PHYSICAL REVIEW D VOLUME 30, NUMBER 1 1 JULY 1984

A PCAC-like picture from a generalized MIT bag model

M. Villani
Dipartimento di Fisica, Università di Bari, Bari, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari, Italy
(Received 28 December 1983)

The MIT bag action is generalized through a new surface term. As a consequence two nonlinear boundary conditions are obtained. The new model shows how a PCAC-like picture can be realized on the basis of confined fermion fields. An interesting symmetry-breaking phenomenon is seen.

I. INTRODUCTION

In this paper we show how a picture very close to the Nambu chiral-symmetry realization[1] can be derived from a model based on confined fermion fields. To this picture, however, we are led through a different symmetry-breaking phenomenon. The phenomenon, shared by the model given here, can be summarized as follows: in some circumstances, a small perturbation which breaks a symmetry realized in the Wigner-Weyl fashion gives rise to a physical situation very close to that described by a Nambu-Goldstone[1] realization.

Our considerations are based on a generalization of the MIT bag model.[2-4] Here we are concerned with the interesting intrinsic theoretical properties of this model. Phenomenological applications are out of the scope of this work. Furthermore, confined vector fields are not considered.

The simplest idealized version of the model given in this paper starts from the MIT bag action for a confined fermion field. We introduce in this action a new surface term, which, among other things, gives some physical reality to the surface of a bag. As a consequence of the new term, instead of one, we have two nonlinear boundary conditions related to the relativistic invariance. In order to satisfy both the conditions we need another fermion field, with the benefit that a charge-conjugation invariance is realized too.

Our model with two fermion fields is formulated in Sec. II. In Sec. III we analyze its spectral properties by considering static spherical bags at rest. As will be clear, besides the relativistic invariance, quantum theory, in the form given in Ref. 5, will have an important role. In Sec. IV we show how a PCAC-like picture can be realized in our model. As is known, in the usual MIT bag model, chiral symmetry is badly broken even if we consider zero-mass fermion fields.[3] Several attempts to improve chiral symmetry have been discussed in the literature.[3,6-9] In the hybrid chiral bags,[3,6] chiral invariance can be restored, but at the cost of the introduction of fundamental pseudoscalar fields. On the other hand, in our model we have two fermion fields. From this and the results of Sec. III, we expect some differences in the meson excited states compared with the standard bag model. In our model there will be more excited levels coming from a quark and antiquark both in the same excited state. However, due to other nonlinear boundary conditions, levels where the quark and the antiquark are in different states (for example, one excited and the other not) will be absent.

In Sec. V we analyze the symmetry properties of our model with regard to the above-mentioned phenomenon and the notion of spontaneous symmetry breaking.[1]

II. THE MIT MODEL AND ITS GENERALIZATION

Let us fix our attention on a massless fermion field $\psi(x)$ of spin $\frac{1}{2}$ confined to a bounded and connected spatial region V. Let Ω be the space-time hypertube swept out by V. The boundary S of V sweeps out, in space-time, the boundary Σ of Ω. In the MIT bag model the equations for $\psi(x)$ are

$$i\gamma^{\mu}\partial_{\mu}\psi=0 \quad (\text{inside } \Omega)\ , \tag{2.1}$$

$$(1+in_{\mu}\gamma^{\mu})\psi=0 \quad (\text{on } \Sigma)\ , \tag{2.2a}$$

$$n^{\mu}\partial_{\mu}(\bar{\psi}\psi)-2B=0 \quad (\text{on } \Sigma)\ , \tag{2.2b}$$

where n^{μ} is the interior unit normal to Σ ($n^{\mu}n_{\mu}=-1$; our metric is $g^{00}=-g^{ii}=+1$), while B is the usual constant positive-energy density. The breaking of chiral symmetry comes from the boundary condition (2.2a), since we have

$$n_{\mu}\bar{\psi}\gamma^{5}\gamma^{\mu}\psi=i\bar{\psi}\gamma^{5}\psi \quad (\text{on } \Sigma)\ , \tag{2.3}$$

and the right-hand term is in general different from zero. Inside Ω the axial-vector current $\bar{\psi}\gamma^{5}\gamma^{\mu}\psi$ is locally conserved.

The equations (2.1) and (2.2) can be deduced from an action principle. If we consider the action[3]

$$A_M=\int_{\Omega}d^4x[L_0(x)-B]-\tfrac{1}{2}\int_{\Sigma}d\sigma\,\bar{\psi}\psi\ , \tag{2.4}$$

where

$$L_0(x)=\frac{i}{2}[\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-(\partial_{\mu}\bar{\psi})\gamma^{\mu}\psi]\ , \tag{2.5}$$

then, by requiring A to be stationary with respect to arbitrary variations of ψ inside Ω and on Σ, we obtain (2.1) and (2.2a). Moreover, the independent variations of Σ give the constraint (2.2b).

