Radiation from vacuum strings and domain walls

Tanmay Vachaspati, Allen E. Everett, and Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155
(Received 9 May 1984)

We calculate the order of magnitude of the emission rate of electromagnetic waves, and of massive scalar particles, from oscillating vacuum strings and domain walls. The energy loss due to electromagnetic radiation is shown to be much smaller than that due to gravitational radiation. Scalar radiation can only damp oscillations of frequency of the order of the scalar particle mass. Our results confirm the assumption made in previous work that gravitational radiation is the dominant energy-loss mechanism for macroscopic vacuum structures.

I. INTRODUCTION

Phase transitions in the early universe can give rise to macroscopic topological structures—vacuum domain walls and strings. The cosmological evolution of these structures has been discussed in Refs. 1—11. Topologically stable domain walls are fatal to cosmological models.[1–3] Unstable walls are probably harmless; they break into pieces bounded by strings and radiate away their energy.[4–6] The strings are potentially very interesting: they can generate density fluctuations sufficient to explain galaxy formation[7–10] and can even lead to some observable effects at present.[3,11] One of the important parameters determining the cosmological evolution of strings is the lifetime of oscillating closed loops. In Refs. 8 and 9 it is assumed that the dominant energy-loss mechanism of the loops is gravitational radiation. It is possible, however, that the loops can lose energy faster by emitting particles other than gravitons. The same problem arises when one tries to estimate the lifetime of pieces of wall bounded by strings.

It is easily seen that, for macroscopically large strings or walls, only emission of massless particles can be important. If R is the size of the loop, then its frequency of oscillation is $\omega \sim R^{-1}$, and the emission of particles with masses greater than R^{-1} is suppressed (the same argument applies to the walls). Assuming that neutrinos have nonvanishing masses, there are only two known massless particles: photon and graviton. Thus, the only process competing with the gravitational radiation is the electromagnetic (EM) radiation from strings and walls.

The main purpose of this paper is to estimate the EM energy loss by oscillating walls and strings. An analytic treatment of the general case of strongly curved walls and strings is very difficult. We shall therefore consider only small perturbations on flat walls and straight strings, with the hope that extension of these results will give correct order-of-magnitude estimates for the general case. We also consider the emission of massive scalar particles by domain walls, where the calculation is easier to do. As noted above, this process is not expected to be an important energy-loss mechanism, but we consider it for the sake of completeness, and because the method of calculation is of some interest.

The fields whose vacuum expectation values (VEV's) are associated with walls and strings are electrically neutral, and do not couple to a single photon field. In the case of strings, there is a VEV of one of the heavy gauge boson fields, coupled to one of the broken symmetry degrees of freedom, in the vicinity of the string. Two-photon emission can occur as a result of the coupling of the photon field to this heavy gauge field; when such coupling is present, this is the dominant radiation mechanism. This is discussed in Sec. II. One problem presents itself here in the perturbation-theoretic calculation. It turns out that there is an infinite set of diagrams all of which are of the same (lowest) order in the small parameters of the calculation, the gauge coupling constant and the ratio of the amplitude of the string oscillations to the radius of the curvature. We calculate only the simplest of these diagrams. The exact answer will be given, in the limit that the coupling constant and amplitude are small, by our result multiplied by a coefficient which has the form of an infinite series of purely numerical terms; hopefully this series is sufficiently convergent that our result gives the correct order of magnitude. This problem is also reflected in the fact that our result is not gauge invariant; only the full lowest-order result obtained by summing the infinite set of lowest-order diagrams can be expected to manifest gauge invariance. The origin of this problem lies in the fact that, although the string thickness is small, the heavy-gauge-boson field is large inside the string. Thus the photon wave functions are appreciably modified inside the string, so that there is no limit in which perturbation theory, in the sense of considering only a few simple diagrams, is strictly applicable.

Electromagnetic radiation from strings can also occur through vacuum polarization effects. We discuss one example of such a diagram, involving the coupling of the Higgs field to a charged fermion, in Sec. III. The results are suppressed relative to the previous mechanism not only by powers of the fine-structure constant, but, more importantly, by powers of the ratio of the width of the string to the wavelength of the perturbation. For this process, in contrast with that discussed in Sec. II, lowest-order perturbation theory is applicable; there is only a single lowest-order diagram, and this diagram yields a gauge-invariant result.

In Sec. IV the emission of massive scalar particles by a domain wall is considered. In Sec. V we discuss EM radiation by a domain wall. Since there is no gauge boson field with a VEV in the case of a domain wall, the dominant mechanism in the case of strings, that of Sec. II, is not present. We consider radiation from walls due to vacuum polarization effects, using the same model which was considered for strings in Sec. III. We conclude briefly in Sec. VI. Our results indicate that in all cases of cosmological interest energy loss by vacuum singularities is completely dominated by gravitational radiation. This confirms the assumption made in Refs. 4 and 8.

