PHYSICAL REVIEW D VOLUME 30, NUMBER 10 15 NOVEMBER 1984

Preferred-frame interactions

E. N. Glass and G. Szamosi
Department of Physics, University of Windsor, Windsor, Ontario, Canada N9B 3P4
(Received 25 June 1984)

A new relativistic coupling, covariantly formulated, with respect to a preferred frame is considered. The new coupling is illustrated here by a scalar field coupled to a particle. The new coupling avoids unphysical features present in the traditional scalar-field—particle coupling.

I. INTRODUCTION

We consider the possibility of a coupling between a relativistic field and a particle, where the coupling is given in a preferred frame. A necessary feature of the preferred-frame coupling is a future-pointing, timelike vector which, when adjoined with a right-handed spacelike triad of vectors, forms a preferred Lorentz frame. Since nature offers a variety of preferred frames, we are motivated to establish a relativistic interaction, covariantly formulated, with respect to such frames.

There are many examples of naturally occurring preferred frames. The frames comoving with the matter of Friedmann-Robertson-Walker cosmologies are preferred frames, and so is the frame in which the blackbody background radiation is isotropic. In the spacetime surrounding a static star, gravitational measurements can determine the timelike Killing vector which generates time translations asymptotically. That vector is the "gravitationally preferred" timelike vector of a preferred frame. Indeed, in any experiment with an externally prepared field, the frame attached to the apparatus which prepared the field is "preferred."

It is our intention here to always consider the relativistic field and the coupled particle as existing in a background of other noninterfering fields. It is the physics of those "other fields" which will specify the preferred frame. We consider the background to be global, encompassing the entire spacetime, so that localized experiments which prepare other fields do not define a preferred frame unless the prepared field is the relativistic field itself.

In this work, we will illustrate the new coupling with the scalar-field interaction. One reason for this choice is that the scalar field is the simplest of all field theories. Another involves the "unphysical" features of the traditional scalar field coupling to a single particle. The traditional coupling gives rise to a field-dependent mass. This led Leiter and Szamosi[1] to find arbitrarily large preacceleration times for a particle moving (with radiational reaction) in a scalar field. Furthermore, in one-particle Dirac theory, a fermion interacting via the traditional coupling with a scalar potential barrier does *not* exhibit the Klein paradox. For this case, there is no hint of the existence of the fermion's antiparticle. A final reason is that classical equations of motion involving the scalar field remain experimentally untested since long-range scalar fields have not been observed.

For the new coupling to be given below, there is a new equation of motion under which particle mass remains constant, and it is clear from the formulation that motion with radiation reaction will not have arbitrarily large preaccelerations. It will also emerge below that, in one-particle Dirac theory, the fermion-potential-barrier problem does exhibit the Klein paradox under the new coupling.

Except for the final section, we consider scalar fields on Minkowski space. Our considerations are based on the use of c-number fields, consistent with the one-particle concept. A preferred frame in Minkowski space can be established by directly preparing the scalar field itself, or by electromagnetic measurements of the isotropy of the blackbody background. Although blackbody background radiation is formulated in a big-bang cosmology, we work in the approximation of system scale size much less than the local radius of curvature. Furthermore, we assume the time scale over which the preferred frame remains unchanged to be much greater than the times of the scalar field processes.

The paper is organized as follows. Notation is established in Sec. II, and then the traditional scalar field coupling is reviewed. The new coupling is introduced in Sec. III. Remarks at the end of this section point out the differences between the two couplings. In Sec. IV, conservation of energy is discussed for both the traditional and new couplings. Preferred frames in curved background spaces are established in Sec. V, and conclusions are presented in Sec. VI.

II. TRADITIONAL COUPLING

Consider a scalar field $\Phi(x)$, which obeys the Klein-Gordon equation on Minkowski space:

$$(\Box+\mu^2)\Phi=0 \ . \tag{2.1}$$

Equation (2.1) can be derived from a variational principle with action[2]

$$S_\Phi=\tfrac{1}{2}\int_R (\eta^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}-\mu^2\Phi^2)d^4x \ . \tag{2.2}$$

($\eta^{\mu\nu}$ is the Minkowski metric with signature -2, partial derivatives are denoted by $\partial_\mu \Phi$ and $\Phi_{,\mu}$, and we work in units such that $c=1$.)

