Application of the Newman-Penrose tetrad scheme to the light-cone gauge

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We adapt the Newman-Penrose tetrad formalism to a systematic and efficient evaluation of Feynman integrals in the light-cone gauge. Our procedure exploits the traditional assumptions of locality and power counting as well as the useful tensor technique. It is shown that this program yields the same results as the much lengthier Feynman parameter method. There are no double poles to one-loop order.

I. INTRODUCTION

The aim of this paper is to apply the tetrad formalism of Newman and Penrose[1] to a systematic study of certain noncovariant Feynman integrals, called light-cone gauge integrals. As the name implies, these integrals are characterized by a constant null vector n_μ. The most important single feature of the Newman-Penrose formalism is a four-dimensional basis spanned entirely by null vectors. The scheme originated in the context of gravitation and cosmology, where null vectors are virtually ubiquitous.

The light-cone gauge[2-10] continues to intrigue particle theorists, because it is free of ghosts and appears to be more effective than other noncovariant gauges in studying the ultraviolet finiteness of certain supersymmetric Yang-Mills theories.[2-4,11,12] The light-cone gauge is specified by the condition $n\cdot A^a = n_\mu A^a_\mu = 0$ and $n^2=0$, $\mu=0,1,2,3$, A^a_μ being a massless Yang-Mills field and n_μ a constant null vector.

The crux of the paper is the following. In the light-cone gauge n_μ has to satisfy $n^2=0$. [We shall take $n_\mu=(n_0,n_3,0,0)$ to simplify matters.] The latter condition implies that the components of n_μ are linearly dependent, with $n_3=+n_0$ or $n_3=-n_0$, $n_0>0$; hence there are two possibilities: $n_\mu\equiv(n_0,+n_0,0,0)$ and $n^*_\mu\equiv(n_0,-n_0,0,0)$. Consider now the integral

$$I_\mu=\int dq\, q_\mu G(q^2,(q-p)^2,q\cdot n)=Ap_\mu+Bn_\mu\ ,$$

which follows from symmetry and Lorentz invariance. If $n^2\neq 0$, this is a perfectly good ansatz giving unique coefficients A,B (see Sec. II). However, if n_μ is a *null* vector with linearly dependent components, the above ansatz fails and has to be replaced by

$$I_\mu=Ap_\mu+Bn_\mu+Cn^*_\mu\ .$$

This is where the tetrad calculus of Newman and Penrose becomes effective. Under ordinary circumstances, the introduction of n^*_μ into the ansatz would seem a bit strange, but in the elegant Newman-Penrose formalism, where any four-dimensional vector can be represented in terms of four null vectors, the appearance of n^*_μ is almost natural. This will be demonstrated in Sec. III.

Application of the Newman-Penrose scheme to the light-cone gauge has a two-fold advantage. Besides offering new insight into the structure of the integrals, the scheme permits, under the traditional assumptions of locality and power counting,[13] an efficient and systematic evaluation of integrals such as

$$\int \frac{d^{2\omega}q\, f(q_\mu,q_\mu q_\nu,\ldots)}{g(q^2,(q-p)^2,q\cdot n)}\ ,\quad n^2=0\ . \tag{1.1}$$

These had been previously computed, albeit with an improved prescription for $(q\cdot n)^{-1}$, by the "safe" but tedious Feynman parameter technique.[7,8] The word "locality" is short here for "locality of the *divergent* part of the integral in the external momenta." We employ dimensional regularization in a space of 2ω dimensions with a Minkowski metric $(+,-,-,-)$.

The article is planned thus: Section II begins with a short review of the well-known tensor method which enables us to calculate integrals by exploiting symmetry arguments as well as Lorentz invariance.[14-16] We then demonstrate the failure of the tensor method if a *conventional* ansatz for light-cone integrals is employed. In Sec. III the Newman-Penrose formalism is summarized and then adapted to the light-cone gauge. The new procedure is explained in Sec. IV by means of three examples. The article concludes with a discussion.