II. ELECTROMAGNETIC RADIATION FROM A STRING

We consider a case in which strings arise when a symmetry group G is spontaneously broken and $\langle\phi\rangle$, the VEV of a scalar field ϕ, acquires a magnitude η; we will designate the unbroken subgroup of G, which includes the electromagnetic gauge group, by H. The existence of a string corresponds to a situation in which, as the angle θ varies from 0 to 2π along a circular path in coordinate space, the vacuum state of the system is given by $g(\theta)|0\rangle$, where $|0\rangle$ is the vacuum state at $\theta=0$, $g(\theta)\in G$, with $g(0)=I$, and $g(2\pi)\in H(0)$, the invariance group of the vacuum at $\theta=0$. Strings arise when the path followed by $g(\theta)$ in the manifold of G cannot be continuously contracted to a point. This may arise either because G is not simply connected, or because H includes a discrete symmetry so that its manifold is disconnected. It is this latter situation which seems potentially of greatest interest cosmologically.[5] The string radius a is expected to be of order $1/(h\eta)$, where h is the magnitude of the scalar-meson self-coupling. The VEV of ϕ differs from η within the string, and vanishes at its center. We define the directions within the manifold of G so that the group elements $g(\theta)$ correspond to rotations generated by the generator τ_1 of G. Outside the string the covariant derivative of ϕ must vanish. This condition will be satisfied if the VEV's of the gauge fields A^1_θ outside the string are given by[12]

$$\begin{aligned}&\langle A^1_\theta\rangle = W_\theta = 1/gr\ ,\\ &\langle A^1_\mu\rangle = W_\mu = 0\ ,\quad \mu\neq\theta\ ,\\ &\langle A^i_\mu\rangle = 0\ ,\quad i\neq 1\end{aligned}\tag{2.1}$$

where r, θ, and z are cylindrical coordinates in a system with the string along the z axis, subscripts and superscripts refer to Lorentz components and components in group space, respectively, with A_θ being the component in the direction of increasing θ, and g is the gauge coupling constant. Equations (2.1) describe a pure gauge field; the VEV's of the generalized electric and magnetic fields vanish outside the string. Within the string, however, where the VEV of ϕ depends on r, the generalized magnetic field in the l direction is nonzero. It follows from Eqs. (2.1) that the generalized Yang-Mills magnetic field $\vec{H}^1=\nabla\times\vec{A}^1$, and hence that a flux of H^1 exists in the string because of the Stokes theorem. Note that, since τ^1 does not leave the vacuum state invariant and hence is not a generator of the unbroken subgroup H, A^1_μ, and $\vec{H}^1$ are massive fields, with mass of order $g\eta$, which is consistent with the confinement of $\vec{H}^1$ within the string.

Since $\langle\phi\rangle$ is invariant under operations of the unbroken subgroup H, including EM gauge transformations generated by the electric charge operator Q, it follows that ϕ is an electrically neutral field and does not couple to the electromagnetic gauge field. Also there is no coupling of the field A^1_μ to a single photon field, since the terms in the Lagrangian which are cubic in the gauge fields are proportional to f^{abc}, the structure constants of the gauge group; these are totally antisymmetric, so that there is no cubic term involving only the fields W_μ and A_μ. Thus there is no term coupling a single photon field to those fields having VEV's associated with the string. There can be, however, a quartic term in the Yang-Mills Lagrangian coupling two photon fields to the string. To see how this arises, let us write the electromagnetic field as $A_\mu(x)n(x)$, where $n(x)$ is a vector of unit magnitude in group space belonging to the adjoint representation of G. The direction of $n(x)$ in group space varies with position because of the variation of the direction of $\langle\phi\rangle$, and hence of the generators of H, along a path enclosing the string. We can now write the contributions of A_μ and W_μ to the Yang-Mills field tensor $F^a_{\mu\sigma}$ as

$$\begin{aligned}F^a_{\mu\sigma} &= \delta^{al}W_{\mu\sigma}+n^aA_{\mu\sigma}+(\partial_\mu n^a)A_\sigma-(\partial_\sigma n^a)A_\mu\\ &\quad+gf^{alc}n^c(W_\mu A_\sigma-W_\sigma A_\mu)\ ,\end{aligned}\tag{2.2}$$

where

$$W_{\mu\sigma}=\partial_\mu W_\sigma-\partial_\sigma W_\mu \tag{2.3}$$

and similarly for $A_{\mu\sigma}$. Equation (2.2) can be rewritten as

$$F^a_{\mu\sigma}=\delta^{al}W_{\mu\sigma}+n^aA_{\mu\sigma}+A_\sigma D_\mu n^a-A_\mu D_\sigma n^a\ , \tag{2.4}$$

where

$$D_\mu n^a=\partial_\mu n^a+gf^{alc}n^cW_\mu\ . \tag{2.5}$$

Hence the interaction term in the Yang-Mills Lagrangian $-F^a_{\mu\sigma}F^{\mu\sigma a}/4$ involving only W_μ and A_μ is

$$L_1=-m^2{}_{\mu\sigma}A^\mu A^\sigma/2\ , \tag{2.6}$$

where

$$m^2{}_{\mu\sigma}=\eta_{\mu\sigma}D_\alpha n^aD^\alpha n^a-D_\mu n^aD_\sigma n^a \tag{2.7}$$

and $\eta_{\mu\sigma}$ is the Minkowski metric tensor.

It is possible that the generator τ^1 commutes with electric charge. Then $f^{alc}n^c=0$, n^a is a constant, and there is no direct coupling between W^μ and A^μ. In this case the radiation mechanism discussed in the present section is absent and electromagnetic radiation can occur only through vacuum polarization diagrams (see Sec. III). Throughout this section we assume that the generators of W^μ and A^μ do not commute.