We also consider a particle of rest mass m and scalar charge g moving on a world line $z^\mu(\tau)$ with tangent $v^\mu = dz^\mu/d\tau$. The free-particle equation of motion is obtained from the action

$$S_m = -\int_{\tau_1}^{\tau_2} m(v^\alpha v_\alpha)^{1/2} d\tau \,. \tag{2.3}$$

The total action for particle plus field with the *traditional* coupling is given by[2,3]

$$S = S_m + S_\Phi + S_{\text{trad}} \,,$$

where

$$S_{\text{trad}} = -g\int_{\tau_1}^{\tau_2}\int_R \Phi(x)\delta_4(x-z)(v^\alpha v_\alpha)^{1/2} d\tau\, d^4x \,. \tag{2.4}$$

Varying Φ yields

$$(\Box + \mu^2)\Phi = -\rho_{\text{trad}}(x) \,, \tag{2.5a}$$

$$\rho_{\text{trad}}(x) = g\int_{\tau_1}^{\tau_2} \delta_4(x-z)(v_\alpha v^\alpha)^{1/2} d\tau \,. \tag{2.5b}$$

Varying the world line z^μ provides the equation of motion

$$(v_\alpha v^\alpha)^{-1/2}\frac{d}{d\tau}[(m+g\Phi)v_\mu (v_\alpha v^\alpha)^{-1/2}] = g\partial_\mu \Phi \,. \tag{2.6}$$

Changing to proper time s, where $ds/d\tau = (v^\alpha v_\alpha)^{1/2}$, Eq. (2.6) can be rewritten as

$$\frac{d}{ds}[(m+g\Phi)u_\mu] = g\partial_\mu \Phi \tag{2.7}$$

with the unit four-velocity $u^\mu = dz^\mu/ds$. The coupling constant g has dimensions such that $\dim(g\Phi) = $ energy.

Note that the effective mass is field dependent. Leiter and Szamosi[1] found arbitrarily large preacceleration times for a particle moving (with radiation reaction) in a scalar field where the coupling was traditional.

III. NEW COUPLING

A new coupling for the scalar field is given by

$$S_{\text{new}} = -g\int_{\tau_1}^{\tau_2}\int_R \Phi(x) t_\mu v^\mu \delta_4(x-z)\, d\tau\, d^4x \,. \tag{3.1}$$

Here, t_μ is dual to the timelike vector of a preferred Lorentz frame, and is given by the exact one-form

$$t_\mu dx^\mu = dt \,. \tag{3.2}$$

In Minkowski space, t_μ is both curl-free and a unit vector.

The action $S = S_\Phi + S_m + S_{\text{new}}$ yields

$$(\Box + \mu^2)\Phi = -\rho_{\text{new}}(x) \,, \tag{3.3a}$$

$$\rho_{\text{new}}(x) = g\int_{\tau_1}^{\tau_2} t_\mu v^\mu \delta_4(x-z)\, d\tau \,, \tag{3.3b}$$

$$m\frac{d}{d\tau}[v_\mu (v_\alpha v^\alpha)^{-1/2}] = gA_{\mu\nu}v^\nu \,, \tag{3.4a}$$

$$A_{\mu\nu} = \Phi_{,\mu}t_\nu - \Phi_{,\nu}t_\mu \,. \tag{3.4b}$$

Upon using proper time s, Eq. (3.4) becomes

$$m\frac{d}{ds}(u_\mu) = g(\Phi_{,\mu}t_\alpha u^\alpha - t_\mu \Phi_{,\alpha}u^\alpha) \,. \tag{3.5}$$

For comparison, Eq. (2.7) is rewritten as

$$(m+g\Phi)\frac{d}{ds}(u_\mu) = g(\Phi_{,\mu} - \Phi_{,\alpha}u^\alpha u_\mu) \,. \tag{3.6}$$

One can argue that Eq. (3.6) can be reformulated as an equation with constant mass by introducing

$$g\tilde{\Phi} = m\ln(m+g\Phi) \,,$$

but $\tilde{\Phi}$ will not obey a Klein-Gordon equation, so that kind of reformulation is not germane.

To compare the new Eq. (3.5) and the traditional Eq. (3.6), consider the case of Φ as a given external field. dt is normal to the $t=$const hypersurfaces of the Lorentz frame in which $\Phi(x)$ is constructed as a solution of the Klein-Gordon equation. We now use

$$u^\mu = (\gamma, \gamma\vec{v}), \quad \gamma^{-2} = 1 - \vec{v}^{\,2} \,, \qquad d/ds = \gamma\, d/dt, \quad \partial_\mu \Phi = (\partial_t \Phi, \nabla\Phi) \,. \tag{3.7}$$

It is readily established that both (3.5) and (3.6) have the usual classical limit

$$m\, d\vec{v}/dt = -g\nabla\Phi \,. \tag{3.8}$$

However, one should be able to solve a valid equation of motion iteratively (to obtain a numerical solution) at all points along the particle's trajectory. Equation (3.6) fails this criterion for a particle moving in a long-range field Φ such that $m+g\Phi$ comes arbitrarily close to zero[1] along points of the particle's orbit, while Eq. (3.5) admits convergent iterations at those same points.