Now we generalize the action (2.4). First, we add a coupling between the spin and some kinematical variables of Σ. Furthermore, as a consequence of this coupling, we introduce two massless spin-$\frac{1}{2}$ fields $\psi_1(x)$ and $\psi_2(x)$ and write

$$\psi(x)=\begin{bmatrix}\psi_1(x)\\ \psi_2(x)\end{bmatrix} . \tag{2.6}$$

Besides the spacelike normal n^μ, we consider the unit timelike tangent t^μ to Σ. If $\vec{\xi}$ are the coordinates of the points of S, then S can be described by a parametric equation of the type

$$S_t:\ \vec{\xi}=\vec{\xi}(\alpha_1,\alpha_2;t) . \tag{2.7}$$

Therefore,

$$t^\mu=\frac{(1,\dot{\vec{\xi}}(\alpha_1,\alpha_2;t))}{(1-\dot{\vec{\xi}}^2)^{1/2}} , \tag{2.8}$$

namely, t^μ is the four-velocity vector of the points of S.

Now we consider the action

$$A=\int_\Omega d^4x\ [L_0(x)-B] -\tfrac{1}{2}\int_\Sigma d\sigma[\alpha\bar\psi\psi+\tfrac{1}{2}\beta\bar\psi\sigma^{\mu\nu}\tau_3(t_\mu n_\nu-t_\nu n_\mu)] , \tag{2.9}$$

where $L_0(x)$ has the same form as in Eq. (2.5), ψ is given by (2.6), and

$$\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu],\quad \tau_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} .$$

In Eq. (2.9), α and β are two dimensionless parameters, which, as will be shown, cannot be independent.

Arbitrary variations of ψ give the Dirac equation inside Ω,

$$i\gamma^\mu\partial_\mu\psi=0 ,$$

and the boundary condition,

$$[in_\mu\gamma^\mu+\alpha+\tfrac{1}{2}\beta\sigma^{\mu\nu}(t_\mu n_\nu-t_\nu n_\mu)\tau_3]\psi=0\quad(\text{on }\Sigma) . \tag{2.10}$$

From (2.10) it follows that

$$n_\mu\bar\psi_i\gamma^\mu\psi_i=0\quad(\text{on }\Sigma)\quad(i=1,2) , \tag{2.11}$$

$$\bar\psi_i[\alpha\pm\tfrac{1}{2}\beta\sigma^{\mu\nu}(t_\mu n_\nu-t_\nu n_\mu)]\psi_i=0 \quad (\text{on }\Sigma)\ (+,\ i=1;\ \ -,\ i=2) . \tag{2.12}$$

Now we consider an arbitrary infinitesimal variation of the region Ω

$$\Omega\to\Omega' .$$

If the points of Ω' have coordinates x'^μ, we can write

$$x'^\mu=x^\mu+(\delta x^\mu)(x)\equiv x^\mu+\delta x^\mu . \tag{2.13}$$

Under the variation (2.13), we have the following variation of the action A:

$$\delta A=\int_\Sigma d\sigma[L_0(x)-B]n_\mu\delta x^\mu-\tfrac{1}{2}\int_\Sigma(\delta d\sigma)[\alpha\bar\psi\psi+\tfrac{1}{2}\beta\bar\psi\sigma^{\mu\nu}\tau_3\psi(t_\mu n_\nu-t_\nu n_\mu)] -\tfrac{1}{2}\int_\Sigma d\sigma[\alpha\partial_\lambda(\bar\psi\psi)+\tfrac{1}{2}\beta(t_\mu n_\nu-t_\nu n_\mu)\partial_\lambda\bar\psi\sigma^{\mu\nu}\tau_3\psi]\delta x^\lambda-\tfrac{1}{2}\beta\int_\Sigma d\sigma\ \bar\psi\sigma^{\mu\nu}\tau_3\psi\delta(t_\mu n_\nu-t_\nu n_\mu) . \tag{2.14}$$

We see that the variation $\delta d\sigma$ is ineffective due to the boundary equation (2.12). Furthermore, we observe that

$$\delta(t_\mu n_\nu)=t'_\mu(x')n'_\nu(x')-t_\mu(x)n_\nu(x)\quad(x\in\Sigma)$$

(with $t'_\mu t'^\mu=+1$, $n'_\mu n'^\mu=-1$, $t'_\mu n'^\mu=0$) can be written in the form

$$\delta(t^\mu n^\nu)=[g^{\mu\rho}\omega^{\nu\sigma}(x)+g^{\nu\sigma}\omega^{\mu\rho}(x)]t_\mu n_\nu ,$$