II. REVIEW OF TENSOR METHOD

As is well known, integrals of the form

$$\int \frac{dq\, F(q_\mu,q_\mu q_\nu,\ldots)}{G(q^2,(q-p)^2)}\ ,\quad dq\equiv d^{2\omega}q \tag{2.1}$$

may either be computed by the Feynman parameter technique, or by the much quicker tensor method (as we shall call it) which exploits the symmetry and Lorentz invariance of the integrals.[14-16] If certain basic, i.e., scalar, integrals are already known, the tensor method allows us to compute (2.1) *without* further integration, as illustrated below.

(a) *Covariant-gauge integrals:* Consider the Euclidean-space integral

$$I_{\mu\nu}=\int dq\, q_\mu q_\nu[q^2(q-p)^2]^{-1}\ , \tag{2.2}$$

which depends on the single parameter p_μ. From symmetry considerations and Lorentz invariance, $I_{\mu\nu}$ has to be a linear combination of the rank-two objects $\delta_{\mu\nu}$ and $p_\mu p_\nu$. The tensor method consists, therefore, of making the ansatz

$$I_{\mu\nu}=A\delta_{\mu\nu}+Bp_\mu p_\nu \tag{2.3}$$

and finding the coefficients (amplitudes) A,B by multiplying (2.3) with $p_\mu p_\nu$, then contracting μ with ν. Thus,

$$\int dq\,(q\cdot p)^2[q^2(q-p)^2]^{-1}=Ap^2+Bp^4\ , \tag{2.4a}$$

$$\int dq\,q^2[q^2(q-p)^2]^{-1}=2\omega A+Bp^2\ . \tag{2.4b}$$

To calculate (2.4a), let $q\cdot p=\frac{1}{2}[q^2+p^2-(q-p)^2]$ and use the scalar integrals[17]

$$\int dq\,[q^2(q-p)^2]^{-1}\equiv\bar{I}=\pi^2[\Gamma(\omega-1)]^2\Gamma(2-\omega)\times[\Gamma(2\omega-2)]^{-1}(p^2)^{\omega-2}\ , \tag{2.5}$$

$$\int dq\,q^{-2}=\int dq\,(q-p)^{-2}=0\ . \tag{2.6}$$

The resulting equations

$$\tfrac{1}{4}p^4\bar{I}=Ap^2+Bp^4\ ,$$

$$0=2\omega A+Bp^2$$

are solved by

$$A=-p^2\bar{I}[4(2\omega-1)]^{-1}\ ,\quad B=2\omega\bar{I}[4(2\omega-1)]^{-1}\ ,$$

so that

$$\int dq\,q_\mu q_\nu[q^2(q-p)^2]^{-1}=(2\omega p_\mu p_\nu-p^2\delta_{\mu\nu})\bar{I}[4(2\omega-1)]^{-1}\ . \tag{2.7}$$

(b) *Axial-gauge integrals:* The axial gauge, specified by $n\cdot A^a=0$ and $n^2\neq 0$, leads to integrals like

$$I_\mu=\int dq\,q_\mu[q^2(q-p)^2q\cdot n]^{-1}\ ,\quad n^2\neq 0\ . \tag{2.8}$$

Since I_μ depends on p_μ and n_μ, the proper ansatz is

$$I_\mu=Ap_\mu+Bn_\mu\ . \tag{2.9}$$

Multiplication of (2.9) by n_μ, p_μ yields, respectively,

$$p\cdot n\bar{I}/n^2=Ap^2+Bn\cdot p\ , \tag{2.10a}$$

$$\bar{I}=Ap\cdot n+Bn^2\ , \tag{2.10b}$$

with[18]

$$\int dq\,[(q-p)^2q\cdot n]^{-1}=2p\cdot n\bar{I}/n^2\ ,\quad n^2\neq 0\ , \tag{2.11}$$

$$\int dq\,[q^2(q-p)^2q\cdot n]^{-1}=0\ , \tag{2.12}$$

where only the divergent part of the last integral is given. Solving Eqs. (2.10) we find $A=0$, $B=\bar{I}/n^2$, so that