For static scalar fields ($\partial\Phi/\partial t = 0$), using (3.7), Eq. (2.7) admits the constant of motion

$$E_{\text{trad}} = \gamma(m+g\Phi) \,, \tag{3.9a}$$

and Eq. (3.5) admits the constant

$$E_{\text{new}} = \gamma m + g\Phi \,. \tag{3.9b}$$

E_{new} and E_{trad} are the total energies (positive) for the respective couplings. Equations (3.9) can be rewritten in the explicit form

$$\vec{v}^{\,2} = 1 - E_{\text{trad}}^{-2}(m+g\Phi)^2 \,, \tag{3.10a}$$

$$\vec{v}^{\,2} = 1 - m^2(E_{\text{new}} - g\Phi)^{-2} \,. \tag{3.10b}$$

In a region of attractive scalar field ($g\Phi$ negative) the traditional coupling admits orbital points such that $m+g\Phi$ approaches zero with $|\vec{v}|$ approaching light speed. Indeed, $m+g\Phi$ can become negative and leave $|\vec{v}|$ undefined. On the other hand, the new coupling allows normal orbital properties [cf. (3.9b) and (3.10b)] for the same $g\Phi$. This criticism is purely aesthetic, since classical scalar fields have not been observed.

For quantum systems, no distinction between the two couplings can be found at the level of the Schrödinger equation, since both couplings have the nonrelativistic limit (3.8). One must therefore search for differences at the relativistic quantum level.

Remarks

(a) Note that if $\Phi=-k/r$ and $\mu=0$ (i.e., Φ is static and long range), then Eq. (3.5) is, with the use of (3.7), the equation of motion for the "Sommerfeld problem", with Hamiltonian

$$H=(m^2+\vec{p}^{\,2})^{1/2}-gk/r\ .$$

The orbit equation admits a well-known exact solution with periapsis advance.[4]

(b) For the Dirac equation, the new scalar field coupling appears the same as an electrostatic potential. The difference is that here Φ can be a function of space and *time*:

$$[\gamma^\mu(p_\mu-g\Phi t_\mu)-m]\psi=0\ .$$

One has the additional possibility of adding $g\gamma^\mu\gamma^\nu A_{\mu\nu}$ [where $A_{\mu\nu}$ is given in (3.4b)] to the mass just as the Pauli moment term is added in electrodynamics.

For a static external scalar field, the Dirac equation for a fermion interacting via the new coupling is formally identical with an electrostatic coupling (for static fields one can, in principle, distinguish between scalar and electric charge). Therefore, a potential barrier problem exhibits the Klein paradox. This is the standard exhibition (within the one-particle model) of the limitation of the external field approximation[5] and a clue to the existence of the fermion's antiparticle.

The Dirac equation for a fermion with traditional coupling to a scalar field is given by

$$(\gamma^\mu p_\mu-m-g\Phi)\psi=0\ .$$

With an external static potential barrier, as above, it has been shown that the traditional coupling does *not* display the Klein paradox.[6]

(c) The traditional and new couplings provide different sources for the Klein-Gordon equation. For the traditional coupling, Eq. (2.5) can be written as

$$(\Box+\mu^2)\Phi=-g\int\delta_4(x-z)ds\ .$$

In an explicit inertial frame, $ds=\gamma^{-1}dt$, and so

$$\begin{aligned}(\Box+\mu^2)\Phi&=-g\int\delta_3(\vec{x}-\vec{z})\delta_0(t-z^0)\gamma^{-1}dt\\&=-g\gamma^{-1}\delta_3(\vec{x}-\vec{z})\ ,\end{aligned}\qquad(3.11)$$

where γ is given in Eq. (3.7). The faster the source particle moves, the less it drives the scalar field.

For the new coupling, Eq. (3.3) yields

$$(\Box+\mu^2)\Phi=-g\int t_\alpha u^\alpha\delta_4(x-z)ds\ .$$

In the preferred frame $t_\alpha u^\alpha=\gamma$, and $ds=\gamma^{-1}dt$, which implies

$$(\Box+\mu^2)\Phi=-g\delta_3(\vec{x}-\vec{z})\ .$$

Here, the source particle drives the scalar field independently of the particle's velocity.