It should be noted that the above definition of n denoted by $n=n_0(\theta)$ cannot be continued all the way to the string axis, $r=0$. $n_0(\theta)$ is rotated by 2π as θ changes from 0 to 2π and is singular at $r=0$. If we require $n(\vec{x})$ to be a continuous function of $\vec{x}$, we are forced to allow $n(\vec{x})$ to point in the directions of the broken symmetry generators inside the string. For example, $n(\vec{x})$ can be defined as

$$n(\vec{x})=f(r)m+g(r)n_0(\theta)\ , \tag{2.8}$$

where m is a constant unit vector such that $m^a n_0^a(\theta)=0$ for all θ. ($m^a=\delta^{al}$ satisfies these conditions.) We must also require that $f^2(r)+g^2(r)=1$, $f(0)=1$, and $f(r)$ vanishes at least like $\exp(-r/a)$ for $r \gg a$. A natural choice of $f(r)$ is a bell-shaped function of width $\sim a$, e.g., $f(r)\sim\exp(-r^2/a^2)$. Since the choice of m^a and $f(r)$ is largely arbitrary, we expect all physical results to be independent of this choice. We have not checked this independence explicitly, since we are unable to do an explicit calculation of the radiation from a string. All we can do is an order-of-magnitude estimation, which is insensitive to the detailed shape of $f(r)$.

To summarize, the quartic coupling of the photon field to the classical gauge field of the string introduces an effective position- and polarization-dependent photon mass term, $m_{\mu\nu}(\vec{x})$. This term is $\sim a^{-2}$ inside the string ($r \lesssim a$) and is exponentially small outside the string ($r \gg a$).

In the case of an oscillating string the photon mass becomes time dependent and leads to two-photon emission via the diagram of Fig. 1, in which solid lines represent the classical field W_μ and wavy lines the photon field A_μ. Two-photon emission can also occur through the diagram of Fig. 2, in which the bar on the exchanged solid line indicates that the exchanged particle must be a heavy boson other than that corresponding to the field A_μ^1 as a result of the antisymmetry of the structure constants. Figure 2 corresponds to the presence of a cubic interaction term L_2 in the Lagrangian, where

$$L_2=-gf^{abc}A_\mu^b A_\nu^c \partial^\mu A^{a\nu}\ . \tag{2.9}$$

We now turn to the calculation of the radiation from an oscillating string, neglecting for the moment the cubic interaction L_2. We take the case of a straight string lying along the z axis. In order to get an order of magnitude estimate of the radiation, which is all we require, we assume that the string oscillates as a whole in the x direction, with the x coordinate of the center of the string being given by

$$x=\xi(t)=A\cos\omega t\ . \tag{2.10}$$

Since we take the string to be straight, our results can be applicable only in the case $A \ll R$, where R is the radius of curvature. Since $\omega\sim R^{-1}$, we require

$$\omega A \ll 1\ . \tag{2.11}$$

For macroscopic strings $R \gg a$, and hence we also have

$$\omega a \ll 1\ . \tag{2.12}$$

We write the effective photon mass squared, $m^2_{\mu\sigma}$, for a static string as $m^2(r)=m^2((x^2+y^2)^{1/2})=m^2(x,y)$ where, as we have discussed, we expect $m^2_{\mu\sigma}(r)$ to vanish rapidly for $r>a$. For an oscillating string we replace $m^2_{\mu\sigma}(x,y)$ by $m^2_{\mu\sigma}(x',y)$, where

$$x'(t)=x-\xi(t)\ . \tag{2.13}$$

The amplitude for the emission of two photons with momentum and polarization vectors k_1, ϵ_1 and k_2, ϵ_2 is

$$S=\left\langle \vec{k}_1,\vec{\epsilon}_1;\vec{k}_2,\vec{\epsilon}_2 \left| T\left[i\int d^4x\, L_I \right] \right| 0 \right\rangle\ . \tag{2.14}$$

The total energy emitted by the string, E, is given by

$$E=\frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^6 4k_{10}k_{20}}(k_{10}+k_{20})\sum_{\epsilon_1\epsilon_2}|S|^2\ , \tag{2.15}$$

where

$$S=-i\epsilon_1^\mu\epsilon_2^\sigma C_{\mu\sigma}(k_1+k_2) \tag{2.16}$$

and

$$C_{\mu\sigma}(k)=\int d^4x\, m^2_{\mu\sigma}(x',y)e^{ikx}\ . \tag{2.17}$$

The integral for $C_{\mu\sigma}(k)$ can be written as

$$\delta(k_{1z}+k_{2z})\int dx\,dy\, m^2_{\mu\sigma}(x,y)\exp[-i(k_x x+k_y y)]\int dt\,\exp(ik_0 t)\exp[ik_x\xi(t)]\ . \tag{2.18}$$

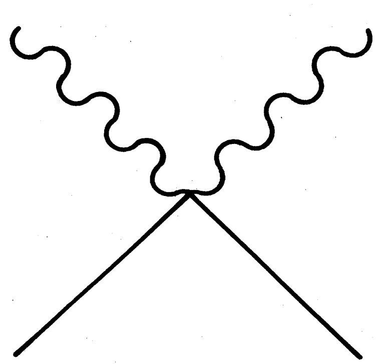

FIG. 1. Simplest diagram for two-photon emission from an oscillating string due to the interaction Lagrangian L_1, Eq. (2.6). The solid lines represent the classical heavy gauge field, W_μ, of the string, and wavy lines represent photons.

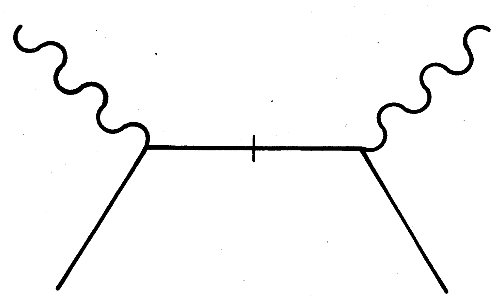

FIG. 2. Simplest diagram for the emission of two photons from a string due to the cubic interaction, Eq. (2.9). The barred exchanged solid line represents a heavy gauge boson other than that corresponding to the field $W_\mu=A_\mu^1$ of the string.