$$I_\mu=\int dq\,q_\mu[(q-p)^2q^2q\cdot n]^{-1}=n_\mu\bar{I}/n^2\ ,\quad n^2\neq 0\ , \tag{2.13}$$

which is identical to the expression obtained by the cumbersome parameter technique. We observe, for later reference, that the divergent part of I_μ is *local*—terms like $p_\mu\bar{I}/p\cdot n$ do *not* occur—and that (2.13) satisfies naive power counting.[13] The axial-gauge integral (2.13) has been tested repeatedly in explicit calculations, both in Yang-Mills theory[19-20] and quantum gravity,[18] and there can be little doubt about its correctness.

The obvious question now is "What happens to (2.13) in the light-cone gauge where $n^2=0$?" If we make the same ansatz as for the axial gauge,

$$I_\mu=\int dq\,q_\mu[q^2(q-p)^2q\cdot n]^{-1}=A_0p_\mu+B_0n_\mu\ , \tag{2.14}$$

multiplication by n_μ gives $\bar{I}=A_0p\cdot n+B_0n^2$, so that $A_0=\bar{I}/p\cdot n$ which is clearly *nonlocal.* On the other hand, if locality is to be preserved, A_0 must be zero, implying $\bar{I}=0$, which contradicts the well-established value of $\bar{I}$ in Eq. (2.5). Accordingly, the *conventional* ansatz (2.14) leads either to nonlocal expressions or to integrals that are inconsistent. In Secs. III and IV we shall propose, and then apply, an improved ansatz for the light-cone integrals (1.1) and (2.14).

III. THE NEWMAN-PENROSE FORMALISM

As mentioned in the Introduction, the tetrad scheme was developed by Newman and Penrose[1] in the context of gravitation. We summarize its main features[21] and then adapt the scheme to the analysis of a certain class of light-cone gauge integrals.

Consider the set of orthonormal vectors $\{\vec{e}_0,\vec{e}_1,\vec{e}_2,\vec{e}_3\}$, where

$$\vec{e}_0=(1,0,0,0)\ ,\quad \vec{e}_1=(0,1,0,0)\ ,\quad \vec{e}_2=(0,0,1,0)\ ,\quad \vec{e}_3=(0,0,0,1)\ , \tag{3.1}$$

satisfying

$$\vec{e}_0\cdot\vec{e}_0=1\ ,\quad \vec{e}_0\cdot\vec{e}_i=0\ ,\quad \vec{e}_i\cdot\vec{e}_j=\delta_{ij}\ ,\quad i,j=1,2,3\ , \tag{3.2}$$

with metric $\eta_{\mu\nu}=(1,-1,-1,-1)$, $\mu,\nu=0,1,2,3$. One of the chief characteristics of the Newman-Penrose (NP) tetrad scheme, and a distinct advantage for our purposes, is its representation of any four-dimensional vector by four null vectors $\vec{\ell}$, $\vec{k}$, $\vec{m}_1$, and $\vec{m}_2$:

$$\vec{\ell}=(\vec{e}_0+\vec{e}_1)/\sqrt{2}=(1/\sqrt{2})\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\ ,$$

$$\vec{k}=(\vec{e}_0-\vec{e}_1)/\sqrt{2}=(1/\sqrt{2})\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}\ , \tag{3.3}$$

$$\vec{m}_1=(\vec{e}_2+i\vec{e}_3)/\sqrt{2}=(1/\sqrt{2})\begin{bmatrix}0\\0\\1\\i\end{bmatrix},$$

$$\vec{m}_2=(\vec{e}_2-i\vec{e}_3)/\sqrt{2}=(1/\sqrt{2})\begin{bmatrix}0\\0\\1\\-i\end{bmatrix}.$$

