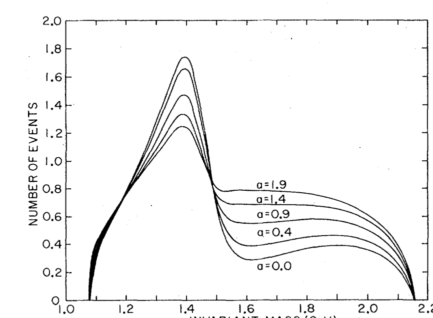

FIG. 9. Plot of $[d\Gamma(\psi \to p\bar{p}\pi^0)/dM_{p\pi^0}]/\Gamma(\psi \to p\bar{p})$, where $M_{p\pi^0}$ stands for the invariant mass of the $p\pi^0$ system. The calculated values take into account the proton and N^*-pole contributions. See the text for details.

from experimental known decay rates $\Gamma(N_\alpha\to p\eta)$ and $\Gamma(N_\beta\to p\eta)$. As for $g_{\eta N\bar{N}}$, we use the value of $g_{\eta N\bar{N}}/g_{\pi N\bar{N}}\simeq 0.05$ corresponding to $d/f=2.7$ in Eq. (1.22). Then, under the same assumption as in the calculation of $\Gamma(\psi\to p\bar{p}\pi^0)/\Gamma(\psi\to p\bar{p})$, we can compute $\Gamma(\psi\to p\bar{p}\eta)/\Gamma(\psi\to p\bar{p})$ and have tabulated various contributions in the second column of Table I. To our surprise, the interference between N_α and N_β is now found to be large in spite of the opposite parity. This is perhaps due to the fact that the phase volume for $\psi\to p\bar{p}\eta$ is small and hence the expected cancellation due to the opposite parity for the interference is imperfectly realized in contrast to the case of $\psi\to\pi^0 p\bar{p}$. Regardless, if we choose a suitable constructive interference between N_α and N_β with $a^2\simeq 0.3$ as before, we calculate

$$\Gamma(\psi\to p\bar{p}\eta)/\Gamma(\psi\to p\bar{p})\simeq 0.64\pm 0.52 \tag{2.26}$$

in comparison to the experimental ratio of 1.01 ± 0.20. In view of the large uncertainties in both experimental and theoretical information, this agreement is not unreasonable. However, if we compute the mass spectrum with respect to $M_{\eta p}$, then we encounter a serious problem as we see from Fig. 10. The flat nature of the calculated spectrum is mainly due to the large interference between N_α and N_β. One possibility is to postulate the existence of a new baryon isobar N^* which couples strongly with $N\eta$ and $N\eta'$ but very weakly with $N\pi$ system: Then, the existence of such a new resonance even in the mass range of 1400–1600 MeV will not affect the discussion of $\psi\to p\bar{p}\pi^0$ decay but with a possibly sizeable effect to $\psi\to p\bar{p}\eta$ and $p\bar{p}\eta'$ modes. Note that it will be very difficult to prove or disprove the existence of such a resonance from the standard phase-shift analysis of pion-nucleon scattering data. However, we must be cautious about any such conjecture. First, we made many assumptions in our calculations such as $F_0=F_0^{(\alpha)}=F_0^{(\beta)}=0$. Second, the peak in the η-p mass plot does not appear[12] to experimentally reflect any particular resonance at all. Therefore, the situation is perhaps less definite both experimentally and theoretically for analysis of the $\psi\to p\bar{p}\eta$ mode at the present time.

We have also computed the contribution from the proton-pole diagram for $\psi\to p\bar{p}\omega$ decay. However, the calculated value amounts only to one-tenth of the experimental value, suggesting again the importance of the baryon-isobar poles. We will reconsider this problem in the next section.