Recalling that $m^2_{\mu\nu} \sim a^{-2}$ inside the string, one finds that the x and y integrations simply yield a factor of order 1. In doing the time integration, we can make use of Eq. (2.11) and the fact that k_x is of order ω to write $\exp[ik_x\xi(t)] \simeq 1 + ik_x\xi(t)$. Thus the time integration yields a factor $\omega A \delta(k_{10}+k_{20}-\omega)$, so that one obtains, for the order of magnitude of $C_{\mu\nu}$

$$C_{\mu\nu} \sim \omega A \delta(k_{1z}+k_{2z})\delta(k_{10}+k_{20}-\omega) . \qquad (2.19)$$

We can now substitute (2.19) into (2.16) and (2.15) and divide by $\delta(0)^2$ to estimate ϵ_γ, the electromagnetic energy radiated per unit length per unit time. One obtains a factor of order ω^4 from the volume element in k space in Eq. (2.15) constrained by the two δ functions in (2.19), and hence

$$\epsilon_\gamma \sim \omega^5 A^2 \sum_{\epsilon_2\epsilon_2} |\epsilon_{1\mu}\epsilon^{2\mu}|^2 \sim \omega^5 A^2 . \qquad (2.20)$$

We have thus far not included the effect of the term L_2 in the interaction Lagrangian. We will not calculate its effect in detail; it is easy to see that it is comparable in order of magnitude to that of L_1, so that it does not change the estimate in Eq. (2.20).

In basing our estimate of ϵ_γ on the diagrams of Figs. 1 and 2 we are, of course, using perturbation theory. Note that the result in Eq. (2.20) is independent of the gauge coupling g. There are also higher-order diagrams, e.g., those in Fig. 3, which are of zeroth order in g, since the extra factors of g are compensated by additional factors of the external field W_μ which is proportional to $1/g$. The number of zeroth-order diagrams is infinite, and strictly speaking we cannot use perturbation theory. The main reason for this is that, although a is small, the classical gauge field W_μ is large inside the string, leading to a substantial modification of the photon and heavy-gauge-boson wave functions for $R < a$. It can be shown that, to the lowest order in ωA, the contribution of each zeroth-order diagram to $C_{\mu\nu}$ has the order of magnitude of Eq. (2.19). The leading term in the expansion of ϵ_γ in powers of g and ωA is, therefore, given by Eq. (2.20) with a coefficient in the form of a numerical series (hopefully convergent). Thus, we expect that for $\omega A \ll 1$ Eq. (2.20) will provide a correct order-of-magnitude estimate.

The result in Eq. (2.20) is, of course, not gauge invariant. This is not surprising, since only the combined contribution of all zeroth-order diagrams is expected to be gauge invariant. Note that the situation with regard to gauge invariance in the present case differs somewhat from that in more familiar situations. We have in effect imposed a choice of electromagnetic gauge by taking A^1_μ as the heavy gauge field of the string, so that the latter points in the same direction in group space at all points in coordinate space. We might call this the "parallel" gauge. Since electromagnetic gauge transformations involve gauge rotations generated by the electric charge operator, they do not leave the gauge field of the string invariant and take one out of the parallel gauge. We expect, therefore, that a change in the polarization four-vector of an emitted photon of the form $\epsilon_\mu \to \epsilon_\mu + ck_\mu$, corresponding to the familiar form of an electromagnetic gauge transformation, will leave even the exact amplitude invariant only when accompanied by the corresponding transformation of the heavy boson field of the string.

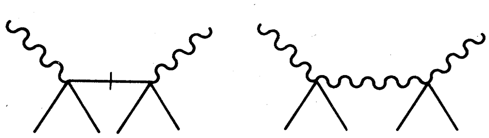

FIG. 3. An example of a diagram more complicated than that of Fig. 1 but of the same order in the small parameters g and ωA which also leads to two-photon production from a string.

Equation (2.20) gives the radiation rate per unit length for an infinite string oscillating coherently as a whole. This result is not directly applicable to radiation from finite strings or from incoherently oscillating infinite strings. Consider, for example, a charged rod of length l and charge density λ oscillating coherently with amplitude A and frequency ω. Then the dipole radiation rate is $E_d \sim \lambda^2 A^2 \omega^4 l^2$, and the rate per unit length, $\epsilon = E_d/l$, grows proportionally to l:

$$\epsilon \sim \lambda^2 A^2 \omega^4 l . \qquad (2.21)$$

On the other hand, for an infinite rod

$$\epsilon \sim \lambda^2 A^2 \omega^3 , \qquad (2.22)$$

and there seems to be no correspondence between (2.21) and (2.22). We note, however, that the dipole approximation is applicable only for $l \ll \omega^{-1}$. A direct calculation shows that in the opposite limiting case, $l \gg \omega^{-1}$, the radiation rate is $\epsilon \sim \lambda^2 A^2 \omega^3$, as for an infinite rod. In the intermediate regime, when $l \sim \omega^{-1}$, both Eqs. (2.21) and (2.22) give the right order of magnitude. This is exactly the case of interest to us, since closed loops of size $\sim R$ oscillate with a frequency $\omega \sim R^{-1}$ and thus we expect Eq. (2.20) to be applicable in this case.