The new basis vectors are indeed null,

$$\vec{\ell}^{\,2}=\vec{k}^{\,2}=\vec{m}_1{}^2=\vec{m}_2{}^2=0\ , \tag{3.4a}$$

with

$$\vec{\ell}\cdot\vec{k}=1\ ,\quad \vec{m}_1\cdot\vec{m}_2=-1\ , \tag{3.4b}$$

$$\vec{\ell}\cdot\vec{m}_1=\vec{\ell}\cdot\vec{m}_2=\vec{k}\cdot\vec{m}_1=\vec{k}\cdot\vec{m}_2=0\ . \tag{3.4c}$$

The *inner product* between two vectors $\vec{u}_1$ and $\vec{u}_2$,

$$\vec{u}_i=\vec{\ell}A_1+\vec{k}B_i+\vec{m}_1C_i+\vec{m}_2D_i\ ,\quad i=1,2\ ,$$

has the form

$$\vec{u}_1\cdot\vec{u}_2=A_1B_2+A_2B_1-C_1D_2-C_2D_1\ , \tag{3.5}$$

while

$$(\vec{u}_i)^2=2(A_iB_i-C_iD_i)\ ,\quad i=1\ \text{ or }\ 2\ . \tag{3.6}$$

In order to adapt the NP formalism to the light-cone gauge, we must relate the components of the vector $p_\mu=(p_0,p_3,p_1,p_2)$, in the old basis $B=\{\vec{e}_0,\vec{e}_3,\vec{e}_1,\vec{e}_2\}$, to its components in the new basis $B'=\{\vec{l},\vec{k},\vec{m}_1,\vec{m}_2\}$. It turns out that

$$p_\mu=\vec{\ell}(p_0+p_3)/\sqrt{2}+\vec{k}(p_0-p_3)/\sqrt{2}+\vec{m}_1(p_1-ip_2)/\sqrt{2}$$
$$+\vec{m}_2(p_1+ip_2)/\sqrt{2}\ ,\quad i=\sqrt{-1}\ , \tag{3.7}$$

$$p_\mu=\vec{\ell}p^++\vec{k}p^-+\vec{m}_1p_\perp+\vec{m}_2\bar{p}_\perp\ ,$$

$$p^2=2(p^+p^--p_\perp\bar{p}_\perp)\ , \tag{3.8}$$

where

$$p^+=(p_0+p_3)/\sqrt{2}\ ,\quad p^-=(p_0-p_3)/\sqrt{2}\ ,$$
$$p_\perp=(p_1-ip_2)/\sqrt{2}\ ,\quad \bar{p}_\perp=(p_1+ip_2)/\sqrt{2}\ . \tag{3.9}$$

Similarly, for the light-cone vector $n_\mu=(n_0,n_3,n_1,n_2)$,

$$n_\mu=\vec{\ell}(n_0+n_3)/\sqrt{2}+\vec{k}(n_0-n_3)/\sqrt{2}+\vec{m}_1(n_1-in_2)/\sqrt{2}$$
$$+\vec{m}_2(n_1+in_2)/\sqrt{2}\ , \tag{3.10}$$

$$n^2=2(n^+n^--n_\perp\bar{n}_\perp)=0\ . \tag{3.11}$$

To simplify the algebra, and without affecting the basic arguments, we may set $n_\mu=(n_0,n_3,0,0)$. The condition $n^2=n_0{}^2-n_3{}^2=0$ is then equivalent to $n_3=\pm n_0$, $n_0>0$, and implies that in the NP-representation (3.3), the vector (3.10) is either given by $\vec{\ell}n_0\sqrt{2}$ or $\vec{k}n_0\sqrt{2}$. These vectors may be distinguished by the notation

$$n_\mu=\vec{\ell}n_0\sqrt{2}\ , \tag{3.12a}$$

$$n^*_\mu=\vec{k}n_0\sqrt{2}\ ; \tag{3.12b}$$

clearly, $n^2=(n^*)^2=0$, while

$$n\cdot n^*=(n^+)^2\ ,\quad n^+=n_0\sqrt{2}=n_3\sqrt{2}\ . \tag{3.13}$$

Confining ourselves, for the moment, to the two-dimensional (n_0,n_3) subspace, we deduce from (3.12) that both n_μ and n^*_μ are needed to span the appropriate subspace. The presence of n_μ is not sufficient. The reason for this is that n_μ possesses linearly dependent components.