where $|\omega^{\mu\nu}(x)|\ll 1$ and $\omega^{\mu\nu}(x)=-\omega^{\nu\mu}(x)$. In other words, $\delta(t_\mu n_\nu)$ can be related to an infinitesimal local rotation in the space-time. Under this local rotation the quantity

$$\bar\psi\sigma^{\mu\nu}\tau_3\psi(t_\mu n_\nu-t_\nu n_\mu)$$

is invariant. Therefore,

$$\begin{aligned}\bar\psi\sigma^{\mu\nu}\tau_3\psi\delta(t_\mu n_\nu-t_\nu n_\mu) &=-(t_\mu n_\nu-t_\nu n_\mu)\delta\bar\psi\sigma^{\mu\nu}\tau_3\psi\\ &=-\frac{i}{4}(t_\mu n_\nu-t_\nu n_\mu)\omega_{\rho\sigma}(x)\bar\psi[\sigma^{\rho\sigma},\sigma^{\mu\nu}]\tau_3\psi .\end{aligned} \tag{2.15}$$

Then, from $\delta A=0$, we obtain the two boundary conditions

$$\alpha\partial^\lambda\bar\psi\psi+\tfrac{1}{2}\beta(t_\mu n_\nu-t_\nu n_\mu)\partial^\lambda\bar\psi\sigma^{\mu\nu}\tau_3\psi+2n^\lambda B=0 \quad (x\in\Sigma) , \tag{2.16}$$

$$\beta(t_\mu n_\nu-t_\nu n_\mu)\bar\psi[\sigma^{\rho\sigma},\sigma^{\mu\nu}]\tau_3\psi=0\quad(x\in\Sigma) . \tag{2.17}$$

Equation (2.16), as in the usual MIT bag model, is related to the energy-momentum conservation. If $T^{\mu\nu}$ is the energy-momentum tensor of our model

$$T^{\mu\lambda}=\frac{i}{2}[\bar\psi\gamma^\mu\partial^\lambda\psi-(\partial^\lambda\bar\psi)\gamma^\mu\psi]+Bg^{\mu\lambda} ,$$

then

$$\partial_\mu T^{\mu\lambda}=0\quad(\text{inside }\Omega)$$

and

$$n_\mu T^{\mu\lambda}=0\quad(\text{on }\Sigma)$$

as a consequence of (2.10) and (2.16). On the other hand, due to (2.17), we have angular momentum conservation:

$$\partial_\mu J^{\mu,\rho\sigma}=0\quad(\text{in }\Omega) ,$$

$$n_\mu J^{\mu,\rho\sigma}=0\quad(\text{on }\Sigma) ,$$

where

$$J^{\mu,\rho\sigma}=x^\rho T^{\mu\sigma}-x^\sigma T^{\mu\rho}+\tfrac{1}{4}\bar\psi(\gamma^\mu\sigma^{\rho\sigma}+\sigma^{\rho\sigma}\gamma^\mu)\psi .$$

We conclude this section by showing that the parameters α and β cannot be arbitrary. Let us write Eq. (2.10) in the form

$$(1+\beta t\tau_3)n\!\!\!/\psi = i\alpha\psi \ .$$

Then,

$$(1+\beta t\tau_3)n\!\!\!/(1+\beta t\tau_3)n\!\!\!/\psi = -\alpha^2\psi \ . \tag{2.18}$$

From (2.18) we have

$$\alpha^2+\beta^2=+1 \ . \tag{2.19}$$

We make a conventional choice on the sign of α and β: both are assumed non-negative. Therefore, $0\le\alpha\le1$, $0\le\beta\le1$. In our model we have a free parameter, for example, α with $\beta=+(1-\alpha^2)^{1/2}$.

III. STATIC SPHERICAL BAG AT REST

In order to analyze the spectral properties of our model, we seek solutions such that the surface of the bag is a sphere of fixed radius R, at rest. In this case,

$$t^0=+1,\ t^i=0,\ n^\mu=(0,-\hat{n}) \ , \tag{3.1}$$

where $\hat{n}$ is the outer normal to S.

The linear boundary condition (2.10) becomes

$$(1+\beta\gamma^0\tau_3)\hat{n}\cdot\vec{\gamma}\psi = i\alpha\psi \ . \tag{3.2}$$