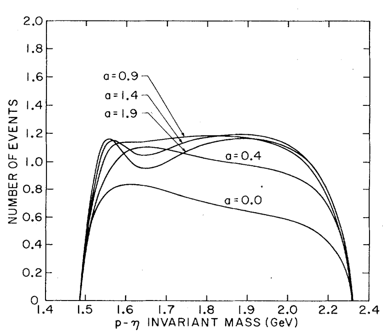

FIG. 10. Plot for $[d\Gamma(\psi\to p\bar{p}\eta)/dM_{p\eta}]/\Gamma(\psi\to p\bar{p})$, where $M_{p\eta}$ refers to the invariant mass of the $p\eta$ system. For details, see the text.

III. MESON-POLE DIAGRAMS

In Sec. I, it has been remarked that we perhaps should not take into account contributions from meson-pole diagrams in addition to those from baryon-pole diagrams because of possible double-counting of the same quark graph. However, we calculate here their contributions for the resulting completeness and comparison. In order to estimate matrix elements due to two step processes $\psi\to V_9P_9\to p\bar{p}P_9$ with $P_9=\pi_0$, η, and η' (see Fig. 2) we introduce the following effective local interactions.

$$H_{\psi VP}=g_{\psi VP}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\psi_\nu(x)\partial_\alpha V_\beta(x)P(x)\ , \tag{3.1}$$

$$H_{\rho NN}=g_{\rho NN}\bar{N}(x)\gamma_\lambda \underline{\tau} N(x)\cdot\underline{\rho}^\lambda(x) + i\frac{1}{4m}\tilde{g}_{\rho NN}\bar{N}(x)[\gamma_\mu,\gamma_\nu]\underline{\tau}N(x)\cdot\partial^\mu\underline{\rho}^\nu(x)\ , \tag{3.2}$$

$$H_{\omega NN}=g_{\omega NN}\bar{N}(x)\gamma_\lambda N(x)\omega^\lambda(x) + i\frac{1}{4m}\tilde{g}_{\omega NN}\bar{N}(x)[\gamma_\mu,\gamma_\nu]N(x)\partial^\mu\omega^\nu(x)\ . \tag{3.3}$$

Note that we need not consider $H_{\phi NN}$ since it should be zero[13] for the ideal ω-ϕ mixing because of the quark-line rule. Then, the decay matrix elements for $\psi\to P_9 p\bar{p}$ are given by

$$S_V(\psi\to P_9 p\bar{p})=-(2\pi)^4 i\delta^{(4)}(q-p+p'-k)\times\left[\frac{m^2}{4k_0q_0p_0\,|p_0'\,|\,V^4}\right]^{1/2}M_V\ , \tag{3.4a}$$

$$M_V=\frac{g_{\psi VP}}{m_V{}^2-(q-k)^2}\epsilon^{\mu\nu\alpha\beta}q_\mu k_\alpha\epsilon_\nu(q)\times\bar{u}(p)[(g_{VNN}+\tilde{g}_{VNN})\gamma_\beta-(1/2m)\tilde{g}_{VNN}(p+p')_\beta]v(p')\ . \tag{3.4b}$$

Since any of ρ, ρ', ρ''... and ω, ω', ω'',... intermediate states could contribute to $\psi\to\pi^0 p\bar{p}$ and $\psi\to\eta(\eta')p\bar{p}$ decays, respectively, the total decay matrix element should be

$$M^{(V)}=M_V+M_{V'}+M_{V''}+,\cdots\ . \tag{3.5}$$

Before going into further detail, we note the following. Take the rest frame $q_\mu=(\vec{0},M)$ of ψ, and consider the zero-momentum limit $\vec{k}\to 0$ (but $k_0\to m_\pi\neq 0$) for the pion. From Eq. (3.4), we see $M^{(V)}\to 0$ in this limit, in accordance with a stronger version of the soft-pion theorem as we have already emphasized in Sec. I.