IV. APPLICATION TO LIGHT-CONE INTEGRALS

The purpose of this section is to apply the results of Sec. III, coupled with the tensor method of Sec. II, to three one-loop integrals in the light-cone gauge.

(a) *Example A.* Consider the integral

$$I_\mu=\int dq\, q_\mu[q^2(q-p)^2q\cdot n]^{-1}\ ,\quad n^2=0\ . \tag{4.1}$$

In view of the discussion in Sec. III, we propose the ansatz

$$I_\mu=\vec{\ell}A_1+\vec{k}A_2+\vec{m}_1A_3+\vec{m}_2A_4\ , \tag{4.2}$$

where $A_i\equiv A_i(p,n,n^*)$, $i=1,2,3,4$. From locality and dimensional arguments—integral (4.1) diverges logarithmically—we deduce that $A_i=A_i(n,n^*)$. Moreover, in the simple frame

$$n_\mu=(n_0,n_3,0,0)=\vec{\ell}n_0\sqrt{2}$$

[cf. Eq. (3.10)], where $n_\perp=\bar{n}_\perp=0$, the coefficients A_3,A_4 vanish and

$$I_\mu=\vec{\ell}A_1+\vec{k}A_2\ . \tag{4.3}$$

A_1,A_2 are easily found by the tensor method, as shown below.

Since

$$q\cdot n=(\vec{\ell}n^+)\cdot(\vec{\ell}q^++\vec{k}q^-+\vec{m}_1q_\perp+\vec{m}_2\bar{q}_\perp)=n^+q^-\ ,$$

multiplication of (4.3) by n_μ yields $I_\mu n_\mu=n^+A_2$, or $A_2=\bar{I}/n^+$, while multiplication by

$$p_\mu=\vec{\ell}p^++\vec{k}p^-+\vec{m}_1p_\perp+\vec{m}_2\bar{p}_\perp$$

gives

$$\int dq\, q\cdot p[q^2(q-p)^2q\cdot n]^{-1}=p^-A_1+p^+A_2\ . \tag{4.4}$$

The left-hand side (LHS) of (4.4) is trivially solved, using $q\cdot p=\frac{1}{2}[q^2+p^2-(q-p)^2]$ and the formulas

$$\int dq[(q-p)^2q\cdot n]^{-1}=2p^+\bar{I}/n^+\ , \tag{4.5a}$$

$$\int dq[q^2(q-p)^2q\cdot n]^{-1}=0\ , \tag{4.5b}$$

which must be computed by the parameter method.[7] Hence, (4.4) becomes $p^+\bar{I}/n^+=p^-A_1+p^+A_2$, implying $A_1=0$. Finally,

$$\int dq\, q_\mu [q^2(q-p)^2 q\cdot n]^{-1} = \vec{k}\bar{I}/n^+ \;,\quad n^2=0\;,$$
$$= n^*_\mu \bar{I}/n\cdot n^* \;, \tag{4.6}$$

where the last step follows from Eqs. (3.12b) and (3.13). It is clear that the integral (4.6) respects locality and naive power counting, just like its axial-gauge counterpart, Eq. (2.13). Notice, also, that in the transition from the axial to the light-cone gauge, n^2 is formally replaced by the new invariant $n\cdot n^*$. Furthermore, multiplication of (4.6) by n_μ correctly gives $\bar{I}=\bar{I}$, in sharp contrast to the embarrassing result $\bar{I}=0$ obtained with a conventional ansatz [cf. Eq. (2.14)].

The last example was discussed in some detail to highlight the interplay between the various Newman-Penrose null vectors. In the next two examples our approach will be more succinct, as well as more general, with $n_\mu=(n_0,n_3,n_1,n_2)$ and $n^*_\mu=(n_0,-n_3,-n_1,-n_2)$ replacing $n_\mu=(n_0,n_3,0,0)$ and $n^*_\mu=(n_0,-n_3,0,0)$, respectively.