We can now compare the rate of energy loss through EM radiation, as given by Eq. (2.20), with that due to gravitational radiation. Let us consider a loop of radius R, so that $\omega \sim R^{-1}$. We take $A \sim R$, and assume that Eq. (2.20) continues to give the right order of magnitude in this case. Since the total rate of energy loss from the string due to gravitational radiation is E_g, where

$$E_g \sim \eta^4/m_P{}^2 \qquad (2.23)$$

and m_P is the Planck mass, we obtain

$$E_\gamma/E_g \sim m_P{}^2/(\eta^4 R^2) . \qquad (2.24)$$

Taking η of the order of 10^{16} GeV, a value suggested by the string theory of galaxy formation,[8,13] one finds from Eq. (2.24) that gravitational radiation dominates for $R > 10^{-27}$ cm. For macroscopic loops of cosmological significance the EM radiation is totally negligible, as assumed in Refs. 8 and 9.

There is another way to look at Eq. (2.20) which is perhaps illuminating. Note that $\omega^2 A \sim a_0$, where a_0 is the proper acceleration of a segment of the string. Hence it follows that the electromagnetic energy emitted in one cycle is of order a_0^2. Since the energy per unit length of the string is η^2, this says that the rate of energy loss per cycle becomes large for $a_0 \sim \eta = m_V/g$, where m_V is the heavy boson mass. This agrees with the estimate given in Ref. 12 for the condition under which nongravitational radiation from a collapsing loop becomes significant.

III. OTHER DIAGRAMS FOR ELECTROMAGNETIC RADIATION FROM A STRING

One can find other diagrams which also lead to electromagnetic radiation from the string. In general these turn out to be suppressed by additional factors of g and/or ωa, although they will dominate if the generators of W^μ and A^μ commute. As an example we consider the diagram of Fig. 4, in which the scalar field ϕ is coupled to a charged fermion field χ. The interaction Lagrangian is

$$L_{\rm int} = g_\chi \bar{\chi}\chi\phi + e\bar{\chi}\gamma_\mu \chi A^\mu \ . \tag{3.1}$$

In the presence of an oscillating string $\langle\phi\rangle$ is position and time dependent, being given by an expression of the form

$$\langle\phi\rangle = \eta[1 - f(\{[x-\xi(t)]^2 + y^2\}^{1/2})] \ , \tag{3.2}$$

where $f(r)$ is a function such that $f(0)=1$, and f vanishes rapidly for $r > a$. Radiation occurs through the coupling of the electromagnetic field to the scalar field by means of an intermediate fermion loop. The amplitude for emission of a single photon vanishes, but two-photon emission occurs via the diagram of Fig. 4. Note that the fermions acquire a mass $m_f \sim g_\chi\langle\phi\rangle$ when ϕ develops a VEV. Thus the fermions in question will have superheavy masses.

The calculation follows that in the previous section. The only difference is that in Eq. (2.15) S is replaced by S', where

$$S' = 2g_\chi e^2 \eta \bar{f}(k_1+k_2) T_R(k_1,k_2) \ , \tag{3.3}$$

where

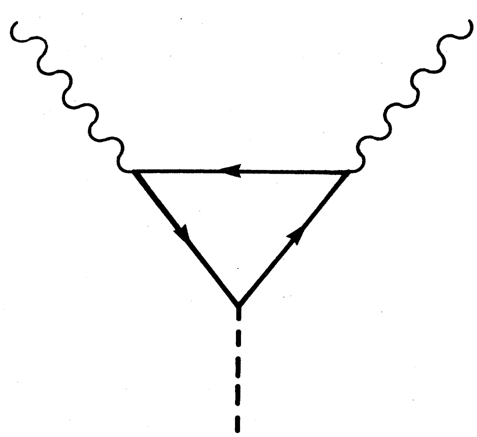

FIG. 4. Lowest-order diagram for two-photon production from a string due to the interaction Lagrangian $L_{\rm int}$ of Eq. (2.1). The dashed line represents the scalar Higgs field of the string, and the directed solid lines represent fermions.

$$T(k,p) = \int \frac{d^4q}{(2\pi)^4} {\rm Tr}\left[\frac{1}{\not q-\mu}\epsilon_1\frac{1}{\not q+\not k-\mu}\epsilon_2\frac{1}{\not q+\not k+\not p-\mu}\right] , \tag{3.4}$$

$$T_R(k,p) = T(k,p) - T(0,0) \ , \tag{3.5}$$

$$\mu = g_\chi \eta \ , \tag{3.6}$$

is the asymptotic value of the fermion mass, and

$$\bar{f}(k) = \int d^4x [f(\{[x-\xi(t)]^2+y^2\}^{1/2}) - f((x^2+y^2)^{1/2})] e^{ik\cdot x} \ . \tag{3.7}$$