The value of $g_{\psi VP}$ can be easily determined as follows. Consider first the $\psi\to\pi\rho$ decay. We then calculate

$$\Gamma(\psi\to\pi\rho)=\frac{1}{4\pi}(g_{\psi\rho\pi})^2k^3\ ,$$

where k is the magnitude of the pion momentum in the rest frame of ψ. From the known decay rate of $\Gamma(\psi\to\pi\rho)$ we estimate

$$|g_{\psi\rho\pi}|\simeq 1.78\times10^{-3}(\mathrm{GeV})^{-1}\ . \tag{3.6}$$

For $g_{\psi\omega\eta}$ and $g_{\psi\omega\eta'}$, we assume the exact validity of both SU(3) and the quark-line rule to find

$$g_{\psi\omega\eta}=S_1\sin(\theta_0-\theta_1)g_{\psi\rho\pi}\ ,$$
$$g_{\psi\omega\eta'}=S_2\cos(\theta_0-\theta_2)g_{\psi\rho\pi}\ . \tag{3.7}$$

Here, we assumed ideal mixing for the ω-ϕ complex. Note that the ratio $g_{\psi\omega\eta'}/g_{\psi\omega\eta}$ is equal to R_0 as in Eqs. (1.12) and (1.13). Values of g_{VNN} and $\tilde{g}_{VNN}$ can be evaluated from the Sakurai's universal ρ-coupling hypothesis[30] together with the ρ/ω dominance model for electric-magnetic form factors of the nucleon. Then, we find

$$g_{\rho pp}=\tfrac{1}{2}f_{\rho\pi\pi}=\tfrac{1}{2}(2.8\times4\pi)^{1/2}\ , \tag{3.8a}$$

$$\tilde{g}_{\rho pp}=(\mu_p-\mu_n)g_{\rho pp} \tag{3.8b}$$

as well as

$$g_{\omega pp}=3g_{\rho pp}\ , \tag{3.9a}$$

$$\tilde{g}_{\omega pp}=(\mu_p+\mu_n)g_{\omega pp}\ , \tag{3.9b}$$

where μ_p and μ_n are anomalous magnetic moments of the proton and neutron, respectively. The introduction of higher radial excited states $\rho',\rho'',\ldots$ and ω', $\omega'',\ldots$, etc., requires knowledge of additional coupling parameters $g_{\rho' p\bar{p}}$, $g_{\omega' p\bar{p}}$, etc. Here, we simply assume that the effect of $V',V'',\ldots$ may be approximated by replacing the V propagator in Eq. (3.4b) as

$$\frac{1}{m_V{}^2-(q-k)^2}\Longrightarrow\frac{1}{m_V{}^2-(q-k)^2}+\frac{\lambda'}{(m_{V'})^2-(q-k)^2}+\frac{\lambda''}{(m_{V''})^2-(q-k)^2}+\cdots \tag{3.10}$$

for suitable parameters $\lambda',\lambda'',\ldots$, etc.

In what follows, we neglect contributions from higher-order radial states ρ'' and ω'', etc., and assume $0\le\lambda'\le1$. The calculated ratios of

$$\Gamma(\psi\to\pi^0 p\bar{p})/\Gamma(\psi\to p\bar{p})$$

and

$$\Gamma(\psi\to\eta p\bar{p})/\Gamma(\psi\to\mathrm{all})$$

are tabulated in Table II for three values $\lambda'=0$, 0.5, and 1.0. From Table II, we see that the contributions from vector poles are smaller by a factor of 10 in comparison to those from baryon-pole diagrams. Moreover, we find also that interferences between vector-pole and baryon-pole diagrams are, in general, very small in magnitude. Therefore, the main results of Sec. II will remain unchanged, even if we have to include vector-meson pole diagrams. Of course, there are exceptions. First of all, the value of λ' may be very large. Second, there may exist ρ'' and ω'' whose masses are very near twice the proton mass, so that their contributions will be significant because of small propagators. However, for the latter case, we expect a large peak for large pion energy E_π in the spectrum, which does not appear to exist in the present experimental data.