We fix first our attention on the Dirac equation and on Eq. (3.2). We limit ourselves to solutions of the Dirac equation with angular momentum $j=\frac{1}{2}$ (Ref. 5). These are given by

$$\psi_{i,n,-1,m}(t,\vec{r}) = N(\omega_{n,-1}^{(i)}) \begin{bmatrix} ij_0(\omega_{n,-1}^{(i)}r)U_m \\ -j_1(\omega_{n,-1}^{(i)}r)\vec{\sigma}\cdot\hat{n}U_m \end{bmatrix} e^{-i\omega_{n,-1}^{(i)}t} \tag{3.3}$$

and

$$\psi_{i,n,1,m}(t,\vec{r}) = N(\omega_{n,1}^{(i)}) \begin{bmatrix} -j_1(\omega_{n,1}^{(i)}r)\vec{\sigma}\cdot\hat{n}U_m \\ ij_0(\omega_{n,1}^{(i)}r)U_m \end{bmatrix} e^{-i\omega_{n,1}^{(i)}t} \quad (r\le R) \tag{3.4}$$

where i ($i=1,2$) is the index of the two spinor fields $\psi_1(x)$ and $\psi_2(x)$, n labels the eigenfrequencies determined by Eq. (3.2), U_m is a two-component Pauli spinor, and $N(\omega_{n,k}^{(i)})$ ($k=\pm1$) is a normalization constant. The solutions (3.3) have a positive state parity (index -1), while the solutions (3.4) have a negative state parity.

If the states (3.3) and (3.4) are normalized to a single fermion in the bag, we have

$$N^2(\omega_{n,k}^{(i)}) = \frac{1}{4\pi R^3}\frac{(x_{n,k}^{(i)})^4}{(x_{n,k}^{(i)})^2-sen^2x_{n,k}^{(i)}} \ , \tag{3.5}$$

where

$$x_{n,k}^{(i)}=\omega_{n,k}^{(i)}R \ .$$

The allowed values of $x_{n,k}^{(i)}$ are determined from Eq. (3.2), which gives, for $i=1$,

$$\tan x_{n,-1}^{(1)} = \frac{x_{n,-1}^{(1)}}{1-gx_{n,-1}^{(1)}} \ , \tag{3.6}$$

$$\tan x_{n,1}^{(1)} = \frac{x_{n,1}^{(1)}}{1+\frac{1}{g}x_{n,1}^{(1)}} \ , \tag{3.7}$$

where

$$g=\frac{\alpha}{1+\beta}=\frac{\alpha}{1+(1-\alpha^2)^{1/2}} \ .$$

The function $g(\alpha)$ is a monotonic increasing function of α, with $g(0)=0$ and $g(1)=1$. For $g=1$, Eqs. (3.6) and (3.7) are the eigenvalue conditions of the usual MIT bag model.

The allowed values of $x_{n,k}^{(2)}$ are determined through the substitution $\beta\to-\beta$ or $g\to1/g$. We have

$$\tan x_{n,-1}^{(2)} = \frac{x_{n,-1}^{(2)}}{1-\frac{1}{g}x_{n,-1}^{(2)}} \ , \tag{3.8}$$

$$\tan x_{n,1}^{(2)} = \frac{x_{n,1}^{(2)}}{1+gx_{n,1}^{(2)}} \ . \tag{3.9}$$

We see that the following relation holds:

$$x_{n,-1}^{(1)} = -x_{-n,+1}^{(2)} \ , \qquad x_{n,1}^{(1)} = -x_{-n,-1}^{(2)} \ . \tag{3.10}$$

We assume that the positive (negative) n label the positive (negative) roots of Eqs. (3.6), ..., (3.9). Due to Eq. (3.10), it is sufficient to consider the positive roots of the above equations.

The positive roots of Eqs. (3.6) and (3.7) are monotonic increasing functions of g. For $g=1$, we have the MIT values

$$x_{1,-1}^{(1)}(1)=2.04,\ x_{2,-1}^{(1)}(1)=5.4,\ \ldots,$$

$$x_{1,1}^{(1)}(1)=3.8,\ \ x_{2,1}^{(1)}(1)=7.0,\ \ldots .$$

It is interesting to consider small values of g. In this case the $x_{n,k}^{(1)}(g)$ can be developed in powers of g. We have

$$x_{1,-1}^{(1)}(g)=3g-\tfrac{9}{5}g^3+\cdots \ ,$$
$$x_{2,-1}^{(1)}(g)=4.5+g+\cdots \ , \tag{3.11}$$
$$x_{1,1}^{(1)}(g)=\pi+g+\ldots,\ x_{2,1}^{(1)}(g)=2\pi+g+\cdots \ .$$

For the positive roots of Eqs. (3.8) and (3.9), which are decreasing functions of g, we have

$$x_{1,-1}^{(2)}(g)=\pi-g+\cdots \quad [x_{1,1}^{(2)}(1)=2.04] \ ,$$
$$x_{2,-1}^{(2)}(g)=2\pi-g+\cdots \quad [x_{2,-1}^{(2)}(1)=5.4] \ , \tag{3.12}$$
$$x_{1,1}^{(2)}(g)=4.5-g+\cdots \quad [x_{1,1}^{(2)}(1)=3.8] \ .$$

It is interesting to note that in the limit $g \to 0$, the roots $x_{n,1}^{(1)}$ and $x_{n,-1}^{(2)}$ $(n>0)$ give the same eigenfrequencies of the nonrelativistic quark model. Furthermore, when $g \to 0$, we have degenerate parity doublets. As we see, the first level (positive or negative) can be as small as we want, by choosing g appropriately.