By power counting the integral in (3.4) is linearly divergent, but calculation shows that the dangerous terms cancel out and the integral is actually finite. Nevertheless a finite renormalization must be carried out by the subtraction of $T(0,0)$ in Eq. (3.3) in order to ensure gauge invariance. (Note that here, in contrast with the diagrams discussed in Sec. III, one does expect the diagram in Fig. 4 to give a result which is gauge invariant in the usual way. The only field with a VEV involved in Fig. 4 is ϕ, which is invariant under gauge rotations generated by Q.) If $T_R(0,0) \neq 0$, it would correspond to a nonzero photon mass. Equivalently, note that $k_1 = k_2 = 0$ corresponds to the presence of a constant EM potential which is gauge equivalent to the vacuum. Such a constant potential will not result in any polarization of the vacuum, and hence will not produce any fermion pairs with which the neutral field ϕ can interact.[14] A straightforward integration gives, for k_{10}, $k_{20} \ll \mu$,

$$T_R = [(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2) - (\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)]/(12\pi^2\mu) \tag{3.8}$$

which is indeed invariant under the gauge transformation $\epsilon_1 \to \epsilon_1 + a_1k_1$, $\epsilon_2 \to \epsilon_2 + a_2k_2$. Summing over the photon polarizations gives

$$\sum_{\epsilon_i} |T_R|^2 = 2(k_1\cdot k_2)^2/(12\pi^2\mu)^2 \ . \tag{3.9}$$

One can now replace S by S' in Eq. (2.15), make use of Eqs. (3.3) and (3.9), and proceeding as in the previous section, estimate ϵ'_γ, the rate of EM radiation from a string as a result of the diagram of Fig. 4. One obtains

$$\epsilon'_\gamma \sim e^4 A^2 \omega^9 a^4 \sim e^4(\omega a)^4 \epsilon_\gamma \ . \tag{3.10}$$

Thus Fig. 4 is suppressed, not only by two powers of the fine-structure constant as a result of the two electromagnetic vertices, but by the factor $(\omega a)^4$, and thus is totally negligible as compared with the diagrams in Figs. 1 and 2.

IV. SCALAR RADIATION FROM A DOMAIN WALL

We turn now to a discussion of radiation from a domain wall. We begin, for the sake of completeness, by discussing the radiation of massive scalar particles. We discuss this in the context of radiation from a domain wall where the calculation is somewhat easier to do. The qualitative conclusions should hold also for scalar radiation from strings.

The simplest model with domain-wall solutions is the scalar field theory described by the Lagrangian

$$\mathscr{L}=\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\ , \tag{4.1}$$

where

$$V(\phi)=-\tfrac{1}{2}\kappa^2\phi^2+(h/4!)\phi^4\ . \tag{4.2}$$

The potential $V(\phi)$ has minima at $\phi=\pm(6\kappa^2/h)^{1/2}\equiv\pm\eta$, and so the symmetry $\phi\to-\phi$ is spontaneously broken. When the theory is quantized around one of the true vacuum states, it describes self-interacting scalar particles of mass $m=(2\kappa^2)^{1/2}$.

The field equation corresponding to the Lagrangian (4.1) is

$$\phi-\tfrac{1}{2}m^2\phi+(1/3!)h\phi^3=0\ , \tag{4.3}$$

and the solution describing a domain wall in the xy plane is[2]

$$\phi_0(z)=-\eta\tanh(mz/2)\ . \tag{4.4}$$

The wall separates regions with $\phi_0=\eta$ at negative z and with $\phi_0=-\eta$ at positive z. The width of the wall is $\sim m^{-1}$. If the wall is perturbed, it will oscillate, and we expect it to emit ϕ particles. Note that here we have a rather interesting situation in which the wall and the radiation are both described by the same scalar field equation (4.3).

The unperturbed wall has $\phi=0$ on the whole plane $z=0$. Suppose that for a perturbed wall the surface of $\phi=0$ is

$$z=\xi(x,y,t)\ . \tag{4.5}$$

We want now to represent ϕ as a sum of two terms describing the wall and the radiation, respectively. Of course, such a representation is not unique. We shall use the decomposition

$$\phi(x,y,x,t)=\phi_0\left[\frac{z-\xi(x,y,t)}{(1-\partial_\mu\xi\partial^\mu\xi)^{1/2}}\right]+\psi(x,y,z,t)\ , \tag{4.6}$$

where ϕ_0 has the functional form of Eq. (4.4) and ψ corresponds to radiation. The argument of ϕ_0 is chosen so that there is no "radiation" for a uniformly moving wall [Eq. (4.6) with $\xi=vt$ and $\psi=0$ is a Lorentz transform of Eq. (4.4)]. Substituting (4.6) in the field equation (4.3) we obtain

$$\Box\psi+\sum_{n=1}^{\infty}(1/n!)\psi^nV^{(n+1)}(\psi_0)=-\phi_0'\alpha\{\Box\xi+\tfrac{1}{2}\alpha^2(z-\xi)\Box(\partial_\mu\xi\partial^\mu\xi)+\tfrac{3}{4}\alpha^4(z-\xi)[\partial_\mu(\partial_\alpha\xi\partial^\alpha\xi)]^2\}$$
$$-\phi_0''\alpha^2(z-\xi)\{2\xi+(z-\xi)[\tfrac{1}{2}\alpha^2\partial_\mu(\partial_\alpha\xi\partial^\alpha\xi)]^2\}\ , \tag{4.7}$$

where $\alpha=(1-\partial_\mu\xi\partial^\mu\xi)^{-1/2}$ and $V(\phi)$ is given by Eq. (3.2).

Despite the rather messy form of Eq. (4.7), one class of solutions is easily found. Indeed, $\psi=0$ is a solution if ξ is such that $\Box\xi=0$ and $\partial_\mu\xi\partial^\mu\xi=0$. These conditions are satisfied if ξ describes a plane wave of arbitrary shape traveling along the wall with the velocity of light, e.g., $\xi=f(x\pm t)$. The conclusion is that there is no radiation if the perturbation of the wall has the form of a traveling plane wave. (Note that here we have not assumed that the perturbation is small.)