(b) *Example B.* We know from the last paragraph in Sec. III that the integral

$$I_\mu = \int dq\, q_\mu [(q-p)^2 q\cdot n]^{-1} \;,\quad n^2=0 \tag{4.7}$$

should be represented by p_μ, n_μ as well as n^*_μ. Hence, we make the ansatz

$$I_\mu = a n_\mu + b n^*_\mu + c p_\mu \tag{4.8}$$

and then apply locality and dimensional arguments to extract the general structure of a,b,c. The last step is not essential but it does simplify subsequent arguments considerably. Also, we shall denote the *dimension* of a quantity H by square brackets: $[H]$. Since $[I_\mu]=[p^2]$, the coefficients in (4.8) must possess the form

$$a=\lambda(p\cdot n^*)^2/(n\cdot n^*)^2\;,\quad c=\tau(p\cdot n^*)/n\cdot n^*\;,$$
$$b=\sigma p^2/n\cdot n^* \quad\text{or}\quad b=\rho p\cdot np\cdot n^*/(n\cdot n^*)^2\;, \tag{4.9}$$

with λ,σ,τ,ρ pure numbers proportional to $\bar{I}$. In deriving (4.9) we made use of the fact that $1/n^2$ ceases to be an acceptable invariant in the light-cone gauge.[5,7] As a result, terms with the dimension $[p/n]$ are necessarily restricted to the form $p\cdot n^*/n\cdot n^*$. Hence, Eq. (4.8) reads

$$I_\mu = \lambda n_\mu (p\cdot n^*)^2/(n\cdot n^*)^2 + \tau p_\mu p\cdot n^*/n\cdot n^* + \left\{\begin{matrix}\sigma p^2/n\cdot n^* \\ \rho p\cdot np\cdot n^*/(n\cdot n^*)^2\end{matrix}\right\} n^*_\mu \;. \tag{4.10}$$

Multiply (4.10) by n_μ in which case

$$0=\tau p\cdot n^* p\cdot n/n\cdot n^* + \left\{\begin{matrix}\sigma p^2 \\ \rho p\cdot n^* p\cdot n/n\cdot n^*\end{matrix}\right. \;,$$

so that either $\tau=\sigma=0$ or $\tau=-\rho$. Since the first solution leads to contradictory integrals, we choose $\rho=-\tau$. Substituting the latter into (4.10) and multiplying the resulting expression by p_μ we obtain

$$\text{LHS}=\bar{I}(n\cdot n^*)^{-1}[2p^2 p\cdot n^* - 3p\cdot n\,(p\cdot n^*)^2/n\cdot n^*]\;, \tag{4.11a}$$
$$\text{RHS}=(n\cdot n^*)^{-1}[(\lambda-\tau)p\cdot n(p\cdot n^*)^2/n\cdot n^* + \tau p^2 p\cdot n^*]\;, \tag{4.11b}$$

or eventually

$$(\tau-2\bar{I})p^2 + (\lambda-\tau+3\bar{I})p\cdot np\cdot n^*/n\cdot n^* = 0\;.$$

Hence $\tau=2\bar{I}$, $\lambda=-\bar{I}$, $\rho=-2\bar{I}$, and

$$\int dq\, q_\mu [(q-p)^2 q\cdot n]^{-1} = \bar{I}(p\cdot n^*/n\cdot n^*)[-n_\mu p\cdot n^*/n\cdot n^* + 2(p_\mu - n^*_\mu p\cdot n/n\cdot n^*)]\;. \tag{4.12}$$

(c) *Example C.* As our last illustration, consider

$$I_{\mu\nu} = \int dq\, q_\mu q_\nu [q^2(q-p)^2 q\cdot n]^{-1}\;,\quad n^2=0\;,$$
$$= A\delta_{\mu\nu} + B(p_\mu n^*_\nu + p_\nu n^*_\mu) + C n^*_\mu n^*_\nu + E n_\mu n_\nu + D(n_\mu n^*_\nu + n_\nu n^*_\mu) + F p_\mu p_\nu + G(p_\mu n_\nu + p_\nu n_\mu)\;. \tag{4.13}$$