Moreover, a charge-conjugation relation links the states $\psi_{1,n,k,m}$ and $\psi_{2,n,k,m}$. From Eq. (3.10) we have

$$\psi_{1,n,k,m}^{c}(t,\vec{r})=\eta\psi_{2,-n,-k,-m}(t,\vec{r}) , \tag{3.13}$$

where η is an arbitrary phase factor and

$$\psi_{1,n,k,m}^{c}(t,\vec{r})=\eta C\bar{\psi}_{1,n,k,m}^{T}(t,\vec{r}) \quad (C=i\gamma^2\gamma^0) .$$

Now we consider the nonlinear boundary conditions (2.16) and (2.17), assuming $0<g<1$. Equation (2.16) becomes

$$\alpha\partial^i\bar{\psi}\psi+i\beta\hat{n}\cdot\partial^i\psi^+\vec{\gamma}\tau_3\psi+2n^iB=0 \quad (i=1,2,3) \tag{3.14}$$

at $r=R$. Owing to Eq. (2.12), we have no condition for $\lambda=0$.

In order to satisfy Eq. (3.14) we should consider the general solution of the previous linear equations,

$$\psi_i(t,\vec{r})=\sum_{n,k,m} a_{inkm}N(\omega_{n,k}^{(i)})\psi_{i,n,k,m}(t,\vec{r}) \quad (i=1,2) \tag{3.15}$$

where a_{inkm} are the amplitudes for each mode in the bag. However, the time independence[5] of Eq. (3.14) requires that only one eigenfrequency can be present for each field. Therefore, we can write

$$\begin{aligned}\psi_1(t,\vec{r})&=N(\omega_{n,k}^{(1)})[a_{1nkm}\psi_{1,n,k,m}(t,\vec{r})\\&\qquad+a_{1nk-m}\psi_{1,n,k,-m}(t,\vec{r})] ,\\ \psi_2(t,r)&=N(\omega_{n',k'}^{(2)})[a_{2n'k'm'}\psi_{2,n',k',m'}(t,\vec{r})\\&\qquad+a_{2n'k'-m'}\psi_{2,n',k',-m'}(t,\vec{r})] .\end{aligned} \tag{3.16}$$

It can be seen that for the $\psi_i(t,\vec{r})$ given by (3.16), Eq. (3.14) is equivalent to

$$-\frac{1}{2}\frac{d}{dr}\bar{\psi}(\alpha+i\beta\gamma^0\vec{\gamma}\cdot\hat{n}\tau_3)\psi=B \quad (r=R) . \tag{3.17}$$

Finally, from (3.16) and (3.17), we obtain

$$\sum_m [x_{n,k}^{(1)}a_{1nkm}^{*}a_{1nkm}+x_{n',k'}^{(2)}a_{2n'k'm}^{*}a_{2n'k'm}]=4\pi R^4B \tag{3.18}$$

as in the usual MIT model.

In Eq. (3.18) the $x_{n,k}^{(1)}$ and $x_{n',k'}^{(2)}$ can be positive or negative. In order to have the left-handed side of Eq. (3.18) positive definite, we treat the amplitudes a_{inkm} as fermionic operators.[5] We set

$$\begin{aligned}&a_{inkm}=b_{inkm},\quad n>0 ,\\&a_{1nkm}\equiv d_{2-n-k-m}^{\dagger},\quad n<0 ,\\&a_{2nkm}\equiv d_{1-n-k-m}^{\dagger}\quad n<0\end{aligned} \tag{3.19}$$

with the well-known anticommutation relations

$$\{b_{inkm},b_{inkm}^{\dagger}\}\equiv\{d_{inkm},d_{inkm}^{\dagger}\}=+1 ,$$

and all other anticommutators zero.

In Eq. (3.19) we have taken into account the relations (3.10) and (3.13).

The left-hand side of Eq. (3.18) is assumed normal ordered: therefore, positive terms like

$$x_{n,k}^{(i)}b_{inkm}^{\dagger}b_{inkm} ,$$

$$|x_{n,k}^{(2)}|d_{1-n-k-m}^{\dagger}d_{1-n-k-m} ,$$

or

$$|x_{n,k}^{(1)}|d_{2-n-k-m}^{\dagger}d_{2-n-k-m}$$

will appear.