To study radiation from the wall, we have to consider perturbations other than traveling waves. From now on we shall assume that the perturbation is small. We do not expect radiation for $\omega<m$, and so we take $m<\omega\ll A^{-1}$. To estimate the order of magnitude of various terms on the right-hand side of (4.7), we note that ϕ_0' and ϕ_0'' are peaked near $z=0$ with a width $\sim m^{-1}$. Thus, $(z-\xi)\sim m^{-1}$, $\xi\sim A$, $\partial_\mu\sim\omega$, and $\alpha\sim1$. Keeping only the first two terms of the expansion in powers of ωA and linearizing in ψ, Eq. (4.7) becomes

$$[\Box+V''(\phi_0)]\psi=-\phi_0'[\Box\xi+\tfrac{1}{2}z\Box(\partial_\mu\xi\partial^\mu\xi)]\ . \tag{4.8}$$

If we neglect the second term on the right-hand side, then (4.8) has a solution $\psi=0$ if ξ satisfies

$$\Box\xi=0\ . \tag{4.9}$$

Equation (4.9) is the linearized equation for small perturbations of the wall. For definiteness, we shall take a solution in the form of a standing wave,

$$\xi=A\sin(\omega x)\sin(\omega t)\ . \tag{4.10}$$

Radiation appears in the next order of perturbation theory.

Equation (4.9) can be rewritten, with the use of (4.8), as

$$(\Box+m^2)\psi=-\tfrac{1}{2}z\phi_0'(z)\Box(\partial_\mu\xi\partial^\mu\xi)+\tfrac{3}{2}m^2f(z)\psi\ , \tag{4.11}$$

where $f(z)=1-\phi_0^2(z)/\eta^2$. The analysis is greatly simplified if we can ignore the last term in (4.11). This is possible when $\omega\gg m$. In this limit the solution of (4.11) is

$$\psi=\int d^4x'\Delta_R(x,x')[-\tfrac{1}{2}z'\phi_0(z')\Box(\partial_\mu\xi\partial^\mu\xi)_{x'}]\ , \tag{4.12}$$

where $\Delta_R(x,x')$ is the retarded Green's function. We are interested only in the asymptotic behavior of ψ at large z, which is relevant for radiation. In the limit $z\to\infty$, Eqs. (4.10) and (4.12) give

$$\psi\to\pi^2\omega^4A^2m^{-2}\eta\exp(-2\pi\omega/m)\cos(\omega z-2\omega t)\ . \tag{4.13}$$

The corresponding energy flux is

$$\bar{T}_{03}=\left\langle\frac{\partial\psi}{\partial t}\frac{\partial\psi}{\partial z}\right\rangle$$
$$=2\pi^4\omega^{10}A^4m^{-4}\eta^2\exp(-4\pi\omega/m)\ , \tag{4.14}$$

where $\bar{T}_{03}$ indicates averaging over the period.

From the result, Eq. (4.14), we see that for $\omega\gg m$ the radiation is exponentially small. Hence only modes with $\omega\sim m$ can be substantially damped. To make an order-of-magnitude estimate of damping in the case, we let $\omega\sim m$ in (4.14). This gives

$$\bar{T}_{03}\sim\omega^6A^4\eta^2\ . \tag{4.15}$$

The characteristic damping time can be found from

$$\tau \sim \frac{\sigma_\omega}{\bar{T}_{03}} , \tag{4.16}$$

where

$$\sigma_\omega \sim \sigma \omega^2 A^2 \tag{4.17}$$

is the energy of the perturbation per unit area and $\sigma \sim \eta^2 m$ is the surface mass density of the wall.[2] From Eqs. (4.15)—(4.17) we find

$$\tau \sim (\omega^3 A^2)^{-1} . \tag{4.18}$$

For larger perturbations with $\omega A \sim 1$ the damping time is comparable to the period of oscillation, ω^{-1}. To summarize, the scalar radiation is negligible for $\omega \gg m$ and $\omega \ll m$. Only modes with $\omega \sim m$ are substantially damped.

V. ELECTROMAGNETIC RADIATION FROM A DOMAIN WALL

Domain walls are not charged, nor, in contrast to strings, do they have associated with them nonvanishing expectation values of the gauge boson fields. Thus EM radiation is possible only due to vacuum polarization effects. We will consider radiation in the same model as that studied for strings in Sec. III and represented by the diagram of Fig. 4. The calculation is very similar, involving only a change in the function $\bar{f}(k)$. We now take the wall to lie parallel to the plane $x=0$, and to oscillate as a whole according to Eq. (2.10). Equation (3.2) is then replaced by

$$\langle \phi \rangle = \eta[1 - f(x - \xi(t))] \tag{5.1}$$

and Eq. (3.7) is replaced by

$$\bar{f}(k) = \int d^4x [f(x-\xi(t)) - f(x)] e^{ik\cdot x} . \tag{5.2}$$

Substitute Eq. (5.2) into Eq. (3.3) for S', and again proceed as in Sec. III but with S replaced by S' in Eq. (2.15). One now obtains δ functions for both the y and z components of the total momentum of the two photons. There is also one less factor of a in S' than in the case of a string, since the region where $f \neq 0$ is now restricted to a range of order a in only one of the three spatial coordinates, rather than two as in the string. Repeating the calculation of Sec. III with these modifications, one obtains for $\epsilon_{\gamma\,\text{wall}}$, the rate of EM radiation per unit area per unit time from a domain wall.

$$\epsilon_{\gamma\,\text{wall}} \sim e^4 A^2 \omega^8 a^2 . \tag{5.3}$$

The result (5.3) differs from the corresponding result in Eq. (3.10) for the radiation from a string due to the process in Fig. 4 by a factor $(\omega a^2)^{-1}$, arising from the two fewer factors of a in $|S'|^2$ in the domain-wall case, and from the additional restriction on the range of photon momenta due to the additional δ function.