TABLE II. Branching ratios for $\Gamma(\psi\to p\bar{p}\pi^0)$ and $\Gamma(\psi\to p\bar{p}\eta)$ calculated on the basis of vector-meson pole diagrams (see Fig. 2).

	$\Gamma(\psi\to p\bar{p}\pi^0)/\Gamma(\psi\to\mathrm{all})$	$\Gamma(\psi\to p\bar{p}\eta)/\Gamma(\psi\to\mathrm{all})$
$\lambda'=0$	0.93×10^{-4}	0.27×10^{-4}
$\lambda'=0.5$	1.57×10^{-4}	0.44×10^{-4}
$\lambda'=1.0$	2.22×10^{-4}	0.62×10^{-4}
Experimental	$(1.1\pm0.1)\times10^{-3}$	$(2.3\pm0.4)\times10^{-3}$

So far, we did not consider the $0^{\pm}$-meson-pole diagrams. The two-step decay $\psi\to\eta\eta'$ followed by η (or $\eta')\to p\bar{p}$ is forbidden by the charge-conjugation invariance. Similarly, the normal 2^+ mesons f and f' will not contribute for $\psi\to\eta p\bar{p}$ decay since $\psi\to\eta f$ and $\eta f'$ are forbidden again by charge-conjugation invariance. If an abnormal scalar meson η_s with negative charge-conjugation parity exists, then two-step mechanics $\psi\to\eta_s\eta$ (or η') followed by $\eta_s\to p\bar{p}$ may contribute to $\psi\to p\bar{p}\eta$ (or η'). Since there is no experimental evidence for such a scalar meson η_s, we have nothing to say about such a possibility.

Concluding this section, we have also calculated the contribution for $\psi\to p\bar{p}\omega$ from the proton-pole diagram where we replace π^0 by ω in Fig. 1. Since the coupling constants $g_{\omega p\bar{p}}$ and $\tilde{g}_{\omega p\bar{p}}$ are known from Eqs. (3.8) and (3.9), the calculation is straightforward. Assuming $F_0=0$ again, we find then

$$\frac{\Gamma(\psi\to p\bar{p}\omega)}{\Gamma(\psi\to p\bar{p})}\simeq 0.061$$

in comparison to the experimental ratio of

$$\left.\frac{\Gamma(\psi\to p\bar{p}\omega)}{\Gamma(\psi\to p\bar{p})}\right|_{\mathrm{experimental}}\simeq 0.73\ .$$

This suggests that the main contribution for $\psi\to\omega p\bar{p}$ must result from other Feynman diagrams, such as N^*-pole diagrams. However, since there is no information available for the N^*-N-ω interaction, we cannot make any definite conclusion. We note that $\psi\to\omega\eta$ (or η') followed by η (or $\eta')\to p\bar{p}$ will give negligibly small contribution because of the same reason as in $\psi\to\eta p\bar{p}$ decay. One possibly important diagram may be the 2^+ tensor pole diagram where we have $\psi\to\omega f\to\omega p\bar{p}$. Since many unknown coupling parameters are involved in the calculation, we will not, however, pursue the issue here.

ACKNOWLEDGMENT

This work is supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ER13065.

APPENDIX

Here, we give explicit expression for the decay rate for $\psi\to\pi^0 p\bar{p}$ due to the proton-pole diagram of Fig. 1. Summing over possible spin states of final baryons and changing the sign of p' into $-p'$ (so that $p_0'>0$ now), we then find

$$\Gamma^{(I)}(\psi\to\pi^0 p\bar{p})=\frac{1}{(2\pi)^5}\frac{(g_{NN\pi})^2}{4M}\int\int\int\frac{d^3k\,d^3p\,d^3p'}{k_0p_0p_0'}\delta^{(4)}(q-p-p'-k)[\,|F_M|^2A_1+|F_0|^2A_2+\mathrm{Re}\,(F_0^*F_M)A_3]\ , \tag{A1}$$