Now we consider the other nonlinear condition. Equation (2.17) is equivalent to

$$n_\mu J^{\mu,\alpha\beta}=0 \quad (\text{on } \Sigma) ,$$

where J^μ is normal ordered. Then, in the case of a static spherical bag, we have the constraints

$$\psi_1^{\dagger}\gamma_5\psi_1+\psi_2^{\dagger}\gamma_5\psi_2=0 \quad (r=R) , \tag{3.20}$$

$$\hat{n}_i\epsilon_{ijl}(\psi_1^{\dagger}\Sigma^l\psi_1+\psi_2^{\dagger}\Sigma^l\psi_2)=0 \quad (r=R) , \tag{3.21}$$

where ψ_1 and ψ_2 are given by (3.16) and (3.19), and

$$\Sigma^l=\begin{pmatrix}\sigma^l & 0\\ 0 & \sigma^l\end{pmatrix} \quad (l=1,2,3) .$$

However, Eq. (3.20) is already satisfied for each field separately, since we have

$$\psi_{inkm}^{\dagger}(t,\vec{r})\gamma_5\psi_{ink\pm m}(t,\vec{r})=0 \quad (i=1,2) . \tag{3.22}$$

Concerning Eq. (3.21), we need the following remark: We have replaced a classical boundary condition with an operator equation; then, a proper correspondence with the classical case leads us to consider the constraint for the operator $n_\mu J^{\mu,\alpha\beta}$ as classical equation for its eigenvalues. So we need a representation in which $n_\mu J^{\mu,\alpha\beta}$ is diagonal.

Since the other nonlinear constraint has already been diagonalized [Eq. (3.18)], in order to have $n_\mu J^{\mu,\alpha\beta}$ diagonal in the same representation we assume that of the two modes a_{inkm} and a_{ink-m} only one can be excited.

Then, Eq. (3.21) becomes

$$N^2(\omega_{n,k}^{(1)})[j_0{}^2(x_{n,k}^{(1)})-j_1{}^2(x_{n,k}^{(1)})]U_m^{\dagger}\sigma^lU_m:a_{1nkm}^{\dagger}a_{1nkm}:+N^2(\omega_{n',k'}^{(2)})[j_0{}^2(x_{n',k'}^{(2)})-j_1{}^2(x_{n',k'}^{(2)})]U_{m'}^{\dagger}\sigma^lU_{m'}:a_{2n'k'm'}^{\dagger}a_{2n'k'm'}:=0 . \tag{3.23}$$

This equation cannot be satisfied by each field separately (except the case $g=1$). Both fields ψ_1 and ψ_2 must be present in the bag at the same time, with the constraint

$$N^2(\omega_{n,k}^{(1)})[(j_0{}^2(x_{n,k}^{(1)})-j_1{}^2(x_{n,k}^{(1)})]=\pm N^2(\omega_{n',k'}^{(2)})[j_0{}^2(x_{n',k'}^{(2)})-j_1{}^2(x_{n',k'}^{(2)})] . \tag{3.24}$$

From our linear boundary condition and the spectral properties (3.10), it follows that Eq. (3.24) can be satisfied only if we take the sign $+$ and $x_{n,k}^{(1)}=-x_{n',k'}^{(2)}$, that is if

$$n'=-n,\quad k'=-k\ . \tag{3.25}$$

Then, by making use of Eq. (3.15), we can write Eq. (3.23) in the two forms

$$U_m^\dagger\sigma^l U_m b_{1nkm}^\dagger b_{1nkm}-U_{m'}\sigma^l U_{m'}d_{1nk-m'}^\dagger d_{1nk-m'}=0$$

or

$$U_m^\dagger\sigma^l U_m d_{2n'k'-m}^\dagger d_{2nk-m}-U_{m'}\sigma^l U_{m'}b_{2n'k'm'}^\dagger b_{2n'k'm'}=0\ .$$

We conclude that the physical states of our model are such that

$$m=m',\quad b_{1nkm}^\dagger b_{1nkm}=d_{1nk-m}^\dagger d_{1nk-m}$$

or

$$b_{2nkm}^\dagger b_{2nkm}=d_{2nk-m}^\dagger d_{2nk-m}\ , \tag{3.26}$$

where $m=\pm\frac{1}{2}$.

Owing to Eq. (3.26), every state is built up only through a particle and its antiparticle with opposite polarization and the same eigenfrequency. We have then pseudoscalar mesons or vector mesons with zero-spin components along the z axis. The properties of the vector mesons will be investigated elsewhere. In the following we will fix our attention on the pseudoscalar mesons.