It is interesting to compare the present result with the radiation rate for an oscillating mirror (reflecting plane surface)[15]

$$\epsilon_{\gamma\,\text{mirror}} \sim A^2 \omega^6 . \tag{5.4}$$

We see that the radiation from a domain wall is weaker by a factor $(e^2 \omega a)^2$. This reflects the fact that a mirror corresponds to a conducting surface, while the diagram in Fig. 4 causes the domain wall to behave like a thin layer of dielectric, so that its coupling to the EM field is much weaker.

Once again, let us compare $\epsilon_{\gamma\,\text{wall}}$ with the rate of energy loss of a wall due to gravitational radiation. We consider a piece of wall of linear dimension R, take $\omega \simeq R^{-1}$, and assume that our results remain valid for $A \simeq R$, so that the total rate of EM energy loss from the wall segment is given by $E_{\gamma\,\text{wall}} \simeq e^4 a^2 R^{-4}$. The rate of gravitational energy loss from a wall is given by[4] $E_{g\,\text{wall}} \simeq \eta^4 R^2 / (a^2 m^2)$ so that

$$E_{\gamma\,\text{wall}} / E_{g\,\text{wall}} \simeq e^4 m_P{}^2 / (h^4 \eta^8 R^6) . \tag{5.5}$$

If $e/h \sim 1$, then (5.5) gives $E_{\gamma\,\text{wall}}/E_{g\,\text{wall}} \ll 1$ for $\eta > 1$ GeV if $R \gg 10^{-8}$ cm. This justifies the assumption made in Ref. 4 that the dominant energy loss mechanism is gravitational radiation.

VI. CONCLUSIONS

The principal results of this paper are the estimates of the rate of energy loss by electromagnetic radiation of domain walls and strings. These estimates were obtained for the case of straight walls and strings oscillating as a whole with frequency and amplitude satisfying $\omega A \ll 1$. We have, however, assumed that these results give the right order of magnitude in the general case of large amplitudes ($\omega A \simeq 1$) and curved walls and strings with curvature radius $\simeq A$ so that our results can be applied to the cosmological evolution of vacuum structures. For strings the dominant mechanism for electromagnetic radiation is the nonlinear coupling of the electromagnetic field to the nonzero vacuum expectation value of a heavy-gauge-boson field in the string; this coupling is represented by the diagrams in Figs. 1 and 2. Our estimate for the rate of radiation from a string is given in Eq. (2.20). The crucial thing about this result is that it confirms, as shown by Eq. (2.24), the expectation that for cosmologically interesting strings the rate of electromagnetic energy loss is completely negligible compared to that due to gravitational radiation.

A direct coupling between the heavy gauge field W^μ and the electromagnetic field is absent if the corresponding generators commute. In this case electromagnetic radiation can occur only through vacuum polarization diagrams, such as that in Fig. 4. The radiation rate in this case is much smaller than (2.20). On the other hand, vacuum polarization diagrams provide the only mechanism for electromagnetic radiation by domain walls. Again, the estimates are in agreement with the expectation that the dominant energy-loss mechanism is gravitational radiation.

We have also considered the radiation of massive scalar particles of mass m. This process is damped by energy conservation for $\omega \ll m$, and by interference effects for $\omega \gg m$, and so is of importance only for perturbations with wavelengths of order m^{-1}. This effect is not of cosmological significance. Note that we have considered only the case where the vacua on the two sides of the domain wall are degenerate. Otherwise the wall accelerates, becomes ultrarelativistic, and scalar radiation may become important.[16]

ACKNOWLEDGMENT

The work of A. V. was supported in part by the National Science Foundation under Grant No. 8206202.

[1]T. W. B. Kibble, J. Phys. A **9**, 1387 (1976); Phys. Rep. **67**, 183 (1980).
[2]Y. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3 (1974) [Sov. Phys.-JETP **40**, 1 (1975)].
[3]A. Vilenkin, Phys. Rev. D **23**, 852 (1981).
[4]A. Vilenkin and A. Everett, Phys. Rev. Lett. **48**, 1867 (1982).
[5]T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Rev. D **26**, 435 (1982).
[6]A. Everett and A. Vilenkin, Nucl. Phys. **B207**, 43 (1982).
[7]Y. B. Zel'dovich, Mon. Not. R. Astron. Soc. **192**, 663 (1980).
[8]A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); **46**, 1946(E) (1981); Phys. Rev. D **24**, 2082 (1981).
[9]T. W. B. Kibble and N. Turok, Phys. Lett. **116B**, 141 (1982).
[10]A scenario in which the cosmological density fluctuations are produced by walls bounded by strings has been discussed by F. Stecker and Q. Shafi, Phys. Rev. Lett. **50**, 928 (1983).
[11]A. Vilenkin, Phys. Lett. **107B**, 47 (1981).
[12]A. Everett, Phys. Rev. D **24**, 858 (1981).
[13]A. Vilenkin and Q. Shafi, Phys. Rev. Lett. **51**, 1716 (1983).
[14]J. Schwinger, Phys. Rev. **82**, 664 (1951).
[15]L. Ford and A. Vilenkin, Phys. Rev. D **25**, 2569 (1982).
[16]R. F. Sawyer, Phys. Rev. D **24**, 1581 (1981).