where we have set

$$A_1=(m^2+pp')[(a^2-b^2)(\epsilon\cdot k)^2+b^2\epsilon^2k^2]-2ab(\epsilon\cdot k)[(p\cdot\epsilon)(p'\cdot k)-(p'\cdot\epsilon)(p\cdot k)]$$
$$-2b^2\{\epsilon^2(p\cdot k)(p'\cdot k)-(\epsilon\cdot k)[(p\cdot k)(p'\cdot\epsilon)+(p\cdot\epsilon)(p'\cdot k)]+k^2(p\cdot\epsilon)(p'\cdot\epsilon)\}\ , \tag{A2}$$

$$A_2=\frac{1}{m^2}[(m^2-p\cdot p')k^2+2(p\cdot k)(p'\cdot k)]\left[\frac{p'\cdot\epsilon}{2p\cdot k+k^2}-\frac{p\cdot\epsilon}{2p'\cdot k+k^2}\right]^2, \tag{A3}$$

$$A_3=4k^2\left[\frac{p'\cdot\epsilon}{2p\cdot k+k^2}-\frac{p\cdot\epsilon}{2p'\cdot k+k^2}\right]^2. \tag{A4}$$

In Eq. (A2), we have set for simplicity,

$$a=\frac{1}{2p\cdot k+k^2}-\frac{1}{2p'\cdot k+k^2}\ ,\qquad b=\frac{1}{2p\cdot k+k^2}+\frac{1}{2p'\cdot k+k^2}\ . \tag{A5}$$

The spin average over the polarization vector $\epsilon_\mu(q)$ can be obtained by substituting, for example,

$$\epsilon^2=-1\ ,\qquad (\epsilon\cdot p)(\epsilon\cdot p')\to\frac{1}{3}\left[\frac{(q\cdot p)(q\cdot p')}{M^2}-(p\cdot p')\right]. \tag{A6}$$

Since $k^2=m_\pi{}^2$, all terms involving k^2 are numerically small, and may be omitted. Especially, we may set $A_3=0$ so that the interference between F_0 and F_M is practically zero for $\psi\to\pi^0 p\bar{p}$ decay. We can further integrate Eq. (A1) in terms of two variables s_1, and s_2 given by

$$s_1=(p+p')^2=(q-k)^2\ ,\qquad s_2=(p'+k)^2=(q-p)^2 \tag{A7}$$

as is given in Ref. 31.