For an observer in an arbitrary Lorentz frame, the motion of the preferred frame is given by

$$t^\alpha=\gamma_V(1,\vec{V})\ .$$

The source particle has four-velocity

$$u^\alpha=\gamma_u(1,\vec{u})\ ,$$

and $ds=\gamma_u{}^{-1}dt$. It follows that

$$(\Box+\mu^2)\Phi=-g\gamma_V(1-\vec{u}\cdot\vec{V})\delta_3(\vec{x}-\vec{z})\ .\qquad(3.12)$$

Equation (3.12) implies that, in principle, a scalar wave detector and a source particle at rest in the observer's frame ($\vec{u}=0$) could be used to measure the relative velocity between the observer and the preferred frame.

Note that when the source particle is at rest in the preferred frame, $\vec{u}=\vec{V}$, and Eq. (3.12) becomes

$$(\Box+\mu^2)\Phi=-g\gamma_u{}^{-1}\delta_3(\vec{x}-\vec{z})\ .\qquad(3.13)$$

Comparison of Eqs. (3.11) and (3.13) shows that, for this special case, the preferred-frame distinction disappears.

IV. CONSERVATION OF ENERGY

A. Traditional coupling

If one considers a system consisting of particle plus field exchanging energy, and if the system is translation invariant, then energy conservation can be expressed with the use of the total energy-momentum tensor for the system

$$T^{\mu\nu}=t^{\mu\nu}+\tau^{\mu\nu}\ ,\qquad(4.1)$$

where $t^{\mu\nu}$ is the field contribution and $\tau^{\mu\nu}$ is the particle energy-momentum tensor. For the scalar field, the canonical and symmetric tensors are identical and are given by

$$t^{\mu\nu}(x)=\Phi^{,\mu}\Phi^{,\nu}+\tfrac{1}{2}\eta^{\mu\nu}(\mu^2\Phi^2-\Phi_{,\alpha}\Phi^{,\alpha})\ .\qquad(4.2)$$

It follows directly that

$$t^{\mu\nu}{}_{,\nu}=\Phi^{,\mu}(\Box+\mu^2)\Phi\ ,\qquad(4.3)$$

and using the traditional equation (2.5)

$$t^{\mu\nu}{}_{,\nu}=-g\Phi^{,\mu}\int\delta_4[x-z(s)]ds\ .\qquad(4.4)$$

With overdots denoting d/ds, the particle contribution has the form

$$\tau^{\mu\nu}(x)=\int M(s)\dot{z}^\mu\dot{z}^\nu\delta_4[x-z(s)]ds\ .\qquad(4.5)$$

Straightforward calculation yields the divergence

$$\tau^{\mu\nu}{}_{,\nu}=\int\frac{d}{ds}(M\dot{z}^\mu)\delta_4(x-z)ds\ .\qquad(4.6)$$

Conservation of total energy-momentum (and translation invariance) is expressed by

$$T^{\mu\nu}{}_{,\nu}=0\ .$$

Substituting Eqs. (4.4) and (4.6) yields the equation of motion

$$\frac{d}{ds}(M\dot{z}_\mu)=g\Phi_{,\mu}\ .\qquad(4.7)$$

Here $\dot{z}_\mu\dot{z}^\mu=1$, and it follows directly that $M=m+g\Phi$. Equation (4.7) is identical with the traditional equation of motion (2.7).

B. New coupling

The change of momentum density for the field can still be expressed with $t^{\mu\nu}{}_{,\nu}$ as above, and Eqs. (4.2) and (4.3) remain valid. The new coupling equation (3.3) implies

$$t^{\mu\nu}{}_{,\nu}=-g\Phi^{,\mu}\int t_\alpha u^\alpha \delta_4(x-z)ds \ . \tag{4.8}$$

We have not found an appropriate energy-momentum tensor for the particle, and so we use the particle's canonical momentum. The actions (2.3) and (3.1) yield

$$L=m(v^\mu v_\mu)^{1/2}+g\Phi t_\mu v^\mu \ . \tag{4.9}$$

The particle momentum is given by

$$p^\mu=\frac{\partial L}{\partial v_\mu}=mv^\mu(v^\alpha v_\alpha)^{-1/2}+g\Phi t^\mu \ ,$$

or, in terms of $u^\mu=dz^\mu/ds$,

$$p^\mu=mu^\mu+g\Phi t^\mu \ . \tag{4.10}$$

Conservation of total energy and momentum is expressed by

$$t^{\mu\nu}{}_{,\nu}+\frac{dp^\mu}{ds}=0 \ . \tag{4.11}$$

With Eq. (4.8) expressing $t^{\mu\nu}{}_{,\nu}$ on the particle's world line, Eq. (4.11) is identical with the equation of motion (3.5).