Proceeding as in example B, we infer that the coefficients $A,\ldots,G$ have the structure

$$A=\alpha p\cdot n^*/n\cdot n^*\;,\quad B=\beta/n\cdot n^*\;,$$
$$C=\gamma p\cdot n/(n\cdot n^*)^2\;, \tag{4.14}$$
$$D=\delta p\cdot n^*/(n\cdot n^*)^2\;,\quad E=F=G=0\;,$$

where $\alpha,\beta,\gamma,\delta$ are again pure numbers, proportional to $\bar{I}$. Let us justify the form of E. Since $[I_{\mu\nu}]=[p/n]$, it follows that

$$[E]=[p/n^3]=[p\cdot n^*/n\cdot n^*][n^{-2}]\;,$$

so the question is how to "handle" $[n^{-2}]$. Terms like $p^2/(p\cdot n)^2$ have the right dimension but are obviously nonlocal, while expressions like $(n^*)^2/(n\cdot n^*)^2$ vanish identically. We conclude that E must be zero. Arguments similar to these may be used to obtain the structure of the other coefficients in (4.14). Hence,

$$I_{\mu\nu} = (n\cdot n^*)^{-1}[\alpha p\cdot n^* \delta_{\mu\nu} + \beta(p_\mu n^*_\nu + p_\nu n^*_\mu) + \gamma(p\cdot n/n\cdot n^*) n^*_\mu n^*_\nu + \delta(p\cdot n^*/n\cdot n^*)(n_\mu n^*_\nu + n_\nu n^*_\mu)]\;. \tag{4.15}$$

Application of the tensor method leads to a quick solution for $\alpha,\ldots,\delta$. Specifically, contracting μ with ν in (4.15) and using (4.5a) we get

$$2\alpha+\beta+\delta=\bar{I}\;. \tag{4.16a}$$

Similarly, multiplication by $n_\mu n_\nu$ gives

$$2\beta+\gamma=(1/2)\bar{I}\;, \tag{4.16b}$$

whereas multiplication by $p_\mu n_\nu$ leads to two equations,

$$\beta=\tfrac{1}{2}\bar{I}\;, \tag{4.16c}$$

$$\alpha+\beta+\gamma+\delta=0\;. \tag{4.16d}$$

The solution of system (4.16) is $\alpha=\beta=\frac{1}{2}\bar{I}$, $\gamma=\delta=-\frac{1}{2}\bar{I}$, and thus,

$$\int dq\, q_\mu q_\nu [q^2(q-p)^2 q\cdot n]^{-1} = (2n\cdot n^*)^{-1}\bar{I}[p\cdot n^*\delta_{\mu\nu}+(p_\mu n^*_\nu+p_\nu n^*_\mu) - (p\cdot n/n\cdot n^*)n^*_\mu n^*_\nu - (p\cdot n^*/n\cdot n^*)(n_\mu n^*_\nu+n_\nu n^*_\mu)] \,. \tag{4.17}$$

This integral agrees exactly with the expression originally derived by the much lengthier and less transparent parameter technique[8] and without imposing any "constraints" such as locality or validity of power counting.

V. DISCUSSION

In this article we have adapted the tetrad formalism of Newman and Penrose to a systematic evaluation of light-cone integrals which are characterized by the null vector n_μ. It was found that the tetrad calculus provided not only a deeper insight into the structure of these integrals, but it also led quite naturally to an improved ansatz. The latter utilizes both n_μ and n^*_μ and is vital for a successful application of the tensor method.

Invoking locality and dimensional arguments, we applied our new procedure to three single-loop integrals [Eqs. (4.6), (4.12), and (4.17)]. We found complete correspondence with the expressions obtained earlier by the cumbersome Feynman parameter method. This agreement between two conceptually different procedures is encouraging: it reinforces our view that even light-cone integrals should respect locality and power counting, just as do integrals in the covariant Feynman-Landau gauges.[17]

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