[1]M. Roos *et al.*, Particle Data Group, Phys. Lett. **111B**, 1 (1982).
[2]J. Applequist, R. M. Barnett, and K. Lane, Annu. Rev. Nucl. Part. Sci. **28**, 387 (1978).
[3]F. E. Close, *An Introduction to Quarks and Partons* (Academic, London, New York, and San Francisco, 1979); H. S. Shopper; in *The Whys of Subnuclear Physics,* Proceedings of the International School of Subnuclear Physics, Erice, 1977, edited by A. Zichichi (Plenum, New York, 1979).
[4]T. Appelquist *et al.*, Phys. Rev. Lett. **34**, 365 (1975); E. Eichten *et al.*, *ibid.* **34**, 369 (1975); E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980); W. Buchmuller and S. Tye, *ibid.* **24**, 132 (1981).
[5]G. Karl, Nuovo Cimento **38**, 315 (1979); H. Fritzsch and J. Jackson, Phys. Lett. **66B**, 4 (1977); F. D. Gault and A. B. Rimmer, Z. Phys. C **8**, 353 (1981); G. W. Intemann, Phys. Rev. D **27**, 2755 (1983).
[6]S. Okubo, Phys. Rev. D **13**, 1994 (1976); **14**, 1809 (1976); S. Okubo and D. Weingarten, *ibid.* **14**, 1803 (1976).
[7]L. J. Clavelli and G. W. Intemann, Phys. Rev. D **28**, 2767 (1983).
[8]S. K. Mtingwa, Phys. Rev. D **15**, 3340 (1977); J. G. Korner and M. Kuroda, *ibid.* **16**, 2165 (1977).
[9]J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981); M. Claudson, S. L. Glashow, and M. B. Wise, *ibid.* **25**, 1345 (1982).
[10]J. Kodaira and K. Sasaki, Phys. Rev. D **17**, 1381 (1978); J. Pasupathy and G. A. Singh, *ibid.* **18**, 791 (1978); K. Terasaki, Prog. Theor. Phys. **61**, 678 (1977).
[11]I. Perruzi *et al.*, Phys. Rev. D **17**, 2901 (1978).
[12]M. W. Eaton *et al.*, Phys. Rev. D **29**, 804 (1984).
[13]S. Okubo, Prog. Theor. Phys. Suppl. **63**, 1 (1978). However, we changed the notation here from $(K_0)^{1/2}$ to R_0.
[14]M. Frank and P. J. O'Donnell, Phys. Rev. D **29**, 921 (1984).
[15]T. Teshima and S. Oneda, Phys. Rev. D **29**, 2067 (1984); C. Rosenzweig, A. Salomone, and J. Schechter, *ibid.* **24**, 2545 (1981); Nucl. Phys. **B206**, 12 (1982); F. D. Gault and A. B. Rimmer, Z. Phys. C **8**, 353 (1981).
[16]S. Okubo, Phys. Lett. **5**, 165 (1963); G. Zweig, CERN Report No. TH-402, 1964 (unpublished); J. Iizuka, Prog. Theor. Phys. Suppl. **37-38**, 21 (1966).
[17]S. Okubo and K. Jaganathan, Phys. Rev. D **15**, 177 (1977).
[18]W. D. Apel *et al.*, Yad. Fiz. **25**, 561 (1977) [Sov. J. Nucl. Phys. **25**, 300 (1977)].
[19]W. R. Stanton *et al.*, Phys. Lett. **92B**, 353 (1980).
[20]J. E. Olsson, in *Proceedings of the Fifth International Workshop on Photon-Photon Collisions, Aachen, 1983,* edited by Ch. Berger (Lecture Notes in Physics, Vol. 171) (Springer, Berlin, 1983); Ch. Berger, in *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High*

Energies, Ithaca, New York, edited by D. G. Cassell and D. L. Kreinick (Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, 1983).
[21]H. Genz, Phys. Rev. D **28**, 1094 (1983).
[22]M. D. Scadron, Phys. Rev. D **29**, 2076 (1984). Note that Φ_P in this paper is related to our θ by $\phi_P=(\pi/2)-\theta_0+\theta$.
[23]A. T. Filippov *et al.*, Yad. Fiz. **29**, 1035 (1979) [Sov. J. Nucl. Phys. **29**, 534 (1979)].
[24]D. Hitten, in *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energies, Ithaca, New York* (Ref. 20).
[25]S. L. Adler and R. F. Dashen, *Current Algebra and Application to Particle Physics* (Benjamin, New York, 1968); B. W. Lee, *Chiral Dynamics* (Gordon and Breach, New York, 1972).
[26]For example, see T. N. Pham, B. Pire, and T. N. Truong, Phys. Lett. **61B**, 183 (1976), and references quoted therein.
[27]For example, P. H. Frampton, *Dual Resonance Models* (Benjamin, Reading, Mass., 1974).
[28]C. Jarlskog and H. Pilkuhn, Phys. Lett. **20**, 428 (1966); B. R. Martin, Phys. Rev. **138**, B1136 (1965).
[29]For example, S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, Illinois, 1961).
[30]J. J. Sakurai, *Meson and Currents* (University of Chicago Press, Chicago, 1969).
[31]E. Byckling and K. Kajantie, *Particle Kinematics* (Wiley, London, 1972).