The radius of these mesons is given by Eq. (3.18). We limit ourselves to the interesting case of small g and call $R_{n,k}^{(i)}$ ($n>0$) the radius of the bag with the field energy $2\omega_{n,k}^{(i)}$ ($n>0$). We have the sequence

$$R_{1,-1}^{(1)}\simeq\left[\frac{3g}{2\pi B}\right]^{1/4},\quad R_{1,-1}^{(2)}\simeq\left[\frac{\pi-g}{2\pi B}\right]^{1/4},$$
$$R_{1,1}^{(1)}\simeq\left[\frac{\pi+g}{2\pi B}\right]^{1/4},\quad R_{1,1}^{(2)}\simeq\left[\frac{4.5-g}{2\pi B}\right]^{1/4},\ \ldots . \tag{3.27}$$

The rest mass of our bags is given by

$$M(R_{n,k}^{(i)})=\frac{2x_{n,k}^{(i)}}{R_{n,k}^{(i)}}+\frac{4\pi B}{3}(R_{n,k}^{(i)})^3$$
$$=\tfrac{4}{3}(4\pi B)^{1/4}(2x_{n,k}^{(i)})^{3/4}\ , \tag{3.28}$$

with

$$M(R_{1,-1}^{(1)})\simeq\tfrac{4}{3}(4\pi B)^{1/4}(6g)^{3/4},\ \text{etc.},\ldots .$$

We see that, in our model, the ground state can have a rest mass and a radius as small as we want by taking g appropriately. Near the limit $g\to 0$, this ground state can be described, in a complete relativistic treatment, through a nearly local massless pseudoscalar field.

IV. PARTIAL CONSERVATION OF THE AXIAL-VECTOR CURRENT

As we can see in Eq. (2.3), the conservation of the axial-vector current is broken badly in the usual MIT bag model. On the contrary, our model allows a partial conservation of the axial-vector current (PCAC) when g is small.

We consider the bags which, in their rest frame, are described by the solutions of the previous section. The bags can have an arbitrary fixed total momentum. However, the explicit construction of these boosted states is not necessary (before the boosting, quantum corrections associated to the center-of-mass fluctuations would be required.[3,7] Each bag sweeps out a space-time hypertube Ω, with boundary Σ. Inside Ω, as we have seen, both ψ_1 and ψ_2 are present at the same time. Let us call $A_\mu^{(i)}(x)$ the axial-vector current inside the bag $|\,\bar{i}i;M(R_{n,k}^{(i)})\rangle$ with rest mass $M(R_{n,k}^{(i)})$.

We consider the only relevant matrix elements of $A_\mu^{(i)}(x)$

$$\langle\,\bar{i}i;M(R_{n,k}^{(i)})\,|\,A_\mu^{(i)}(x)\,|\,0\rangle\ , \tag{4.1}$$

where by $|\,0\rangle$ we denote the vacuum. Owing to the structure of our solutions, we see that an axial-vector current term of the type $\bar\psi_i\gamma^5\gamma^\mu\psi_i$ gives no contribution to the matrix element (4.1). Let us fix our attention on the axial-vector current $A_\mu^{(1)}(x)$. The proper candidate for $A_\mu^{(1)}(x)$, with nonvanishing matrix element (4.1), is

$$A_\mu^{(1)}(x)=\bar\psi_1\gamma^5\gamma_u\psi_2\ , \tag{4.2}$$

where ψ_1 (ψ_2) contains only positive (negative) frequencies. The current (4.2) is locally conserved,

$$\partial^\mu A_\mu^{(1)}(x)=0\quad(\text{inside }\Omega)\ . \tag{4.3}$$

However, the conservation of a current in a confined model[2] would require, besides Eq. (4.3), the further condition,

$$n^\mu A_\mu^{(1)}(x)=0\quad(\text{on }\Sigma)\ .$$

Now, from Eq. (2.10), we have the boundary condition

$$(in_\mu\gamma^\mu+\alpha-\beta t_\mu n_\nu\sigma^{\mu\nu})\psi_2=0\quad(\text{on }\Sigma)\ .$$

From this, it follows that

$$in_\mu\bar\psi_1\gamma^5\gamma^\mu\psi_2+\alpha\bar\psi_1\gamma^5\psi_2-\beta t_\mu n_\nu\bar\psi_1\sigma^{\mu\nu}\gamma^5\psi_2=0\quad(\text{on }\Sigma)\ .$$

On the other hand, we have the boundary condition for the field ψ_1,

$$i\bar\psi_1 n_\mu\gamma^\mu-\alpha\bar\psi_1=\beta t_\mu n_\nu\bar\psi_1\sigma^{\mu\nu}\quad(\text{on }\Sigma)\ .$$

Then, we conclude that

$$n^\mu A_\mu^{(1)}(x)=\alpha\bar\psi_1 i\gamma^5\psi_2$$
$$=\frac{2g}{1+g^2}\bar\psi_1 i\gamma^5\psi_2\quad(\text{on }\Sigma)\ . \tag{4.4}$$