V. CURVED BACKGROUNDS

Here, test fields and particles moving in curve spacetimes are considered. Each background, with curved metric $g_{\mu\nu}$, will offer a particular preferred frame. The field equations and equations of motion for the particles are obtained from the actions of Secs. II and III with the substitutions (minimal coupling)

$$\eta^{\mu\nu}\rightarrow g^{\mu\nu}, \ \ d^4x\rightarrow\sqrt{-g}\ d^4x \ ,$$
$$\delta_4(x-z)d^4x\rightarrow\delta_4(x-z)d^4x \ . \tag{5.1}$$

Thus, using Eq. (2.2), the source-free scalar-field equation on a curved background is

$$(\nabla_\alpha\nabla^\alpha+\mu^2)\Phi=0 \ , \tag{5.2}$$

where ∇_α is the covariant derivative.

The traditional scalar coupling yields, from (2.3) and (2.4),

$$u^\alpha\nabla_\alpha[(m+g\Phi)u_\mu]=g\nabla_\mu\Phi \ , \tag{5.3}$$

where $u^u=dz^\mu/ds$. The new coupling on a curved background, from (2.3), (3.1), and (5.1) gives rise to

$$mu^\alpha\nabla_\alpha u_\mu=gA_{\mu\nu}u^\nu \ ,$$
$$A_{\mu\nu}=\Phi_{,\mu}t_\nu-\Phi_{,\nu}t_\mu \ . \tag{5.4}$$

We will consider two classes of background spacetimes: static spacetimes, where the timelike Killing vector selects the preferred frame, and cosmologies where the preferred frame is comoving with the matter.

A. Static spacetimes

Consider a static spacetime with hypersurface-orthogonal timelike Killing vector ξ^μ. The covariant derivative of ξ^μ is given by

$$\nabla_\nu\xi_\mu=\alpha^{-1}{}_{[\mu}\nabla_{\nu]}\alpha \ , \tag{5.5}$$

where

$$\alpha=\xi^\mu\xi_\mu, \ \ \text{and} \ \ \xi^\mu\partial_\mu\alpha=\partial_t\alpha=0 \ . \tag{5.6}$$

There is a closed one-form $\alpha^{-1}\xi_\mu dx^\mu$ which is locally exact and hence associated with $t=$const hypersurfaces:

$$dt=\alpha^{-1}\xi_\mu dx^\mu \ . \tag{5.7}$$

Closure follows from Eq. (5.5), and $t_\mu t^\mu=\alpha^{-1}$.

B. Friedmann-Robertson-Walker cosmologies

Consider a Friedmann-Robertson-Walker (FRW) spacetime. The cosmological time t is related to the matter four-velocity u^μ by

$$dt=u_\mu dx^\mu \ , \tag{5.8}$$

where

$$\nabla_\nu u_\mu=\tfrac{1}{3}(g_{\mu\nu}-u_\mu u_\nu)\Theta, \ \ \Theta=\nabla_\mu u^\mu \ .$$

Closure of (5.8) follows since u^μ is tangent to an irrotational flow.

VI. CONCLUSION

A preferred-frame interaction for the scalar field has been introduced and extensively compared with the traditional scalar-field—particle coupling. Difficulties with the traditional one-particle equation of motion have been discussed, and it has been shown that those difficulties are not present for the new coupling.

Since differences between the two couplings can be distinguished only at the relativistic level, and since classical scalar fields are not observed, a natural next step is to attempt quantization of the preferred-frame—scalar-field interaction. The recent work of Hartle and Kuchar[17] on path-integral quantization of parametrized theories provides an appropriate starting point.

ACKNOWLEDGMENTS

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1]D. Leiter and G. Szamosi, Phys. Rev. D **13**, 2762 (1976).

[2]A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (Dover, New York, 1980).

[3]J. L. Anderson, *Principles of Relativity Physics* (Academic, New York, 1967).

[4]R. D. Sard, *Relativistic Mechanics* (Benjamin, New York, 1970).

[5]J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons,* 2nd ed. (Springer, New York, 1976), p. 312.

[6]P. E. Kunin and I. M. Taksar, Zh. Eksp. Teor. Fiz. **5**, 506 (1957) [Sov. Phys. JETP **5**, 426 (1957)].

[7]J. B. Hartle and K. V. Kuchar, J. Math. Phys. **25**, 57 (1984).