Similar results are obtained for $A_\mu^{(2)}=\psi_2\gamma^5\gamma_u\psi_1$,

$$\partial^\mu A_\mu^{(2)}(x)=0\quad(\text{in }\Omega)\ ,$$
$$n^\mu A_\mu^{(2)}(x)=\frac{2g}{1+g^2}\bar\psi_2 i\gamma^5\psi_1\quad(\text{on }\Sigma)\ . \tag{4.5}$$

As we see, in the limit $g\to 0^+$, the axial-vector currents $A_\mu^{(i)}(x)$ are conserved. For small g, we interpret Eqs. (4.4) and (4.5) as versions of PCAC in a confined model. This point of view is supported by Eq. (3.28), which shows that the amount of violation of the axial-current conservation is related to the lowest rest mass of our bags.

V. CONCLUSIONS

As can be expected, the conservation of $A_\mu^{(1)}$ and $A_\mu^{(2)}$ when $g=0$ comes from a symmetry property of our model. In fact, when $\alpha=0$, the action (2.9) is invariant under the following global symmetry groups:

$$[\mathrm{U}(1)]_{A2}:\ \psi \to e^{-i\lambda \tau_2 \gamma^5}\psi\ ,$$
$$[\mathrm{U}(1)]_{A1}:\ \psi \to e^{-i\phi \tau_1 \gamma^5}\psi\ , \qquad (5.1)$$

where λ and ϕ are arbitrary real constants. The associated conserved currents are

$$j_{5\mu}^{(1)} = \bar{\psi}\gamma^5\gamma_\mu\tau_1\psi,\quad j_{5\mu}^{(2)} = \bar{\psi}\gamma^5\gamma_\mu\tau_2\psi\ ,$$

which are linear combinations of $A^{(1)}$ and $A^{(2)}$.

Let us see what happens in our model, where $g=0$. The positive roots $x_{n,k}^{(i)}(0)$ are given by Eqs. (3.11) and (3.12), for $g=0$. The root $x_{1,-1}^{(1)}(0)=0$ is a spurious one, since the corresponding state is meaningless. The lowest eigenfrequency is associated with $x_{1,1}^{(1)}(0)=x_{1,-1}^{(2)}(0)=\pi$. Besides Eq. (3.10), we now have a further relation

$$\begin{aligned} x_{n,1}^{(1)}(0) &= x_{n,-1}^{(2)}(0) \\ x_{n+1,-1}^{(1)}(0) &= x_{n,1}^{(2)}(0) \end{aligned} \quad (n>0)\ . \qquad (5.2)$$

Of course, the above relations are a consequence of the symmetry properties given before.

Now, owing to Eq. (5.2), our nonlinear boundary conditions allow a new set of solutions, besides the massive pseudoscalar mesons given in Sec. III for $g=0$.

It can be seen that in the new set of solutions there are scalar mesons which are built up through the fields ψ_1 and ψ_2, both with positive or negative frequencies.

Then, to a given rest mass of our bags, we have associated states with opposite parity. We can say that, when $g=0$, the symmetry of our model is manifested in the Wigner-Weyl mode. Now, when g is different from zero, but as small as we want, the situation changes in some respects drastically. The scalar mesons disappear and a pseudoscalar meson appears with radius and rest mass finite, but both as small as we want. Such a feature is very close to the Nambu-Goldstone realization of a symmetry. With respect to the spontaneous-symmetry-breaking scheme, there is a difference: the vacuum of our model is not degenerate. This, of course, is related to the property that we have a finite (even in arbitrarily small) lowest rest mass in our spectrum.

The above results lead us to speak of stable or unstable Wigner-Weyl modes. Furthermore, the following suggestion emerges: spontaneous symmetry breaking can be considered as a limiting case of physical situations in which a perturbation breaks an unstable Wigner-Weyl realization.

In the idealized model given in this paper, we have only mesons. Further aspects and generalizations of our model, including fermions, will be given in a future work.

[1]K. Huang, *Quarks, Leptons and Gauge Fields* (World Scientific, Singapore, 1982).

[2]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974).

[3]R. L. Jaffe, in *Pointlike Structure Inside and Outside Hadrons,* proceedings of the Seventeenth International School of Subnuclear Physics, Erice, 1979, edited by A. Zichichi (Plenum, New York, 1982).

[4]E. J. Squires, Rep. Prog. Phys. **42**, 1187 (1979).

[5]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D **10**, 2599 (1974).

[6]A. W. Thomas, CERN Report No. TH3368, 1982 (unpublished).

[7]J. Donoghue and K. Johnson, Phys. Rev. D **21**, 1975 (1980).

[8]T. Goldman and R. Haymaker, Phys. Rev. D **24**, 724 (1981).

[9]Q. Ho-Kim and H. C. Lee, Phys. Rev. D **29**, 1455 (1984).